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# A Dynamic Model of Sectoral Agglomeration Effects \*

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#### Abstract

This note derives a theoretical model that justifies the dynamic specification used in empirical works investigating the impact of agglomeration effects on regional industry-specific labour productivity. It extends the seminal multiregional framework of Ciccone (2002) to allow for sectoral disaggregation and a temporal dimension. As a result, present productivity becomes a function of past productivity and other contemporaneous and lagged control variables.

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<sup>1</sup> 

## 1 Introduction

This short paper provides a theoretical basis for the empirical estimation of the impact of agglomeration effects on regional labour productivity in Brülhart and Mathys (2006). The framework is directly inspired by two trends in the recent literature on agglomeration effects.

First, the seminal papers by Ciccone (2002) and Ciccone and Hall (1996) analyse theoretically and empirically the effect of agglomeration on aggregate regional labour productivity. We build on their work and add two extensions: our unit of observation is defined as a particular *sector* in a given region, and the model is couched in a *dynamic* setting. This enables us to derive theoretically a dynamic sector-specific equation that can directly be taken to the data.

Second, explicit consideration of the time dimension is a response to the recent concern in this literature to control for unobserved and persistent variables that may be driving agglomeration as well as productivity, by using dynamic panel estimation techniques (see Combes et al., (2004), for an example relating to employment dynamics). The critical empirical issue in our context is that causality between density and productivity could run in both directions and hence we have to control for endogeneity. This is done by exploiting the relevant moment conditions coming from the dynamic panel representation.

In sum, this work proposes a theoretical foundation for an estimable dynamic equation of the productivity - agglomeration link controlling for endogeneity. Section 2 develops the model, and section 3 concludes.

## 2 Model Setup

Consider a world composed of small open regions, where firms locate and produce goods and services demanded by the representative consumer. There is perfect competition in all sectors, perfect factor mobility and no barriers to trade, so that product prices, wages and capital rental rates are equal across regions. Firms maximise profits, and the representative consumer decides intertemporally in which region and how much to consume, to invest and to work. The corresponding maximisation problems are analysed in detail below.

#### 2.1 Firms

Firms belong to a given industry, are located in a given region and maximise profits:<sup>1</sup>

$$\begin{aligned} Max \quad \Pi_{fdst} &= p_{st}Y_{fdst} - T_{fdst} \\ &= p_{st}\Omega_{dst} \left[ \left( N_{fdst}H_{fdst} \right)^{\nu} \left( K_{fdst} \right)^{1-\nu} \right]^{\alpha} \\ &\quad \left( \frac{Y_{dst}}{A_d} \right)^{\left( \frac{\lambda_s - 1}{\lambda_s} \right)} \left( \frac{\sum_{-s}Y_{d-st}}{A_d} \right)^{\left( \frac{\lambda_{-s} - 1}{\lambda_{-s}} \right)} N_{dt}^{\gamma} \end{aligned} \tag{1}$$
$$-w_t N_{fdst} - r_t K_{fdst}, \end{aligned}$$

with  $0 \leq \nu \leq 1, 0 < \alpha \leq 1$ ,

<sup>&</sup>lt;sup>1</sup>The production function differs slightly from that used in Ciccone (2002). We do not consider the role of land as an explicit input, because we are interested in labour productivity at the sectorregion level without concern for how rents are distributed across factors. Land therefore features only insofar as it determines proximity spillovers.

where  $\Pi$  is the profit of the firm, p is the price of the consumption good, Y stands for output, T is total cost,  $\Omega$  is TFP, N is employment, H stands for human capital, K for physical capital, A is land area in square meters, wis the wage rate, and r is the interest rate. Subscript d stands for region, s for sector, -s for all sectors except s, t for the time period and f for the firm.

The production function can display constant returns ( $\alpha = 1$ ) or decreasing returns ( $\alpha < 1$ ) to factor inputs. It allows for three kinds of externalities. First, what we shall refer to as the *localisation effect* is captured by the regional production density of the own sector, with the corresponding elasticity given by  $\frac{\lambda_s-1}{\lambda_s}$ .<sup>2</sup> A postive localisation externality implies that  $\lambda_s > 1$ . Second, what we term *urbanisation effect* is represented by the regional density of the sum of other sectors' production, with an elasticity of  $\frac{\lambda_{-s}-1}{\lambda_{-s}}$ . Again, for this externality to be positive, we must have that  $\lambda_{-s} > 1$ . Third, we call *scale effect* the impact of a region's total employment, the relevant elasticity being  $\gamma$ . A positive scale effect, requires that  $\gamma > 0$ .<sup>3</sup>

Assuming that the individual firm is too small to influence regional sectorlevel production or regional employment, the first-order conditions with respect to capital and labour are given by:

$$\frac{\partial \Pi_{fdst}}{\partial K_{fdst}} \Rightarrow \frac{(1-\nu)\alpha p_{st} Y_{fdst}}{K_{fdst}} = r_t, \tag{2}$$

<sup>&</sup>lt;sup>2</sup>The elasticity is denoted in this way in order to allow for simpler expressions for labour productivity.

<sup>&</sup>lt;sup>3</sup>Hence, we decompose and extend the sources of externalities compared to Ciccone (2002), who, by aggregating across sectors, focused on a combination of the localisation effect and the urbanisation effect.

<sup>4</sup> 

$$\frac{\partial \Pi_{_{fdst}}}{\partial N_{_{fdst}}} \Rightarrow \frac{\nu \alpha p_{st} Y_{_{fdst}}}{N_{_{fdst}}} = w_t.$$

These conditions would remain unchanged if expressed for aggregate output and inputs over all firms in a given sector and region (i.e., the f subscripts may be dropped).

#### 2.2 Workers and capital

Agents choose freely where and in which sector to work, to invest and to consume. We assume that consumers have a taste for product diversity and asymmetric preferences across sectors but are neutral with respect to goods' region of origin. Specifically, agents maximise expected utility of the following form:

$$G = E \sum_{t=0}^{\infty} \beta^t \left( \sum_d \sum_s \sigma_s \log(C_{dst}) \right),$$

where  $\beta$  is a discount factor and  $\sigma_s$  represents tastes for goods from different sectors.

Given that the mass of workers equals 1 ( $\sum_d \sum_s N_{dst} = 1$ ), the aggregate budget constraint is:

$$w_t + r_t \sum_d \sum_s K_{dst} = \sum_d \sum_s p_{st} C_{dst} + \sum_d \sum_s I_{dst}.$$

We furthermore assume the following capital accumulation function:

$$K_{dst+1} = BK_{dst}^{1-\delta} I_{dst}^{\delta}, \tag{3}$$

where  $\delta$  represents the depreciation rate (0 <  $\delta \leq 1$ ), and B is a scale parameter (1 < B <  $\infty$ ).<sup>4</sup>

#### 2.3 Solution

The Lagrangian can then be written as:

$$\begin{aligned} \mathcal{L} &= E \sum_{t=0}^{\infty} \beta^t \left( \sum_d \sum_s \sigma_s \log(C_{dst}) \right) \\ &+ E \sum_{t=0}^{\infty} \beta^t \mu_t \left( w_t + r_t \sum_d \sum_s K_{dst} \right) \\ &- E \sum_{t=0}^{\infty} \beta^t \mu_t \left( \sum_d \sum_s p_{st} C_{dst} + \sum_d \sum_s B^{-\frac{1}{\delta}} K_{dst}^{\frac{\delta-1}{\delta}} K_{dst+1}^{\frac{1}{\delta}} \right), \end{aligned}$$

resulting in the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial C_{dst}} : \frac{\sigma_s}{p_{st}C_{dst}} = \mu_t, \tag{4}$$

$$\frac{\partial \pounds}{\partial K_{dst+1}} : \mu_t \frac{1}{\delta} \frac{I_{dst}}{K_{dst+1}} = \beta E \left( \mu_{t+1} \left[ r_{t+1} - \left( 1 - \frac{1}{\delta} \right) \frac{I_{dst+1}}{K_{dst+1}} \right] \right).$$
(5)

After some manipulations, we obtain our structural equation:<sup>5</sup>

<sup>4</sup>The law of motion for the capital stock (3) is common in macroeconomic theory as it allows for closed-form solutions. See e.g. Hercowitz and Sampson (1991), and Neusser (2001). 5Con A and 1 A for a second balance balance in the second state of the second st

<sup>&</sup>lt;sup>5</sup>See Appendix A for a complete derivation.

$$\ln (p_{st}Y_{dst}) - \ln (N_{dst}) = (1 - \delta [1 - \alpha\lambda_s(1 - \nu)]) \ln y_{dst-1} + (\alpha\lambda_s\nu - 1) [\ln N_{dst} - \ln A_d] + ((1 - \delta) + \alpha\lambda_s(\delta - \nu)) [\ln N_{dst-1} - \ln A_d] + \lambda_s \left(\frac{\lambda_{-s} - 1}{\lambda_{-s}}\right) \left[\ln \left(\sum_{-s} Y_{d-st}\right) - \ln A_d\right] + \lambda_s \left(\frac{\lambda_{-s} - 1}{\lambda_{-s}}\right) (\delta - 1) \left[\ln \left(\sum_{-s} Y_{d-st-1}\right) - \ln A_d\right] + \gamma\lambda_s \ln N_{dt} + (\gamma\lambda_s(\delta - 1)) \ln N_{dt-1} + \delta\lambda_s(\alpha - 1) \ln A_d + \alpha\lambda_s\nu \ln (H_{dst}) + \alpha\lambda_s\nu (\delta - 1) \ln (H_{dst-1}) + \ln \left(B\varpi_{ds2}^{\delta}\right)^{(1-\nu)\alpha\lambda_s} + \ln \left(p_{st}p_{st-1}^{\delta-1}\right) + \ln(\Omega_{dst}\Omega_{dst-1}^{\delta-1})^{\lambda_s},$$

where  $y_{dst-1} = \frac{p_{st-1}Y_{dst-1}}{N_{dst-1}}$  and  $\varpi_{ds2}^{\delta}$  represents the constant share of investment in the total value of output in sector s and region d.<sup>6</sup>

For ease of exposition and as a basis for estimation, we rewrite equation (6) in the following reduced form:

$$P_{dst} = \alpha_1 P_{ds,t-1} + \alpha_2 D_{dst} + \alpha_3 D_{ds,t-1} + \alpha_4 U_{dst} + \alpha_5 U_{ds,t-1} + \alpha_6 E_{dst} + \alpha_7 E_{ds,t-1} + \alpha_8 A_d + \alpha_9 H_{dst} + \alpha_{10} H_{ds,t-1} + \omega_{dst},$$
(7)

 $<sup>^6 \</sup>mathrm{See}$  Appendix B for the proof.

where (all variables in logs), P stands for productivity, D for employment density (localisation effect), U for "other" sectors' output density (urbanisation effect), E for regional employment (scale effect), A for regional area, Hfor regional human capital, and  $\omega$  captures remaining, possibly time variant, effects. The following constraints are imposed by the theoretical model:

$$\begin{aligned} \alpha_1 &= 1 - \delta \left[ 1 - \alpha \lambda_s (1 - \nu) \right], \\ \alpha_2 &= \alpha \lambda_s \nu - 1, \\ \alpha_3 &= (1 - \delta) + \alpha \lambda_s (\delta - \nu), \\ \alpha_4 &= \lambda_s \left( \frac{\lambda_{-s} - 1}{\lambda_{-s}} \right), \\ \alpha_5 &= \lambda_s \left( \frac{\lambda_{-s} - 1}{\lambda_{-s}} \right) (\delta - 1), \\ \alpha_6 &= \gamma \lambda_s, \\ \alpha_7 &= \gamma \lambda_s (\delta - 1), \\ \alpha_8 &= \delta \lambda_s (\alpha - 1), \\ \alpha_9 &= \alpha \lambda_s \nu, \\ \alpha_{10} &= \alpha \lambda_s \nu (\delta - 1), \\ \omega &= \ln \left( B \varpi_{ds2}^{\delta} \right)^{(1 - \nu) \alpha \lambda_s} + \ln \left( p_{st} p_{st-1}^{\delta - 1} \right) + \ln (\Omega_{dst} \Omega_{dst-1}^{\delta - 1})^{\lambda_s} \end{aligned}$$

The model thus implies that current productivity at the sector-region level is a function of lagged productivity, current and lagged human capital and current and lagged externalities, possibily arising from three different sources (localisation, urbanisation and scale). Productivity furthermore depends on the total area of the region and on random sector-region specific features.

This equation can be taken to the data. A dynamic panel GMM estimation of equation (7) is provided in Brülhart and Mathys (2006).<sup>7</sup> We find that

<sup>&</sup>lt;sup>7</sup>Data and econometric constraints made it impossible to include human capital and area ex-

the elasticity between aggregate density and labour productivity is around 13 percent.

## 3 Conclusion

We present an extension of the model by Ciccone (2002) to a dynamic and sector-level setting, providing a formal underpinning for the empirical specification used in Brülhart and Mathys (2006). More precisely, we outline a model where factors of production are perfectly mobile between sectors and regions and the representative consumer allocates production to consumption and investment in an intertemporal setting. This simple framework leads to an ADL (1,1) specification for labour productivity which can directly be taken to the data and which allows, from an econometric point of view, to account for endogeneity problems and adjustment dynamics.

plicitly.

# Appendix A: Model Derivation

Combining (4) and (5) yields:

$$\frac{1}{p_{st}C_{dst}} \frac{1}{\delta} \frac{I_{dst}}{K_{dst+1}} = \beta E \left\{ \frac{1}{p_{st+1}^* C_{dst+1}} \left[ r_{t+1} - \left(1 - \frac{1}{\delta}\right) \frac{I_{dst+1}}{K_{dst+1}} \right] \right\}.$$

Replacing the interest rate by the marginal product of capital and multiplying both sides by  $\delta$  allows us to write:

$$\frac{1}{p_{st}C_{dst}}\frac{I_{dst}}{K_{dst+1}} = \beta E \left\{ \frac{1}{p_{st+1}C_{dst+1}} \left[ \delta \frac{(1-\nu)\alpha p_{st}Y_{dst+1}}{K_{dst+1}} - (\delta-1) \frac{I_{dst+1}}{K_{dst+1}} \right] \right\}.$$

Multiplying by  $K_{dst+1}$  yields:

$$\frac{I_{dst}}{p_{st}C_{dst}} = \beta E \left\{ \frac{1}{p_{st+1}C_{dst+1}} \left[ \delta(1-\nu)\alpha p_{st+1}Y_{dst+1} - (\delta-1)I_{dst+1} \right] \right\}.$$
 (8)

The following accounting identity must hold for each region and sector:

$$p_{st}Y_{dst} \equiv p_{st}C_{dst} + I_{dst} + V_{dst},\tag{9}$$

where  $V_{dst}$  (positive or negative) is net investment abroad.<sup>8</sup>

Next, define the following shares, which remain constant over time:<sup>9</sup>

$$\frac{p_{st}C_{dst}}{p_{st}Y_{dst}} = \varpi_{ds1}, \ 0 \le \varpi_{ds1} \le 1,$$

$$\frac{I_{dst}}{p_{st}Y_{dst}} = \varpi_{ds2}, \ 0 \le \varpi_{ds2};$$

$$\frac{V_{dst}}{p_{st}Y_{dst}} = 1 - \varpi_{ds1} - \varpi_{ds2}$$

Using these definitions in (8) yields:

$$\frac{\overline{\omega}_{ds2}p_{st}Y_{dst}}{\overline{\omega}_{ds1}p_{st}Y_{dst}} = \beta E \left[ \frac{1}{\overline{\omega}_{dst1}p_{st+1}Y_{dst+1}} \delta(1-\nu)\alpha p_{st+1}Y_{dst+1} \right] -\beta E \left[ \frac{1}{\overline{\omega}_{dst1}p_{st+1}Y_{dst+1}} \left(\delta-1\right)\overline{\omega}_{ds2}p_{st+1}Y_{dst+1} \right].$$

Simplifying leads to:

$$\varpi_{ds2} = \frac{\beta \delta(1-\nu)\alpha}{1-(1-\delta)\beta}.$$

<sup>&</sup>lt;sup>8</sup>One could also write  $p_{st}Y_{dst} \equiv p_{st}C_{dst} + PI_{dst}$  where  $PI_{dst}$  is potential investment. Then,  $p_{st}Y_{dst} \equiv p_{st}C_{dst} + I_{dst} + (PI_{dst} - I_{dst})$ , with  $PI_{dst} - I_{dst} = V_{dst}$ , where  $I_{dst}$  is total investment in sector s and region d.

<sup>&</sup>lt;sup>9</sup>The proof is given in Appendix B.

Taking the regional version of the production function in (1) and solving for labour productivity allows us to write:

$$\frac{p_{st}Y_{dst}}{N_{dst}} = y_{dst} = p_{st}\Omega_{dst}^{\lambda_s} \left[ H_{dst}^{\nu} \left(\frac{K_{dst}}{N_{dst}}\right)^{1-\nu} \right]^{\alpha\lambda_s}$$

$$\left(\frac{A_d}{N_{dst}}\right)^{1-\alpha\lambda_s} \left(\frac{\sum_{-s}Y_{d-st}}{A_d}\right)^{\lambda_s \left(\frac{\lambda_{-s}-1}{\lambda_{-s}}\right)} A_d^{\lambda_s (\alpha-1)} N_{dt}^{\gamma\lambda_s},$$
(10)

and using the law of motion for capital in (3) results in:

$$y_{dst} = p_{st} \Omega_{ds}^{\lambda_s} \left[ H_{dst}^{\nu} N_{dst}^{\nu-1} \left( B K_{dst-1}^{1-\delta} I_{dst-1}^{\delta} \right)^{1-\nu} \right]^{\alpha \lambda_s} \\ \left( \frac{A_d}{N_{dst}} \right)^{1-\alpha \lambda_s} \left( \frac{\sum_{-s} Y_{d-st}}{A_d} \right)^{\lambda_s \left( \frac{\lambda_{-s}-1}{\lambda_{-s}} \right)} A_d^{\lambda_s (\alpha-1)} N_{dt}^{\gamma \lambda_s}.$$

Next, replacing investment and capital by:

$$I_{dst-1} = \varpi_{ds2} y_{dst-1} N_{dst-1}$$
, where  $y_{dst-1} = \frac{p_{st-1} Y_{dst-1}}{N_{dst-1}}$ ,

$$\begin{split} K_{dst-1}^{(1-\nu)\alpha\lambda_s} &= \frac{y_{dst-1}}{p_{st-1}\Omega_{dst-1}^{\lambda_s} \left[H_{dst-1}^{\nu}N_{dst-1}^{\nu-1}\right]^{\alpha\lambda_s} \left(\frac{A_d}{N_{dst-1}}\right)^{1-\alpha\lambda_s} \left(\frac{\sum_{-s}Y_{d-st}}{A_d}\right)^{\lambda_s\widetilde{\lambda}_{-s}} A_d^{\lambda_s(\alpha-1)}N_{dt-1}^{\gamma\lambda_s}},\\ \text{where } \widetilde{\lambda}_{-s} &= \frac{\lambda_{-s}-1}{\lambda_{-s}}, \end{split}$$

leads to:

$$y_{dst} = p_{st} p_{st-1}^{\delta-1} \left( \Omega_{dst} \Omega_{dst-1}^{(\delta-1)} \right)^{\lambda_s} \left( B \varpi_{ds2}^{\delta} \right)^{(1-\nu)\alpha\lambda_s} \left( H_{dst} H_{dst-1}^{\delta-1} \right)^{\nu\alpha\lambda_s} \left( \frac{N_{dst}}{A_d} \right)^{\alpha\lambda_s\nu-1} \\ \left( \frac{N_{dst-1}}{A_d} \right)^{(1-\delta)+\alpha\lambda_s(\delta-\nu)} N_{dt}^{\gamma\lambda_s} N_{dt-1}^{\gamma\lambda_s(\delta-1)} \left( \frac{\sum_{-s} Y_{d-st}}{A_d} \right)^{\lambda_s \left( \frac{\lambda_{-s}-1}{\lambda_{-s}} \right)} \\ \left( \frac{\sum_{-s} Y_{d-st-1}}{A_d} \right)^{\lambda_s \left( \frac{\lambda_{-s}-1}{\lambda_{-s}} \right)(\delta-1)} A_d^{\delta\lambda_s(\alpha-1)} y_{dst-1}^{1-\delta[1-\alpha\lambda_s(1-\nu)]}.$$

Taking logs finally yields equation (6).

# Appendix B: Proof

This proof shows that shares of investment, net investment abroad and consumption expenditure are constant.

Using equations (8) and (9), we obtain:

$$\frac{I_{dst}}{p_{st}C_{dst}} = E\left\{\frac{\beta(1-\nu)\alpha\delta}{p_{st+1}C_{dst+1}}\left(p_{st+1}C_{dst+1} + \left(1 - \frac{1-\frac{1}{\delta}}{(1-\nu)\alpha}\right)I_{dst+1} + V_{dst+1}\right)\right\}.$$

Simplifying yields:

$$\frac{I_{dst}}{p_{st}C_{dst}} = \beta\delta(1-\nu)\alpha + \beta\delta(1-\nu)\alpha E\left(\frac{V_{dst+1}}{p_{st+1}C_{dst+1}}\right) \qquad (11)$$

$$+\beta\left[\delta(1-\nu)\alpha + (1-\delta)\right] E\left(\frac{I_{dst+1}}{p_{st+1}C_{dst+1}}\right).$$

Solving equation (11) forward results in:

$$\frac{I_{dst}}{p_{st}C_{dst}} = \Theta \sum_{l=0}^{\infty} \Psi^l \left[ 1 + E\left(\frac{V_{dst+1+l}}{p_{st+1+l}C_{dst+1+l}}\right) \right] + \lim_{l \to \infty} \Psi^l E\left(\frac{I_{dst+l}}{p_{st+l}C_{dst+l}}\right),\tag{12}$$

where:

$$\Theta = \beta \delta (1 - \nu) \alpha,$$

$$\Psi = \beta \left[ \delta(1-\nu)\alpha + (1-\delta) \right] < 1.$$

The last term in (12) vanishes (since  $\Psi < 1$ ). Assuming that  $E\left(\frac{V_{dst+1+l}}{p_{st+1+l}C_{dst+1+l}}\right) = E(Z_{dst+1+l})$  is bounded above implies that  $\Psi^l E(Z_{dst+1+l})$  converges to zero and therefore the sum will converge to a constant (since  $\Theta \sum_{l=0}^{\infty} \Psi^l E(Z_{max}) = \Theta E(Z_{max}) \sum_{l=0}^{\infty} \Psi^l$ ), say  $Z_{ds}$ :

$$\frac{I_{dst}}{p_{st}C_{dst}} = \frac{\Theta}{1 - \Psi} + Z_{ds} \equiv \Lambda_{ds}.$$
(13)

Plugging this back into the Euler equation yields:

$$\Lambda = \beta E \left\{ \left[ \delta(1-\nu) \alpha \frac{p_{st+1} Y_{dst+1}}{p_{st+1} C_{dst+1}} - (\delta - 1) \Lambda_{ds} \right] \right\},\,$$

and simplifying allows us to write:

$$\frac{p_{st+1}C_{dst+1}}{p_{st+1}Y_{dst+1}} = \frac{\Theta}{\Lambda_{ds}\left[1 + \beta\left(\delta - 1\right)\right]} = \frac{p_s C_{ds}}{p_s Y_{ds}}.$$
(14)

Combining (13) and (14), we get:

$$\frac{I_{dst+1}}{p_{st+1}Y_{dst+1}} = \frac{\Theta}{1+\beta\left(\delta-1\right)} = \frac{I_{ds}}{p_s Y_{ds}},$$

and therefore also:

$$\frac{V_{dst+1}}{p_{st+1}Y_{dst+1}} = 1 - \frac{\Theta\left(1 + \Lambda_{ds}\right)}{\Lambda_{ds}\left[1 + \beta\left(\delta - 1\right)\right]} = \frac{V_{ds}}{p_s Y_{ds}}.$$

Note that if net investment abroad is added up over all regions and sectors, it should cancel out, which means that we have:

$$\sum_{d} \sum_{s} V_{ds} = 0 = \sum_{d} \sum_{s} \left[ 1 - \frac{\Theta \left( 1 + \Lambda_{ds} \right)}{\Lambda_{ds} \left[ 1 + \beta \left( \delta - 1 \right) \right]} \right] p_{s} Y_{ds}.$$

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