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Enhanced optical Anderson localization effects in modulated Bloch lattices

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Abstract – We study Anderson localization dynamics in periodically modulated optical Bloch arrays. Using an effective model, we show that, in such arrangements, even a weak disorder may play an important role and can lead to enhanced Anderson localization effects.

Introduction. – Anderson localization (AL) is an important quantum-mechanical process that describes the absence of diffusive electronic behavior in random lattices arising from destructive quantum interference effects [1]. In fact, AL can explain the absence of conductivity in certain highly disordered systems [1–3]. Since the seminal paper by Anderson, work in this field has been intensive. However, experimental observation of such effects remained very much elusive due to many-body interactions that naturally arise in solid-state physics. In order to circumvent this difficulty, optical realizations of AL were proposed [2]. Recently AL has been observed in both 1D and 2D systems of coupled waveguide arrays [4,5]. These structures offer considerably more control over the experimental parameters and are also immune to the presence of weak optical losses that happen to be detrimental to AL in other settings [1].

On the other hand there has been considerable effort in understanding the dynamical behavior of quantum systems with time-dependent parameters. For example, time-dependent Hamiltonians are used to describe the corresponding quantum evolution under the action of an applied time-varying force [6]. Most often, the effect of such a force is to mix the system's initial state into a quantum-mechanical superposition and depending on the nature of the time variation, the system may eventually evolve into a new final state. The ability to utilize such external control parameters to create certain desired final states has been a subject of intense research investigation in the past decades [7]. A profound example of such processes is coherent control of chemical reactions where laser pulses are synthesized and used in order to steer reactions to favor certain possible outcomes over others [7]. A closely relavant example is the use of engineered laser pulses to induce coherent population transfer between energy levels [8] and its related Landau-Zener problem [9]. Another important class of time-dependent systems is that of quantum kicked rotors which in the classical limit are known to exhibit chaotic behavior [10]. It is also worth noting that the intriguing effects of geometric phase arise as a result of adiabatic evolution of Hamiltonians having time-dependent parameter space with dimensions larger than one [11].

Given the variety of physical phenomena associated with time-dependent Hamiltonians, it is interesting to note that most studies on Anderson localization are carried out for time-independent systems. It would be of interest to bridge this gap and investigate localization effects when the Hamiltonian varies with time.

Along these lines, the process of Anderson localization in one dimension dimer model under the influence of ac (alternating current) electric field was previously investigated and it was shown that it can lead to enhanced localization effects, even for relatively weak disorder [12]. The explanation of this phenomenon is closely related to another process known as dynamic localization (DL) [13,14]. Under DL conditions, an applied ac field (superimposed on a periodic potential) will result in a localization of the electronic wave function. For certain conditions however, this localization ceases to exist and the electron

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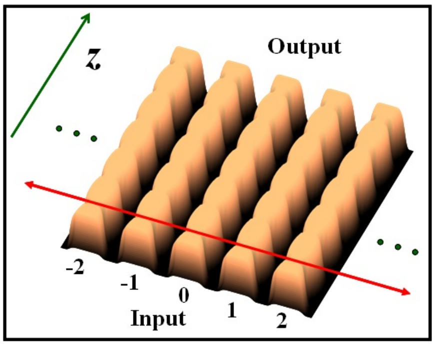

Fig. 1: (Colour on-line) Possible realization of a periodically modulated Bloch lattice. The periodic coupling is obtained by periodic modulation of the waveguides' width while randomness is also introduced by changing the waveguides' width (diagonal disorder).

wavepacket spreads all over the lattice sites. If the applied field parameters are adjusted so that the diffraction is weak, it follows that even a weak disorder may lead to AL effects. Dynamic localization has recently been experimentally realized in the context of optics, *e.g.* in coupled curved optical arrays where the curvature effectively translates into periodic tilting of the array index [15–17]. Another interesting process in dynamic Bloch oscillator arrays is that of resonant delocalization (RD) [18]. In this case, because of resonance effects, a beam continuously undergoes diffraction in spite of the fact that it propagates in a Bloch array.

In this letter we study a closely related problem, namely AL in conjunction with resonant delocalization. We show that in such configurations even a weak disorder could lead to enhanced Anderson localization effects. Before we proceed, we note that observing these effects in solid-state physics is problematic due to many-body effects. A possible experimental route towards such an observation would be to investigate similar effects in linear photonic systems where photon-photon interactions are absent. In general, RD can be realized in optics by periodically modulating the coupling constants between adjacent sites in a waveguide array while also linearly ramping the effective refractive index —see fig. 1. Different configurations with modulated coupling constants have been also considered in previous works [19].

Localization effects: analysis and results. – To analyze the aforementioned system we consider an optical array configuration that is described by the following coupled mode equation:

$$i\frac{\mathrm{d}\phi_n}{\mathrm{d}z}+\kappa\left(z\right)\left(\phi_{n+1}+\phi_{n-1}\right)+\left(nf_0+\delta_n\right)\phi_n=0. \qquad (1)$$

In eq. (1), ϕ_n represents the optical modal field amplitude at waveguide site n, and $\kappa\left(z\right)$ is the z periodic coupling constant between any two adjacent channels. On the other hand f_0 is the ramping strength of this Bloch oscillator array. In addition, δ_n represents a lattice site dependent random perturbation, added to the original value of the ramping function, and here it accounts for the disorder (on-diagonal disorder). It is important to note that if the coupling constant is periodic, $\kappa\left(z\right)=\kappa\left(z+\Lambda\right)$, eq. (1) will be invariant under the translation $z\rightarrow z+\Lambda$. Hence one can always look for solutions to eq. (1) in the form of *Floquet-Bloch* modes. In other words, the solution can take the form $\phi_n\left(z\right)=\exp\left(i\mu z\right)H_n\left(z\right)$ where the system's eigenmode obeys $H_n\left(z\right)=H_n\left(z+\Lambda\right)$ and μ is the quasi-eigenvalue (analogous to quasi-momentum in solid-state periodic potentials). Also note that the configuration described by eq. (1) is periodic in the transverse direction only when $f_0=\delta_n=0$. Clearly any non-zero refractive index ramping destroys this periodicity. Thus, in general the stationary or the Bloch modes of the system (when $f_0\neq 0$) are not necessarily extended all over the lattice. In fact, for a constant coupling κ, these modes correspond to the so-called Wannier-Stark ladder spectrum and they are indeed localized [20]. Using the transformation

$$\varphi_n\left(z\right)=u_n\left(z\right)\exp\left(i\,nf_0\,z\right),$$

the above equation reduces to

$$i\frac{\mathrm{d}u_n}{\mathrm{d}z}+\kappa\left(z\right)\left(e^{if_0z}u_{n+1}+u_{n-1}e^{-if_0z}\right)+\delta_nu_n=0. \qquad (2)$$

Here we first consider the case of a perfect lattice without any disorder, *i.e.* $\delta_n=0$. Under these conditions, eq. (2) is now periodic in the transverse direction and hence one might be tempted to look for extended states. However the analysis is now complicated by the fact that the lattice is no longer invariant under longitudinal translations $z\rightarrow z+\Lambda$. It is only under the resonant condition $\Lambda=2\pi\,m/f_0$, with m being an integer, that the lattice invariance under discrete z translations is restored. For concreteness, let us consider the case where $\kappa(z)=\kappa_0+\varepsilon\cos(\Gamma z)$. The above resonance condition then translates into $f_0=m\Gamma$. For $m=1$, this becomes exactly the resonant delocalization condition derived in [18] and under this latter condition, the impulse response (single channel excitation) was shown to diffract unboundedly upon propagation. In what follows, we investigate the system under this physical condition of $m=1$. Using the ansatz $u_n\left(z\right)=\exp\left(i\int_0^z\lambda\left(z'\right)\mathrm{d}z'\right)\exp\left(iqn\right)$ in eq. (2) when $\delta_n=0$, we find that $\lambda\left(z\right)=2\kappa\left(z\right)\cos(q+f_0z)$. If we substitute in this latter expression the particular choice of $\kappa(z)=\kappa_0+\varepsilon\cos(\Gamma z)$, we finally obtain

$$\lambda\left(z\right)=2\kappa_0\cos(q+f_0z)+\varepsilon\cos(q)+\varepsilon\cos(q+2f_0z).$$

In other words, the solution for (2) takes the form

$$u_n\left(z\right)=\exp\left(iqn\right)\exp\left(i\varepsilon\cos(q)\;z\right)\,G\left(z\right), \qquad (3)$$

where

$$G\left(z\right)=\exp\left(i\int_0^z\left\{2\kappa_0\cos(q+f_0z')+\varepsilon\cos(q+2f_0z')\right\}\mathrm{d}z'\right).$$

Note that, in the transverse direction, the above solution behaves in the same way as if the lattice was uniform, namely it varies like exp (iqn). In the longitudinal dimension however, it is of the form of Floquet-Bloch modes, *i.e.* exp $(i\varepsilon \cos(q)\, z)\; G\,(z)$, where $G\,(z) = G\,(z + 2\pi/f_0)$. It thus follows that under this resonant condition, $\varepsilon \cos(q)$ can be considered as the quasi-eigenvalue characterizing light propagation in such geometries. The solutions presented in eq. (3) are also known as Houston modes [21] and under the above-mentioned resonant condition of $f_0 = \Gamma$ coincide with the Bloch modes of eq. (1) [21,22]. An immediate consequence of the above analysis is that the system's eigenvalue is renormalized to a smaller value. In other words it was reduced from $2\kappa_0 \cos(q)$, as in the case of a uniform array, to $\varepsilon \cos(q)$. This mathematical statement translates into a weaker effective hopping constant and hence weaker diffraction. This renormalization of the coupling between adjacent sites has been also predicted for different systems in solid-state physics involving periodic potentials superimposed with both dc and ac applied electric fields [12,13]. The reduction in the effective coupling renders the structure more sensitive to any disorder and hence enhanced localization effects are expected under these conditions.

At this point, in order to demonstrate this enhancement of localization effects, we insert back the random disorder parameter δ_n and carry out numerical simulations for eq. (1) for two different scenarios. In the first case, we take $\varepsilon = f_0 = 0$, corresponding to a disordered uniform array and we compare these results with those of a modulated Bloch structure with $\kappa_0 = f_0 = \Gamma = 1$ and $\varepsilon = 0.2$. In both cases, the random disorder is selected from a uniform distribution $\delta \in [-\Delta,\ \Delta]$ and will be characterized by the disorder parameter $\eta = \Delta/\kappa_0$.

Figure 2 shows simulation results for two different disordered uniform waveguide arrays. Figures 2(a) and (b) illustrate the dynamics for a relatively small disordered parameter of $\eta = 1$.

A top view of single input propagation is depicted in (a) while (b) shows the output intensity in log scale. The blue line represents the output for one array while the red line is the average intensity over 40 different realizations of the random lattice. Similarly, figs. 2(c) and (d) depict the same set of data for a stronger randomized lattice of $\eta = 2$. Clearly, linear variations of intensity (in log scale) with waveguide site are observed in both figs. 2(b) and (d) —indicating Anderson localization. The slope in (d) is clearly steeper than that of (b), indicating tighter localization effects as anticipated for larger disorder parameter.

Figure 3 shows numerical results for a modulated Bloch lattice using the previously mentioned parameters. Discrete diffraction under resonant delocalization conditions is shown in fig. 3(a) while localization effects for single channel input when $\eta = 0.4$ and $\eta = 0.6$ are depicted in figs. 3(b) and (c), respectively. Finally, fig. 3(d) illustrates the output intensity as well as the ensemble average output intensity in log scales. Few comments are in place here. First, in fig. 3(d), a linear slope indicating Anderson-like localization is observed. However, in general, when the numerical experiment was repeated several times, it was found that in occasions, the best fit was polynomial rather than linear. Second, the average intensity curve is considerably less steeper than the single output curve shown in fig. 3(d). In some sense, this can be observed also in

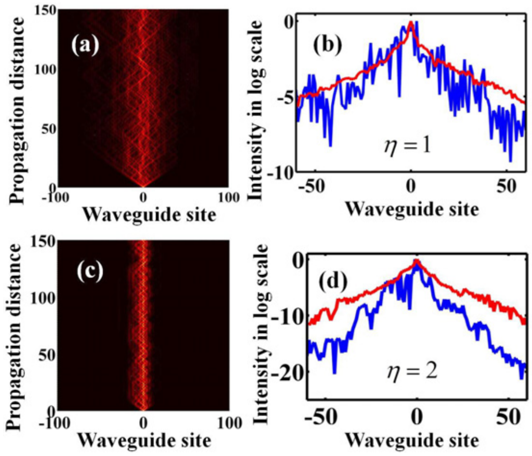

Fig. 2: (Colour on-line) (a) and (b): Localization dynamics in uniform waveguide arrays when the disorder parameter is $\eta = 1$. A top view of the propagation in one of the lattice realizations is depicted in (a) while (b) shows the output intensity in log scale (log to the base e). Panels (c) and (d) depict similar information for an array with disorder parameter $\eta = 2$. Note the linear slope of the intensities in the log scale graph —a sign of Anderson-type localization.

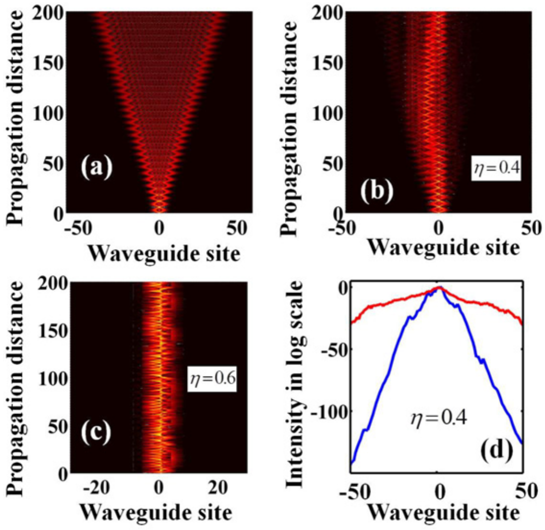

Fig. 3: (Colour on-line) (a) Diffraction under resonant delocalization conditions when no disorder is present. Panels (b) and (c) depict localization effects for single channel input when $\eta = 0.4$ and $\eta = 0.6$, respectively. (d) Output intensity (blue steeper curve) and average output over 100 lattice realizations (red line). Linear slopes can still be observed.

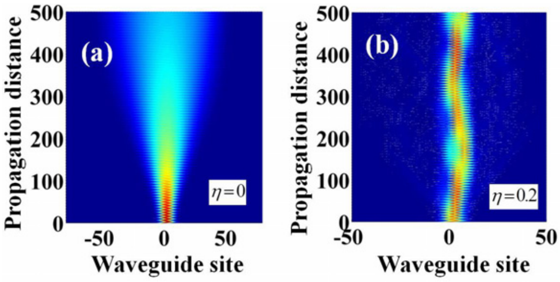

Fig. 4: (Colour on-line) (a) Diffraction of Gaussian beam in a Bloch array in the absence of any disorder. (b) Localization of the same input when $\eta = 0.2$.

fig. 2(d) for uniform lattices; however it is more profound in modulated Bloch arrays.

The above two observations can be roughly understood by noting that eq. (2) indicates that the structure has a different localization length scale at each propagation distance z. Thus, in contrast with the uniform lattice case, the input beam in the modulated Bloch lattice is expected to experience some breathing and bending upon propagation. For example, see fig. 3(b).

Note that in Bloch arrays, strong localization effects can be observed for disorder parameters almost 2–5 times lower than that used for the uniform array, thus confirming our previous analysis and remarks concerning the renormalization of coupling constant.

Finally, we explore how these localization effects will be manifested when the input beam is Gaussian as opposed to a single channel excitation. Again, as before, the examined array has the parameters $\kappa_0 = f_0 = \Gamma = 1$ and $\varepsilon = 0.2$. For the sake of comparison, fig. 4(a) shows propagation dynamics of a Gaussian beam input in the absence of any disorder. On the other hand, fig. 4(b) depicts the Gaussian beam evolution under a disorder parameter of $\eta = 0.2$. Evidently, strong localization effects can be observed, even for such weak randomness. This can be understood by noting that Gaussian inputs are wider than single waveguide excitations, hence they experience slower diffraction which makes them more susceptible to an even weaker disorder. Similar effects to those in fig. 4(b) can be also observed in uniform lattices for $\eta = 0.5$.

Comparison with localization effects in Hamiltonians exhibiting dynamic localization. – Finally, it is also instructive to compare the above-discussed scenarios with localization effects due to random changes in Hamiltonians having the form

$$i\frac{\mathrm{d}\phi_n}{\mathrm{d}z} + \kappa_0\left(\phi_{n+1} + \phi_{n-1}\right) + \left(nf_0\cos(\Omega\, z) + \delta_n\right)\phi_n = 0. \tag{4}$$

As has been shown, under certain conditions, this system (in the absence of any random variations) gives rise to the phenomenon of dynamic localization [14]. By using the transformation [12] $\varphi_n(z) = u_n(z)\exp\left(i\, n\frac{f_0}{\Omega}\sin(\Omega z)\right)$, we obtain

$$i\frac{\mathrm{d}u_n}{\mathrm{d}z} + \kappa_0\left(e^{i\frac{f_0}{\Omega}\sin(\Omega z)}u_{n+1} + u_{n-1}e^{-i\frac{f_0}{\Omega}\sin(\Omega z)}\right) + \delta_n u_n = 0. \tag{5}$$

As we did before, in order to gain insight into the renormalization of the tunnelling rate from one channel to another, we consider the case where $\delta_n = 0$. By assuming a solution of the form $u_n(z) = \exp\left(i\int_0^z \lambda(z')\mathrm{d}z'\right)\exp\left(iqn\right)$ for eq. (5), we find that $\lambda(z) = 2\kappa_0\cos(q + \frac{f_0}{\Omega}\sin(\Omega z))$. The periodic term in the previous expression can be expanded using Fourier series: $\cos(q + \frac{f_0}{\Omega}\sin(\Omega z)) = a_0 + \sum_1^\infty [a_n\cos(n\Omega z) + b_n\sin(n\Omega z)]$. The first term in such an expansion is the dc component while the rest of the summation is a periodic function with period equal to $2\pi/\Omega$. In order to proceed, we note that the dc term can be calculated analytically: $a_0 = \frac{\Omega}{2\pi}\int_{-\pi/\Omega}^{\pi/\Omega}\cos(q + \frac{f_0}{\Omega}\sin(\Omega z))\,\mathrm{d}z = J_0(\frac{f_0}{\Omega})\cos(q)$, where J_0 is the Bessel function of the first kind and zero order. Substituting back, we finally find that $u_n(z) = \exp(iqn)\exp(i\,2\kappa_0 J_0(\frac{f_0}{\Omega})\cos(q)\, z)\, Q(z)$, where in this case the function $Q(z)$ takes the form $Q(z) = \exp\left(i\int_0^z \{\,2\kappa_0 \sum_1^\infty [a_n\cos(n\Omega z') + b_n\sin(n\Omega z')]\,\}\mathrm{d}z'\right)$. We note here that $Q(z) = Q\left(z + \frac{2\pi}{\Omega}\right)$. In other words, the solutions take the form of Houston modes with a quasi-eigenvalue $2\kappa_0 J_0\left(\frac{f_0}{\Omega}\right)\cos(q)$. Similar to the resonant delocalization case, and by comparing this quasi-eigenvalue to the corresponding eigenvalue for uniform lattice $2\kappa_0\cos(q)$, we find that here also the coupling constant is effectively normalized to its new value of $\kappa_{eff} = \kappa_0 J_0\left(\frac{f_0}{\Omega}\right)$ [12,14]. It is interesting to note that when the quantity $\frac{f_0}{\Omega}$ coincides with one of the zeros of the Bessel function $J_0(x)$, the coupling constant effectively vanishes and localization effects take place, even without introducing any disorder —thus leading to the so-called dynamic localization effect [14]. We also remark that as opposed to the resonant delocalization situation, where the effective hopping depends only on the coupling constant modulation depth ε, here the tunneling between adjacent waveguide channels is a function of the modulation period and the refractive index ramping as well. Thus, when random effects are included, the degree of localization enhancement will be determined by the interplay between these two parameters. Depending on the ratio f_0/Ω, Anderson-like localization in waveguide arrays (or their analogous quantum systems) described by eq. (4) can supersede or precede localization effects obtained for optical lattices obeying eq. (1) when both exhibit the same degree of randomness.

Conclusions. – In conclusion, we have studied the effect of disorder on wave propagation in modulated Bloch arrays. These configurations may provide better understanding to similar dynamics occuring in time-dependent quantum systems. We derived the quasi-stationary Houston modes of the structure under resonant delocalization conditions, and showed that the hopping constant between two adjacent channels is renormalized, and for the specific case under investigation, to weaker values. By investigating single channel excitation in such geometries under random uniform disorder, we demonstrated localization dynamics, occurring for much weaker disorder parameters, compared with those needed to observe similar effects in uniform lattices. We have also studied localization effects in modulated Bloch arrays under Gaussian beam excitation conditions. In this latter scenario, we found that even weaker disorder is enough to observe beam localization effects. Finally, we have compared our results with those obtained for Bloch waveguide arrays having a periodic modulation in their propagation constants. In this case, we have demonstrated that enhancing Anderson-like localization effects is still possible and we have highlighted the difference between this process and localization effects associated with resonant delocalization regimes described by eq. (1).

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