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Abstract

This paper develops a North-South trade model with heterogeneous labour and horizontally differentiated products and compares the implications of two policies: Southern intellectual property rights (IPRs) and Northern immigration policy, with the latter aiming to attract Southern talent as means of preempting imitation. Individuals self-select into becoming entrepreneurs and innovate (imitate) in the North (South). The likelihood of imitation depends on product quality, imitator's talent, and IPRs strength. Several interrelated channels of competition are identified. Allowing high-talent migration when IPRs protection in the South is weak shifts imitation to low-quality and innovation to high-quality products. The outcome is in stark contrast to the policy of strengthening Southern IPRs, which limits low-talent imitation in the South and encourages low-quality innovation in the North. Migration also increases the income of low-talent entrepreneurs, as well as the average quality of products imitated by high-talent entrepreneurs in the South. Global income rises with migration, but is not guaranteed to rise with stronger Southern IPRs.

Keywords: Intellectual property rights; High-skilled migration; Imitation; Talent; Innovation; Product quality; Entrepreneur ability; North-South trade.

JEL classification: F22, O31, O34, J24, K37, O38.

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1 Introduction

While the advanced economies (the North) may view imitation as a threat to innovation activity, imitation remains vital to technological progress in a large number of non-industrialized developing economies (the South). To limit imitation, the North actively pushes to raise the global standards of intellectual property rights (IPRs), while the South firmly resists this push, fearing that the further reforms would limit its access to Northern innovative products and technologies and thereby, stifle its development. Meanwhile, the South is concerned with an alarming loss of talent ("brain drain") as high-talent individuals migrate to the North, where intellectual assets have stronger protection and career prospects are better.[1] Using data from Miguelez and Fink (2013) and Park (2008), Figure 1 shows that inventor emigration rates are highest among countries with weak IPRs. This negative association persists over time, even as the strength of IPRs rises around the world to meet global standards. But as the talent pool declines in the South, so does the imitation threat. Rather than pushing for stronger IPRs in the South, the North can use its immigration policy to preempt imitation. Industry leaders in the North have long emphasized the importance of attracting high-talent individuals from the South, arguing that a more open high-skilled migration policy would allow the Northern firms to limit competitive pressure from the South and also strengthen the North's innovation capacity. In countries where the enforcement of IPRs continues to pose a serious concern, the competitive pressure from imitators is of utmost importance for the Northern firms.[2]

[Figure 1 here]

This paper explores the role of migration policy as a tool to battle imitation. The key questions in this respect are: What are the consequences of global talent flows? How do the two policies—i.e., strengthening Southern IPRs and opening the North to high-talent immigration— compare or

[1]Many advanced countries (e.g., Canada, Australia and the UK) rely on points-based immigration systems to select high-skilled immigrants. A potential immigrant with high education, language fluency, experience in a skill-based occupation will receive more points and so, is more likely to be admitted. In these countries, the government acts as the main gatekeeper. In the U.S., by contrast, high-skilled immigration is primarily employment based. Firms directly select high-skilled immigrants whom they want to employ and use immigration to gain access to high talent and rely on this talent to unlock innovation (Kerr, 2018).

[2]Pushing for expansion of H-1B visa—a non-immigrant visa that allows U.S. companies to temporarily employ foreign workers in specialty occupations that require theoretical or technical expertise,—an American billionaire entrepreneur Mark Cuban said on Fox News in 2017: "If American companies can't go out and hire the best talent, then those American companies are still gonna have to compete with those smarter people in the global economy, no matter what." When the South's innovative capability is weak, this competition is between innovating firms in the North and imitating firms in the South. Both imitation and innovation require knowledge and since high-talent migrants are carriers of knowledge, they are a competitive asset. Access to top global talent is the key determinant of nations' competitive advantage and as Hansen and Niedomysl (2009) underscore, this is particularly true in today's international economy where the time span from innovation to imitation of goods and services is rapidly declining.

interact in their impact on imitation activity in the South and innovation activity in the North? We introduce an occupational choice model of an innovative North and an imitative South with two dimensions of heterogeneity: product varieties spread over a continuum of quality and individuals differing in their entrepreneurial ability. This framework allows us to study the impact of high-talent migration and IPRs policies on the composition of innovated and imitated products, while accounting for competition between entrepreneurs and different types of varieties. An individual in each region can become an entrepreneur or a production worker. Entrepreneurs earn rents associated with their ability. In the North, a higher ability entrepreneur innovates a higher quality variety and earns more rents. In the South, entrepreneurs imitate innovated varieties: imitation is easier for high-talent entrepreneurs but for a given talent, higher quality varieties are more difficult to imitate. Accordingly, the imitator's expected rents rise with product quality initially but fall eventually as the likelihood of successful imitation falls. The endogenous entrepreneurship decisions determine the policies' impacts.

Strengthening IPRs in the South reduces the expected rents of imitators, forcing some low-talent entrepreneurs to exit imitation. Thus, the policy restrains imitation but only of low-quality varieties. In the North, the innovators' rents rise, because the competitive pressure from imitation in the Southern market falls, and low-talent individuals enter into innovation of low-quality products. Opening the North to high-talent immigration as an alternative policy implies the reallocation of high-talent Southern entrepreneurs from high-quality imitation in the South to high-quality innovation in the North, which creates three effects. First is a direct "brain drain" effect: as the mass of high-talent entrepreneurs in the South falls, the set of high-quality varieties that are imitated contracts. At the same time, the mass of high-quality varieties innovated (by the migrants) rises and with that, the spending on each individual variety falls in each region. This negative competition effect reduces the rents of entrepreneurs in each region, which discourages low-quality imitation and innovation. But low-talent entrepreneurs in the South also benefit from a positive competition effect, which arises because high-quality imitation contracts and innovation shifts away from low-quality products. This effect is particularly strong in the South where IPRs protection is weak and wage rate is low, in which case the rents of low-talent Southern entrepreneurs rise, encouraging low-quality imitation.

The two policies differ critically in their impact on the entrepreneurial activity in each region. The North's high-talent immigration policy targets imitation of high-quality products (due to "brain drain") and shifts innovation towards high-quality varieties. While migration creates competition for North-born innovators, it also allows them to better exploit the Southern market by preempting high-quality imitation. Migration increases average quality of products innovated in the North and also, of products imitated by high-talent entrepreneurs in the South. These results are in sharp contrast to the policy of strengthening Southern IPRs, which affects low-quality imitation most and promotes low-quality innovation.

Further, the policies differ in their impact on income and its distribution within each region. With migration, the aggregate income in the production sector falls in the South and rises in the North. But a strengthening of IPRs in the South has the opposite effect. These results suggest that openness of the North to high-talent migration is more appealing from a global development perspective as it helps the South reduce its reliance on the production sector and promote transition towards a more entrepreneurial economy. With respect to the entrepreneurial rents in each region, strengthening IPRs only affects rents in the Southern market, where it transfers rents from the entrepreneurs in the South to the entrepreneurs in the North. Migration, by contrast, affects rents in both markets. For the North-born entrepreneurs, the loss of rents in the Northern markets is offset by the gain of rents in the Southern market; and for the non-migrant entrepreneurs in the South, the loss of rents in high-quality imitation is offset by the gain in low-quality imitation.

Overall, we find that when the migrants' earnings count towards the Southern income, allowing high-talent migration to North increases income in the South and might also increase income in the North, particularly so when the South's IPRs are weak. When the migrants' earnings count towards the Northern income, by contrast, income always rises in the North and falls in the South. But global income necessarily rises with high-talent migration to the North. With strengthening IPRs in the South, by contrast, income rises in the North and falls in the South; and global income is not guaranteed to rise.

The paper proceeds as follows. Section 2 discusses the relevant literature. Section 3 describes the basic North-South model of trade with full enforcement of IPRs in the South. Section 4 focuses on the imitation and innovation decisions when IPRs in the South are partially enforced, describes the model equilibrium, and examines the impact of strengthening IPRs. In Section 5, we assume the North introduces a migration quota for the entry of high-talent individuals and study how this policy impacts the model equilibrium. Section 6 compares the policies' impacts on income and welfare. Section 7 concludes.

2 Literature

The underlying purpose of IPRs is to encourage innovation by protecting innovators' intellectual assets from imitation. The literature on trade-related IPRs has uncovered the important role of IPRs protection in stimulating technology transfer via international trade and multinational firm activity (see, e.g., Helpman, 1993; Maskus and Penubarti, 1995; Lai, 1998; Branstetter et al., 2006; Branstetter et al., 2007; Ivus, 2010, 2015; Canals and Sener, 2016; Ivus et al., 2017; Ivus and Park, 2019; Ivus and Saggi, 2020). The migration literature has also uncovered the important role of skilled immigration in promoting innovation in host countries by offering expertise and entrepreneurial skills (Ganguli et al., 2020).[3] Bosetti et al. (2015) argues that policies aimed at attracting skilled migrants could boost innovation in Europe. Miguelez and Moreno (2015) investigate the effectiveness of European policies aimed at attracting foreign researchers and examines preconditions under which migrant researchers help foster EU competitiveness in innovation. Stuen et al. (2012) find that international doctoral students contribute to knowledge production at scientific laboratories in the US, and argues that visa restrictions on the entry of high-quality students are particularly costly for academic innovation. Kerr and Lincoln (2010) show that total science and engineering employment and invention increases with more H-1B visa admissions in the US, mainly due to immigrants' direct contributions. More recently, Akcigit et al. (2017) and Morrison et al. (2018) find, using historical data, that immigrant inventors to the US substantially contribute to innovation in the country and generate positive knowledge spillovers that benefit the productivity of US inventors.

The above two literatures have been pursued in isolation of each other. Few studies to date have studied IPRs and migration in a unified framework. Mondal and Gupta (2008) introduce migration into the Helpman (1993) model of trade-related IPRs and show that a strengthening of IPRs in the South decreases the share of imitated products, shifts labour from the South to the North, and promotes innovation. In Kuhn and McAusland (2009), skilled emigrants improve the quality of goods to the benefit of the source country consumers; the increment in quality is large when the source and host countries have more disparate IPRs. McAusland and Kuhn (2011) consider whether governments looking to attract migrant innovators have a greater incentive to protect intellectual property. The paper shows that while advanced developing countries pass overreaching IPRs (i.e., relative to globally efficient levels), poorer developing countries with large innovator emigration find it optimal to under-protect IPRs, assuming goods produced abroad are valued less at home. In Naghavi and Strozzi (2015), IPRs protection works as a moderating factor between migration and innovation. Knowledge acquired by emigrants flows back to the source country through diaspora networks, and this flow-back generates brain gain when the source country's IPRs are strong.

An important question in the study of high-skilled migration flows is: What type of entrepreneurial activities would the migrants have undertaken in their origin countries had they not had the opportunity to emigrate? The majority of innovative activity is concentrated in high-income economies; while in many low-income countries, entrepreneurs engage in imitative research and development activities because economic institutions, regulatory environment, infrastructure,

[3] Also, innovator mobility is one channel through which technologies are diffused worldwide (e.g., Hunt and Gauthier-Loiselle, 2010; Moser et al., 2014). Ganguli (2015), for example, draws upon the influx of Russian scientists to the US after the end of the Soviet Union to show how high-skilled immigrants contribute to knowledge diffusion as a basis for innovation and economic growth.

and market and business sophistication are not conducive to innovation (Dutta et al., 2016).[4]

And yet, despite the rich array of evidence on the relationship between high-skilled immigration and innovation, there has been practically no discussion of imitation in the international migration literature. One notable exception is Kerr (2008), which outlines a leader-follower model of technology transfer through ethnic networks and then empirically evaluates the role of U.S. ethnic scientific and entrepreneurial communities for international technology transfer to the countries of origin. While the model developed in our paper is novel in that the occupational decisions of entrepreneurs are endogenously determined in the presence of labour and product market heterogeneity, it resembles Kerr (2008) in three respects. First, the focus in Kerr (2008) is on the steady-state equilibrium where the leading economy does not imitate and the follower's economy does not innovate. Similarly, our framework is of innovative North and imitative South. Kerr (2008) argues that such equilibrium is not limited to extremely poor regions but may also arise in emerging economies with a small inventive stock. Secondly, similar to our paper, Kerr (2008) assumes that entrepreneurial scientists imitate foreign inventions for domestic use only, and a larger stock of foreign inventions provides a larger pool of technologies available for imitation. Last, its assumption that the follower's expatriates work only in the research sector is also akin to ours.

This paper highlights the importance of studying skilled migration through the framework of entrepreneurial decisions and in this respect, contributes to two recent important approaches in the literature. The first approach is to analyze skilled migration from the perspective of the firm. Kerr et al. (2015a) argue that firms are mostly absent from the literature on the impact of skilled migration, stressing that many skilled immigrant admissions are driven by firms themselves (e.g., the H-1B visa). Kerr et al. (2015b) and Kerr et al. (2016) underscore the need for greater clarity in understanding the heterogeneity in firm employment choices and the impact of migration on reallocation across firms and aggregate productivity. In our paper, we abstract from the migration decisions and instead, focus on the heterogeneity in the entrepreneurial decisions. Immigration creates competitive pressure and affects the occupational choice and productivity of native entrepreneurs in the North and the remaining entrepreneurs in the South.

The second approach is to consider the impact of skilled migration on labour-market outcomes when knowledge and skills are specialized. Borjas and Doran (2012) examine the productivity effects of the influx of Soviet mathematicians after 1992 on the American (and global) mathematics community. The paper finds that the influx into the U.S. created competitive pressure in both the job market and in the market for codified knowledge; consequently, marginal U.S. mathematicians became much more likely to move to lower quality institutions and exit knowledge production

[4]Talent and IPRs protection are not sufficient for a developing South to transition to an innovating economy. It must also possess a critical level of complementary research, technological, and marketing assets to ensure that its entrepreneurs can absorb the transferred technologies, introduce new products, and exploit innovations.

altogether. In Peri and Sparber (2011), immigration affects occupations and the associated skill content of native-born employees. The paper finds that immigrants with graduate degrees specialize in occupations demanding quantitative and analytical skills, while native-born employees with the same educational attainment move to occupations requiring interactive and communication skills. Wadhwa et al. (2012) find that during 2006-2012, immigrants founded 24% of engineering and technology companies in the U.S. and 44% of high-tech companies in Silicon Valley.[5]

In line with this literature, our paper emphasizes that the heterogeneous talent and the diverse range of quality among products available for imitation are important considerations when studying the impacts of migration from the developing South. The labour and product market heterogeneity affects the type of activities entrepreneurs engage in, as well as their productivity, and also determines the degree of competitive pressure that migration creates in labor and product markets.

3 The Basic Set-up

Suppose the world consists of two regions: imitative South ($i = S$) and innovative North ($i = N$). The North invents new products because of its comparative advantage in research and development (R&D). The South imitates products because the regulation and enforcement of IPRs is not so strong as to make imitation prohibitively expensive. The mass of individuals is L_N in the North and L_S in the South, normalized to $L_N = 1$ and $L_S = l$. An individual can be an entrepreneur or a production worker in each region. An entrepreneur is an innovator in the North and an imitator in the South. Each worker provides one unit of labour, and labour is the only factor of production.

There exists a homogeneous good υ_0, which is treated as the numeraire. Suppose that innovation introduces a continuum of differentiated product varieties, denoted by $\upsilon \in \Upsilon$. The instantaneous utility function of the representative agent in region $i = \{S, N\}$ is given by

$$U_i = c_i(\upsilon_0) + \left[\int_{\Upsilon} z(\upsilon)^{1-\theta} c_i(\upsilon)^{\theta} d\upsilon \right]^{\frac{1}{\theta}},$$

where $c_i(\upsilon_0)$ is the consumption of the homogeneous good; $z(\upsilon)$ and $c_i(\upsilon)$ respectively denote the quality and consumption level of variety υ; and $\theta = (\sigma - 1)/\sigma$, with $\sigma > 1$ being the constant elasticity of substitution in consumption.[6]

[5]Kahn et al. (2017) finds that the documented higher rates of entrepreneurship among immigrants (an immigrant entrepreneurship premium) are specific to science-based entrepreneurship, which is consistent with immigrants having larger endowments of entrepreneurial skills or greater alertness to entrepreneurial opportunities. Kerr and Kerr (2016) quantifies immigrant contributions to new firm creation, and finds that immigrant-founded businesses have faster employment growth than native businesses, particular in high-tech sectors.

[6]Product quality is modelled as a utility shifter in, for example, Hummels and Klenow (2005) and Hallak (2006).

3.1 A Closed Northern Economy

Individuals in the North are spread over a continuum of ability $a \in [0, 1]$, distributed with density $g_N(a)$ and cumulative distribution $G_N(a)$. The North innovates and produces differentiated product varieties, the market for which is monopolistically competitive with free entry and exit into each variety. An individual can become an entrepreneur who innovates a variety υ of quality $z(\upsilon)$. High ability leads to high-quality variety and thus, ability a and quality z are the same. Consequently, the density of entrepreneurs that innovate varieties of quality $z(\upsilon)$ is given by $f(z) = L_N g_N(z) = g_N(z)$, with cumulative distribution denoted by $F(z)$.[7]

Individuals who do not become entrepreneurs become production workers.[8] Workers are homogeneous in their production ability. One unit of labour produces one unit of the innovated variety or w (> 1) units of the homogeneous good. The homogeneous good market is competitive. We normalize the price of the homogeneous good to one. Thus, the North's wage rate is w.[9]

Assume for now that the Northern economy is closed. The representative individual in the North faces the following budget constraint: $Y_N = c_N(\upsilon_0) + E_N$, where Y_N denotes total income that is equal to total expenditure; $E_N = \int_\Upsilon p_N(\upsilon)c_N(\upsilon)d\upsilon$ is the aggregate expenditure on the differentiated varieties that is exogenously given; and $p_N(\upsilon)$ and $c_N(\upsilon)$ are the price and consumption of variety υ. Maximizing U_N subject to Y_N yields the following "quality-adjusted" demand for each differentiated variety in the North:

$$c_N(\upsilon) = z(\upsilon) \left[\frac{p_N(\upsilon)}{P_N}\right]^{-\sigma} \left(\frac{E_N}{P_N}\right), \tag{1}$$

where $P_N \equiv \left[\int_\Upsilon z(\upsilon)p_N(\upsilon)^{1-\sigma}d\upsilon\right]^{1/(1-\sigma)}$ is the price-quality index. Assuming an interior solution, consumption of the homogeneous good is determined by the residual income $c_N(\upsilon_0) = Y_N - E_N$.[10]

An entrepreneur that innovates variety υ enjoys a monopoly in that variety because varieties of the same quality are horizontally differentiated. The entrepreneur charges the monopoly price $p_N(\upsilon) = p_N = w/\theta$, which equals a fixed markup above marginal cost w, and sells $c_N(\upsilon)$. This

[7]With this assumption, the intensive margin of innovation of each quality is fixed. In the Appendix A.1, we revise the model to allow for endogenous intensive margin, so that we can examine the policies' impacts on the intensive margin of innovation.

[8]We assume no imitation in the North. Thus, no one becomes an imitator.

[9]In each region, the homogeneous good sector works as a buffer to absorb all residual labour.

[10]With quasi-linear preferences and a given aggregate expenditure on the differentiated varieties, prices determine the demand for the differentiated varieties and the residual income determines the homogeneous good demand. The consumption of the homogeneous good adjusts to fully absorb a change in income; there is no income effect on the consumption of differentiated varieties. Alternatively, one could fix the income share of the differentiated varieties. This specification would embed the income effect on the consumption of differentiated varieties, as it would imply that the aggregate expenditure on differentiated varieties adjusts in a fixed proportion to the change in income.

decision is the same for all varieties of the same quality z. The entrepreneurial rents are given by

$$\pi(z) = (p_N - w)c_N(z) = \frac{z}{\sigma}\left(\frac{p_N}{P_N}\right)^{1-\sigma} E_N. \tag{2}$$

The rents rise with product quality: $\pi'(z) > 0$. With free entry into innovation, there must exist a cutoff $\hat{z}$ such that the entrepreneurs with quality (ability) $z = \hat{z}$ are indifferent between working as production workers or innovators: $\pi(\hat{z}) = w$. Entrepreneurs with $z > \hat{z}$ earn rents $\pi(z) > w$. From (2), the implicit solution for the cutoff $\hat{z}$ is

$$G(\hat{z}) \equiv \hat{z}E_N - \sigma w \int_{\hat{z}}^{1} z dF(z) = 0. \tag{3}$$

Thus when the Northern economy is closed, the quality set of innovated varieties is $\Phi = [\hat{z}, 1]$.

3.2 Open Economy with Full IPRs Enforcement in the South

We consider the global economy with free trade. In the South, individuals become either production workers or entrepreneurs who imitate Northern varieties. One unit of labour produces one unit of the imitated differentiated variety or one unit of the homogeneous good. The homogeneous good is traded at the price of $p(\upsilon_0) = 1$. Thus, the Southern wage rate is equal to one.

The likelihood of imitation depends on the strength of IPRs protection in the South, measured by $\Omega \in [0, 1]$. As in Grossman and Lai (2004), Ω is the probability that a patent is enforced by the Southern government. In the North, IPRs are fully enforced.

In this subsection, we consider the full enforcement of IPRs in the South: $\Omega = 1$. In this case, the Northern entrepreneurs can sell their varieties in the global market without risking imitation, and all individuals in the South produce the homogeneous good.

Let $c_X(\upsilon)$ denote the demand for variety υ in the South and Υ^F denote the set of innovated varieties when $\Omega = 1$. The representative individual in the South faces this budget constraint: $Y_S = c_S(\upsilon_0) + E_S$, where Y_S denotes total income that is equal to total expenditure; $c_S(\upsilon_0)$ is the consumption of the homogeneous good; $E_S = \int_{\Upsilon^F} p_X(\upsilon)c_X(\upsilon)d\upsilon$ is the aggregate expenditure on the imported varieties; and $p_X(\upsilon)$ and $c_X(\upsilon)$ are the price and consumption of variety υ. Maximizing U_S subject to Y_S, we obtain the South's "quality-adjusted" demand for each variety

$$c_X(\upsilon) = z(\upsilon)\left[\frac{p_X(\upsilon)}{P_X}\right]^{-\sigma}\left(\frac{E_S}{P_X}\right),$$

where $P_X \equiv \left[\int_{\Upsilon^F} z(\upsilon)p_X(\upsilon)^{1-\sigma}d\upsilon\right]^{1/(1-\sigma)}$ is the price-quality index, and $c_S(\upsilon_0) = Y_S - E_S$.

A Northern entrepreneur that innovates variety υ sets the monopoly price $p_X(\upsilon) = p_N(\upsilon) = w/\theta$

in both regions and earns the global rents given by

$$\pi_N(z) = (p_N - w)[c_N(z) + c_X(z)] = \frac{z}{\sigma}\left(\frac{p_N}{P_N}\right)^{1-\sigma}(E_N + E_S), \tag{4}$$

where $P_N = P_X$. As $\pi'_N(z) > 0$ and entry into innovation is free, there exists a cutoff $\bar{z}$ such that entrepreneurs with quality $z = \bar{z}$ are indifferent between working as production workers or innovators: $\pi_N(\bar{z}) = w$. Using (4), we find the following implicit solution for the cutoff $\bar{z}$:

$$G^F(\bar{z}) \equiv \bar{z}(E_N + E_S) - \sigma w \int_{\bar{z}}^{1} z dF(z) = 0. \tag{5}$$

Thus in an open global economy with full IPRs enforcement in the South, the quality set of innovated varieties is $\Phi^F = [\bar{z}, 1]$. Based on the above analysis, we establish the following result:

Proposition 1. *A unique equilibrium $\hat{z} \in (0,1)$ exists in a closed Northern economy, such that individuals with ability $a \geq \hat{z}$ choose to become entrepreneurs innovating varieties of quality $z \geq \hat{z}$ and individuals with ability $a < \hat{z}$ choose to become production workers. This equilibrium is instead $\bar{z} \in (0, \hat{z})$ in an open global economy with full IPRs enforcement in the South.*
Proof: See Online Appendix.

Since $\bar{z} < \hat{z}$, trade enlarges the quality set of innovated varieties.

4 Imitation, Innovation, and IPRs

In this section, we consider partial enforcement of IPRs in the South: $\Omega < 1$. A Northern entrepreneur who sells an innovated variety of quality z in the global market enjoys a monopoly in the South until this variety is imitated. An imitated variety is a copy of the Northern variety that infringes on Northern patents; hence while it is sold to consumers in the South, it is not exported to the North, where IPRs are fully enforced.

4.1 Imitation

Each individual in the South can enter imitation and become an entrepreneur to compete with imports of the Northern variety that it imitates. Imitators have the same production technology as innovators but lower marginal cost (labour cost). We assume that competition is on the basis of price with $w > 1/\theta$, where a large North-South marginal cost differential allows the Southern entrepreneur to charge the monopoly price $p_S = 1/\theta$ and earn the rents $\pi_S(z) = (1/\theta - 1)c_S(z)$.[11]

[11] This is referred to as the wide-gap case in Grossman and Helpman (1993). The alternative is the narrow-gap case, when $w < 1/\theta$. In this case, a Southern entrepreneur enjoys a relatively small cost advantage over its Northern

The quality set of innovated varieties is $\Phi^F = [\bar{z}, 1]$ in an open global economy with full IPRs enforcement in the South ($\Omega = 1$) and $\Phi = [\hat{z}, 1]$ in the closed Northern economy case. In an open global economy with partial IPRs enforcement in the South ($0 < \Omega < 1$), the quality set of innovated varieties will include all varieties with quality $z \geq \hat{z}$ and possibly some varieties with quality $z \in (\bar{z}, \hat{z})$. We let Φ^P denote this set and determine it below.

Let Υ^P be the set of innovated varieties and $\Upsilon^M \subset \Upsilon^P$ be the set of imitated varieties when $\Omega \in (0, 1)$. Then, the representative agent in the South faces the following budget constraint: $Y_S = c_S(v_0) + E_S$, where the aggregate expenditure on the differentiated varieties is $E_S = \int_{\Upsilon^P \setminus \Upsilon^M} p_X(v) c_X(v) dv + \int_{\Upsilon^M} p_S(v) c_S(v) dv$. That is, E_S is the sum of the expenditure on the imported innovated and domestic imitated varieties, which are priced at $p_X(v)$ and $p_S(v)$, respectively. Maximizing U_S subject to Y_S, we obtain the South's "quality-adjusted" demand for each innovated and imitated variety, respectively,

$$c_X(v) = z(v) \left[\frac{p_X(v)}{P_S} \right]^{-\sigma} \left(\frac{E_S}{P_S} \right), \qquad c_S(v) = z(v) \left[\frac{p_S(v)}{P_S} \right]^{-\sigma} \left(\frac{E_S}{P_S} \right), \tag{6}$$

where $P_S \equiv [\int_{\Upsilon^P \setminus \Upsilon^M} z(v) p_X(v)^{1-\sigma} dv + \int_{\Upsilon^M} z(v) p_S(v)^{1-\sigma} dv]^{1/(1-\sigma)}$ is the price-quality index, and consumption of the homogeneous good determined by the residual income $c_S(v_0) = Y_S - E_S$. Since the two markets are segmented, the consumption decision in the North is the same as that in Section 2.1, except that the set of innovated varieties is now Υ^P rather than Υ.

Next, we need to determine the quality set of innovated varieties Φ^P and the quality set of imitated varieties Φ^M when $\Omega \in (0, 1)$. To simplify our exposition, we first examine the imitation decision when all varieties $z \geq \bar{z}$ are available for imitation (which arises when $\Omega = 1$), and study the innovation decision in the following section.

All Southern individuals have the same ability in production but differ in talent when it comes to imitation: they are divided into low-talent L and high-talent H types, with their respective mass given by g_S^L and g_S^H. With a large enough g_S^L, the L individuals face an occupational choice decision, with some ending up working in production while others becoming entrepreneurs in imitation. The rents in imitation are equal to the wage in production for the threshold L individual, who is thus indifferent between the two occupations. The mass of L entrepreneurs, denoted by f_S^L, is endogenously determined and satisfies $f_S^L < l g_S^L$. In contrast, assuming a small enough mass of high-talent individuals in the South, all H individuals become entrepreneurs in imitation, each earning more than an L entrepreneur or a production worker. Thus the mass of H entrepreneurs is $f_S^H = l g_S^H$.

rival and is unable to set the monopoly price without fear of being undercut. Thus, the Southern entrepreneur sets the price $p_S = w$ and earns the rents $\pi_S(z) = (w - 1) c_S(z)$.

The expected rents of a Southern j imitator of a variety with quality z are given by

$$\pi_S^j(z) = \mu^j(z)\,(p_S - 1)\,c_S(z) = \frac{z\mu^j(z)}{\sigma}\left(\frac{p_S}{P_S}\right)^{1-\sigma} E_S, \tag{7}$$

where $\mu^j(z)$ is the likelihood of the innovated variety with quality z being imitated by $j \in \{L, H\}$ entrepreneur. We define this likelihood as follows:

$$\mu^j(z) \equiv (1 - \Omega)m^j(z), \tag{8}$$

where $m^j(z)$ is the rate of imitation that depends on the quality z and the entrepreneur's type j. It is natural to assume that for a given quality z, imitation is easier for high-talent entrepreneurs: $m^L(z) < m^H(z)$; but for any given entrepreneur, higher quality varieties could be more difficult to imitate: $dm^j(z)/dz \leq 0$.[12] A Southern entrepreneur earns more rents from imitating a higher quality variety, but above a certain quality level, higher quality varieties become more difficult to imitate, and the expected rents from imitation start falling. We assume that the rents $\pi_S^j(z)$ reach maximum at a_j, so that a_j is the optimal quality for a j entrepreneur to imitate. This optimal quality is lower for the low-talent type: $a_L < a_H$.

To model the difficulty of imitating high-quality varieties, we adopt this simple linear specification for the imitation rate:

$$m^j(z) = \begin{cases} \alpha_j & \text{for } z \leq a_j, \\ \alpha_j - \beta_j z & \text{for } z > a_j. \end{cases} \tag{9}$$

[Figure 2 here]

Figure 2 shows that $m^j(z)$ stays at α_j until it reaches quality level a_j, after which point it falls (at a rate β_j) as quality z rises. In other words, each entrepreneur can conveniently imitate up to a certain quality, after which imitation becomes increasingly difficult.[13] This occurs at a later stage for H imitators: $a_H > a_L$. The parameter α_j represents the absorptive capacity of j imitators, and the difference between a_L and a_H reflects the degree of heterogeneity between the two types of individuals in terms of their ability to imitate higher quality varieties. Lemma 1 summarizes how entrepreneurs' rents change with quality:

[12] For simplicity, we use probability to model the difficulty of imitating high-quality varieties. Alternatively, we could assume that imitating a higher quality variety requires more resources (i.e., is more costly). We consider this alternative formulation in the Appendix A.2. We thank an anonymous reviewer for this comment.

[13] High-quality varieties would be more difficult to imitate if, for example, their underlying technology is too complex. Teece (1986) argued that the complex, tacit nature of knowledge reduces appropriability hazards which result from technological leakage of information leading to imitation by rivals. Also, high-quality varieties could be better protected from imitation by market-made technology 'masquing' (Taylor, 1993), fragmented to discourage imitation (Zhao, 2006), or covered by trade secret protection (Donoso, 2014).

Lemma 1. *$d\pi_S^j(z)/dz > 0$ for $z \leq a_j$ and $d\pi_S^j(z)/dz < 0$ for $z > a_j$, where $a_j = \alpha_j/(2\beta_j)$.*
Proof: The maximization of (7) with respect to z given (9) yields $a_j = \alpha_j/(2\beta_j)$.

Figure 3 shows the imitators' rent functions in relation to the imitation rate functions.

[Figure 3 here]

Suppose that the L and H entrepreneurs do not compete for imitating the same variety so that the L and H imitation sets do not overlap. This is true for a large enough gap between the best varieties for each type to imitate (low a_L and high a_H), accentuating heterogeneity in entrepreneurial talent to imitate, which is the focus of our paper.[14] All Southern entrepreneurs of the same type are *ex-ante* identical. If a sufficient number of Southern entrepreneurs imitates varieties of quality z (within Φ^P), then the entrepreneurs will be randomly allocated starting from a_j on both sides until the density of Southern entrepreneurs imitating the varieties is equal to the density of Northern entrepreneurs innovating the varieties, $f(z)$.

The quality set of varieties imitated by the H entrepreneurs is continuous and therefore, we can denote it as $[z_{H0}, z_{H1}]$, with $a_H \in (z_{H0}, z_{H1})$. In equilibrium, the mass of Southern H entrepreneurs must equal the mass of Northern entrepreneurs over the range $[z_{H0}, z_{H1}]$, such that:

$$lg_S^H = \int_{z_{H0}}^{z_{H1}} dF(z). \tag{10}$$

A low-talent individual enters imitation as long as rents $\pi_S^L(z)$ exceed the Southern wage rate. We can denote the quality set of varieties imitated by the L entrepreneurs as $[z_{L0}, z_{L1}]$, with $a_L \in (z_{L0}, z_{L1})$. With free entry into imitation, the equilibrium requires that $\pi_S^L(z) \geq 1$ for any $z \in [z_{L0}, z_{L1}]$, where the rents at the two end points are equal to the Southern wage rate. Then the mass of L entrepreneurs is endogenously determined as follows: $f_S^L = \int_{z_{L0}}^{z_{L1}} dF(z)$.

Thus, the quality set of imitated varieties is $\Phi^M = [z_{L0}, z_{L1}] \cup [z_{H0}, z_{H1}]$. To simplify the exposition, we restrict the range to $z_{L0} \geq \hat{z}$ and $z_{H1} < 1$. Ruling out $z_{H1} = 1$ avoids the corner solution where the South is able to imitate even the most sophisticated high-quality varieties. And restricting $z_{L0} \geq \hat{z}$ ensures that the imitation set is a subset of the innovation set in a closed Northern economy.[15] The L entrepreneurs will not engage in imitation of an innovated variety with quality z below z_{L0} if such imitation does not lead to sufficiently high rents, above the wage

[14] In the Appendix A.4, we discuss our results in the full overlapping case, which reduces the framework to a unique type of entrepreneurs in the South. In addition, in the Online Appendix, we provide the solution for the more general overlapping case.

[15] As we show below, this assumption in turn implies that with partial enforcement of IPRs, the equilibrium innovation cutoff z^P is in the range $(\bar{z}, \hat{z})$, where $[\bar{z}, 1]$ is the quality set of innovative varieties in the open economy with full IPRs enforcement in the South and $[\hat{z}, 1]$ is the quality set of innovative varieties in the closed Northern economy. Allowing $z_{L0} < \hat{z}$ would not qualitatively change our results, as we discuss in the Appendix A.3.

of one in production. Both of these low-quality varieties, $z < z_{L0}$ and z_{L0}, are equally easy to imitate; but the lower-quality variety $z < z_{L0}$ faces a lower demand and so, leads to lower rents.[16]

Lemma 2. *Given $z_{L0} \geq \hat{z}$, $z_{H1} < 1$ ($\int_{a_H}^{1} dF(z) > f_S^H$), and that $[z_{L0}, z_{L1}]$ and $[z_{H0}, z_{H1}]$ do not overlap ($z_{L1} < z_{H0}$), the end points defining the two intervals, z_{j0} and z_{j1}, are determined by (10), and*

$$\pi_S^L(z_{L0}) = \pi_S^L(z_{L1}) = 1 \quad \textit{and} \quad \pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1. \tag{11}$$

Proof: See Online Appendix.

Figure 4 shows one equilibrium. There are five non-overlapping subsets: least-quality set $[\hat{z}, z_{L0})$ with no imitation; low-quality set $[z_{L0}, z_{L1}]$ with imitation by the L entrepreneurs; middle-quality set (z_{L1}, z_{H0}) with no imitation; high-quality set $[z_{H0}, z_{H1}]$ with imitation by the H entrepreneurs; and highest-quality set $(z_{H1}, 1]$ with no imitation. The expected rents of a j imitator of a variety of quality $z \in \Phi^M$ follow from (7) and are given by

$$\pi_S^j(z) = \frac{z\mu^j(z)}{\sigma}\left[\frac{w^{\sigma-1}E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right], \tag{12}$$

where $P_S = \left[\psi_N p_N^{1-\sigma} + \psi_S(p_S^{1-\sigma} - p_N^{1-\sigma})\right]^{1/(1-\sigma)}$ and $\psi_N \equiv \int_{\bar{z}}^{1} z dF(z)$, $\psi_S \equiv \sum_j \int_{z_{j0}}^{z_{j1}} z\mu^j(z)dF(z)$.

[Figure 4 here]

4.2 Innovation

So far, we have assumed that all varieties with $z \geq \bar{z}$ are available for imitation. We now determine the quality set of innovated varieties Φ^P as follows:

Proposition 2. *Suppose that IPRs are partially enforced in the South. Then, there exists a unique equilibrium $z^P \in (\bar{z}, \hat{z})$, such that individuals with ability $a \geq z^P$ choose to become entrepreneurs innovating varieties of quality $z \geq z^P$ and individuals with ability $a < z^P$ choose to become production workers. The quality set of innovated varieties is $\Phi^P = [z^P, 1]$.*

Proof: See Online Appendix.

The cutoff z^P is implicitly defined by

$$G^P(z^P) \equiv z^P(E_N + \xi E_S) - \sigma w \int_{z^P}^{1} z dF(z) w = 0, \tag{13}$$

[16] Some examples of industries containing very low- or very high-quality varieties for which we do not observe imitation are nails (low quality) and aircraft (high quality). The conjecture is also consistent with the observed difference in industries' dependence/reliance on patent protection as a means of limiting imitation. Low-quality products in "Basic metals," "Non-metallic mineral products," and "Rubber and plastic products" industries do not depend much on patent protection because they face low imitation by rivals (Cohen et al., 2000).

where ψ_N is now defined as follows: $\psi_N \equiv \int_{z^P}^1 z dF(z)$, and $\xi \equiv \psi_N/[\psi_N + (w^{\sigma-1} - 1)\psi_S] < 1$.

Innovators $z \in \Phi^P \setminus \Phi^M$ do not risk imitation, whereas innovators $z \in \Phi^M$ are priced out of the Southern market by the j imitator with probability $\mu^j(z)$. Thus, from (4), with (1) and (6), $P_N^{1-\sigma} = \psi_N p_N^{1-\sigma}$ and $P_S^{1-\sigma} = \psi_N p_N^{1-\sigma} + \psi_S(p_S^{1-\sigma} - p_N^{1-\sigma})$, we obtain the innovators' global rents

$$\left\{ \begin{array}{ll} \pi_N(z) = \frac{z}{\sigma}\left[\frac{E_N}{\psi_N} + \frac{E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right], & \text{for} \quad z \in \Phi^P \setminus \Phi^M \\ \pi_N(z) = \frac{z}{\sigma}\left[\frac{E_N}{\psi_N} + \frac{[1-\mu^j(z)]E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right], & \text{for} \quad z \in \Phi^M \end{array} \right\}. \tag{14}$$

It follows that $\pi'_N(z) > 0$ for z in the set $\Phi^P \setminus \Phi^M$, which is a union of the three continuous sets: $[z^P, z_{L0})$, (z_{L1}, z_{H0}), and $(z_{H1}, 1]$. Also, $\pi'_N(z) > 0$ for z in the set Φ^M, which is a union of the two continuous sets: $[z_{L0}, z_{L1}]$ and $[z_{H0}, z_{H1}]$. The competition with imitators reduces the innovators' rents in the Southern market but does not affect the rents in the North. Thus, $\pi_N(z) > w$ for any $z > \hat{z}$, where $\hat{z}$ is the closed Northern economy cut-off, such that $\hat{z} < z_{L0}$. Hence, the cutoff z^P must be below $\hat{z}$. The cutoff z^P must also be above $\bar{z}$, which is the open economy cutoff when IPRs are fully enforced in the South. Innovators with $z \in (\bar{z}, \hat{z}]$ do not risk imitation but a partial enforcement of IPRs in the South still reduces their rents, because it enables competitive pressure from imitated varieties (that are priced relatively low).[17]

4.3 IPRs Protection in the South

Stronger IPRs in the South reduce the likelihood of the innovated variety of quality z being imitated: $d\mu^j(z)/d\Omega = -m^j(z) < 0$. The j imitator's expected rents $\pi_S^j(z)$ thus fall, and the quality sets of imitated and innovated varieties adjust as follows:

Proposition 3. *Strengthening IPRs in the South expands the innovation set (z^P falls), contracts the L imitation set (z_{L0} rises and z_{L1} falls), and has no effect on the H imitation set (z_{H0} and z_{H1} do not change).*

Proof: See Online Appendix.

Figure 5 illustrates the effects of strengthening IPRs in the South. The direct effect is given by a proportionate shift in $\pi_S^j(z)$. As the L imitators' rents $\pi_S^L(z)$ fall below the Southern wage rate of one, the L entrepreneurs to the immediate right of z_{L0} and left of z_{L1} exit imitation, causing the cutoff z_{L0} to rise and the cutoff z_{L1} to fall. A reduction in the H imitators' rents $\pi_S^H(z)$, by contrast, does not affect the cutoffs z_{H0} and z_{H1}, as long as $\pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1$ as in Lemma 2. Intuitively, the H and L entrepreneurs are impacted differently by a strengthening of IPRs because the H entrepreneurs enjoy the talent premium (i.e., earn higher rents relative to L imitators for any $\Omega < 1$), as is standard in occupational choice model. In this context, the impact

[17] We note that $\xi < 1$ in the equation (13) when $\Omega < 1$.

of stronger IPRs is absorbed by the L entrepreneurs who exit imitation and become production workers; the H entrepreneurs experience a decline in rents but continue to engage in H imitation and enjoy the talent premium.

[Figure 5 here]

When the L imitation set contracts, the imitated varieties are replaced with the imported innovated varieties at the margins. The competitive pressure on the Northern innovators in the Southern market thus falls (because ψ_S falls) and their rents $\pi_N(z)$ rise. This incentivizes Northern individuals with ability to the immediate left of z^P to enter innovation and with such entry, the innovation set expands into lower-quality varieties (z^P falls). As the mass of innovated varieties available for consumption rises, consumer spending on each imitated variety in the South falls (because ψ_N rises), reducing imitators' rents $\pi_S^j(z)$. This negative competition effect of stronger IPRs exacerbates the contraction in the L imitation set.

In our model, strengthening IPRs expands the extensive margin of innovation on the low-quality side but does not impact it on the high-quality side. This is reasonable given that IPRs are strengthened *in the South* and higher-quality varieties are increasingly difficult to imitate. The strength of South's IPRs affects the innovators' competition with imitators in the Southern market only, and it is low-quality innovation that is most acutely impacted by such competition. High-quality varieties above $\bar{z}_{H1}$, by contrast, face no imitation in the South.[18] The innovation of high-quality goods thus only depends on *the North's* IPRs, which are already fully enforced.

5 High-Talent Migration

Suppose now that the North introduces a migration quota M for the entry of high-talent individuals from the South. The quota is restrictive: $M < lg_S^H$, and high-talent individuals are randomly selected to fill it. The high-talent individuals who migrate become innovators in the North, introducing varieties with quality a_H. With zero fixed cost of migration, a high-talent individual will migrate as long as their entrepreneurial rents rise following migration. This requires that $\pi_N(a_H) > \pi_S^H(a_H)$, which holds under the following sufficient condition:[19]

$$\frac{E_N}{E_S} > w^{\sigma-1}. \tag{15}$$

[18] As mentioned in footnote 13, high-quality varieties in "Commercial Machinery" tend not to rely on patent protection to limit imitation because technological complexity or other non-patent methods of protection act as barriers to imitation in this industry (Ivus et al., 2016).

[19] Using (12) and (14), we rewrite $\pi_N(a_H) > \pi_S^H(a_H)$ as $E_N/(\xi E_S) > (w^{\sigma-1}+1)\mu^H(a_H) - 1$. The condition (15) follows because $\xi < 1$ and $\mu^H(a_H) < 1$.

The migration quota is 100% filled provided the North's relative aggregate expenditure on the differentiated varieties (E_N/E_S) is high, the North's wage rate (w) is low, or the elasticity of substitution in consumption (σ) is low.[20] When w and σ are low, the South's expenditure on each imitated variety is low relative to its expenditure on each innovated variety; consequently, the entrepreneurial rents of the high-talent individuals rise significantly following migration.

After migration, the mass of H entrepreneurs in the South falls while the mass of entrepreneurs in the North rises by M and so, the condition (10) becomes:

$$lg_S^H - M = \int_{z_{H0}}^{z_{H1}} dF(z) + M. \tag{16}$$

The migration also increases the mass of innovated varieties of quality a_H available for consumption and also, for imitation. The conditions (11) and (14), which together with (16) define the equilibrium cutoffs, are unchanged, but the (quality-adjusted) numbers of innovated varieties and imitated varieties are now respectively given by:

$$\psi_N \equiv \int_{z^P}^{1} zdF(z) + a_H M \quad \text{and} \quad \psi_S \equiv (1-\Omega)\left[\sum_j \int_{z_{j0}}^{z_{j1}} zm^j(z)dF(z) + a_H m^H(a_H)M\right]. \tag{17}$$

The North's migration policy has two effects on the H imitation activity. First is the direct "brain drain" effect: the mass of H entrepreneurs in the South falls, and the high-quality imitation set contracts in response, as z_{H0} rises and z_{H1} falls to ensure that $\pi_S^H(z_{H0}) = \pi_S^H(z_{H1})$ and condition (10) holds. Second is the negative competition effect. The mass of a_H varieties innovated by the migrants and subsequently imitated in the South rises and with that, the competitive pressure on the H imitators goes up, as spending on each individual variety in the South falls. The price-quality index P_S falls (due to "love of variety"), pushing the rents $\pi_S^H(z)$ down but the H imitation cutoffs z_{H0} and z_{H1} do not change, as long as $\pi_S^H(z) > 1$.

The policy also changes the competitive pressure on the L imitators. Competition coming from high-talent migrants rises as the mass of a_H varieties rises, and the L imitators' rents $\pi_S^L(z)$ fall in response. But the competitive pressure from the H entrepreneurs remaining in the South falls as the H imitation set contracts and high-quality imitated varieties are replaced with more expensive imported innovated varieties at the H margins. The price-quality index P_S rises and so, the L imitators' rents $\pi_S^L(z)$ also rise. This is the positive impact of "brain drain."[21] The L imitators also face competition from North-born innovators. As we discuss below, the cutoff z^P rises following

[20] This follows since $p_X c_X(v)/[p_S c_S(v)] = w^{1-\sigma}$.

[21] In the long run, emigration of high-talent entrepreneurs could also encourage low-talent entrepreneurs to invest in talent acquisition, further mitigating the concern of "brain drain." We thank an anonymous reviewer for this comment.

migration. The innovation set contracts and with that, the competitive pressure on the L imitators falls, and the rents $\pi_S^L(z)$ go up.[22]

Importantly, the overall impact of high-talent migration on the L imitation activity depends on the strength of IPRs in the South, Ω. We show in the Online Appendix (proof of Proposition 4) that the value of Ω influences the weight of competitive pressure from the South-born high-talent individuals who migrate vis-à-vis those who stay in the South, as determined by:

$$\Lambda(\Omega) = (1-\Omega)(w^{\sigma-1}-1)\alpha_H(2z_{H0}-a_H)-a_H. \tag{18}$$

According to (18), the positive impact of "brain drain" (lower competitive pressure on the L imitators due to a contraction in the H imitation set) is significant when IPRs protection is weak in the South (Ω is low). In this case, the overall competition from South-born H entrepreneurs falls following migration. But as Ω rises, the negative competition effect (higher competitive pressure on the L imitators due to an increase in the mass of a_H varieties) starts to dominate the benefit of "brain drain."

The extent of competition from the North-born innovators depends on

$$\Lambda^P(\Omega) = -\frac{a_H\varepsilon^P}{1+\varepsilon^P+\xi E_S/E_N}, \quad \text{where} \quad \varepsilon^P \equiv -\frac{d\psi_N}{dz^P}\frac{z^P}{\psi_N}. \tag{19}$$

Intuitively, according to (19), the positive competition effect caused by a contraction in the innovation set is strong when the quality-adjusted number of innovated varieties ψ_N is highly elastic with respect to the cutoff z^P. The function $\Lambda^P(\Omega)$ is increasing in Ω because stronger IPRs protection in the South counterbalances the impact of migration, as it expands the innovation set into lower-quality varieties (Proposition 3) and shifts Southern expenditure toward innovated varieties (increases ξE_S).

Suppose $\Lambda(0) \geq 0$ or equivalently, the North-South wage gap w is high enough to satisfy

$$(w^{\sigma-1}-1)\alpha_H(2z_{H0}-a_H) \geq a_H. \tag{20}$$

This condition gives a high enough cost advantage of producing in the South that makes imitation worthwhile.[23] Condition (20) also implies from (15) that E_S/E_N is low.

We establish that migration expands the L imitation set when IPRs are sufficiently weak such that $\Lambda(\Omega) > \Lambda^P(\Omega)$, and contracts it when $\Lambda(\Omega) < \Lambda^P(\Omega)$:

Proposition 4 (Migration and imitation). *Opening the North to high-talent migration con-*

[22]The L imitation cutoffs z_{L0} and z_{L1} respond to a change in the L imitators' rents $\pi_S^L(z)$, since $\pi_S^L(z_{L0}) = \pi_S^L(z_{L1}) = 1$.

[23]For this condition to hold it is sufficient that $(w^{\sigma-1}-1)\alpha_H(2\tilde{z}_{H0}-a_H)-a_H \geq 0$, where $\tilde{z}_{H0} < z_{H0}$ is defined by $\pi_S^H(\tilde{z}_{H0}) = \pi_S^H(1)$, which implies that $\tilde{z}_{H0} = (\alpha_H - \beta_H)/\alpha_H$.

tracts the H imitation set (z_{H0} rises and z_{H1} falls). Given (20), there exists a unique critical IPRs strength in the South, $\bar{\Omega}$, such that the L imitation set expands (z_{L0} falls and z_{L1} rises) if $\Omega < \bar{\Omega}$ and contracts (z_{L0} rises and z_{L1} falls) if $\Omega > \bar{\Omega}$.

Proof: See Online Appendix.

Figure 6 plots $\Lambda(\Omega)$ and $\Lambda^P(\Omega)$ and shows the two possible scenarios outlined in Proposition 4. The two functions intersect at a single point, $\Omega = \bar{\Omega}$.[24] When $\Omega < \bar{\Omega}$, emigration of high-talent individuals significantly lessens the competitive pressure from H imitation. In other words, the positive impact of "brain drain" (caused by a contraction in the H imitation set) is strong when IPRs are weak in the South. Weak IPRs also reinforce the positive competition effect due to a contraction in the innovation set in the North. Competitive pressure from imported innovated varieties is weak in the South when IPRs are weak, because the Southern expenditure on imported innovated varieties is relatively low in this case. As a result, migration increases the L imitators' rents and expands the L imitation set when IPRs protection is weak. When $\Omega > \bar{\Omega}$, by contrast, the contraction in H imitation and innovation sets only has a weak positive effect on the L imitators, so that the negative competition effect due to an increase in the mass of a_H varieties in the Southern market dominates. Consequently, the L imitators' rents fall, and the L imitation set contracts when IPRs protection is strong in the South.[25]

[Figure 6 here]

With regard to innovation, the impact on the cutoff z^P is not immediately clear because the North's migration policy affects the innovators' competitive environment in both markets. Competition from the new a_H varieties heightens in both markets, lowering innovators' rents $\pi_N(z)$. In the Southern market, competitive pressure also rises when L imitation expands, but falls when H imitation contracts. Here, we find that as long as the North's relative aggregate expenditure on the differentiated varieties is sufficiently high for immigration to arise (i.e., inequality (15) holds), the overall competitive pressure on innovators rises and their rents $\pi_N(z)$ fall, pushing the cutoff z^P up for any strength of IPRs protection in the South, Ω:[26]

Proposition 5 (Migration and innovation). *Opening the North to high-talent migration contracts the innovation set (z^P rises).*

[24] As Ω rises from zero to one, $\Lambda(\Omega)$ falls at a constant rate from $\Lambda(0) \geq 0$ to $\Lambda(1) = -a_H$ while $\Lambda^P(\Omega)$ rises from $\Lambda^P(0) < 0$ to $\Lambda^P(1) > -a_H$.

[25] For a sufficiently high Southern wage rate (a low w), this negative competition effect could dominate over the entire range of Ω, so that $\Lambda(\Omega) < \Lambda^P(\Omega) < 0$. We discuss this case in the Online Appendix.

[26] High-talent migrants from the South might not immediately become innovator in the North and instead, end up working in production in the North. As we discuss in the Appendix A.5, migration could expand the innovation set in this case.

Proof: See Online Appendix.

We can also examine the two policies' impacts on the composition of innovated and imitated varieties. Strengthening IPRs in the South and allowing high-talent migration to North both increase the quality-adjusted number of innovated varieties (ψ_N), given in (17). This number rises with stronger IPRs because the innovation set expands (z^P falls). It also rises with migration, but this is due to an increase in the mass of entrepreneurs in the North (in proportion to a_H), and occurs despite a narrower innovation set (z^P rises). In the South, the quality-adjusted expected number of imitated products (ψ_S) falls under both policies. With stronger IPRs, this is because the likelihood of the innovated variety being imitated falls and the L imitation set contracts (z_{L0} rises and z_{L1} falls). The impact of migration on ψ_S is more complex: the mass of a_H varieties available for imitation rises but the H imitation set contracts, and the L imitation set also adjusts. We find that a contraction in the H imitation set has a strong enough effect, that ψ_S falls overall.

These results thus imply that a weak IPRs protection in the South serves to counterbalance the migration-driven change in competitive pressure on entrepreneurs. As ψ_S rises following migration, for example, the competitive pressure on innovators rises in both markets, pushing the innovators' rents $\pi_N(z)$ down. But this effect would be less of a concern if the North's migration policy targeted the South with weak IPRs.

While two policies have similar effects on the quality-adjusted numbers of innovated and imitated varieties, they differ in the impacts on the average quality of innovated and imitated varieties. We examine discuss these effects in detail in the Appendix A.6. Briefly, we find that the average quality of innovated varieties falls with stronger IPRs in the South and if the density of low-quality varieties is relatively high, rises with migration into the North. The expected average quality of the L imitated varieties rises with stronger IPRs and falls with migration; and that of the H imitated varieties remains the same with stronger IPRs and rises with migration.

We can thus conclude that stronger IPRs protection in the South targets imitation of all varieties z but it affects low-quality imitation most and promotes innovation of low-quality varieties. Allowing high-talent migration, by contrast, targets imitation of high-quality products (due to "brain drain") and shifts innovation towards high-quality varieties.

6 Policy, Income, and Welfare

In this section, we compare the impacts of an IPR policy versus a migration policy to attract high-talent workers on income, and welfare.[27] A key consideration for this analysis is whether the

[27] We provide the preliminary detail on the composition of labour and income in the two regions in the Online Appendix.

rents of migrant entrepreneurs count towards the Northern or the Southern income. These rents are given by:

$$\pi_N(a_H)M = \frac{a_H}{\sigma}\left\{\frac{E_N}{\psi_N} + [1-\mu^H(a_H)]\frac{E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right\}M,$$

where ψ_N and ψ_S are given in (17). We consider both cases.

Without the migrants' earnings, the Northern income Y_N consists of the earnings of production workers given by $w\left[1-\int_{z^P}^{1} dF(z)\right]$, where the term in the square brackets is the mass of production workers in the North, and the rents of native entrepreneurs given by $\int_{\Phi^P\setminus\Phi^M}\pi_N(z)dF(z) + \int_{\Phi_M}\pi_N^j(z)dF(z)$, or equivalently:

$$\frac{1}{\sigma}\left[\frac{E_N}{\psi_N} + \frac{E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right]\int_{z^P}^{1} zdF(z) - \frac{1}{\sigma}\frac{E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\sum_j\int_{z_{j0}}^{z_{j1}} z\mu^j(z)dF(z).$$

Also ignoring the migrants' earnings, the Southern income Y_S consists of the earnings of production workers given by $l(1-g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z)$, which is the mass of production workers in the South, and the expected rents of non-migrant entrepreneurs given by $\sum_j\int_{z_{j0}}^{z_{j1}}\pi_S^j(z)dF(z) + \pi_S^H(a_H)M$, which are the rents from imitating the varieties innovated by the Northern native and the Southern migrant entrepreneurs, respectively.

The global income is as follows:

$$Y_G = Y_N + Y_S = w\left[1-\int_{z^P}^{1} dF(z)\right] + \left[l(1-g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z)\right] + \frac{E_N + E_S}{\sigma}. \tag{21}$$

Proposition 6 (Income).
(i) Strengthening Southern IPRs raises (reduces) total income in the North (South).
(ii) Allowing high-talent migration to North raises (reduces) total income in the North (South) when the migrants' earnings count towards the Northern income. When the migrants' earnings count towards the Southern income, income rises in the South, and might also rise in the North provided the South's IPRs are sufficiently below $\bar{\Omega}$ or the H imitation rate is high.
(iii) Global income rises with high-talent migration to North. It also rises with stronger Southern IPRs when the L imitation is widespread.
Proof: See Online Appendix.

The two policies differ critically in their impact on the distribution of income within each region. Consider the impact on the earnings of production workers first. The production income falls in the North and rises in the South when the South strengthens its IPRs. This occurs as the innovation set expands into low-quality varieties (z^P falls) and the L imitation set contracts (z_{L0} rises and z_{L1} falls). When the North opens to high-talent migration, by contrast, the production income rises in the North (as the innovation set contracts away from low-quality varieties) and falls in the South (as the L imitation set expands).

The relative strength of the impact on the production income in each region matters for the policies' impact on the global income, Y_G. This follows from (21), where the two terms in the square brackets work against each other. Allowing high-talent migration to the North increases Y_G when the innovation set contracts away from low-quality varieties but decreases it when the low-quality imitation set expands. The innovation impact of migration is more important, since it results from the change in the competitive environment in both markets. And consequently, global income rises with high-talent migration to the North, even if such migration is from the South with weak IPRs.

With strengthening IPRs in the South, by contrast, global income is not guaranteed to rise. It rises when the low-quality imitation set contracts, but falls when the innovation set expands into low-quality varieties. Both of these effects result from the change in the competitive environment in the South only. Their overall impact is ambiguous and depends on the shape of the L imitation rate function and the distribution of the Northern individuals' ability. Assume that Northern individuals' ability a is Pareto distributed over the range $[a_0, 1]$ with the tail index k, i.e., $G(a) = [1-(a_0/a)^k]/[1-(a_0)^k]$ and so, $F(z) = [1-(z_0/z)^k]/[1-(z_0)^k]$. Then we find that global welfare rises with stronger IPRs under the following sufficient condition:

$$\left(\frac{\bar{z}}{a_L}\right)^k > \alpha_L.$$

This condition is more likely to hold when $\bar{z}$ is high, in which case only entrepreneurs with high enough ability engage in innovation; and a_L and α_L are low, in which case a wide set of innovated varieties is imitated by the L entrepreneurs.[28]

Next, consider the impact on the rents of native entrepreneurs in the North. Stronger IPRs protection in the South increases the rents of Northern entrepreneurs selling to the Southern market. This is because it lowers both the likelihood of the innovated variety z being imitated and the competitive pressure from the L imitated varieties (as the L imitation set contracts). At the same time, stronger IPRs have no effect on the aggregate rents that Northern entrepreneurs earn in the domestic market. Here, while the entry of low-talent entrepreneurs into innovation increases the aggregate rents, it also increases the competitive pressure on the incumbent innovators, reducing the individual rents to fully offset the positive impact. Migration, in turn, reduces rents of the North-born entrepreneurs in the domestic market, as the competitive pressure from the new a_H varieties rises and low-talent entrepreneurs exit innovation. The rents that North-born entrepreneurs earn in the Southern market go up, as the high-quality imitation set contracts (even

[28] When α_L is low, the rents of L entrepreneurs fall (rise) slowly as z falls below (rises above) a_L.

if the low-quality imitation set expands). This positive impact of "brain drain" is particularly strong when the South's IPRs are weak and the H imitation rate is high but still, it is not strong enough so that in aggregate, the rents of the North-born entrepreneurs fall following migration.

Last is the impact on the rents of non-migrant entrepreneurs in the South. These rents falls with stronger IPRs as the likelihood of the innovated variety z being imitated falls and the competitive pressure from the innovated varieties rises. This negative impact of stronger IPRs is most pronounced in low-quality imitation, because the L imitation set contracts. With migration, the entrepreneurial rents fall in high-quality imitation (as the mass of H individuals falls and the H imitation set contracts); the rents in low-quality imitation rise (as the competitive pressure from the H imitated varieties falls and the L imitation set expands), but not strongly enough.

From Proposition 6, strengthening Southern IPRs raises total income in the North and reduces total income in the South. Allowing high-talent migration to North has the same impact, but only when the migrants' earnings fully count towards the Northern income. When instead the migrants' earnings count towards the Southern income, income rises in the South following migration. This is because the increase in the rents of migrant entrepreneurs more than compensates for the loss in the rents of those who stay in the South. Importantly, income might also rise in the North in this case, provided the South's IPRs are weak and the H imitation rate is high. Specifically, the following condition is necessary (but not sufficient) for $dY_N/dM > 0$:

$$\Omega < \bar{\Omega} < 1 - \frac{1}{\alpha_H(1+\bar{\varepsilon})},$$

where $\bar{\varepsilon}$ is the elasticity of ψ_N with respect to $\bar{z}$.

The discussion above highlights a key difference across the two policies on how they impact entrepreneurial rents in each region. Strengthening IPRs only affects rents in the Southern market, where it transfers rents from the entrepreneurs in the South to the entrepreneurs in the North. High-talent migration, by contrast, affects rents in both markets. For the North-born entrepreneurs, the loss of rents in the Northern markets is offset by the gain of rents in the Southern market; and for the non-migrant entrepreneurs in the South, the loss of rents in the high-quality imitation is offset by the gain of rents in the low-quality imitation.

Finally, welfare in each region is given by the indirect utility functions $V_N = Y_N - E_N + E_N/P_N$ and $V_S = Y_S - E_S + E_S/P_S$, where $P_N = p_N\psi_N^{1/(1-\sigma)}$ and $P_S = p_N[\psi_N + (w^{\sigma-1} - 1)\psi_S]^{1/(1-\sigma)}$. Strengthening Southern IPRs raises total income and welfare in the North and lowers total income and welfare in the South. Welfare rises in the North because income Y_N rises and the price-quality index P_N falls, whereas it falls in the South because income Y_S falls and the price-quality index P_S rises. Income and welfare also rise in the North and fall in the South when the migrants' earnings count towards the Northern income. But when the migrants' earnings count towards the

Northern income, welfare rises in the South following migration as income Y_S rises, but falls as the price-quality index P_S rises. We show in the Online Appendix that $dV_S/dM > 0$ when this sufficient condition holds:

$$w^{\sigma-1} > \frac{\psi_N - \psi_S}{\psi_S^{\frac{\sigma-1}{\sigma}} - \psi_S},$$

which requires a large North-South price differential (a high $w^{\sigma-1}$) and a weak strength of South's IPRs (a low Ω, in which ψ_N is low and ψ_S is high. In the North, the price-quality index P_S falls following migration, and this has an additional positive impact on welfare.

7 Conclusion

This paper introduces an occupational choice model of innovative North and imitative South with two dimensions of heterogeneity: product quality and entrepreneurial ability. This framework allows us to study how two policies (strengthening of IPRs in the South and opening the North to migration of high-talent individuals as means of preempting imitation) impact innovation activity in the North and imitation activity in the South. The policies' impacts depend on the endogenous entrepreneurship decisions and the intensity of competition between innovated and imitated varieties.

The model predicts that the two policies have critically different implications for the entrepreneurial activity and income distribution in each region. Opening the North to migration directly limits the imitation of high-quality products; but in the South, where IPRs protection is weak and wage rate is low, it also promotes the imitation of low-quality products. A strengthening of IPRs in the South, by contrast, limits low-quality imitation and does not affect the set of high-quality imitated products. Furthermore with migration from the South, the aggregate production income falls in the South and rises in the North, whereas a strengthening of IPRs in the South has the opposite effect. With respect to the entrepreneurial rents in each region, strengthening IPRs only affects rents in the Southern market, where it transfers rents from the entrepreneurs in the South to the entrepreneurs in the North. Migration, by contrast, affects rents in both markets. For the North-born entrepreneurs, the loss of rents in the Northern markets is offset by the gain of rents in the Southern market; and for the non-migrant entrepreneurs in the South, the loss of rents in high-quality imitation is offset by the gain in low-quality imitation.

The model also show that the two policies have a degree of complementarity. Migration increases the rents that North-born entrepreneurs earn in the Southern market, and this positive impact of "brain drain" is particularly strong when the South's IPRs are weak. When the migrants' earnings count towards the Southern income, migration increases income in the South and when the South's IPRs are weak, might also increase income in the North.

The findings imply that it is not in the interest of the South to strengthen its IPRs. But the results also suggest that the North's migration policy could be an attractive alternative to the policy of imposing stronger IPRs in developing economies when the goal is to combat imitation and promote innovation. Improved migration prospects for high-talent entrepreneurs is not a zero-sum game: the rents of low-talent entrepreneurs could also rise in the South, as well as the average quality of products in the high-talent imitation sector. Also with high-talent migration to the North, both the South and the North could be made better off, as global income necessarily rises. This could be achieved by sharing in the migrants' earnings. The South could permit emigration only if the North ensures that some of the migrants' earnings flow back home.[29] The North would find it optimal to agree to such requirement if the South's IPRs are weak or the high-quality imitation is widespread.

In further research, the model could be extended to allow for innovation in the South. Such model would be better suited to study the policies' impacts on an emerging economy (e.g., China) which possesses a critical level of complementary research, technological, and marketing assets to enable the local entrepreneurs to absorb foreign technology. The strength of IPRs in this "more advanced" Southern economy would still be below the global optimal level, and the North would have an incentive to push for global IPRs reforms (Lai and Qiu, 2003).

High-talent migrants could transfer superior knowledge acquired in the North back to their countries of origin, preparing the ground for transforming the South into a more innovative economy. This channel of brain gain can materialize under a sound IPRs environment for the Southern innovators to be active and remain in the South (see for example, Naghavi and Strozzi, 2015). A transition of South from imitation to innovation could diminish the incentives of high-talent entrepreneurs to migrate to the North and of firms or governments to use migration as a tool to limit imitation.

On the empirical side, a first step would be to test our model's predictions using data on inventor migration flows in Miguelez and Finks (2013) and the IAB Brain Drain data in Brüker et al. (2013).

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[29] A work permit in Israel is one example of a migration policy with an income transfer requirement. It requires employers of foreign workers in technology units make monthly deposits for their foreign employees to be collected when permanently leaving the country. A portion of migrant earnings can also flow back to the South as migrant remittances.

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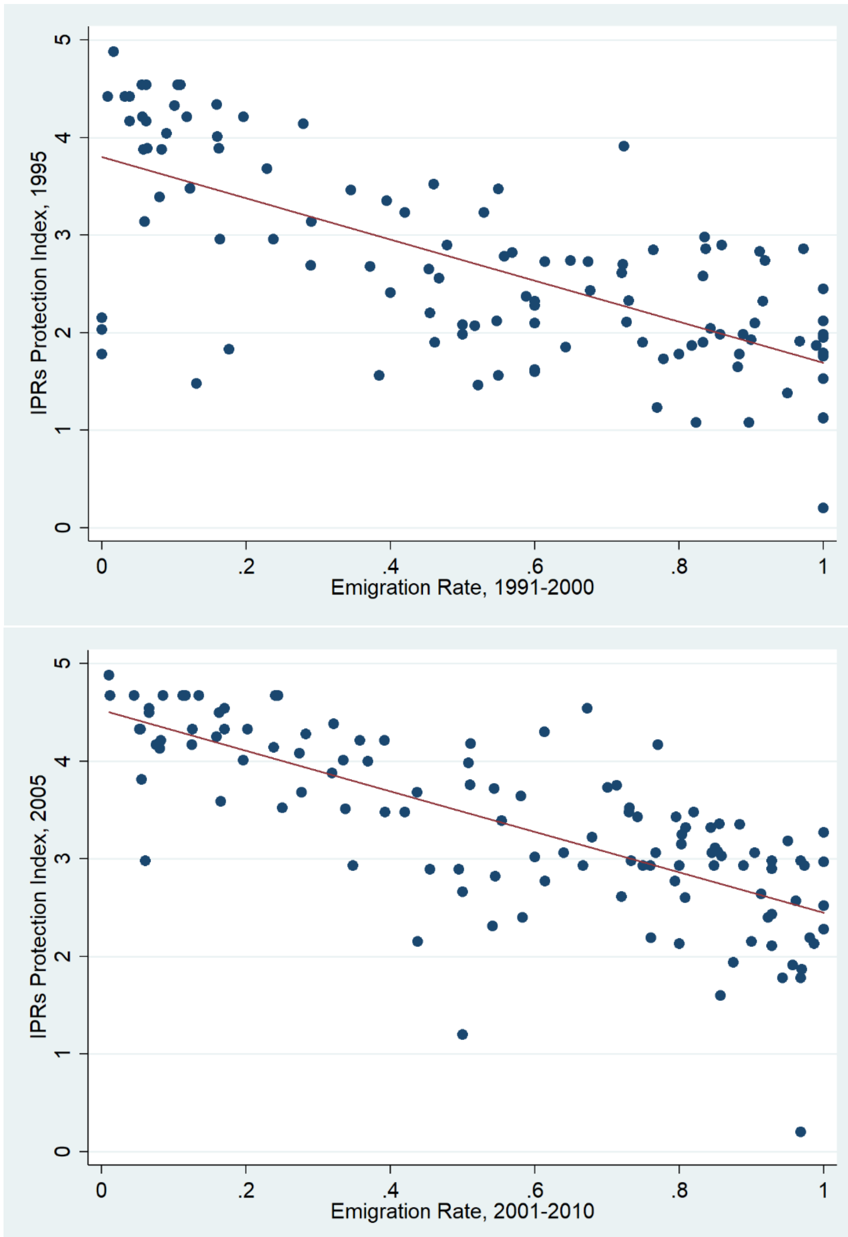
Figure 1: **IPRs protection and inventor emigration rate**



Notes: The inventor emigration rate of origin country i is defined as $diaspora_i/(diaspora_i + residents_i)$, where $diaspora_i$ is the number of national inventors of country i residing abroad and $residents_i$ is the number of inventors residing in country i (including national of country i and immigrants). These data are from Miguelez and Finks (2013). The index of IPRs protection measures the stringency of patent rights, based on five measures of patent laws (coverage, membership in international patent treaties, provisions against losses of protection, enforcement mechanisms, and duration of protection). These data are from Park (2008).

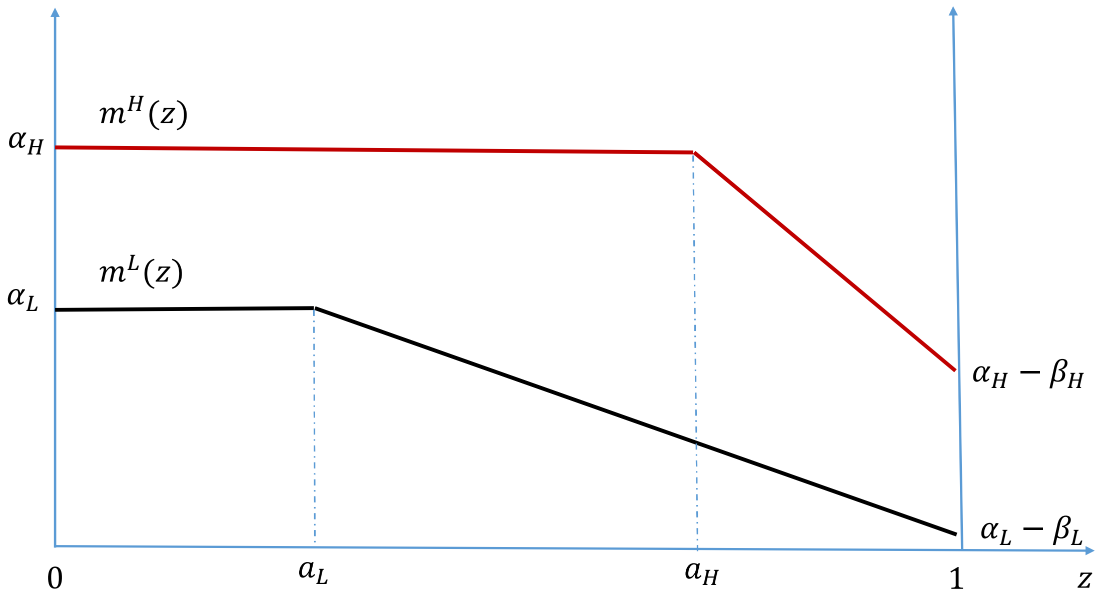


Figure 2: **Imitation rates**

Notes: The figure shows the imitation rate $m^j(z)$ as a function of quality z. The imitation rate stays constant up to quality a_j, after which it falls as quality z rises. This occurs at a later stage for H imitators: $a_H > a_L$.

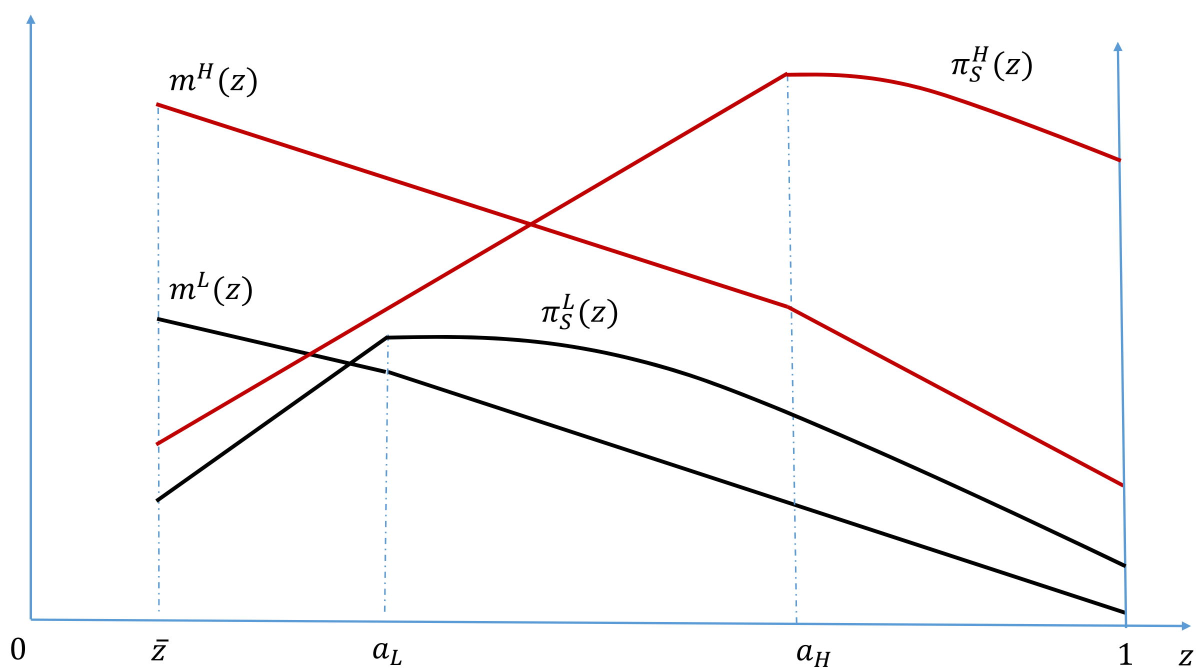


Figure 3: **Imitators' rents**

Notes: The figure shows the imitators' rent functions in relation to the imitation rate functions.

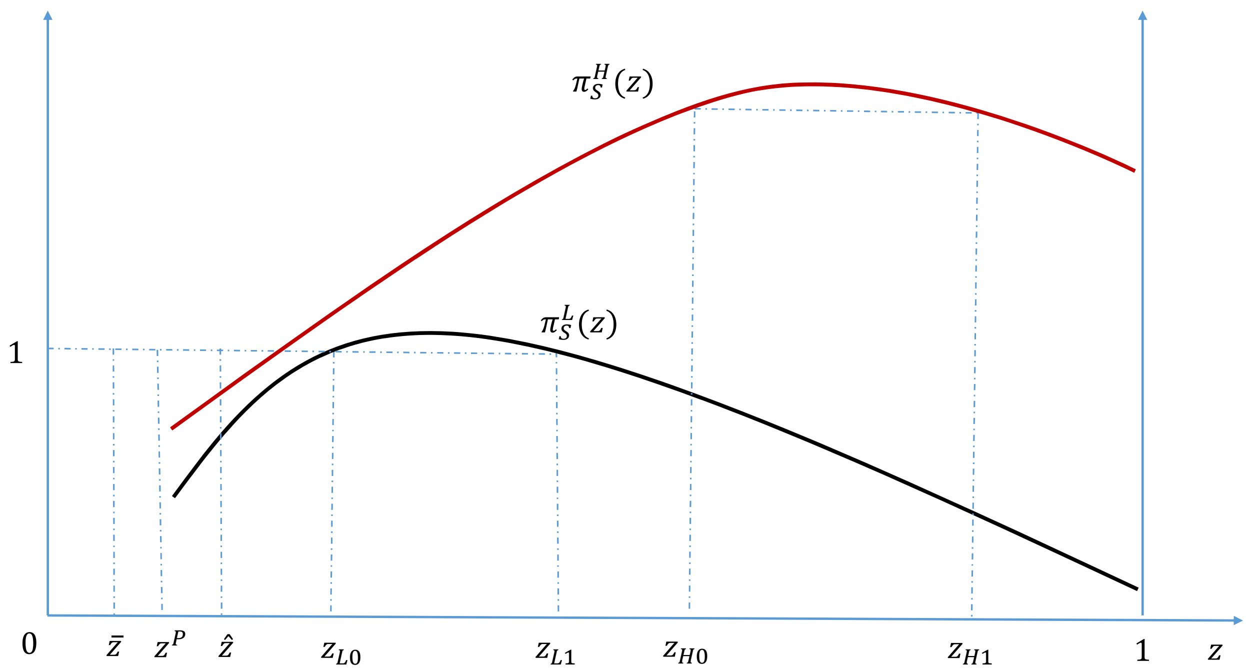


Figure 4: **Equilibrium with no overlap in imitation**

Notes: The figure shows five non-overlapping subsets: least-quality set $[\hat{z}, z_{L0})$ with no imitation; low-quality set $[z_{L0}, z_{L1}]$ with imitation by the L entrepreneurs; middle-quality set (z_{L1}, z_{H0}) with no imitation; high-quality set $[z_{H0}, z_{H1}]$ with imitation by the H entrepreneurs; and highest-quality set $(z_{H1}, 1]$ with no imitation.

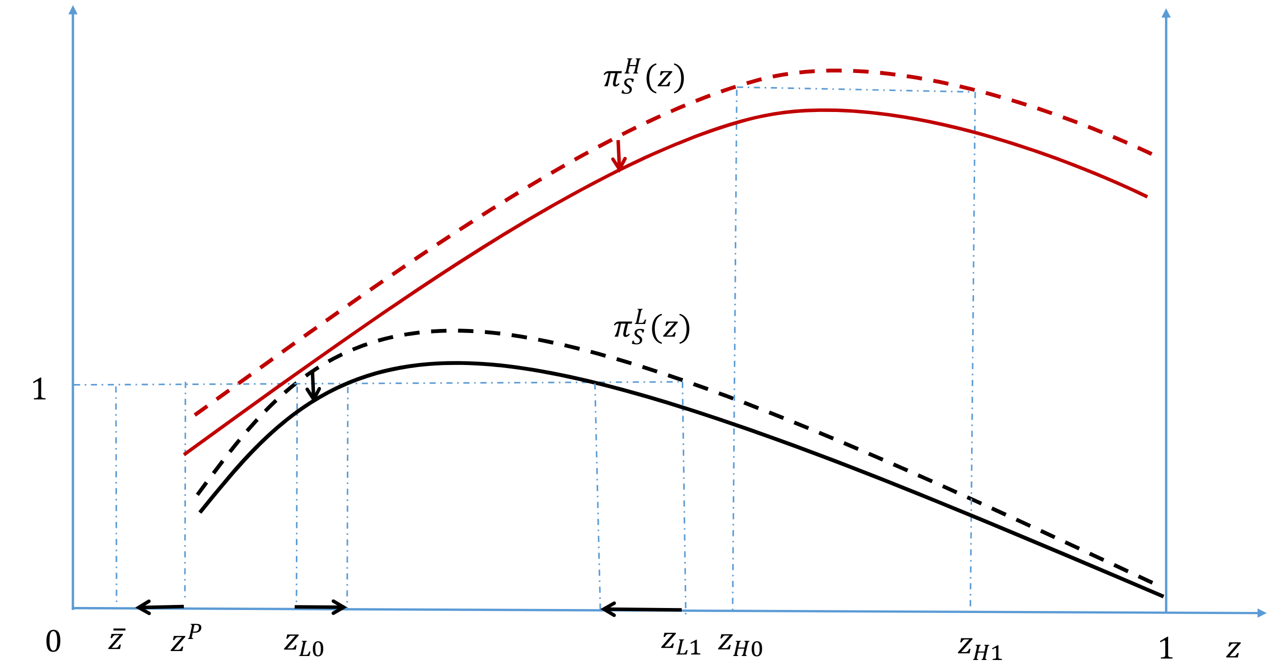


Figure 5: **Strengthening IPRs in the South**

Notes: The figure illustrates the effects of strengthening IPRs in the South. The innovation set expands (z^P falls), the L imitation set contracts (z_{L0} rises and z_{L1} falls), the H imitation set does not change (z_{H0} and z_{H1} do not change).

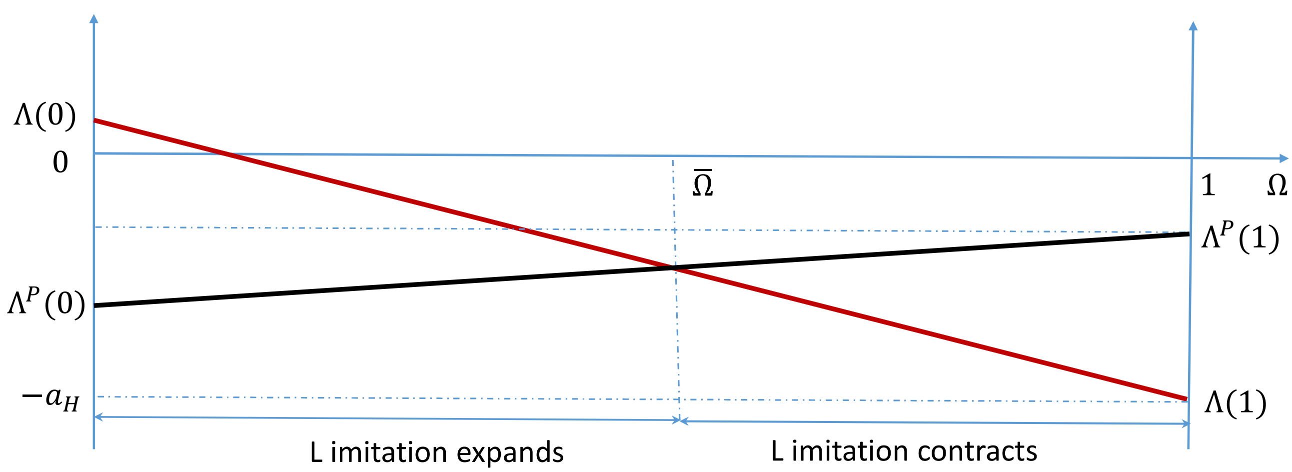


Figure 6: **The impact on the L imitation set when w is high**

Notes: The overall impact of opening the North to high-talent migration on the L imitation activity depends on the strength of IPRs in the South: the L imitation set expands (z_{L0} falls and z_{L1} rises) if $\Omega < \bar{\Omega}$ and contracts if (z_{L0} rises and z_{L1} falls) if $\Omega > \bar{\Omega}$.

Appendix

In this Appendix, we discuss several extensions of the model and their implications for the results.

A.1. Intensive Margins of Innovation

We now modify the model so that it allows us to examine the intensive margin of innovation. For simplicity, we put aside the issue of extensive margin. Specifically, we suppose that all individuals in the North have the same ability, and let Z denote the set of quality that potential innovated products may have, with a specific level of quality denoted by z. We assume that Z (the extensive margin) is fixed and focus on the intensive margin.

A higher quality product requires more resources (entrepreneurs) to innovate. Assume that to innovate quality z, we need $n(z) \geq 1$ entrepreneurs, with $n'(z) \geq 0$. Each individual in the North can choose to become an entrepreneur (innovator) or a production worker, earning wage income w in the latter case. In the former case, the individual needs to choose the quality: form a firm of z people to innovate a variety of quality z. Varieties of the same quality are horizontally differentiated. Firms producing the same quality complete Cournot-style. Assume the Northern population is sufficiently large and the quality set Z is sufficiently small. Then in equilibrium, the expected profit per capita (of any quality) is equal to w. The intensive margin is defined by the number of firms (varieties) of quality z each, denoted by $v(z)$. In the main model, $n(z) = 1$ and v is exogenously given by the ability distribution function $f(z)$; as such, the intensive margin of each quality is fixed.[30]

We keep all the assumptions about the South as in the main model.

With partially enforced IPRs in the South, we can characterize certain aspects of the equilibrium and the policy effects as follows.[31] There exist five non-overlapping subsets (some could be empty though) of Z: least-quality set (z_0, z_1), low-quality set (z_1, z_2), middle-quality set (z_2, z_3), high-quality set (z_3, z_4), and highest-quality set (z_4, z_5), with $z_0 \leq z_1 \leq z_2 \leq z_3 \leq z_4 \leq z_5$. There is no imitation in the least-quality set because the expected return is lower than wage rate of 1 in the South; products in the low-quality set are imitated by the L individuals; no imitation of products in the middle-quality set; products in the high-quality set are imitated by the H individuals, and no imitation of products in the highest-quality set. The intuition is the same as that for the equilibrium derived in the main model. Facing potential imitation, Northern individuals choose the type of product to innovate, which leads to an equilibrium $v(z)$ in each quality, i.e., the intensive margin of innovation.

[30] In our original model, in the case of migration, because high-talent migrants from the South become innovators in the North, the innovation intensity margins of those increased varieties do change, but are given exogenously.

[31] The proof is tedious and thus, not provided in the Appendix. It is available upon request from the authors.

We can examine the effects of strengthening IPRs in the South as in Proposition 3. The main effects are as follows. First, the least-quality set expands, the low-quality set shrinks, the middle-quality set expands, the high- and highest-quality sets remain unchanged, while the entire innovation set Z remains unchanged. Specifically, z_0 does not change, z_1 rises, z_2 falls, and z_3, z_4 and z_5 do not change. These changes in the extensive margins of imitation of various subsets are similar to those in the main model, with the same intuition.

Second, the intensive margins of innovation do not change in the highest- and lowest-quality sets, where there is no imitation. In the high-quality set, even though the extensive margin does not change (z_3 and z_4 do not change), the probability of being successfully imitated falls, and this incentivizes Northern individuals to introduce those products, i.e., the intensive margins of those products increase. In the middle-quality set, the Northern individuals at the lowest end (right to z_2) no longer face imitation (since z_2 falls) and so, the intensive margins of those products increases; whereas the intensive margins of the other products in this set do not change as there is no imitation there. Finally, in the low-quality set, the probability of being successfully imitated falls and thus, the intensive margins of those products increase.

In sum, strengthening IPRs in the South weakly increases innovation intensive margins. We identify the types of innovations that have strictly higher intensive margins.

We can also examine the effects of migration on innovation intensive margin (corresponding to Proposition 5 on extensive margin). Migration of H individuals from the South directly shrinks the imitation of products in the high-quality set. The first-order effect of this reduction in imitation on innovation intensive margin is as follows: increasing intensive margins of the products at the two ends—the lowest (right to z_3) and the highest (left to z_4)—of the high-quality set, because they no longer face imitation, whereas the intensive margins of the other products do not change.

A.2. Costly Imitation

In the main model, imitation requires one individual, independent of the quality of the product, but we assumed that higher quality products are more difficult to imitate. We used probability rather than resources (or cost) to model this assumption. Specifically, we assumed that as product quality rises, the probability of successful imitation falls. In this subsection, we discuss how the analysis and results might change if instead, we model the assumption using resources.

We keep our discussion of innovation as in the main model. Suppose that imitating a higher quality product requires more resources but with the same (fixed) probability of success. Specifically, let $r^j(z)$ be the number of $j \in \{L, H\}$ individuals required to imitate an innovated variety with quality z, with $dr^j(z)/dz > 0$ and $r^L(z) > r^H(z)$. That is, higher-quality varieties require more individuals to imitate but for a given quality z, imitation requires a fewer high-talent entrepreneurs than low-talent ones. Let ξ denote the probability of successful imitation for all qualities. Then, the expected rents of each j imitator of a variety with quality z are as follows:

$$\pi_S^j(z) = \frac{(1-\Omega)\xi}{r^j(z)} (p_S - 1)\, c_S(z) = \frac{z(1-\Omega)\xi}{\sigma r^j(z)} \left(\frac{p_S}{P_S}\right)^{1-\sigma} E_S.$$

Comparing the above $r^j(z)$ and $\pi_S^j(z)$ to $\mu^j(z)$ in (8) and $\pi_S^j(z)$ in (7), we can see that this revised model is equivalent to our original model if we set $\xi/r^j(z) = m^j(z)$. Thus, all the results in our original model also hold in this revised model with costly imitation.

A.3. Imitation of Lowest Quality Varieties

Here we consider a case in which imitation starts from the lowest-quality innovated variety, namely z^P. We have shown in Proposition 3 that strengthening IPRs reduces the L imitation on both sides of the spectrum, for lower quality varieties (as z_{L0} rises) and for higher quality varieties (as z_{L1} falls). In this revised model, the imitation set will likewise contract with stronger IPRs, and the cutoffs z^P and z_{L0} will diverge, as z^P will fall and z_{L0} will rise. Thus, the impact of stronger IPRs on innovation and imitation will remain the same as in our original model. The same is true when the North opens up to high-talent migration and the L imitation set contracts in response. But if migration causes an expansion in the L imitation set (when the South's IPRs are sufficiently weak), then such expansion will only be due to a rise in the cutoff z_{L1}; the cutoff z_{L0} will not fall, because it will be constrained by z^P (i.e., corner solution). Nonetheless, the model's qualitative predictions will remain the same.

A.4. Homogeneous Southern Entrepreneurs

In the main model, Southern individuals differ in their imitation talent: they are divided into low-talent L and high-talent H types. Suppose instead that all Southern individuals are of the same L type. Then in equilibrium, there will be a single L imitation set $[z_{L0}, z_{L1}]$. We refer to this outcome as "full overlap."

In this revised model, strengthening IPRs will contract the L imitation set, as z_{L0} will rise and z_{L1} will fall. Our original model delivered the same result, but with the additional finding that the H imitation set does not change. And consequently, the impact of stronger IPRs on the quality-adjusted number of imitated varieties and the average quality of imitated varieties in this revised model will be the same as in the original model.

Allowing for migration of entrepreneurs from the South in this revised model will produce the same two effects as in our original model: (i) the imitation set $[z_{L0}, z_{L1}]$ will contract due to the brain drain effect; and (ii) the rents $\pi_N(z)$ and $\pi_S^L(z)$ will fall due to competitive pressure from the varieties of quality a_L introduced by migrants in the North. Competitive pressure on Northern innovators from imitators will fall when the imitation set contracts but will rise when a_L varieties are introduced by migrants in the North. This latter effect is strong when the South's IPRs are weak. Since there is a unique imitation set, this implies that migration could be used as a joint policy alongside stronger IPRs in the South to discourage imitation without impeding innovation. Importantly though, it is low-talent (rather than high-talent) entrepreneurs who migrate in this revised model; such migrants might not become innovators in the North.

A.5. Migrants Not Becoming Innovators

High-talent migrants from the South might not immediately become innovators in the North and instead, end up working in production in the North. In this case, the negative competition effect from the original model (which arises when the varieties of quality a_H are introduced by migrants in the North) will not arise here. As in the original model, the L imitation set will expand when IPRs are weak in the South. This is due to lower competitive pressure from the H imitators caused by the brain drain. But the innovation set could now expand on the extensive margin (as opposed to an increase in a_H varieties).

A.6. Composition Effect of IPR versus Migration Policy

Let $\tilde{\psi}_N \equiv \int_{z^P}^{1} dF(z) + M$ denote the (unadjusted) number of innovated varieties, and $\tilde{\psi}_S^L \equiv \int_{z_{L0}}^{z_{L1}} \mu^L(z)dF(z)$ and $\tilde{\psi}_S^H \equiv \int_{z_{H0}}^{z_{H1}} \mu^H(z)dF(z) + \mu^H(a_H)M$ denote the (unadjusted) number of the L and H imitated varieties, respectively. Then $\psi_N^q \equiv \psi_N/\tilde{\psi}_N$ measures the average quality of innovated varieties, and $\psi_S^{Lq} \equiv \psi_S^L/\tilde{\psi}_S^L$ and $\psi_S^{Hq} \equiv \psi_S^H/\tilde{\psi}_S^H$ respectively measure the expected average quality of the L and H imitated varieties. Assuming that Northern individuals' ability a is Pareto distributed over the range $[a_0, 1]$, with the tail index k, we have: $F(z) = [1-(z_0/z)^k]/[1-(z_0)^k]$. We can establish the following Proposition:

Proposition A (Varieties). *The two policies differ in their effects on the average quality as follows: (i) for innovated varieties, ψ_N^q, falls with stronger IPRs and if the tail index k is high, rises with migration; (ii) for the H imitated varieties, ψ_S^{Hq}, does not change with stronger IPRs and rises with migration; (iii) for the L imitated varieties, ψ_S^{Lq}, rises with stronger IPRs and falls with migration.*

Proof: See Online Appendix.

The results show that the two policies differ in their effect on the composition of innovation and imitation. With stronger IPRs in the South, the average quality of innovated varieties (ψ_N^q) falls, because the innovation set expands into the low-quality varieties. With migration into the

North, ψ_N^q rises as the innovation set contracts away from low-quality varieties. But migration also increases the mass of innovated varieties of quality a_H. Because a_H is an intermediate quality in the innovation set $(z^P, 1)$, whether ψ_N^q rises or falls in response depends on the tail of Pareto distribution: it rises when the tail of Pareto distribution is thin (k is high).[32] With stronger IPRs, the expected average quality of the L imitated varieties rises, as the L imitation set contracts, but remains the same for the H imitated varieties (since the H imitation set does not change). With migration, by contrast, the expected average quality of the L imitated varieties falls when the L imitation set expands, and that of the H imitated varieties rises (because the mass of a_H imitated varieties rises and the H imitation set contracts).

[32] A sufficient condition for $d\psi_N^q/dM > 0$ is $k \geq 3$. This condition is consistent with the literature. In Buera et al. (2011), for example, the tail index of the entrepreneurial ability distribution is set to 4.84 to match the employment share of the largest 10% of the establishments in the U.S. In Allub and Erosa (2019), the tail index of the entrepreneurial ability is set to 5.4 to match data on the occupation structure in Brazil.

Online Appendix

Proof of Proposition 1

From $G(z) \equiv zE_N - \sigma w \int_z^1 x dF(x)$, we have $G(0) < 0$, $G(1) > 0$, and $dG(z)/dz > 0$. Thus, there exists a unique equilibrium with $0 < \hat{z} < 1$ such that $G(\hat{z}) = 0$. From $G^F(z) \equiv z(E_N + E_S) - \sigma w \int_z^1 x dF(x)$ and above, we obtain $G^F(0) < 0$, $G^F(\hat{z}) > 0$, and $dG^F(z)/dz > 0$. Thus, there exists a unique equilibrium with $0 < \bar{z} < \hat{z}$ such that $G^F(\bar{z}) = 0$.

Proof of Lemma 2

Ruling out $z_{H1} = 1$ avoids the corner solution where z_{H0} is determined by $\int_{z_{H0}}^1 dF(z) = f_S^H$ and $\pi_S^H(z_{H0}) > \pi_S^H(1)$.

The cutoffs z_{j0} and z_{j1} for $j \in \{L, H\}$ are defined by the following four conditions:

$$\int_{z_{H0}}^{z_{H1}} dF(z) - lg_S^H = 0, \tag{A1}$$

$$\alpha_H z_{H0} - (\alpha_H - \beta_H z_{H1}) z_{H1} = 0, \tag{A2}$$

$$\alpha_L z_{L0} - (\alpha_L - \beta_L z_{L1}) z_{L1} = 0, \tag{A3}$$

$$(1 - \Omega)\alpha_L z_{L0} \frac{w^{\sigma-1} E_S}{\psi_N + (w^{\sigma-1} - 1)\psi_S} - \sigma = 0, \tag{A4}$$

where (A2)-(A3) follow from $\pi_S^j(z_{j0}) = \pi_S^j(z_{j1})$ and (A4) follows from $\pi_S^L(z_{L0}) = 1$.

We need to find the sufficient condition for $z_{L1} < z_{H0}$. Since $z_{H1} < 1$, it follows from (A2) that $z_{H0} > \tilde{z}_{H0}$, where $\tilde{z}_{H0}$ solves $\alpha_H \tilde{z}_{H0} - (\alpha_H - \beta_H) = 0$. Also since $z_{L0} > \hat{z}$, it follows from (A3) that $z_{L1} < \tilde{z}_{L1}$, where $\tilde{z}_{L1}$ solves $(\alpha_L - \beta_L \tilde{z}_{L1})\tilde{z}_{L1} - \alpha_L \hat{z} = 0$. Thus, $z_{L1} < z_{H0}$ if $\tilde{z}_{L1} < \tilde{z}_{H0}$, which requires a sufficiently high $a_H = \alpha_H/(2\beta_H)$ and low $a_L = \alpha_L/(2\beta_L)$, for a given $E_N/(\sigma w)$.

The Overlapping Case

Lemma A.1 establishes the sufficient conditions for the overlapping case.

Lemma A.1. *There exists $\tilde{g}_S^H$ such that in equilibrium, the L entrepreneurs imitate varieties $z \in [z_{L0}, z_{L1}]$, the H entrepreneurs imitate varieties $z \in [z_{H0}, z_{H1}]$, and the two imitation sets overlap, $z_{L1} > z_{H0}$, if $g_S^H \geq \tilde{g}_S^H$, a_L is high and Ω is low.*

Proof. These two conditions are sufficient for the overlapping case: (i) $g_S^H \geq \tilde{g}_S^H$, where $\tilde{g}_S^H \equiv \int_{a_L}^1 dF(z)/l$ and (ii) $[1 - \mu^H(a_L)]\pi_S^L(a_L) > 1$. Under these conditions, the H imitation set, given by $[z_{H0}, 1]$ with $z_{H0} < a_L$, overlaps with the L imitation set, given by $[z_{L0}, z_{L1}]$ with $a_L \in (z_{L0}, z_{L1})$. Substituting for $\pi_S^L(a_L)$, we rewrite $[1-\mu^H(a_L)]\pi_S^L(a_L) > 1$ as follows: $[1-\mu^H(a_L)]a_L\mu^L(a_L)w^{\sigma-1}E_S > \sigma[\psi_N+(w^{\sigma-1}-1)\psi_S]$. Since $\alpha_L < \alpha_H$ and $\psi_N+(w^{\sigma-1}-1)\psi_S < w^{\sigma-1}\psi_N$, this inequality holds if

$$[1-(1-\Omega)\alpha_H]a_L(1-\Omega)\alpha_L w^{\sigma-1}E_S > \sigma\psi_N, \tag{A5}$$

where $\psi_N = \bar{z}(E_N + E_S)/(\sigma w)$ from $G^F(\bar{z}) = 0$. If $\Omega < 1 - 1/(2\alpha_H)$, the left hand side in (A5) rises as Ω rises and so, (A5) holds if it holds for $\Omega = 0$, i.e., if $(1-\alpha_H)a_L\alpha_L$ is high. Thus, $[1-\mu^H(a_L)]\pi_S^L(a_L) > 1$ requires a high a_L and a low Ω, for a given α_H, E_N/E_S and w. ■

Define z'_{j0} and z'_{j1} as in the non-overlapping case and use $\pi_S^j(z)$ to denote j's rents in the entire range $z > \bar{z}$. We have $\pi_S^H(z'_{H0}) = \pi_S^H(z'_{H1})$ and $\pi_S^L(z'_{L0}) = \pi_S^L(z'_{L1}) = 1$. If the two imitation sets overlap, then $z'_{H0} < z'_{L1}$ and L and H entrepreneurs face competition in imitating variety $z \in [z'_{H0}, z'_{L1}]$. This competition reduces the imitators' expected rents below $\pi_S^j(z)$. Suppose, for example, that for any $z \in [z'_{H0}, z'_{L1}]$, one H and one L entrepreneur try to imitate the same variety. If only one entrepreneur succeeds, that imitator will compete with the innovator and will receive the *ex-post* expected rents $\pi_S^j(z)$. If both entrepreneurs succeed, they will also compete in Bertrand between themselves, driving their *ex-post* rents to zero (since their marginal cost is the same). When the likelihood of imitation is $\mu^j(z)$, the expected rents in the range $[z'_{H0}, z'_{L1}]$ are equal to $[1-\mu^L(z)]\pi_S^H(z)$ for the H imitator and $[1-\mu^H(z)]\pi_S^L(z)$ for the L imitator. From the H imitator's perspective, the expected rents rise with quality in the range of low-quality varieties, because $\mu^L(z)$ falls while $\pi_S^H(z)$ rises as z rises. From the L imitator's perspective, by contrast, the expected rents fall with product quality in the range of high-quality varieties, because $\pi_S^L(z)$ falls as z rises while $\mu^H(z)$ is constant for $z \le a_H$. From Lemma A.1 and its proof, we obtain Proposition A.1.

Proposition A.1. *In the overlapping case, where $\Phi^M = [z_{L0}, z_{H1}]$ and $z_{L1} > z_{H0}$, the end points defining the two intervals, z_{j0} and z_{j1}, are determined by (10), $\pi_S^L(z_{L0}) = [1-\mu^H(z_{L1})]\pi_S^L(z_{L1}) = 1$ and $[1-\mu^L(z_{H0})]\pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1$.*

Figure A.1 shows one overlapping equilibrium. The following "thought experiment" provides intuition to this equilibrium allocation. Suppose the L and H entrepreneurs enter the market in a sequential manner. In equilibrium, the H and L entrepreneurs will never compete in the entire range $[z'_{H0}, z'_{L1}]$. Consider an H entrepreneur outside the no-competition range $[z'_{L1}, z'_{H1}]$. This entrepreneur can either imitate a variety to the immediate right of z'_{H1}, say $z_h^+ (> z'_{H1})$, or a variety within $[z'_{H0}, z'_{L1}]$, say z_h^-. In the latter case, it will choose the highest possible quality, i.e., z_h^- will

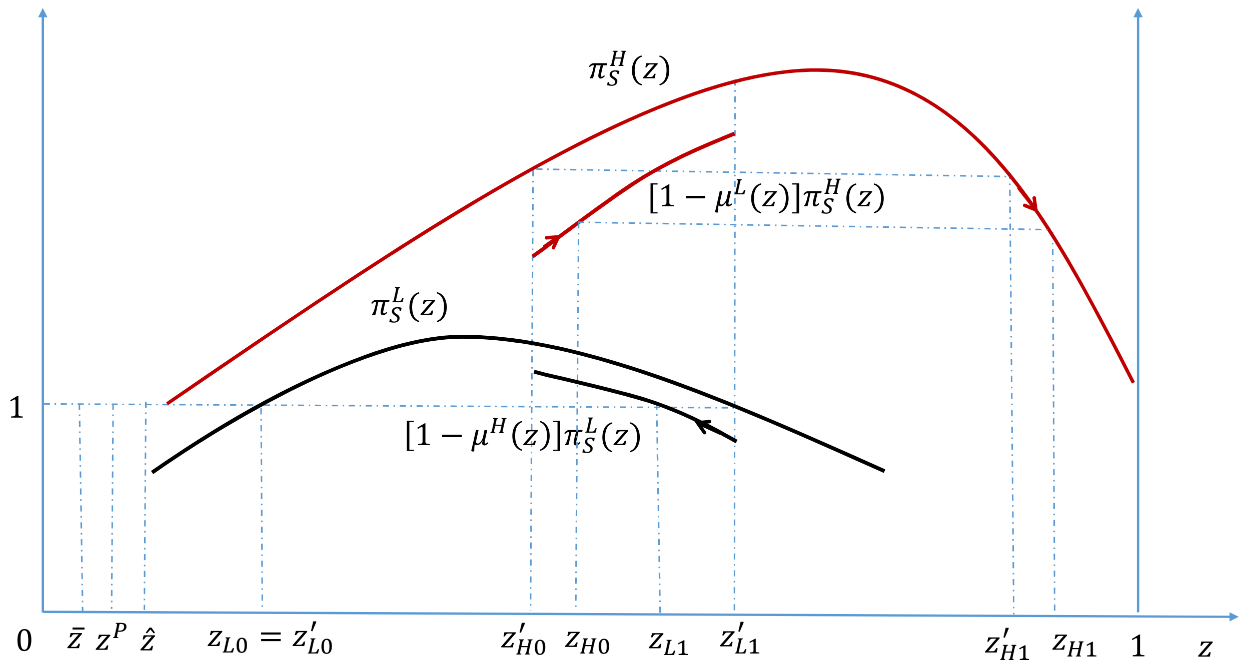


Figure A1: **Equilibrium with overlap in imitation**

be to the immediate left of z'_{L1}, because $[1-\mu^L(z)]\pi_S^H(z)$ is an increasing function of z. The H entrepreneurs will give up some varieties in $[z'_{H0}, z'_{L1}]$ to ensure that the mass of H imitators is equal to the mass innovators over the H imitation range. Due to the monotonicity of rents within this range, the H entrepreneurs will give up varieties to the immediate right of z'_{H0}. The cutoffs z_{H0} and z_{H1} are determined by (10) and $[1-\mu^L(z_{H0})]\pi_S^H(z_{H0}) = \pi_S^H(z_{H1}) > 1$. Next, consider an L entrepreneur outside the non-competition range $[z'_{L0}, z'_{H0}]$. This entrepreneur can imitate a variety within $[z'_{H0}, z'_{L1}]$, say z_l^+. It will choose the lowest possible quality, i.e., z_l^+ will be to the immediate right of z'_{H0}, because $[1-\mu^L(z)]\pi_S^L(z)$ is a decreasing function of z. Due to the monotonicity of rents within $[z'_{H0}, z'_{L1}]$, the L entrepreneurs will give up varieties to the immediate left of z'_{L1} to ensure that the L imitators' expected rents are not below the wage rate of one. The cutoffs z_{L0} and z_{L1} are determined by $\pi_S^L(z_{L0}) = [1-\mu^H(z_{L1})]\pi_S^L(z_{L1}) = 1$. The innovator $z \in [z_{H0}, z_{L1}]$ competes with both L and H imitators and so, earns rents given by

$$\pi_N^{LH}(z) = \frac{z}{\sigma}\left\{\frac{E_N}{\psi_N} + \frac{[1-\mu^L(z)][1-\mu^H(z)]E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right\}, \quad \text{where} \quad \pi_N^{LH\prime}(z) > 0 \quad \text{and} \quad \pi_N^{LH}(z) > w.$$

Proof of Proposition 2

Since $\hat{z} < z_{L0}$, innovator $z \in (\bar{z}, \hat{z}]$ does not face imitation and earns the rents given by

$$\pi_N(z) = \frac{z}{\sigma}\left[\frac{E_N}{\psi_N} + \frac{E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right], \quad \text{where} \quad \xi \equiv \frac{\psi_N}{\psi_N + (w^{\sigma-1}-1)\psi_S} < 1. \tag{A6}$$

Define $G^P(z) \equiv z(E_N + \xi E_S) - \sigma w \int_z^1 x dF(x)$. Since $dG^P(z)/dz > 0$, $G^P(\bar{z}) < 0$ and $G^P(\hat{z}) > 0$ (from Proposition 1), there must exist a unique equilibrium with $z^P \in (\bar{z}, \hat{z})$ such that $G^P(z^P) = 0$.

Innovator z in the set $\Phi^P \setminus \Phi^M = [z^P, z_{L0}) \cup (z_{L1}, z_{H0}) \cup (z_{H1}, 1]$ also earns the rents (A6). Innovator $z \in \Phi^M$ risks imitation with probability $\mu^j(z)$ and earns the expected rents given by

$$\pi_N(z) = \frac{z}{\sigma}\left\{\frac{E_N}{\psi_N} + \frac{[1-\mu^j(z)]E_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right\}. \tag{A7}$$

It is true that $\pi_N(z) > w$ for any $z > \hat{z}$.

Labour and Income Composition

In a closed Northern economy, the mass of individuals $L_N = 1$ is composed of $\int_{\hat{z}}^1 dF(z)$ entrepreneurs, $L_N^D = \int_{\hat{z}}^1 c_N(z)dF(z) = E_N/p_N$ workers who produce differentiated varieties and $L_N^H = (Y_N - E_N)/w$ workers who produce the homogeneous good.[33] The total income is given by $Y_N = w(L_N^D + L_N^H) + \int_{\hat{z}}^1 \pi(z)dF(z) = w[L_N - \int_{\hat{z}}^1 dF(z)] + E_N/\sigma$.

Consider now an open trading economy. In the South, L_S is composed of $f_S^L + lg_S^H$ imitators, L_S^D workers who produce imitated varieties, and L_S^H and L_X^H workers who produce the homogeneous good for domestic consumption and exports, respectively. We have $L_S^H = Y_S - E_S$ and

$$L_S^D = \sum_j \int_{z_{j0}}^{z_{j1}} \mu^j(z)c_S(z)dF(z) = \left[\frac{w^{\sigma-1}\psi_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right]\frac{E_S}{p_S}.$$

Balanced trade requires the value of Southern homogeneous good exports to be equal to the value of its innovated goods imports: $L_X^H = \int_{z^P}^1 p_X c_X(z)dF(z) - \int_{\Phi_M} \mu^j(z)p_X c_X(z)dF(z)$ or

$$L_X^H = \left[\frac{\psi_N - \psi_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right]E_S. \tag{A8}$$

[33] The homogeneous good supply is equal to $c_N^s(v_0) = wL_N^H$ and its demand is equal to $c_N^d(v_0) = Y_N - \int_{\hat{z}}^1 p_N c_N(z)dF(z) = Y_N - E_N$. In equilibrium, $L_N^H = (Y_N - E_N)/w$.

The total income in the South is given by $Y_S = (L_S^D + L_S^H + L_X^H) + \sum_j \int_{z_{j0}}^{z_{j1}} \pi_S^j(z)dF(z)$ or

$$Y_S = \left[L_S(1-g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z)\right] + \left[\frac{\psi_S w^{\sigma-1}}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right]\frac{E_S}{\sigma}. \tag{A9}$$

In the North, $L_N = 1$ is composed of $\int_{z^P}^1 dF(z)$ innovators, L_N^H workers who produce the homogeneous good, and L_N^D and L_X^D workers who produce innovated varieties for domestic consumption and exports. We have $L_N^H = (Y_N - E_N - L_X^H)/w$, $L_N^D = \int_{z^P}^1 c_N(z)dF(z) = E_N/p_N$, and

$$L_X^D = \int_{z^P}^1 c_X(z)dF(z) - \sum_j \int_{z_{j0}}^{z_{j1}} \mu^j(z)c_X(z)dF(z) = \left[\frac{\psi_N - \psi_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right]\frac{E_S}{p_N}.$$

The Northern income is $Y_N = w(L_N^H + L_X^D + L_N^D) + \int_{\Phi^P\setminus\Phi^M} \pi_N(z)dF(z) + \int_{\Phi_M} \pi_N^j(z)dF(z)$ or

$$Y_N = w\left[L_N - \int_{z^P}^1 dF(z)\right] + \left[\frac{\psi_N - \psi_S}{\psi_N + (w^{\sigma-1}-1)\psi_S}\right]\frac{E_S}{\sigma} + \frac{E_N}{\sigma}. \tag{A10}$$

Proof of Proposition 3

The equilibrium is defined by (A1)-(A4) and the following condition:

$$G^P \equiv z^P\left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi}\right) - \sigma w = 0, \quad \text{where} \quad \Psi \equiv \psi_N + (w^{\sigma-1}-1)\psi_S \tag{A11}$$

It follows from (A1)-(A2) that $dz_{H0}/d\Omega = dz_{H1}/d\Omega = 0$. Next, we show that $dz^P/d\Omega < 0$, $dz_{L0}/d\Omega > 0$ and $dz_{L1}/d\Omega < 0$. Totally differentiating (A3), (A4), and (A11), we obtain:

$$\begin{bmatrix} F_P^L & F_{L0}^L & F_{L1}^L \\ 0 & G_{L0}^L & G_{L1}^L \\ G_P^P & G_{L0}^P & G_{L1}^P \end{bmatrix}\begin{bmatrix} dz^P/d\Omega \\ dz_{L0}/d\Omega \\ dz_{L1}/d\Omega \end{bmatrix} = \begin{bmatrix} -F_\Omega^L \\ 0 \\ -G_\Omega^P \end{bmatrix}, \quad \text{where}$$

$$F_\Omega^L = -z_{L0}\mu^L(z_{L0})\frac{w^{\sigma-1}E_S}{\Psi^2}\frac{\psi_N}{1-\Omega} < 0, \tag{A12}$$

$$F_P^L = z_{L0}\mu^L(z_{L0})\frac{w^{\sigma-1}E_S}{\Psi^2}\left(-\frac{d\psi_N}{dz^P}\right) > 0, \tag{A13}$$

$$F_{L0}^L = \frac{d[z_{L0}\mu^L(z_{L0})]}{dz_{L0}}\frac{w^{\sigma-1}E_S}{\Psi} + z_{L0}\mu^L(z_{L0})(w^{\sigma-1}-1)\frac{w^{\sigma-1}E_S}{\Psi^2}\left(-\frac{d\psi_S}{dz_{L0}}\right) > 0, \tag{A14}$$

$$F_{L1}^L = -z_{L0}\mu^L(z_{L0})(w^{\sigma-1}-1)\frac{w^{\sigma-1}E_S}{\Psi^2}\left(\frac{d\psi_S}{dz_{L1}}\right) < 0, \tag{A15}$$

$$G^P_\Omega = z^P(w^{\sigma-1}-1)\frac{E_S}{\Psi^2}\frac{\psi_S}{1-\Omega} > 0, \tag{A16}$$

$$G^P_P = \frac{E_N}{\psi_N} + \frac{E_S}{\Psi} + z^P\left(\frac{E_N}{\psi_N^2} + \frac{E_S}{\Psi^2}\right)\left(-\frac{d\psi_N}{dz^P}\right) > 0, \tag{A17}$$

$$G^P_{L0} = z^P(w^{\sigma-1}-1)\frac{E_S}{\Psi^2}\left(-\frac{d\psi_S}{dz_{L0}}\right) > 0, \tag{A18}$$

$$G^P_{L1} = -z^P(w^{\sigma-1}-1)\frac{E_S}{\Psi^2}\left(\frac{d\psi_S}{dz_{L1}}\right) < 0, \tag{A19}$$

$$G^L_{L0} = \alpha_L > 0, \tag{A20}$$

$$G^L_{L1} = 2\beta_L z_{L1} - \alpha_L > 0. \tag{A21}$$

We find that $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$, where the terms in the brackets are positive from (A13)-(A19) and $G^L_{L1} > 0$ and $G^L_{L0} > 0$ from (A20)-(A21). We have

$$\frac{dz^P}{d\Omega} = \frac{1}{D}\begin{bmatrix} -F^L_\Omega & F^L_{L0} & F^L_{L1} \\ 0 & G^L_{L0} & G^L_{L1} \\ -G^P_\Omega & G^P_{L0} & G^P_{L1} \end{bmatrix}, \quad \frac{dz_{L0}}{d\Omega} = \frac{1}{D}\begin{bmatrix} F^L_P & -F^L_\Omega & F^L_{L1} \\ 0 & 0 & G^L_{L1} \\ G^P_P & -G^P_\Omega & G^P_{L1} \end{bmatrix}, \quad \frac{dz_{L1}}{d\Omega} = \frac{1}{D}\begin{bmatrix} F^L_P & F^L_{L0} & -F^L_\Omega \\ 0 & G^L_{L0} & 0 \\ G^P_P & G^P_{L0} & -G^P_\Omega \end{bmatrix}.$$

It follows that $dz^P/d\Omega = [F^L_\Omega(G^L_{L1}G^P_{L0} - G^L_{L0}G^P_{L1}) - G^P_\Omega(F^L_{L0}G^L_{L1} - F^L_{L1}G^L_{L0})]/D < 0$; $dz_{L0}/d\Omega = G^L_{L1}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D > 0$; and $dz_{L1}/d\Omega = -G^L_{L0}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D < 0$, where $F^L_\Omega < 0$, $G^P_\Omega > 0$, $G^L_{L1} > 0$, $G^L_{L0} > 0$ and all terms in the round brackets are positive.

Proof of Proposition 4

When $M > 0$, the equilibrium is defined by the following conditions:

$$F^H \equiv \int_{z_{H0}}^{z_{H1}} dF(z) + 2M - lg^H_S = 0, \tag{A22}$$

$$G^H \equiv \alpha_H z_{H0} - (\alpha_H - \beta_H z_{H1})z_{H1} = 0, \tag{A23}$$

$$G^L \equiv \alpha_L z_{L0} - (\alpha_L - \beta_L z_{L1})z_{L1} = 0, \tag{A24}$$

$$F^L \equiv z_{L0}\mu^L(z_{L0})\frac{w^{\sigma-1}E_S}{\Psi} - \sigma = 0, \tag{A25}$$

$$G^P \equiv z^P\left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi}\right) - \sigma w = 0, \tag{A26}$$

where $\Psi \equiv \psi_N + (w^{\sigma-1} - 1)\psi_S$. Totally differentiating (A22)-(A23), we obtain

$$\begin{bmatrix} F^H_{H0} & F^H_{H1} \\ G^H_{H0} & G^H_{H1} \end{bmatrix} \begin{bmatrix} dz_{H0}/dM \\ dz_{H1}/dM \end{bmatrix} = \begin{bmatrix} -F^H_M \\ 0 \end{bmatrix}.$$

Hence, $dz_{H0}/dM = -F^H_M G^H_{H1}/D^H > 0$ and $dz_{H1}/dM = F^H_M G^H_{H0}/D^H < 0$, where $D^H = F^H_{H0}G^H_{H1} - F^H_{H1}G^H_{H0} < 0$, $F^H_{H0} < 0$, $F^H_{H1} > 0$, $F^H_M > 0$ from (A22), and $G^H_{H0} > 0$ and $G^H_{H1} > 0$ from (A23).

Next, totally differentiating (A25)-(A26), we obtain

$$\begin{bmatrix} F^L_P & F^L_{L0} & F^L_{L1} \\ 0 & G^L_{L0} & G^L_{L1} \\ G^P_P & G^P_{L0} & G^P_{L1} \end{bmatrix} \begin{bmatrix} dz^P/dM \\ dz_{L0}/dM \\ dz_{L1}/dM \end{bmatrix} = \begin{bmatrix} -F^L_M \\ 0 \\ -G^P_M \end{bmatrix}, \tag{A27}$$

$$\text{where} \quad F^L_M = \Lambda(\Omega) z_{L0}\mu^L(z_{L0})\frac{w^{\sigma-1}E_S}{\Psi^2}, \quad G^P_M = z^P\left[\Lambda(\Omega)\frac{E_S}{\Psi^2} - a_H\frac{E_N}{\psi_N^2}\right], \quad \text{and} \tag{A28}$$

$$\Lambda(\Omega) = (1-\Omega)(w^{\sigma-1} - 1)\alpha_H(2z_{H0} - a_H) - a_H, \tag{A29}$$

since $z_{H0}\mu^H(z_{H0}) = z_{H1}\mu^H(z_{H1})$.[34] Thus, $dz_{L0}/dM = G^L_{L1}(F^L_P G^P_M - F^L_M G^P_P)/D$ and $dz_{L1}/dM = -G^L_{L0}(F^L_P G^P_M - F^L_M G^P_P)/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$ from the proof of Proposition 3. Since $G^L_{L0} > 0$ and $G^L_{L1} > 0$, we have $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ if $F^L_P G^P_M < F^L_M G^P_P$. Using (A13), (A17), (A28), and $\xi = \psi_N/\Psi$ and simplifying, we rewrite $F^L_P G^P_M < F^L_M G^P_P$ as follows:

$$\Lambda(\Omega)\left(1 + \frac{\xi E_S}{E_N}\right) + [\Lambda(\Omega) + a_H]\varepsilon^P > 0, \quad \text{where} \quad \varepsilon^P \equiv -\frac{d\psi_N}{dz^P}\frac{z^P}{\psi_N} = z^P f(z^P)\frac{z^P}{\psi_N}. \tag{A30}$$

Define $\Lambda^P(\Omega)$ by $\Lambda^P(\Omega)(1 + \xi E_S/E_N) + [\Lambda^P(\Omega) + a_H]\varepsilon^P = 0$. That is,

$$\Lambda^P(\Omega) = -\frac{a_H\varepsilon^P}{1 + \varepsilon^P + \xi E_S/E_N}. \tag{A31}$$

Then, $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ if $\Lambda(\Omega) > \Lambda^P(\Omega)$, and $dz_{L0}/dM \geq 0$ and $dz_{L1}/dM \leq 0$ if $\Lambda(\Omega) \leq \Lambda^P(\Omega)$. Assuming $F(z) = [1 - (z_0/z)^k]/(1 - z_0^k)$, we have

$$\varepsilon^P = \frac{k(k-1)z_0^k(z^P)^{1-k}}{kz_0^k(z^P)^{1-k} - 1 + (k-1)(1-z_0^k)a_H M}$$

[34] Since $z_{H0}\mu^H(z_{H0}) = z_{H1}\mu^H(z_{H1})$, we have:

$$-\frac{d\psi_S}{dz_{H0}}\frac{dz_{H0}}{dM} - \frac{d\psi_S}{dz_{H1}}\frac{dz_{H1}}{dM} = 2z_{H0}\mu^H(z_{H0}).$$

and so $d\varepsilon^P/dz^P > 0$ at $M \to 0$. This in turn implies that $d\Lambda^P(\Omega)/d\Omega > 0$, since $d\xi/d\Omega > 0$ and $dz^P/d\Omega < 0$. Thus, as Ω rises from zero to one, Λ^P rises from $-a_H < \Lambda^P(0) < 0$ to $\Lambda^P(1) < 0$, where

$$\Lambda^P(1) = -\frac{a_H\bar{\varepsilon}}{1+\bar{\varepsilon}+\xi E_S/E_N}, \quad \text{where} \quad \bar{\varepsilon} \equiv \frac{\bar{z}^2 f(\bar{z})}{\int_{\bar{z}}^1 z dF(z) + a_H M},$$

because $\xi = 1$, $z^P = \bar{z}$ and $\varepsilon^P = \bar{\varepsilon}$ when $\Omega = 1$.

In Figure 6, $\Lambda(0) \geq 0 > \Lambda^P(0)$ and $\Lambda^P(1) > \Lambda(1) = -a_H$. We assume that $(w^{\sigma-1}-1)\alpha_H(2z_{H0} - a_H) \geq a_H$, which implies that $\Lambda(0) \geq 0$. This assumption requires that the term in the square brackets is positive, which in turn implies that as Ω rises from zero to one, Λ falls at a constant rate. The functions $\Lambda^P(\Omega)$ and $\Lambda(\Omega)$ intersect at a unique point in this case. There exists $\bar{\Omega}$ such that $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$ if $\Omega < \bar{\Omega}$; and $dz_{L0}/dM > 0$ and $dz_{L1}/dM < 0$ if $\bar{\Omega} < \Omega$.

Proof of Proposition 5

From (A27), we find that $dz^P/dM = [F_M^L(G_{L0}^P G_{L1}^L - G_{L0}^L G_{L1}^P) - G_M^P(F_{L0}^L G_{L1}^L - F_{L1}^L G_{L0}^L)]/D$, where $D \equiv G_{L1}^L(F_{L0}^L G_P^P - F_P^L G_{L0}^P) + G_{L0}^L(F_P^L G_{L1}^P - F_{L1}^L G_P^P) > 0$ from the proof of Proposition 3. Thus, $dz^P/dM > 0$ if $F_M^L(G_{L0}^P G_{L1}^L - G_{L0}^L G_{L1}^P) > G_M^P(F_{L0}^L G_{L1}^L - F_{L1}^L G_{L0}^L)$, which using (A14)-(A15), (A18)-(A19) and (A28) we simplify to obtain $\Lambda\xi^2 E_S/E_N < a_H(1+\Upsilon H)$, where

$$\Upsilon \equiv \frac{w^{\sigma-1}-1}{\Psi}\frac{z_{L0}m^L(z_{L0})}{d[z_{L0}m^L(z_{L0})]/dz_{L0}}, \qquad H \equiv -\frac{d[z_{L0}m^L(z_{L0})]/dz_{L0}}{d[z_{L1}m^L(z_{L1})]/dz_{L1}}\left(\frac{d\psi_S}{dz_{L1}}\right) - \frac{d\psi_S}{dz_{L0}} > 0. \quad \text{(A32)}$$

Because $z_{L0}\mu^L(z_{L0}) = z_{L1}\mu^L(z_{L1})$ and $(w^{\sigma-1}-1)\psi_S/\Psi = 1-\xi$, we have

$$\Upsilon H = (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \varepsilon_{L0}\right), \quad \text{where} \quad \text{(A33)}$$

$$\varepsilon_{L1} \equiv \frac{d\psi_S}{dz_{L1}}\frac{z_{L1}}{\psi_S}, \qquad \varepsilon_{L0} \equiv -\frac{d\psi_S}{dz_{L0}}\frac{z_{L0}}{\psi_S}, \qquad \epsilon_{L1} \equiv -\frac{d[z_{L1}m^L(z_{L1})]}{m^L(z_{L1})dz_{L1}}.$$

Now, $\Lambda(\Omega)\xi^2 E_S/E_N < a_H(1+\Upsilon H)$ can be rewritten as follows:

$$\Lambda(\Omega) < \frac{a_H E_N}{\xi^2 E_S}\left[1 + (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \varepsilon_{L0}\right)\right].$$

Define

$$\Lambda^L(\Omega) \equiv \frac{a_H E_N}{\xi^2 E_S}\left[1 + (1-\xi)\left(\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \varepsilon_{L0}\right)\right]. \quad \text{(A34)}$$

Then $dz^P/dM > 0$ if $\Lambda(\Omega) < \Lambda^L(\Omega)$. Assuming $\pi_N(a_H) > \pi_S(a_H)$, which requires that $E_N/(\xi E_S) > (w^{\sigma-1}+1)\mu^H(a_H) - 1$, it is true that $\Lambda(\Omega) < \Lambda^L(\Omega)$. From (A34) we know that for any $\Omega < 1$, $\Lambda^L(\Omega) > a_H E_N/(\xi E_S) > a_H E_N/E_S = \Lambda^L(1)$. Thus $\Lambda(\Omega) \geq \Lambda^L(\Omega)$ if and only if

$(w^{\sigma-1}-1)[2z_{H0}\mu^H(z_{H0})-a_H\mu^H(a_H)]-a_H \geq a_H[(w^{\sigma-1}+1)\mu^H(a_H)-1]$, which is not true since $z_{H0}<a_H$. Hence, $\Lambda(\Omega)<\Lambda^L(\Omega)$ and so, $dz^P/dM>0$.

Proof of Proposition 6(i)

We show that $dY_N/d\Omega>0$ and $dY_S/\Omega<0$. We have

$$Y_N = w\left[1-\int_{z^P}^{1} dF(z)\right]+\frac{\psi_N-\psi_S}{\Psi}\frac{E_S}{\sigma}+\frac{E_N}{\sigma}; \tag{A35}$$

$$Y_S = \left[l(1-g_S^H)-\int_{z_{L0}}^{z_{L1}} dF(z)\right]+\frac{w^{\sigma-1}\psi_S}{\Psi}\frac{E_S}{\sigma}. \tag{A36}$$

Since $\sigma w = z^P(E_N+\xi E_S)/\psi_N$ and $z^Pf(z^P)=\varepsilon^P\psi_N/z^P$, we find that $dY_N/d\Omega>0$ iff

$$w^{\sigma-1}\frac{\psi_S}{\Psi}\left(\frac{d\psi_N}{d\Omega}\frac{1}{\psi_N}-\frac{d\psi_S}{d\Omega}\frac{1}{\psi_S}\right)>\left(\frac{E_N}{\xi E_S}+1\right)\frac{\varepsilon^P}{z^P}\left(-\frac{dz^P}{d\Omega}\right).$$

Using (A47)-(A49), we simplify this inequality to $w^{\sigma-1}+\varepsilon^P>0$, which is true.
Next, we find that $dY_S/d\Omega<0$ iff

$$\frac{w^{\sigma-1}E_S}{\sigma}\frac{\psi_S\psi_N}{\Psi^2}\left(\frac{d\psi_N}{d\Omega}\frac{1}{\psi_N}-\frac{d\psi_S}{d\Omega}\frac{1}{\psi_S}\right)>f(z_{L0})\frac{dz_{L0}}{d\Omega}-f(z_{L1})\frac{dz_{L1}}{d\Omega}. \tag{A37}$$

Using (A49), we obtain

$$f(z_{L0})\frac{dz_{L0}}{d\Omega}-f(z_{L1})\frac{dz_{L1}}{d\Omega}=-\frac{1}{z_{L0}\mu^L(z_{L0})}\left(\frac{\psi_S}{1-\Omega}+\frac{d\psi_S}{d\Omega}\right)=\frac{\psi_S}{1-\Omega}\frac{w^{\sigma-1}E_S}{\sigma\Psi}\frac{(1+\varepsilon^P)\xi}{1-\xi}\left(1+\frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}}.$$

since $z_{L0}\mu^L(z_{L0})=\sigma\Psi/(w^{\sigma-1}E_S)$. Now using (A48)-(A49) and simplifying, we find that the inequality (A37) holds.

Proof of Proposition 6(ii)

The migrants' earnings are given by:

$$\pi_N(a_H)M=\frac{a_H}{\sigma}\left\{\frac{E_N}{\psi_N}+\frac{[1-\mu^H(a_H)]E_S}{\Psi}\right\}M. \tag{A38}$$

(1) Migrants' Earnings Count Towards the Northern Income

When the migrants' earnings count towards the Northern income, Y_N and Y_S are given by (A35) and (A36), where $\Psi \equiv \psi_N+(w^{\sigma-1}-1)\psi_S$. Since $dz^P/dM>0$, $d\psi_N/dM>0$, $d\psi_S/dM<0$, and

$d\Psi/dM < 0$, we have $dY_N/dM > 0$. Also, $dY_S/dM < 0$ when $dz_{L0}/dM < 0$ and $dz_{L1}/dM > 0$, since $d(\psi_S/\Psi)/dM < 0$.

(2) Migrants' Earnings Count Towards the Southern Income

The Impact on Southern Income

When the migrants' earnings count towards the Southern income, we have:

$$Y_S = \left[l(1-g_S^H) - \int_{z_{L0}}^{z_{L1}} dF(z) \right] + \frac{w^{\sigma-1}\psi_S}{\Psi}\frac{E_S}{\sigma} + \frac{a_H}{\sigma}\left\{ \frac{E_N}{\psi_N} + \frac{[1-\mu^H(a_H)]E_S}{\Psi} \right\} M. \tag{A39}$$

We find that dY_S/dM, evaluated at $M = 0$, is given by the sum of these three components:

$$A \equiv \frac{a_H}{\sigma}\left\{ \frac{E_N}{\psi_N} + \frac{[1-\mu^H(a_H)]E_S}{\Psi} \right\} > 0; \tag{A40}$$

$$B \equiv f(z_{L0})\frac{dz_{L0}}{dM} - f(z_{L1})\frac{dz_{L1}}{dM} < 0; \tag{A41}$$

$$C \equiv w^{\sigma-1}\frac{E_S}{\sigma\Psi}\left[\frac{d\psi_S}{dM} - \frac{\psi_S}{\Psi}\frac{d\Psi}{dM} \right] = w^{\sigma-1}\frac{E_S}{\sigma\Psi^2}\left[\psi_N\frac{d\psi_S}{dM} - \psi_S\frac{d\psi_N}{dM} \right] < 0, \tag{A42}$$

where $d\psi_S/dM < 0$ and $d\Psi/dM < 0$. Using the results in the proof of Proposition 6(i) and $z_{L0}\mu^L(z_{L0}) = z_{L1}\mu^L(z_{L1}) = \sigma\Psi/(w^{\sigma-1}E_S)$, we obtain:

$$\frac{d\psi_S}{dM} = \frac{d\psi_S}{dz_{L1}}\frac{dz_{L1}}{dM} + \frac{d\psi_S}{dz_{L0}}\frac{dz_{L0}}{dM} - \frac{\Lambda(\Omega)+a_H}{w^{\sigma-1}-1} = \frac{\sigma\Psi}{w^{\sigma-1}E_S}(-B) - \frac{\Lambda(\Omega)+a_H}{w^{\sigma-1}-1}. \tag{A43}$$

Substituting this result into C and simplifying, we find that:

$$B + C = w^{\sigma-1}\frac{E_S}{\sigma\Psi}\left[-\frac{\Lambda(\Omega)+a_H}{w^{\sigma-1}-1} - \frac{\psi_S}{\Psi}\frac{d\Psi}{dM} \right]$$

Since $d\Psi/dM < 0$, the sufficient condition for $A + B + C > 0$ is given by:

$$\frac{a_H}{\sigma}\left\{ \frac{E_N}{\psi_N} + \frac{[1-\mu^H(a_H)]E_S}{\Psi} \right\} > w^{\sigma-1}\frac{E_S}{\sigma\Psi}\left[\frac{\Lambda(\Omega)+a_H}{w^{\sigma-1}-1} \right],$$

which simplifies to the following:

$$\frac{E_N}{E_S} > w^{\sigma-1}\frac{\psi_N}{\Psi}\frac{1}{a_H}\left[\frac{\Lambda(\Omega)+a_H}{w^{\sigma-1}-1} \right],$$

which is true when $E_N/E_S > w^{\sigma-1}$, since $\psi_N/\Psi < 1$ and $\Lambda(\Omega) + a_H < (w^{\sigma-1}-1)a_H\mu^H(a_H)$ from (A29). Therefore $dY_S/dM > 0$ when $E_N/E_S > w^{\sigma-1}$.

The Impact on Southern Welfare

Differentiating $V_S = Y_S - E_S + E_S/P_S$, where $P_S = p_N \Psi^{1/(1-\sigma)}$, with respect to M, we find that $dV_S/dM > 0$ when this is true:

$$\frac{dY_S}{dM} > \frac{E_S}{\sigma w} \frac{\Psi^{1/(\sigma-1)}}{\Psi} \left(-\frac{d\Psi}{dM} \right). \tag{A44}$$

From the proof in part (2) above, we know that

$$\frac{dY_S}{dM} > w^{\sigma-1} \frac{E_S}{\sigma \Psi} \frac{\psi_S}{\Psi} \left(-\frac{d\Psi}{dM} \right).$$

So the inequality (A44) holds under this sufficient condition: $w^\sigma \psi_S > \Psi^{\sigma/(\sigma-1)}$, or equivalently:

$$w^{\sigma-1} > \frac{\psi_N - \psi_S}{\psi_S^{\frac{\sigma-1}{\sigma}} - \psi_S},$$

which requires high w and ψ_S and low ψ_N.

The Impact on Northern Income and Welfare

When the migrants' earnings count towards the Southern income, we have:

$$Y_N = w \left[1 - \int_{z^P}^{1} dF(z) \right] + \frac{\psi_N - \psi_S}{\Psi} \frac{E_S}{\sigma} + \frac{E_N}{\sigma} - \frac{a_H}{\sigma} \left\{ \frac{E_N}{\psi_N} + \frac{[1 - \mu^H(a_H)] E_S}{\Psi} \right\} M. \tag{A45}$$

We find that dY_N/dM, evaluated at $M = 0$, is given by

$$\frac{dY_N}{dM} = w f(z^P) \frac{dz^P}{dM} - (A + C) = \frac{w}{z^P} \left(a_H - \frac{d\psi_N}{dM} \right) - A - C,$$

where $A > 0$ and $C < 0$ are respectively defined in (A40) and (A42). The first term is the increase in production income and the term $-(A + C) < 0$ is the loss in the rents of North-born entrepreneurs. The second equality sign follows because $z^P f(z^P) dz^P/dM = a_H - d\psi_N/dM$.

Using (A40) and (A42) and simplifying, we obtain:

$$\frac{dY_N}{dM} = a_H \mu^H(a_H) \frac{E_S}{\sigma \Psi} - \frac{1}{\sigma} \left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi} \right) \frac{d\psi_N}{dM} - w^{\sigma-1} \frac{E_S}{\sigma \Psi^2} \left(\psi_N \frac{d\psi_S}{dM} - \psi_S \frac{d\psi_N}{dM} \right),$$

which follows since

$$\frac{w}{z^P} = \frac{1}{\sigma} \left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi} \right);$$

$$\frac{w}{z^P}a_H - A = a_H\mu^H(a_H)\frac{E_S}{\sigma\Psi} > 0.$$

We find that $dY_N/dM > 0$ if and only if this is true:

$$a_H\mu^H(a_H) + w^{\sigma-1}\frac{\psi_N}{\Psi}\left(-\frac{d\psi_S}{dM}\right) - \left(\frac{E_N}{\xi E_S} + 1 - w^{\sigma-1}\frac{\psi_S}{\Psi}\right)\frac{d\psi_N}{dM} > 0,$$

where

$$-\frac{d\psi_S}{dM} = \frac{1}{w^{\sigma-1}-1}\left\{\Lambda(\Omega) + a_H - a_H\varepsilon^P\left[1 - \frac{\Lambda(\Omega)}{\Lambda^P(\Omega)}\right]\frac{\Upsilon H}{\tilde{D}}\right\}.$$

When $\Omega \leq \bar{\Omega}$, the maximum of $-d\psi_S/dM$ is equal to $a_H\mu^H(a_H)$. The minimum of $d\psi_N/dM$ is equal to $a_H/(1+\varepsilon^P)$. So the maximum value of dY_N/dM is positive iff

$$\mu^H(a_H)(1+\varepsilon^P)\left(1 + w^{\sigma-1}\frac{\psi_N}{\Psi}\right) - \left(\frac{E_N}{\xi E_S} + 1 - w^{\sigma-1}\frac{\psi_S}{\Psi}\right) > 0,$$

or equivalently:

$$\mu^H(a_H)(1+\varepsilon^P) + w^{\sigma-1}\left[\frac{\psi_N}{\Psi}\mu^H(a_H)(1+\varepsilon^P) + \frac{\psi_S}{\Psi}\right] > \frac{E_N}{\xi E_S} + 1$$

Since $E_N/(\xi E_S) > w^{\sigma-1}$, this condition requires that $\mu^H(a_H)(1+\varepsilon^P) > 1$, or equivalently:

$$\Omega < \bar{\Omega} < 1 - \frac{1}{\alpha_H(1+\bar{\varepsilon})},$$

where $\bar{\varepsilon} = (\bar{z})^2 f(\bar{z})/\int_{\bar{z}}^1 z dF(z)$ when $M \to 0$. It also follows that this condition is sufficient for $dY_N/dM < 0$:

$$1 - \frac{1}{\alpha_H(1+\bar{\varepsilon})} < \Omega < \bar{\Omega}.$$

Proof of Proposition 6(iii)

(1) The Impact of IPRs

We have:

$$\frac{dY_G}{d\Omega} = wf(z^P)\frac{dz^P}{d\Omega} + f(z_{L0})\frac{dz_{L0}}{d\Omega} - f(z_{L1})\frac{dz_{L1}}{d\Omega},$$

where from the proof of Proposition 6(i):

$$f(z_{L0})\frac{dz_{L0}}{d\Omega} - f(z_{L1})\frac{dz_{L1}}{d\Omega} = \frac{\psi_S}{1-\Omega}\frac{w^{\sigma-1}E_S}{\sigma\Psi}\frac{(1+\varepsilon^P)\xi}{1-\xi}\left(1 + \frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}}.$$

Substituting for $\sigma w = z^P(E_N + \xi E_S)/\psi_N$ and $z^P f(z^P) = \varepsilon^P \psi_N / z^P$, and using (A35), we find that $dY_G/d\Omega > 0$ iff

$$w^{\sigma-1}(1+\varepsilon^P)\Upsilon H > (w^{\sigma-1}-1)\varepsilon^P(1-\xi+\Upsilon H).$$

Using (A33), this inequality simplifies to:

$$\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \frac{\varepsilon_{L0}}{\epsilon_{L0}} > \varepsilon^P \left(\frac{w^{\sigma-1}-1}{w^{\sigma-1}+\varepsilon^P} \right),$$

for which it is sufficient that:

$$\frac{\varepsilon_{L1}}{\epsilon_{L1}} + \frac{\varepsilon_{L0}}{\epsilon_{L0}} > \varepsilon^P. \tag{A46}$$

Using $z_{L0}\mu^L(z_{L0}) = z_{L1}\mu^L(z_{L1})$ and (A33), we rewrite this inequality as follows:

$$\left[\frac{z_{L0}\mu^L(z_{L0})}{z^P}\right]^2 \left[\frac{f(z_{L1})}{-d[z_{L1}m^L(z_{L1})]/dz_{L1}} + \frac{f(z_{L0})}{d[z_{L0}m^L(z_{L0})]/dz_{L0}}\right] > (1-\Omega)\frac{\psi_S}{\psi_N} f(z^P).$$

Since $\psi_S/\Psi_N < 1$, $\Omega < 1$, $d[z_{L0}m^L(z_{L0})]/dz_{L0} = \alpha_L$ and

$$\frac{z_{L0}\mu^L(z_{L0})}{z^P} = \frac{1}{w^\sigma}\left(\frac{E_N}{\xi E_S} + 1\right) > 1,$$

the following is sufficient for (A46) to hold:

$$\frac{f(z_{L0})}{\alpha_L} > f(z^P).$$

Assuming $F(z) = [1-(z_0/z)^k]/(1-z_0^k)$, we have: $f(z^P)/f(z_{L0}) = (z_{L0}/z^P)^k$. Since $\hat{z} < z_{L0} < a_L$ and $z^P > \bar{z}$, the following is sufficient for (A46) to hold: $(\bar{z}/a_L)^k > \alpha_L$.

(2) The Impact of Migration

We find that dY_G/dM, evaluated at $M \to 0$, is given by

$$\frac{dY_G}{dM} = w f(z^P)\frac{dz^P}{dM} + B = \frac{w}{z^P}\left(a_H - \frac{d\psi_N}{dM}\right) + B,$$

where

$$\frac{w}{z^P} = \frac{1}{\sigma}\left(\frac{E_N}{\psi_N} + \frac{E_S}{\Psi}\right);$$

$$a_H - \frac{d\psi_N}{dM} = a_H \frac{\varepsilon^P}{\tilde{D}}\left(1 - \frac{\xi^2 E_S}{E_N} + \Upsilon H\right);$$

$$B = -\frac{w^{\sigma-1}}{w^{\sigma-1}-1} a_H \varepsilon^P \frac{E_S}{\sigma\Psi} \left[1 - \frac{\Lambda(\Omega)}{\Lambda^P(\Omega)}\right] \frac{\Upsilon H}{\tilde{D}} < 0.$$

It follows that $dY_G/dM > 0$ if this is true:

$$\left(\frac{E_N}{\xi E_S} + 1\right)\left(1 - \frac{\xi^2 E_S}{E_N} + \Upsilon H\right) > \frac{w^{\sigma-1}}{w^{\sigma-1}-1}\left[1 - \frac{\Lambda(\Omega)}{\Lambda^P(\Omega)}\right]\Upsilon H,$$

for which the following is sufficient:

$$\frac{E_N}{\xi E_S} + 1 > \frac{w^{\sigma-1}}{w^{\sigma-1}-1}\left[1 - \frac{\Lambda(\Omega)}{\Lambda^P(\Omega)}\right].$$

This condition holds when $\Omega \geq \bar{\Omega}$, in which case the right hand side term is negative. When $\Omega < \bar{\Omega}$, the condition holds when $w^{\sigma-1} - 1 > 1$, which is true.

Proof of Proposition 7(i)

The Impact of IPRs

We show that $d\psi_N/d\Omega > 0$, $d\psi_S/d\Omega < 0$, and $d\Psi/d\Omega < 0$, where $\Psi \equiv \psi_N + (w^{\sigma-1} - 1)\psi_S$. We find

$$\frac{d\psi_N}{d\Omega} = -z^P f(z^P)\frac{dz^P}{d\Omega},$$

$$\frac{d\psi_S}{d\Omega} = -\frac{\psi_S}{1-\Omega} + z_{L1}\mu^L(z_{L1})f(z_{L1})\frac{dz_{L1}}{d\Omega} - z_{L0}\mu^L(z_{L0})f(z_{L0})\frac{dz_{L0}}{d\Omega}.$$

From the proof of Proposition 3, we have $dz^P/d\Omega = [F^L_\Omega(G^L_{L1}G^P_{L0} - G^L_{L0}G^P_{L1}) - G^P_\Omega(F^L_{L0}G^L_{L1} - F^L_{L1}G^L_{L0})]/D$; $dz_{L0}/d\Omega = G^L_{L1}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D$; and $dz_{L1}/d\Omega = -G^L_{L0}(F^L_P G^P_\Omega - F^L_\Omega G^P_P)/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$. Using (A12)-(A21), substituting for $z^P f(z^P) = \varepsilon^P \psi_N / z^P$ from (A30) and simplifying, we obtain

$$\frac{dz^P}{d\Omega} = -\frac{z^P}{1-\Omega}\left(\frac{\xi E_S}{E_N}\right)\frac{1-\xi+\Upsilon H}{\tilde{D}} < 0; \tag{A47}$$

$$\frac{d\psi_N}{d\Omega} = \frac{\varepsilon^P \psi_N}{1-\Omega}\left(\frac{\xi E_S}{E_N}\right)\frac{1-\xi+\Upsilon H}{\tilde{D}} > 0; \tag{A48}$$

$$\frac{d\psi_S}{d\Omega} = -\frac{\psi_S}{1-\Omega}\left[1 + (1+\varepsilon^P)\frac{\xi}{1-\xi}\left(1 + \frac{\xi E_S}{E_N}\right)\frac{\Upsilon H}{\tilde{D}}\right] < 0; \tag{A49}$$

where Υ and H are given in (A32) and $\tilde{D} \equiv (1+\varepsilon^P)(1+\Upsilon H) + (1 + \xi\varepsilon^P + \Upsilon H)\xi E_S/E_N$. Now using (A48)-(A49) and simplifying, we find that $d\Psi/d\Omega = d\psi_N/d\Omega + (w^{\sigma-1}-1)d\psi_S/d\Omega < 0$.

The Impact of Migration

We show that $d\psi_N/dM > 0$, $d\psi_S/dM < 0$, and $d\Psi/dM < 0$. We find that $d\psi_N/dM > 0$ iff $a_H > z^P f(z^P)(dz^P/dM)$. From (A27), we find that $dz^P/dM = [F^L_M(G^P_{L0}G^L_{L1} - G^L_{L0}G^P_{L1}) - G^P_M(F^L_{L0}G^L_{L1} - F^L_{L1}G^L_{L0})]/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$ from the proof of Proposition 3. Substituting for dz^P/dM, we simplify $a_H > z^P f(z^P)(dz^P/dM)$ to obtain: $1 + (1 + 2\xi\varepsilon^P)\xi E_S/E_N + \Upsilon H\,(1 + \xi E_S/E_N) > 0$, which is true. Next, $d\psi_S/dM < 0$ if

$$\mu^H(a_H)a_H + \frac{d\psi_S}{dz_{H1}}\frac{dz_{H1}}{dM} + \frac{d\psi_S}{dz_{H0}}\frac{dz_{H0}}{dM} + \frac{d\psi_S}{dz_{L1}}\frac{dz_{L1}}{dM} + \frac{d\psi_S}{dz_{L0}}\frac{dz_{L0}}{dM} < 0.$$

From (A29), we have

$$\mu^H(a_H)a_H + \frac{d\psi_S}{dz_{H1}}\frac{dz_{H1}}{dM} + \frac{d\psi_S}{dz_{H0}}\frac{dz_{H0}}{dM} = -\frac{\Lambda(\Omega) + a_H}{w^{\sigma-1} - 1}.$$

From the proof of Proposition 4, we have $dz_{L0}/dM = G^L_{L1}(F^L_P G^P_M - F^L_M G^P_P)/D$ and $dz_{L1}/dM = -G^L_{L0}(F^L_P G^P_M - F^L_M G^P_P)/D$, where $D \equiv G^L_{L1}(F^L_{L0}G^P_P - F^L_P G^P_{L0}) + G^L_{L0}(F^L_P G^P_{L1} - F^L_{L1}G^P_P) > 0$. Using (A13)-(A21), (A28), (A31) and (A32), we obtain

$$\frac{d\psi_S}{dz_{L1}}\frac{dz_{L1}}{dM} + \frac{d\psi_S}{dz_{L0}}\frac{dz_{L0}}{dM} = \frac{a_H\varepsilon^P}{w^{\sigma-1} - 1}\left[1 - \frac{\Lambda(\Omega)}{\Lambda^P(\Omega)}\right]\frac{\Upsilon H}{\tilde{D}}. \tag{A50}$$

It follows that

$$(w^{\sigma-1} - 1)\frac{d\psi_S}{dM} = -[\Lambda(\Omega) + a_H] + a_H\varepsilon^P\left[1 - \frac{\Lambda(\Omega)}{\Lambda^P(\Omega)}\right]\frac{\Upsilon H}{\tilde{D}}. \tag{A51}$$

Substituting for $\Lambda^P(\Omega)$ from (A31) and simplifying, we find that $d\psi_S/dM < 0$ if $[\Lambda(\Omega) + a_H][1 + \varepsilon^P + (1 + \xi\varepsilon^P)\xi E_S/E_N] + a_H\Upsilon H(1 + \xi E_S/E_N) > 0$, which is true since $\Lambda(\Omega) + a_H > 0$ in Figure 6.

Last, $d\Psi/dM < 0$. This follows from $d\Psi/dM = d\psi_N/dM + (w^{\sigma-1} - 1)d\psi_S/dM$, using the result in (A51) and

$$\frac{d\psi_N}{dM} = a_H\left[1 - \frac{\varepsilon^P}{\tilde{D}}\left(1 - \frac{\xi^2 E_S}{E_N} + \Upsilon H\right)\right]. \tag{A52}$$

Proof of Proposition 7(ii)

The Impact of IPRs

(1) $d\psi^q_N/d\Omega < 0$ since $(d\tilde{\psi}_N/d\Omega)/\tilde{\psi}_N > (d\psi_N/d\Omega)/\psi_N$, which simplifies to $\psi_N > z^P\tilde{\psi}_N$.

(2) $d\psi_S^{Lq}/d\Omega > 0$ iff $(-d\tilde{\psi}_S^L/d\Omega)/\tilde{\psi}_S^L > (-d\psi_S^L/d\Omega)/\psi_S^L$, which requires the following:

$$\psi_S^L\left[f(z_{L0})\frac{dz_{L0}}{d\Omega} - \frac{m^L(z_{L1})}{m^L(z_{L0})}f(z_{L1})\frac{dz_{L1}}{d\Omega}\right] > z_{L0}\tilde{\psi}_S^L\left[f(z_{L0})\frac{dz_{L0}}{d\Omega} - f(z_{L1})\frac{dz_{L1}}{d\Omega}\right],$$

since $z_{L0}m^L(z_{L0}) = z_{L1}m^L(z_{L1})$. Substituting for $m^L(z_{L1})/m^L(z_{L0}) = z_{L0}/z_{L1}$, $f(z_{L1})/f(z_{L0}) = q^{k+1}$ where $q \equiv z_{L0}/z_{L1}$ and $e \equiv (-dz_{L1}/d\Omega)/(dz_{L0}/d\Omega) = \alpha_L/(2\beta_L z_{L1} - \alpha_L) > 0$, we obtain

$$\frac{\psi_S^L}{z_{L0}} - \tilde{\psi}_S^L\left(\frac{1+eq^{k+1}}{1+eq^{k+2}}\right) > 0, \tag{A53}$$

where

$$\frac{\psi_S^L}{z_{L0}} = \frac{(z_0/z_{L0})^k}{1-z_0^k}\left[\frac{k}{k-1}\alpha_L(1-q^{k-1}) + \frac{k}{k-2}\beta_L z_{L0}q^{k-2})\right],$$

$$\tilde{\psi}_S^L = \frac{(z_0/z_{L0})^k}{1-z_0^k}\left[\alpha_L(1-q^k) + \frac{k}{k-1}\beta_L z_{L0}q^{k-1})\right].$$

Note that the left hand side in (A53) is equal to zero when $z_{L1} \to a_L$, where $a_L = \alpha_L/(2\beta_L)$, in which case $z_{L0} = a_L$ and $e \to \infty$. Further, the left hand side in (A53) rises as z_{L1} rises and correspondingly, z_{L0}, q and e fall. Thus, (A53) always holds.
(3) $d\psi_S^{Hq}/d\Omega = 0$ since $(-d\psi_S^H/d\Omega)/\psi_S^H = (-d\tilde{\psi}_S^H/d\Omega)/\tilde{\psi}_S^H = 1/(1-\Omega)$.

The Impact of Migration

(1) $d\psi_N^q/dM > 0$ iff $(d\psi_N/dM)/\psi_N > (d\psi_N^o/dM)/\tilde{\psi}_N$, which requires the following:

$$(\psi_N - z^P\tilde{\psi}_N)f(z^P)\frac{dz^P}{dM} + \int_{z^P}^1 (a_H - z)dF(z) > 0.$$

The first term is positive. The second is positive iff $a_H[(z^P)^{-k} - 1] - [(z^P)^{1-k} - 1]k/(k-1) > 0$, which is true if $k \geq 3$. This is because $\int_{z^P}^1 (a_H - z)dF(z) > 0$ for $k = 3$ when $a_H > 1/2$ (which is true since $a_H = \alpha_H/(2\beta_H)$ and $m^H(1) = \alpha_H - \beta_H > 0$) and it further rises as k rises.
(2) First, we show that $d\psi_S^{Lq}/dM > 0$ if the L imitation set contracts. We note that $d\psi_S^{Lq}/dM > 0$ iff $(-d\tilde{\psi}_S^L/dM)/\tilde{\psi}_S^L > (-d\psi_S^L/dM)/\psi_S^L$, which requires the following:

$$\psi_S^L\left[f(z_{L0})\frac{dz_{L0}}{dM} - \frac{m^L(z_{L1})}{m^L(z_{L0})}f(z_{L1})\frac{dz_{L1}}{dM}\right] > z_{L0}\tilde{\psi}_S^L\left[f(z_{L0})\frac{dz_{L0}}{dM} - f(z_{L1})\frac{dz_{L1}}{dM}\right],$$

where $dz_{L0}/dM > 0$ and $dz_{L1}/dM < 0$. Since $e \equiv (-dz_{L1}/d\Omega)/(dz_{L0}/d\Omega) = (-dz_{L1}/dM)/(dz_{L0}/dM)$, we obtain (A53), which always holds.

Second, $d\psi_S^{Lq}/dM < 0$ if the L imitation set expands. Note that $d\psi_S^{Lq}/dM < 0$ iff $(-d\tilde{\psi}_S^L/dM)/\tilde{\psi}_S^L <$

$(-d\psi_S^L/dM)/\psi_S^L$, which requires the following:

$$\psi_S^L \left[-f(z_{L0})\frac{dz_{L0}}{dM} + \frac{m^L(z_{L1})}{m^L(z_{L0})} f(z_{L1})\frac{dz_{L1}}{dM} \right] > z_{L0}\tilde{\psi}_S^L \left[-f(z_{L0})\frac{dz_{L0}}{dM} + f(z_{L1})\frac{dz_{L1}}{dM} \right],$$

where $dz_{L0}/dM < 0$ and $dz_{L1}/dM < 0$. Again, we obtain (A53), which always holds.
(3) $d\psi_S^{Hq}/dM > 0$ if $(d\psi_S^H/dM)/\psi_S^H - (d\tilde{\psi}_S^H/dM)/\tilde{\psi}_S^H > 0$, which requires the following:

$$\psi_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - \frac{m^H(z_{H1})}{m^H(z_{H0})} f(z_{H1})\frac{dz_{H1}}{dM} \right] >$$

$$z_{H0}\tilde{\psi}_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - f(z_{H1})\frac{dz_{H1}}{dM} \right] + (\psi_S^H - a_H\tilde{\psi}_S^H)\frac{m^H(a_H)}{m^H(z_{H0})}.$$

We note that

$$\psi_S^H - a_H\tilde{\psi}_S^H = -\frac{(z_0/z_{H0})^k}{1-z_0^k} \left[a_H(1-q^k) - \frac{k}{k-1} z_{H0}(1-q^{k-1}) \right] < 0,$$

where the term in square brackets is positive when $k = 2$ (since $a_H > z_{H0}$) and rises as k rises. Thus, it remains to show the following:

$$\psi_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - \frac{m^H(z_{H1})}{m^H(z_{H0})} f(z_{H1})\frac{dz_{H1}}{dM} \right] > z_{H0}\tilde{\psi}_S^H \left[f(z_{H0})\frac{dz_{H0}}{dM} - f(z_{H1})\frac{dz_{H1}}{dM} \right].$$

Substituting for $m^H(z_{H1})/m^H(z_{H0}) = z_{H0}/z_{H1}$, $f(z_{H1})/f(z_{H0}) = q^{k+1}$, where $q \equiv z_{H0}/z_{H1}$ and $e \equiv (-dz_{H1}/dM)/(dz_{H0}/dM) = \alpha_H/(2\beta_H z_{H1} - \alpha_H) > 0$, we obtain (A53) where $j = L$ is replaced with $j = H$. We have shown that (A53) always holds when $j = L$; likewise, it holds when $j = H$.