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Abstract This paper examines a set of necessary and sufficient conditions under which equilibrium involves mixing multiple types of workers in cities. Multi-skilled cities emerge if workers gain more from labor complementarities than they lose if they cannot consume their most preferred local amenities. A review of the different approaches to the presence of equilibrium in local public good economies is also provided.

JEL Classification D71 · H40 · R13

1 Introduction

It is well known in the public goods literature that a competitive equilibrium may not be efficient due to preference revelation problems. Tiebout (1956) argues that many public goods are local rather than pure and that people's benefits of sharing the cost of public goods will eventually be offset by congestion. As a consequence, under some hypotheses emphasizing free mobility, unlike a pure public goods economy, an efficient equilibrium may exist. Yet, the original Tiebout model is highly restrictive, as it stipulates that consumers live only on dividend income.

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This paper shows that whenever there is a jurisdiction-specific labor market, there is a Tiebout equilibrium where jurisdictions are formed by consumers with different labor skills, despite anonymous crowding in tastes. Berglas (1976) and McGuire (1991) address this issue, but they fail to provide consistent models where an equilibrium can be shown to exist. In particular, as Luque (2013) demonstrates, Berglas's (1976) naive integer assumption on population stratification forecloses many different types of heterogeneous populated jurisdictions. Pauly (1970) and Wooders (1978) observe that a differential techniques' approach is inappropriate when population and locations are considered discrete sets.

Luque (2013) shows equilibrium existence and core decentralization in an economy that allows for anonymous crowding, different labor skills among workers, and labor complementarities among worker types in the production of private goods. That research does not provide conditions that guarantee the formation of multi-skilled cities. We show that multi-skilled cities develop if workers' gains from labor complementarities compensate for their utility loss when they do not consume their most preferred local amenities.

For this result, we do not need to assume that workers' utilities depend on identities of other workers that live in their jurisdiction, i.e., "non-anonymous crowding" ; see Conley and Wooders (1997), Allouch et al. (2009), and Ellickson et al. (1999). Nor must we consider negative externalities within a firm, such as poor working conditions and uncongenial co-workers; see Ellickson et al. (2006). The scenario assumes anonymous crowding, so externalities within a community of these types do not enter into consumer preferences.

Our goal is to demonstrate that heterogeneous Tiebout communities exist and are optimal in equilibrium. This analysis is important in several respects. First, it refutes Bewley's (1981, pp. 725, 733) argument against extension of Tiebout's analysis to include heterogeneous communities. Second, we demonstrate that Tiebout's theory does not contravene the empirical evidence on heterogeneous communities in places that offer both public goods and employment. Glaeser and Saiz (2004) give empirical support to the idea that cities with skilled workers are growing because cities are becoming more economically productive, not because they are becoming more attractive places to live.

This paper is organized as follows. Section 2 explains the main elements of Luque (2013) model with its equilibrium existence and core decentralization results. It explains the differences between this approach to prove the theorems of equilibrium existence and core decentralization and previous approaches to equilibrium existence in the local public goods literature. The comparison is also useful to clarify the notion of a price-taking competitive equilibrium for this economy. We believe our review of the different approaches to equilibrium existence in club/local public good economies is novel and useful for pedagogical purposes. Section 3 builds on the model to identify necessary and sufficient conditions that lead to the formation of heterogeneous populated jurisdictions in equilibrium. Section 4 provides a simple example of this type of equilibrium.

2 The model

We consider a model with only one good traded in the international market. To consume local public goods, a consumer must belong to a jurisdiction, which means the consumer has to acquire a jurisdiction membership. This membership also allows the consumer to work in the jurisdiction industry and thus to obtain a wage.

The jurisdictions economy relies on a conception of how the world is articulated: jurisdictions are "small" in size. Negligible jurisdictions are modeled following Allouch et al. (2009) and Ellickson et al. (1999). Population is divided into city districts, municipalities, villages, or counties, and not into "big" countries. Big cities such as London or New York are seen as conglomerates of jurisdictions, such as Knightsbridge (London) or Soho (New York). Both are examples of jurisdictions with heterogeneous consumers. In our economy, there are a continuum of consumers who are sorted into *finitely populated* jurisdictions. Thus, there are a continuum of negligible (measure zero) jurisdictions (that is, jurisdictions have a finite number of consumers, but their size is negligible with respect to the economy as a whole).

Spatial considerations are not addressed in this model, although space could be incorporated by assigning a location to every possible jurisdiction following Konishi (2008). Then, we could define a city as a set of neighbor jurisdictions that share similar geographic characteristics. If the city has a positive measure of heterogeneous (different labor skills) populated jurisdictions, we say that it is a multi-skilled city. In such a spatial economy, any two jurisdictions that are close enough geographically may have some residents in one jurisdiction that benefit from the public goods of the other jurisdiction. Such a condition has been avoided in the previous Tiebout literature by assuming that every jurisdiction has the capacity to expel "illegal consumers" (exclusion). We require this assumption for our purposes in order to guarantee efficiency. If this assumption were not so, the pricing would be inefficient, as it would not capture the externalities imposed by consumers living in other jurisdiction commuting and consuming the public goods of another jurisdiction.

2.1 Workers and jurisdictions

There are a continuum of consumers in this economy (also called workers). Each consumer has an associated type, which is a complete description of her endowment of the good, $e(h_\theta)$, where $e(h_\theta) = e^\theta > 0$ for all h_θ, skills to carry out a certain type of work (whether professionals or otherwise), and preferences (described below). The two-dimensional space of consumer types is $\mathbf{\Theta}$.[1] Worker types are observable, and we refer to a worker of type $\theta \in \mathbf{\Theta}$ by h_θ.[2]

A jurisdiction provides its consumers a set of local amenities (or public projects) and a place of employment, and it also dictates the profile of workers, i.e., the number of workers of each type. Formally, we describe a *jurisdiction type* $\omega \equiv (g_\omega, y_\omega, \mathcal{C}_\omega)$ by

[1] Our result of equilibrium existence holds for any finite set of consumer types.

[2] We assume that if agents have different skill indexes, then agents must have different labor skills.

its local amenities g_ω,[3] a single-output production technology y_ω (described below), and its profile of workers $\mathcal{C}_\omega \equiv (n_\omega^\theta)_{\theta\in\Theta}$, where $n_\omega^\theta \in \mathbb{N}$ denotes the number of type θ workers in a jurisdiction type ω. $n_\omega = \sum_{\theta\in\mathbf{\Theta}} n_\omega^\theta$ denotes the level of congestion of jurisdiction ω. There are a finite number of local amenities and production technologies, so the set of possible jurisdiction types, denoted by $\mathbf{\Omega} = \{1, ..., \omega, ..., \Omega\}$, is also finite.

The single-output *production technology* y_ω maps labor inputs into output (in terms of the private good). Each worker in a jurisdiction ω supplies a unit of labor inelastically to the local industry, so the relevant vector of labor inputs is expressed by the jurisdiction's profile of workers $\mathcal{C}_\omega$. Each jurisdiction produces at its maximum capacity $y_\omega(\mathcal{C}_\omega)$. Any surplus is shared among the workers of the jurisdiction according to a rule s_ω that assigns a fraction $s_\omega^\theta \in (0, 1)$ to workers of type θ in jurisdiction ω, such that $\sum_\theta s_\omega^\theta = 1$. The wage of a type θ worker in jurisdiction ω is given by $\alpha_\omega^\theta = s_\omega^\theta y_\omega(\mathcal{C}_\omega)/n_\omega^\theta$. We say that the profile of workers $\mathcal{C}_\omega$ exhibits labor complementarities if production declines when we change the profile of workers by making at least one type of worker in ω equal to 0.[4]

As is customary in a theory of local public goods, a consumer must pay a membership price (a transfer and a poll fee) to get access to a jurisdiction and to consume its local amenities. We can embed these taxes into wages by letting $\tilde{\alpha}_\omega^\theta$ denote consumer θ's income associated with jurisdiction ω, i.e., the wage α_ω^θ net of the membership price paid to access jurisdiction ω. For our purposes, we look only at the price variable $\tilde{\alpha}_\omega^\theta$ to demonstrate how heterogeneous communities may form in equilibrium with a wage system $\tilde{\alpha}$. The consumer h_θ's budget constraint in jurisdiction ω is

$$x^{h_\theta} \leq \tilde{\alpha}_\omega^\theta + e^\theta \quad \mathrm{BC}_{h_\theta}(\omega)$$

The preferences of worker h_θ in jurisdiction ω are represented by a utility function $\tilde{u}^{h_\theta}(x, g_\omega, n_\omega)$ that increases with the level of private good consumption (x), declines with the level of congestion n_ω of the jurisdiction, and also depends on the local amenities g_ω provided by the jurisdiction.

We consider a price-taking equilibrium to be an efficient summary of the equilibrium corresponding to a competitive theory of jurisdictions formation. That is, an equilibrium consists of a vector of private good consumption, jurisdiction membership, and wages, so that consumers choose optimally the jurisdiction where they work and live, subject to their respective binding budget constraints; the surplus of each jurisdiction is exhausted (i.e, a jurisdiction's production of the private good minus the cost to provide the local public good must equal the salaries paid by the jurisdiction industry to its workers); the international private good market clears; and the aggregate jurisdiction membership vector is consistent for the whole economy, i.e., there

[3] Local amenities are interpreted here as public projects in the sense of Mas-Colell (1980), i.e., a public project consists of a discrete set of public goods, e.g., schools and parks.

[4] Formally, let a profile of consumers $\mathcal{C}_\omega = (n_\omega^\theta)_{\theta\in\mathbf{\Theta}}$ and consider the set of alternative profiles $\tilde{\mathbf{C}}_\omega$ where all $\tilde{\mathcal{C}}_\omega = (\tilde{n}_\omega^\theta)_{\theta\in\Theta} \in \tilde{\mathbf{C}}_\omega$ is such that $\tilde{n}_\omega^\theta \in \{n_\omega^\theta, 0\}$ for all θ, and $\tilde{n}_\omega^\theta = 0$ for at least one θ. Then, we say that the production technology y_ω exhibits labor complementarities if $y_\omega(\mathcal{C}_\omega) > y_\omega(\tilde{\mathcal{C}}_\omega)$, for all $\tilde{\mathcal{C}}_\omega \in \tilde{\mathbf{C}}_\omega$.

is a consistent matching of consumers in terms of the aggregate of choices [see also Kaneko and Wooders (1986) and Ellickson et al. (1999)].

2.2 Results

Luque (2013) shows there is an equilibrium outcome (existence) and that it passes the standard test of perfect competition (coincidence of the core with the set of equilibrium allocations). The notion of the core is the standard one in the literature of club theory. First, we define a *state* as a vector of private good consumption and jurisdiction membership. Then, we say that a *state* is *feasible* for a measurable set of consumers H if (1) the state satisfies the budget constraints for each consumer $h \in H$; (2) the aggregate membership vector is consistent for H; and (3) the private good international market clears for H. A state is in the core if there is no feasible state s for a subset of consumers H with positive measure, such that no consumer in H is worse off by consuming s, and there are consumers in H (a subset with positive measure) who are strictly better off by consuming s.[5] There are two main results in Luque (2013):

Theorem 1 (Existence) *There exists an equilibrium.*

Theorem 2 (Core equivalence) *The core coincides with the set of equilibria.*

The type of utility function considered in Luque's model assumes that *all consumers of the same type have the same utility function*, i.e., $\tilde{u}^{h_\theta}(x, g_\omega, n_\omega) = \tilde{u}^{\theta}(x, g_\omega, n_\omega)$, for all $h_\theta \in H(\theta)$. This assumption also appears in the club/local public goods literature, including Berglas (1976), and is essential to prove the core decentralization theorem. The intuition is that in this model, skills act as an efficient economic discriminatory device among consumers, i.e., consumer skills are linked in a one-to-one relation with the consumer's tastes for public goods.

This assumption might be justified in certain cases. For example, one could think that individuals who fill a certain type of job in an industry are prone to demand specific health services. Individuals with different working conditions (say, miners and engineers) will demand different public health services and thus will be taxed differently. If signaling (through skills or endowments) is not possible, then equilibrium for this economy would not be efficient. A condition of efficiency would make skills an endogenous choice as in Conley and Wooders (2001). We leave this possibility for future research. We instead note here that the existence of an equilibrium can be proved under the weaker assumption that *consumers of the same type are assigned the same endowments, skills, and tastes for public goods, but may have different preferences for private goods.* In this case, Schmeidler's (1973) result cannot be used in Luque (2013) to purify the mixed strategies equilibrium.

[5] The details of the formal definitions of a price-taking equilibrium and the core can be found in Luque (2013).

2.3 Review of different approaches to equilibrium existence in local public good economies

We now explain in detail Luque's proof of existence of equilibrium, its novelty with respect to other approaches to prove equilibrium existence in a local public goods economy, and how one could purify the equilibrium in an economy with the above weaker assumption on the consumer's utility function.

Luque (2013) investigates the problem of existence of a price-taking equilibrium by transforming it into a problem of existence of a social system equilibrium (Debreu 1952). The approach is by simultaneous optimization. As Arrow and Debreu (1954) assert, in a generalized game, we can test more clearly the consistency of the equations that describe the model. In a simultaneous optimization approach, each player maximizes a payoff function on a constraint set. Both the payoff function and the constraint set may be parameterized by the other players' actions. This second dependence does not occur in games. The extension is a mathematical object referred to as a generalized game by Debreu (1952). In the generalized game, there are auctioneers that mimic the market by choosing prices optimally in order to make the excess demand zero (if it is positive (negative) for some commodities, the auctioneer chooses higher (lower) prices for those goods).

A fixed point theorem: Both the non-excess demand approach and the simultaneous optimization approach are applications of Kakutani's (1941) fixed point theorem. Debreu (1982) clearly exposes the two parallel approaches in a rigorous way. The non-excess demand approach constructs an excess demand correspondence and finds a price vector that makes it equal to zero. To assure this, it uses the theorems of Gale (1955), Nikaido (1956), and Debreu (1956). The simultaneous optimization approach, on the other hand, is an application of Nash (1950) theorem to a generalized game, known as Debreu's (1952) theorem. Arrow and Debreu (1954) then apply Debreu's (1952) theorem to a social economic system, where agents simultaneously seek to maximize their respective payoff functions.[6]

The continuum extension: The three approaches (core, non-excess demand, and simultaneous optimization approach) were conceived initially for an exchange economy with a finite number of agents. Kaneko and Wooders (1986) extend the core approach to an atomless local public good economy. Since then, Wooders and coauthors have further enriched the model in many different ways using the f-core notion. Ellickson et al. (1999) apply the non-excess demand approach to an atomless club economy following the pioneering works of Aumann (1964, 1966). Luque (2013) not only provides the first equilibrium existence proof for a club/local public good economy using a generalized game, but also carries out this analysis in a con-

[6] Shafer and Sonnenschein (1975) provide a more recent version of the Debreu (1952) social existence theorem. They prove that a competitive equilibrium exists also for abstract economies with interdependent, price-dependent, and non-ordered preferences. Related work by Konishi (1996) considers an abstract political economy with a collective choice rule and strategic interactions among players. Konishi proves there exists an equilibrium by synthesizing an equilibrium existence theorem by Shafer and Sonnenschein (1975).

tinuum of agents framework.[7] Most of Luque's extensions follow by application of Hildebrand's (1974) results.

Purification: Luque's purification result also departs from earlier works on club/local public good economies. The discreteness of the set of public projects, in the sense of Ellickson (1979) and Mas-Colell (1980), requires a continuum of agents in order to convexify agents' strategy sets. However, purification of players' mixed strategies cannot be achieved using Schmeidler's (1973) result if agents of the same type have the same tastes with regard to public projects, but can differ in their preferences as to private goods. This implies that their demands for commodities may be different among consumers of the same type. Instead, Luque (2009) applies a particular result of Pàscoa (1998), used by Araujo and Páscoa (2002, Lemma 2) in an incomplete markets economy, which says that purification can be possible if in the extended generalized game, players' mixed strategies depend only on finitely many indicators, one for each type (a statistical indicator). The details of this exercise can be found in Luque (2009).

3 Multi-skilled cities

A single-skilled jurisdiction ω has only one type of worker (i.e., $n_\omega^\theta > 0$ for some θ, and $n_\omega^{\theta'} = 0, \forall \theta' \neq \theta$), but in a multi-skilled jurisdiction there are two types of workers, θ and θ', such that $n_\omega^\theta > 0$ and $n_\omega^{\theta'} > 0$. We now identify necessary and sufficient conditions on the primitives of our economy for the emergence of a multi-skilled city.

It is easy to see that, for our economy, there must be two *necessary conditions on the primitives* for the existence of a mixed community (multiple types), as identified by Berglas (1976):

(NC1) *Labor skills are different among workers.*

If all workers had identical labor skills but different preferences for public goods, then these workers could not increase their utility (through wages) by producing in an industry with a production technology based on labor complementarities (e.g., Leontief or Cobb-Douglas functions). Therefore, these workers will be better off in the standard homogeneous populated Tiebout communities.

(NC2) *There are jurisdiction types with a private good production technology that exhibit labor complementarities.*

In the absence of collaborative production, homogenous groups will coalesce around like individuals in an economy with anonymous crowding; see Scotchmer and Wooders (1987).

There are *sufficient conditions* that assure the emergence of multi-skilled jurisdictions with two types of workers, θ and θ', moving from single-skilled jurisdictions ω and ω' (for types θ and θ', respectively) to a multi-skilled jurisdiction $\bar{\omega}$.

[7] It is worth noting that proving the existence of equilibrium in a non-atomic economy using a generalized game has been useful in the incomplete markets literature (see, for example, Araujo et al. 2000 and Araujo and Páscoa 2002). The technique has never been applied to a local public goods non-atomic economy.

Proposition 1 *Assume that NC1 and NC2 hold. Then, workers of types θ and θ' are better off in a multi-skilled jurisdiction than in their respective single-skilled jurisdictions if there is a budget-feasible consumption increment $\varepsilon > 0$ in the multi-skilled jurisdiction that satisfies two conditions:*

(SC.1) *ε is such that $\tilde{u}^{h_{\tilde{\theta}}}(e^{\tilde{\theta}} + \alpha_{\tilde{\omega}}^{\tilde{\theta}} - \frac{inp(g_{\tilde{\omega}})}{n_{\tilde{\omega}}} + \varepsilon, g_{\tilde{\omega}}, n_{\tilde{\omega}}) > \tilde{u}^{h_{\tilde{\theta}}}(e^{\tilde{\theta}} + \alpha_{\tilde{\omega}}^{\tilde{\theta}} - \frac{inp(g_{\tilde{\omega}})}{n_{\tilde{\omega}}}, g_{\tilde{\omega}}, n_{\tilde{\omega}})$ for all $h_{\tilde{\theta}}$ in $\bar{\omega}$ and $(\tilde{\theta}, \tilde{\omega}) \in \{(\theta, \omega), (\theta', \omega')\}$.*

(SC.2) *$\Delta \equiv n_{\omega'}^{\theta'}(\alpha_{\tilde{\omega}}^{\theta'} - \alpha_{\omega'}^{\theta'}) + n_{\omega}^{\theta}(\alpha_{\tilde{\omega}}^{\theta} - \alpha_{\omega}^{\theta}) \geq (n_{\omega}^{\theta} + n_{\omega'}^{\theta'})(\varepsilon + \frac{inp(g_{\tilde{\omega}})}{n_{\tilde{\omega}}}) - inp(g_{\omega}) - inp(g_{\omega'})$.*

The conditions in Proposition 1 are intuitive and easy to check, given the parameters of the economy. Condition (SC.1) requires the consumption increment ε to be large enough to compensate the two types of workers for not consuming their most preferred local amenities (g_{ω} and $g_{\omega'}$) in the multi-skilled jurisdiction $\bar{\omega}$.[8] Condition (SC.2) says that added wages (Δ) must be enough to cover the consumption increment $(n_{\omega}^{\theta} + n_{\omega'}^{\theta'})\varepsilon$ and the net cost of changing the local amenities $\left((n_{\omega}^{\theta} + n_{\omega'}^{\theta'})\frac{inp(g_{\tilde{\omega}})}{n_{\tilde{\omega}}} - inp(g_{\omega}) - inp(g_{\omega'})\right)$. Conditions (SC.1) and (SC.2) depend only on the primitives of our economy.

Proposition 1 above and Theorems 1 and 2 in Luque (2013) then imply that:

Proposition 2 *In a local public goods economy with anonymous crowding, multi-skilled cities are optimal and exist in equilibrium if conditions (NC1), (NC2), (SC1), and (SC2) hold.*

Proposition 1 asserts that, under certain necessary and sufficient conditions, consumers experience gains from labor complementarities (through higher wages) that are enough to finance more consumption that compensates them for the lack of their most preferred local public goods in their jurisdictions. Proposition 2 then describes how multi-skilled cities emerge in equilibrium. More concretely, under assumptions (NC1), (NC2), (SC1), and (SC2), we can prove that the core involves mixing types of consumers in jurisdictions. We then apply the core decentralization Theorem 2 of Luque (2013) and claim that a decentralized price-taking equilibrium will also be characterized by heterogeneous populated jurisdictions. Finally, we apply the equilibrium existence Theorem 1 in Luque (2013), which asserts that a price-taking equilibrium exists, and conclude, given Theorem 2 above, that the core is non-empty.

Glaeser and Saiz (2004) provide an empirical justification for the formation of multi-skilled cities due to the labor complementarities. They show that skilled cities are growing because they are becoming more economically productive, not because these cities are becoming more attractive places to live.

4 Example

An example illustrates how labor complementarities lead to the formation of a jurisdiction with heterogeneous worker types.

[8] Notice that the amount $e^{\theta} + \alpha_{\omega}^{\theta} - \frac{inp(g_{\omega})}{n_{\omega}}$ is the consumption that saturates the type θ consumer's budget constraint at the single-skilled jurisdiction ω (similarly for consumer type θ' in jurisdiction ω').

Let there be two types of consumers, $\theta = 1, 2$, with the same measure for each type, i.e., $\lambda^1 = \lambda^2 = \lambda$. Commodity endowments are $e^1 = 3$ and $e^2 = 2$, respectively. Let us consider three types of jurisdiction, $\omega_1 = (g_1, y_S, (2, 0))$, $\omega_2 = (g_2, y_S, (0, 2))$, and $\omega_3 = (g_3, y_C, (1, 1))$. For tractability, we assume $n_\omega = 2$, for all $\omega = \omega_1, \omega_2, \omega_3$, so congestion is not a decisive factor for discriminating among jurisdiction types. The production technology y_C is collaborative and of the following form: $y_C(n^1, n^2) = 12\sqrt{n^1 n^2}$. The labor inputs of the production technology y_S are perfect substitutes, with functional form $y_S(n^1, n^2) = 3n^1 + n^2$. Production surplus is shared equally among consumers, so at the production frontier wages are $\alpha^1_{\omega_1} = 3, \alpha^2_{\omega_2} = 1$, and $\alpha^1_{\omega_3} = \alpha^2_{\omega_3} = 6$. The three public goods cost the same, $inp(g_\omega) = 2, \omega = \omega_1, \omega_2, \omega_3$, although all differ in terms of their characteristics (say, in their location). Utilities are defined as follows: $\tilde{u}^1(x, g_1, 2) = \frac{1}{3}\sqrt{x}$; $\tilde{u}^1(x, g_3, 2) = \frac{1}{4}\sqrt{x}$; $\tilde{u}^2(x, g_2, 2) = \frac{1}{4}\sqrt{x}$; and $\tilde{u}^2(x, g_3, 2) = \frac{1}{5}\sqrt{x}$. It is easy to see that, for these parameters, there is an $\varepsilon \in (3.88, 4)$ that satisfies the sufficient conditions (SC.1) and (SC.2). In fact, $\Delta = 16$ and the cost of providing the new public project g_{ω_3} minus the cost of providing the old ones (g_{ω_1} and g_{ω_2}) is 0.

Prices are $\tau_\omega = 1$ for all $\omega = \omega_1, \omega_2, \omega_3, t^1_{\omega_1} = t^2_{\omega_2} = 0$ (since jurisdictions ω_1 and ω_2 are composed of homogeneous consumers) and $t^1_{\omega_3} = -t^2_{\omega_3} = -1.14$.[9] Indirect utilities are $U^1(\omega_1; 1, 0, 1, 3) = 0.74$, $U^1(\omega_3; 1, -1.14, 1, 6) = 0.75$, $U^2(\omega_2; 1, 0, 1, 1) = 0.35$, and $U^2(\omega_3; 1, 1.14, 1, 6) = 0.48$. Both types of consumers are better off in jurisdiction ω_3, even if both consumer types consume their less preferred public good (g_3). This is because wages are higher in ω_3 than in both ω_1 and ω_2, because of the labor complementarities in the production technology y_C.

Thus, we can conclude that heterogeneous populated jurisdictions of type ω_3 and profile $(n^1, n^2) = (1, 1)$ make both types of consumers better off than in the homogeneous populated jurisdictions ω_1 and ω_2. The equilibrium is characterized by heterogeneous communities of type ω_3 with profile $(n^1, n^2) = (1, 1)$. The consistent matching of consumers in terms of the aggregate of choices follows from equality $n^1/n^2 = \lambda^1/\lambda^2$.

5 Appendix

Proof of Proposition 1 Let us first write the wage $\tilde{\alpha}^\theta_\omega$ as a function of the gross α^θ_ω and the membership price paid to access jurisdiction ω : $\tilde{\alpha}^\theta_\omega = \alpha^\theta_\omega - (t^\theta_\omega + \tau_\omega)$. The poll fee, $\tau_\omega = inp(g_\omega)/n_\omega$, common to all workers in the jurisdiction, covers the cost $inp(g_\omega) \geq 0$ of providing the various local amenities. The transfers $t_\omega \in \mathbb{R}^\Theta$ can be positive, negative, or zero, as transfers internalize the externalities among the workers in the jurisdiction, given workers' tastes for the local amenities, wealth, and share of the jurisdiction's production surplus. All workers of type θ in jurisdiction ω pay the same transfer t^θ_ω. Transfers must be such that $\sum_{\theta \in \mathbf{\Theta}} t^\theta_\omega = 0$.

[9] For ω_3, we choose $t^1_{\omega_3}$ to maximize $\sum_{\theta=1,2} \tilde{u}^\theta(x, g_\omega, n_\omega)$ and make $t^2_\omega = -t^1_\omega$ (since $n^1 = n^2 = 1$) and the budget constraints hold with equality.

We can now rewrite worker h_θ's budget constraint in jurisdiction ω as follows:

$$x^{h_\theta} + t_\omega^\theta + \tau_\omega \leq \alpha_\omega^\theta + e^\theta \qquad \text{BC}_{h_\theta}(\omega)$$

Let $\bar{\omega} = (g_{\bar{\omega}}, y_{\bar{\omega}}, (n_{\bar{\omega}}^\theta, n_{\bar{\omega}}^{\theta'}))$ be a jurisdiction type with two types of workers, θ and θ', satisfying (NC1) and (NC2). Let us associate the consumption vector $\bar{x}$ with an economy with multi-skilled jurisdictions formed by worker types θ and θ'. We also associate the consumption vector x with an economy with only single-skilled cities. As in Luque (2013), consistency of population matching is guaranteed by choosing jurisdiction type measures $\gamma(\bar{\omega})$, $\gamma(\omega)$, and $\gamma(\omega')$ so that $\gamma(\bar{\omega}) = \gamma(\omega)n_\omega^\theta/n_{\bar{\omega}}^\theta = \gamma(\omega')n_{\omega'}^{\theta'}/n_{\bar{\omega}}^{\theta'}$.

Now, let $\bar{x}^{h_{\tilde{\theta}}} = x^{h_{\tilde{\theta}}} + \varepsilon$, with $\varepsilon > 0$, for every worker $h_{\tilde{\theta}}$ with type $\tilde{\theta} = \theta, \theta'$, where ε is such that, for any $(x, x') \in BC_{h_\theta}(\omega) \times BC_{h_{\theta'}}(\omega')$ and any $(\omega, \omega') = ((g_\omega, y_\omega, n_\omega^\theta), (g_{\omega'}, y_{\omega'}, n_{\omega'}^{\theta'}))$, we have that $\tilde{u}^{h_\theta}(x + \varepsilon, g_{\bar{\omega}}, n_{\bar{\omega}}) > \tilde{u}^{h_\theta}(x, g_\omega, n_\omega)$ and $\tilde{u}^{h_{\theta'}}(x' + \varepsilon, g_{\bar{\omega}}, n_{\bar{\omega}}) > \tilde{u}^{h_{\theta'}}(x', g_{\omega'}, n_{\omega'})$ for all $h_\theta, h_{\theta'}$. Then, to contradict that a state (x, μ) with only single-skilled jurisdictions does not belong to the core, we need to show that ε is budget feasible for both worker types, given the new wages.

Consumer $h_{\theta'}$'s budget constraints $\text{BC}_{h_{\theta'}}(\bar{\omega})$ and $\text{BC}_{h_{\theta'}}(\omega')$ are, respectively, $\bar{x}^{h_{\theta'}} + inp(g_{\bar{\omega}})/n_{\bar{\omega}} + t_{\bar{\omega}}^{\theta'} = e^{\theta'} + \alpha_{\bar{\omega}}^{\theta'}$ and $x^{h_{\theta'}} + inp(g_{\omega'})/n_{\omega'} + t_{\omega'}^{\theta'} = e^{\theta'} + \alpha_{\omega'}^{\theta'}$, where $\alpha_{\bar{\omega}}^{\theta'}$ is the wage that accommodates consumption increment ε in $\text{BC}_{h_{\theta'}}(\bar{\omega})$. Subtracting $\text{BC}_{h_{\theta'}}(\omega')$ from $\text{BC}_{h_{\theta'}}(\bar{\omega})$ and using $\bar{x}^{h_\theta} = x^{h_\theta} + \varepsilon$ and $t_{\omega'}^{\theta'} = 0$ (since all workers are homogeneous in ω'), we have $\alpha_{\bar{\omega}}^{\theta'} - \alpha_{\omega'}^{\theta'} = \varepsilon + inp(g_{\bar{\omega}})/n_{\bar{\omega}} + t_{\bar{\omega}}^{\theta'} - inp(g_{\omega'})/n_{\omega'}$. Similarly, for a type θ worker, we can write: $t_{\bar{\omega}}^\theta = \alpha_{\bar{\omega}}^\theta - \alpha_\omega^\theta - \varepsilon - inp(g_{\bar{\omega}})/n_{\bar{\omega}} + inp(g_\omega)/n_\omega$. Using $t_{\bar{\omega}}^{\theta'} = (n_{\bar{\omega}}^\theta/n_{\bar{\omega}}^{\theta'})t_{\bar{\omega}}^\theta$ and after a bit of algebra (noticing that $n_\omega^\theta = n_\omega$ and $n_{\omega'}^{\theta'} = n_{\omega'}$), we find condition (SC.2). We conclude that there is no state in the core made up only of single-skilled jurisdictions.

We now can use Theorem 2 in Luque (2013) to show that a core state is an equilibrium, and Theorem 1 to show that an equilibrium exists. □

References

Allouch N, Conley J, Wooders MH (2009) Anonymous price taking equilibrium in Tiebout economies with a continuum of agents: existence and characterization. J Math Econ 45:492–510

Araujo A, Orrillo J, Páscoa M (2000) Equilibrium with default and endogenous collateral. Math Financ 10:1–21

Araujo A, Páscoa M (2002) Bankruptcy in a model of unsecured claims. Econ Theory 20:455–481

Arrow K, Debreu G (1954) Existence of an equilibrium for a competitive economy. Econometrica 22: 265–289

Aumann RJ (1966) Existence of competitive equilibria in markets with a continuum of traders. Econometrica 34:1–17

Aumann RJ (1964) Markets with a continuum of traders. Econometrica 32:39–50

Berglas E (1976) Distribution of tastes and skills and the provision of local public goods. J Pub Econ 6:409–423

Bewley T (1981) A critique of tiebout's theory of local public expenditures. Econometrica 49:713–740

Conley J, Wooders MH (2001) Tiebout economies with differential genetic types and endogenous chosen crowding characteristics. J.Econ Theory 98:261–294

Conley J, Wooders MH (1997) Equivalence of the core and competitive equilibrium in a Tiebout economy with crowding types. J Urban Econ 41:421–440
Debreu G (1952) A social equilibrium existence theorem. In: Proceedings of the National Academy of Sciences
Debreu G (1956) Market equilibrium. In: Proceedings of the National Academy of Science of the USA
Debreu G (1982) Existence of competitive equilibrium. In: Arrow K, Intriligator M (eds) Handbook of mathematical economics, vol II, Chapter 15
Ellickson B (1979) Competitive equilibrium with local public goods. J Econ Theory 21:46–61
Ellickson B, Grodal B, Scotchmer S, Zame WR (2006) The organization of production, consumption and learning. In: Vind K (ed) The birgit grodal symposium. Springer, Berlin
Ellickson B, Grodal B, Scotchmer S, Zame WR (1999) Clubs and the market. Econometrica 67:1185–1217
Gale D (1955) The law of supply and demand. Math Scand 3:155–169
Glaeser E, Saiz A (2004) The rise of the skilled city. Brookings-Wharton Pap Urban Aff 5:47–94
Hildebrand W (1974) Core and equilibrium in a large economy. Princeton University Press, Princeton
Kakutani S (1941) A generalization of Brower's fixed point theorem. Duke Math J 18:457–459
Kaneko M, Wooders MH (1986) The core of a game with a continuum of players and finite coalitions: the model and some results. Math Soc Sci 12:105–137
Konishi H (2008) Tiebout's tale in spatial economies: entrepreneurship, self-selection and efficiency. Reg Sci Urban Econ 38:461–471
Luque J (2013) Heterogeneous Tiebout communities with private production and anonymous crowding. Reg Sci Urban Econ 43:117–123
Luque J (2009) Essays in equilibrium theory: repo markets and formation of jurisdictions with public goods. PhD dissertation, Universidade Nova de Lisboa, Portugal
Mas-Colell A (1980) Efficiency and decentralization in the pure theory of public goods. Q J Econ 94: 625–641
McGuire M (1991) Group composition, collective consumption and collaborative production. Am Econ Rev 81:1391–1407
Nash J (1950) Equilibrium points in N-person games. In: Proceedings of the National Academy of Science of the USA, vol 36, pp 48-49
Nikaido H (1956) On the classical multilateral exchange problem. Metronomica 8:135–145
Páscoa M (1998) Nash equilibrium and the law of large numbers. Int J Game Theory 27:83–92
Pauly MV (1970) Cores and clubs. Pub Choice 14:53–65
Schmeidler D (1973) Equilibrium points of non-atomic games. J Stat Phys 7:295–301
Scotchmer S, Wooders MH (1987) Competitive equilibria and the core in economies with anonymous crowding. J Pub Econ 34:159–174
Shafer W, Sonnenschein H (1975) Equilibrium in abstract economies without ordered preferences. J Math Econ 2:345–348
Tiebout C (1956) A pure theory of local public expenditures. J Polit Econ 64:416–424
Wooders MH (1978) Equilibria, the core, and jurisdiction structures in economies with a local public good. J Econ Theory 18:328–348