Spring April 17, 2022

Layered Circuits in the Economy

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4/18/22

Layered Circuits in the Economy

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Introduction

Classical economics recognizes only two relations among firms, independence or collusion. Collusion, the absence of competition, enables the colluding firms to share a monopoly profit with the colluding firms. These alternatives distort the view of an actual economy that has many types of relations among commercial enterprises. A layered structure offers a more accurate picture of an economy. Structures often involve short term, renewable arrangements. Their variety promotes social welfare and should not to be mistaken for monopoly profit. Layered structures have more than one arrow of a given type. Recall that a Hamiltonian circuit with only one arrow of each type among its members limits its variety.

Structure have a purpose. Scholars learn about them by studying them. They do not invent structures. That is the job of innovators, bold, timid, or lucky While trial and error is a common tactic, it does not explain great success. Those with deeper insight into the implications of the computer age obtain the bigger rewards. Three examples stand out above the crowd, search engines, the cell phone, and logistics for online retailing. The computer age had its greatest effect on the economy by lowering the costs of communication.

A circuit is represented algebraically by the product of its Nöther matrics. The value of a circuit is the product of the values of its Nöther matrics. Each matric represents an arrow that belongs to a circuit. An arrow may belong to many circuits. This inspires layers made of circuits that share selected arrows. A structure is not confined to one copy of an arrow. Arrows also appear in organization diagrams to describe their arrangements.

A set of circuits that share arrows is a layered structure. The Mathematica procedure Join shows the layers in the collection.

Cooperation describes relations among members of a circuit that is also a coalition. Cooperation is present but weaker among circuits in a layered set. For example, vertical integration short of merger is a layered structure.

Open Problem; Core status of layered structure. The payoff to a shared arrow in a layered structure depends on its layered structure membership, as well as on its member circuits. Survival of a layered structure depends on the loyalty of its members at its various levels. Competition among structures has more complicated effects on payoffs to its members than competition among unlayered structures.

Note: In Mathematica an edge links two vertexes. If the edge is an arrows, then it is a directed edge. If the edge shows no arrow, then it is an undirected edge. An undirected edge between two arrows belonging to two different circuits gets oriented depending which circuit uses it. Join and union are two procedures in Mathematica for mergers. Union removes duplicates, join retains duplicates. Join can show two oppositely oriented arrows between two vertexes in two different circuits. This is a shared edge and a layer.

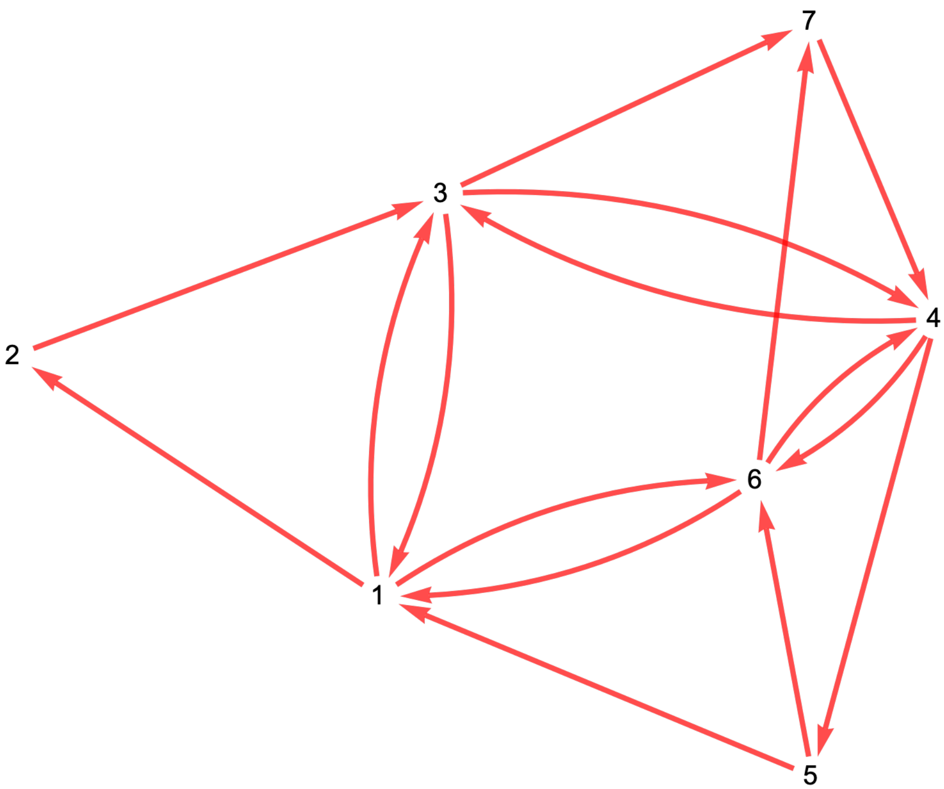
Examples of Layered Structures

The following adjacency matrix of $\lambda 7$ describes a shared structure.

```
In[•]:= λ7 := {{0, 1, 1, 0, 0, 1, 0}, {0, 0, 1, 0, 0, 0, 0},
   {1, 0, 0, 1, 0, 0, 1}, {0, 0, 1, 0, 1, 1, 0},
   {1, 0, 0, 0, 0, 1, 0}, {1, 0, 0, 1, 0, 0, 1},
   {0, 0, 0, 1, 0, 0, 0}}
```

In[•]:= makeGrph[λ7, 3]

Out[•]=



{1 ⟷ 3, 3 ⟷ 4, 4 ⟷ 6, 6 ⟷ 1} is the inner 4-circuit of the structure λ7. The graph shows it shares edges with five different 3-circuits.

In[•]:= MatrixForm[findAllCycles[λ7]]

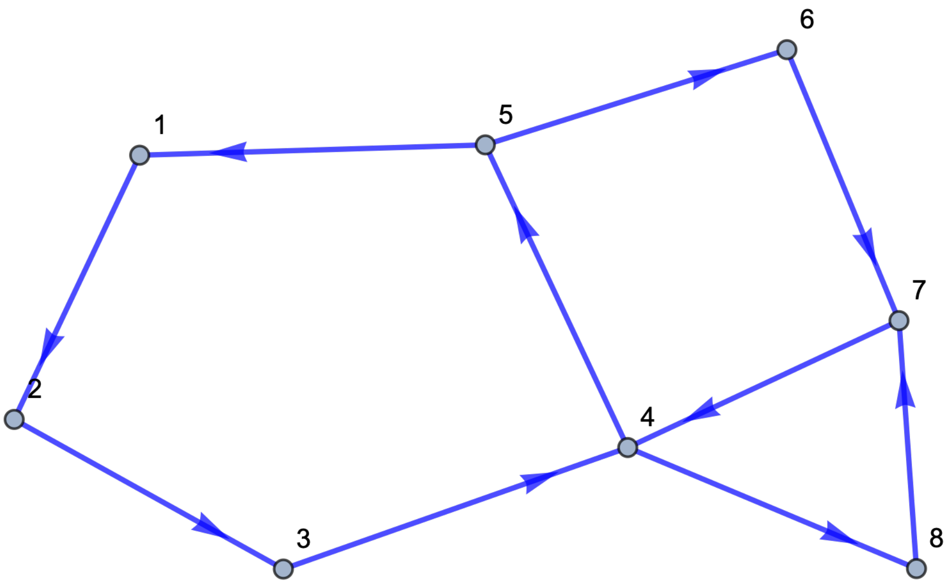
Out[•]//MatrixForm=

$$\begin{pmatrix} \{4\to 6,\ 6\to 4\} \\ \{3\to 4,\ 4\to 3\} \\ \{1\to 6,\ 6\to 1\} \\ \{1\to 3,\ 3\to 1\} \\ \{4\to 6,\ 6\to 7,\ 7\to 4\} \\ \{4\to 5,\ 5\to 6,\ 6\to 4\} \\ \{3\to 7,\ 7\to 4,\ 4\to 3\} \\ \{1\to 2,\ 2\to 3,\ 3\to 1\} \\ \{4\to 5,\ 5\to 6,\ 6\to 7,\ 7\to 4\} \\ \{1\to 6,\ 6\to 4,\ 4\to 5,\ 5\to 1\} \\ \{1\to 6,\ 6\to 4,\ 4\to 3,\ 3\to 1\} \\ \{1\to 3,\ 3\to 4,\ 4\to 6,\ 6\to 1\} \\ \{1\to 3,\ 3\to 4,\ 4\to 5,\ 5\to 1\} \\ \{1\to 6,\ 6\to 7,\ 7\to 4,\ 4\to 5,\ 5\to 1\} \\ \{1\to 6,\ 6\to 7,\ 7\to 4,\ 4\to 3,\ 3\to 1\} \\ \{1\to 3,\ 3\to 7,\ 7\to 4,\ 4\to 6,\ 6\to 1\} \\ \{1\to 3,\ 3\to 7,\ 7\to 4,\ 4\to 5,\ 5\to 1\} \\ \{1\to 3,\ 3\to 4,\ 4\to 5,\ 5\to 6,\ 6\to 1\} \\ \{1\to 2,\ 2\to 3,\ 3\to 4,\ 4\to 6,\ 6\to 1\} \\ \{1\to 2,\ 2\to 3,\ 3\to 4,\ 4\to 5,\ 5\to 1\} \\ \{1\to 3,\ 3\to 7,\ 7\to 4,\ 4\to 5,\ 5\to 6,\ 6\to 1\} \\ \{1\to 2,\ 2\to 3,\ 3\to 7,\ 7\to 4,\ 4\to 6,\ 6\to 1\} \\ \{1\to 2,\ 2\to 3,\ 3\to 7,\ 7\to 4,\ 4\to 5,\ 5\to 1\} \\ \{1\to 2,\ 2\to 3,\ 3\to 4,\ 4\to 5,\ 5\to 6,\ 6\to 1\} \\ \{1\to 2,\ 2\to 3,\ 3\to 7,\ 7\to 4,\ 4\to 5,\ 5\to 6,\ 6\to 1\} \end{pmatrix}$$

The fig[λ] shows two layers. The first layer is a square, {4→5, 5→6, 6→7, 7→4}. Arrow 4→5 in the pentagon links the same two vertex in the square. The arrow 7 → 4 in the triangle {7 → 4, 4 → 8, 8 → 7} links the same two vertexes in the square. The graph does not show which arrows if any are shared with the square. Either Union or Join must be used in the graph to show the shared arrows.

In[∘]:= `fig[λ, 2]`

Out[∘]=



Graph of λ

One version of the Nöther matric of λ is the product of the matrics for each arrow depending on its position in λ. In this version each arrow appears once. Another version is the Nöther matric of the layered circuits given by its three circuits. Now an arrow can appear more than once depending on its layer membership. The graph of λ joins three circuits: **Λ⟦1⟧**, pentagon; **Λ⟦2⟧**, square; and **Λ⟦3⟧** triangle. The arrow 4→5 belongs to both the pentagon and the square. The square contributes to the pentagon. The arrow 7→4 belongs to the square and the triangle. Hence the triangle contributes to the square, the square contributes to the pentagon and, indeed at one remove, the triangle contributes to the pentagon. The pentagon is the final member composed of contributions by other circuits.These graphs illustrate a simplified structure of the complicated elements in the actual economy. This description is familiar in game theory. Chess has three outcomes, win, lose, or draw but the moves causing these outcomes in the extensive chess game are complicated. The real economy described by circuits has complicated relations among circuits resembling the extensive level of a game.

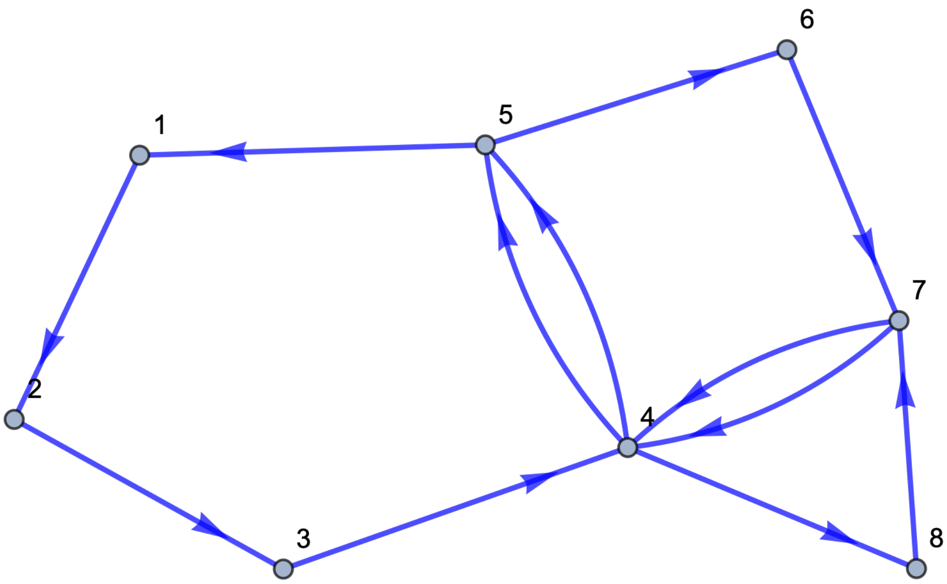
A more elaborate layered structure is next, Λ. It uses three circuits from λ.

In[∘]:=
```
Λ := {{1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 1},
   {5 → 6, 6 → 7, 7 → 4, 4 → 5}, {4 → 8, 8 → 7, 7 → 4}}
```

Join shows two shared arrows forming layers.

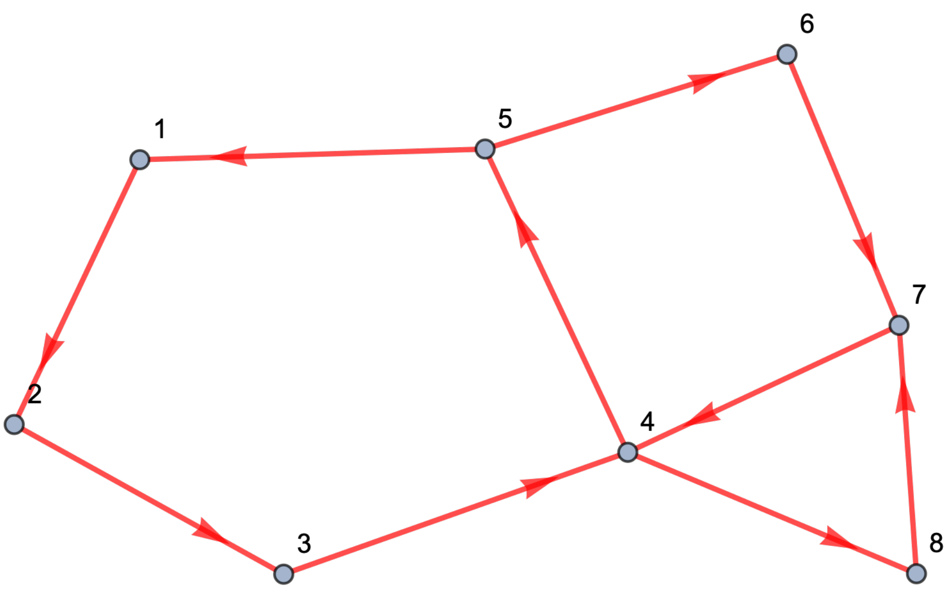
In[∘]:=
```
fig[Join[Λ⟦1⟧, Λ⟦2⟧, Λ⟦3⟧], 2]
```

Out[∘]=



In[∘]:=
```
fig[Λ⟦1⟧ ⋃ Λ⟦2⟧ ⋃ Λ⟦3⟧, 3]
```

Out[∘]=



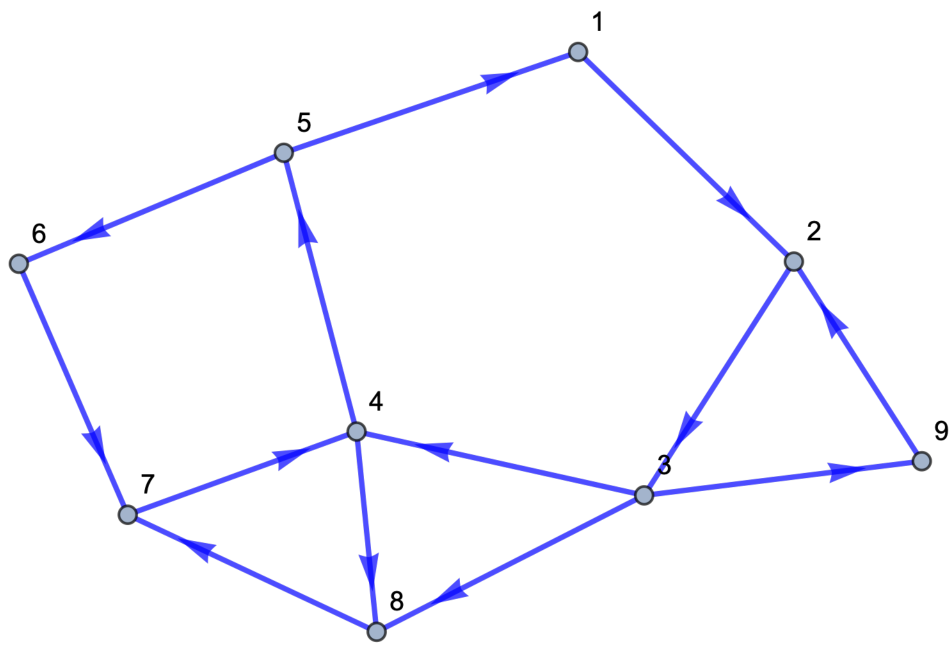
Join shows layers better than Union because Union removes duplicate arrows while Join shows all the arrows.

The next set of 13 arrows, μ, can form different structures. One is Ψ.

In[∘]:=
```
μ := {1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 1, 5 → 6, 6 → 7,
   7 → 4, 4 → 8, 8 → 7, 3 → 8, 9 → 2, 3 → 9}
```

In[∘]:=
```
fig[μ, 2]
```

Out[∘]=



Graph of Ψ = Graph of μ

Graph Ψ is a layered structure of μ. The arrows 3→4 and 4→5 are two sides of the pentagon that could make another circuit different from the two now using these arrows.

In[∘]:=
```
Ψ := {{1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 1},
   {5 → 6, 6 → 7, 7 → 4, 4 → 5}, {4 → 8, 8 → 7, 7 → 4},
   {3 → 4, 4 → 8, 8 → 3}, {4 → 5, 5 → 6, 6 → 7, 7 → 4}}
```

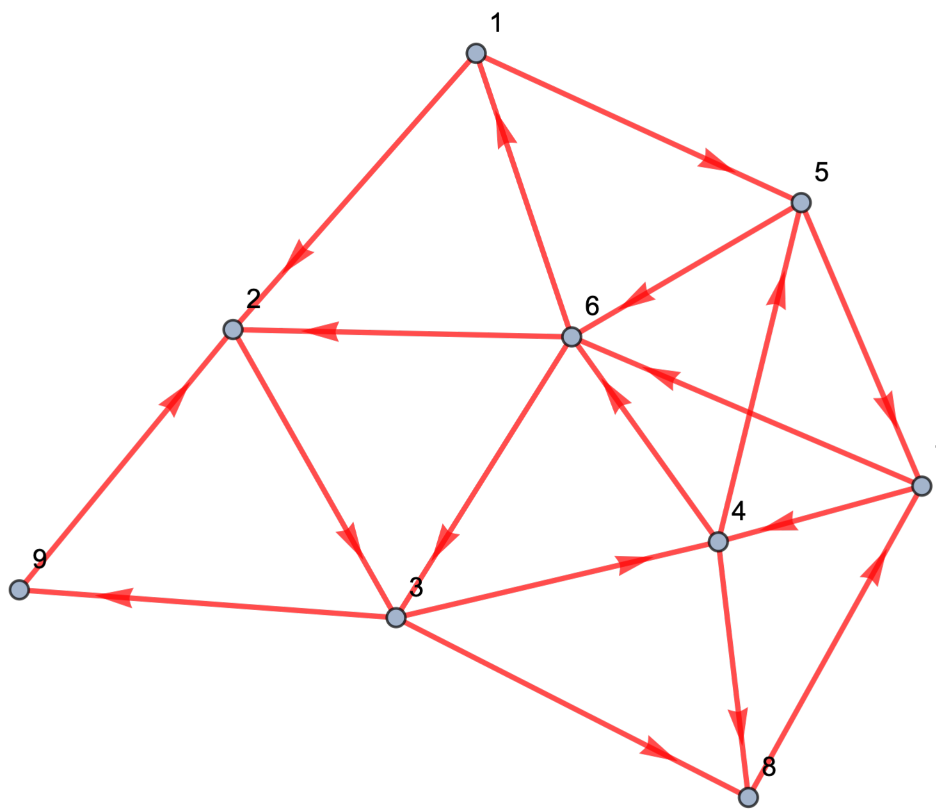
The pentagon in Ψ joins two more circuits than the pentagon in Λ. These two circuits share edges 3→4 and 4→5 with the pentagon.

In[∘]:=
```
ρ := {1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 7, 6 → 1, 1 → 5,
   6 → 2, 6 → 3, 4 → 6, 5 → 6, 7 → 6, 7 → 4, 4 → 8,
   8 → 7, 3 → 8, 9 → 2, 3 → 9}
```

In[∘]:=
```
Θ := {{1 → 2, 2 → 3, 3 → 4, 4 → 5, 5 → 6, 6 → 1},
   {6 → 2, 2 → 3, 3 → 9, 9 → 2},
   {1 → 5, 5 → 7, 7 → 4, 4 → 6, 6 → 1},
   {6 → 3, 3 → 8, 8 → 7, 7 → 6}, {4 → 8, 8 → 7, 7 → 4}}
```

In[∘]:=
```
fig[Θ⟦1⟧ ⋃ Θ⟦2⟧ ⋃ Θ⟦3⟧ ⋃ Θ⟦4⟧ ⋃ Θ⟦5⟧, 3]
```

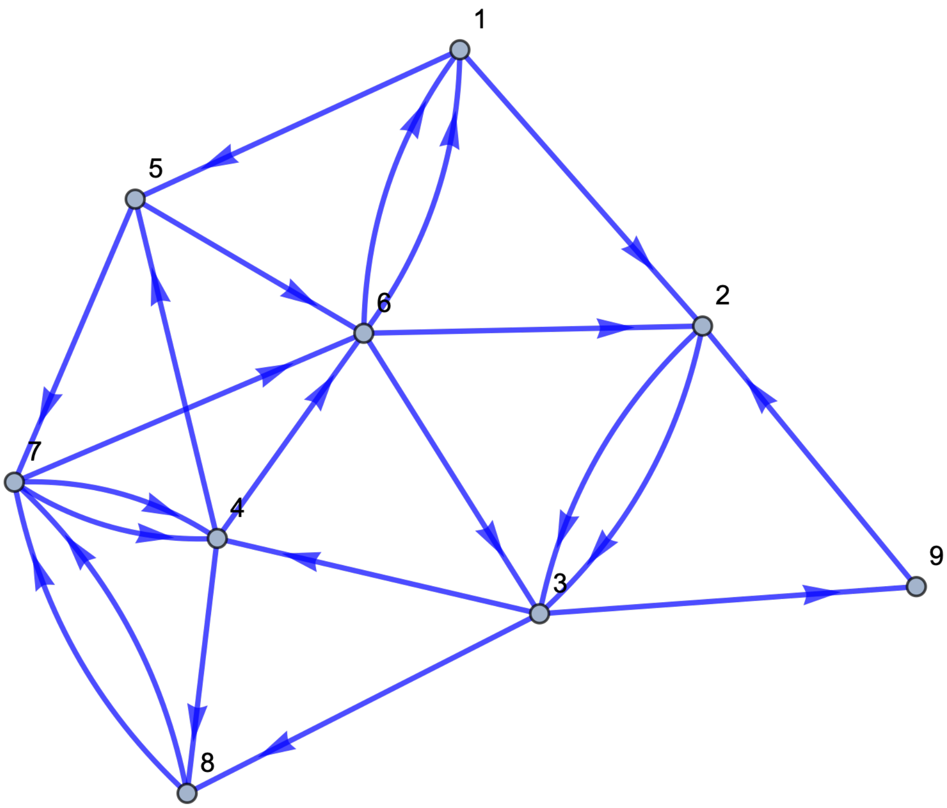
Out[∘]=



Graph of Θ uses Union that removes duplicates.

In[•]:=
```
fig[Join[Θ⟦1⟧, Θ⟦2⟧, Θ⟦3⟧, Θ⟦4⟧, Θ⟦5⟧], 2]
```

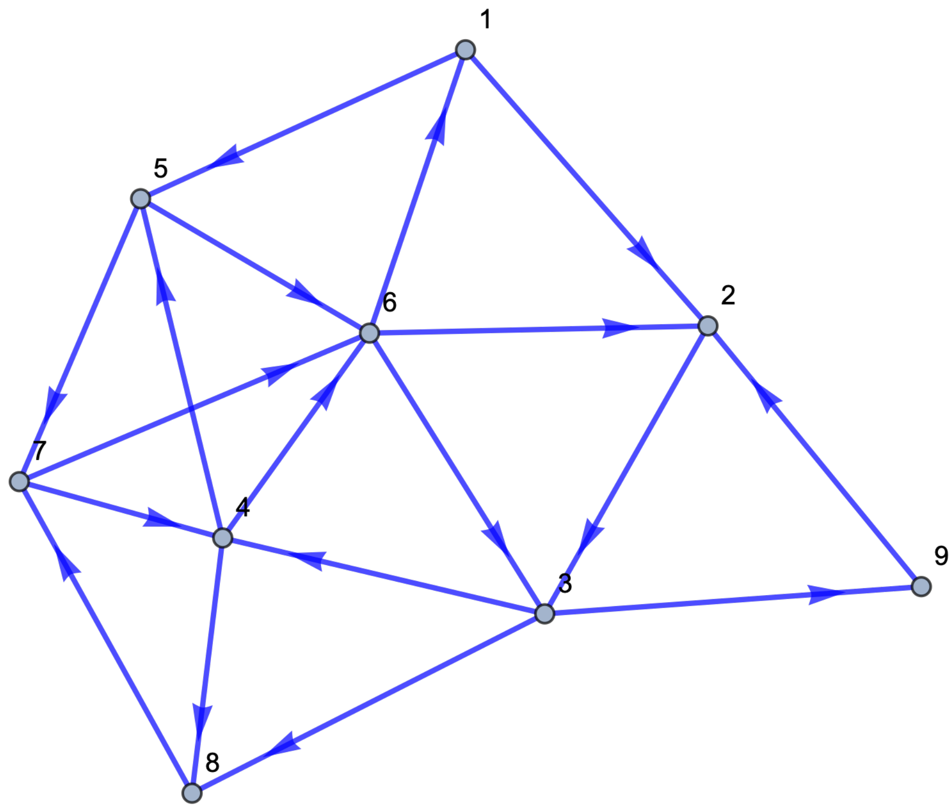
Out[•]=



Graph of Θ using Join that includes duplicates. Join clearly shows are shared edges of layers.

In[•]:= fig[ρ, 2]

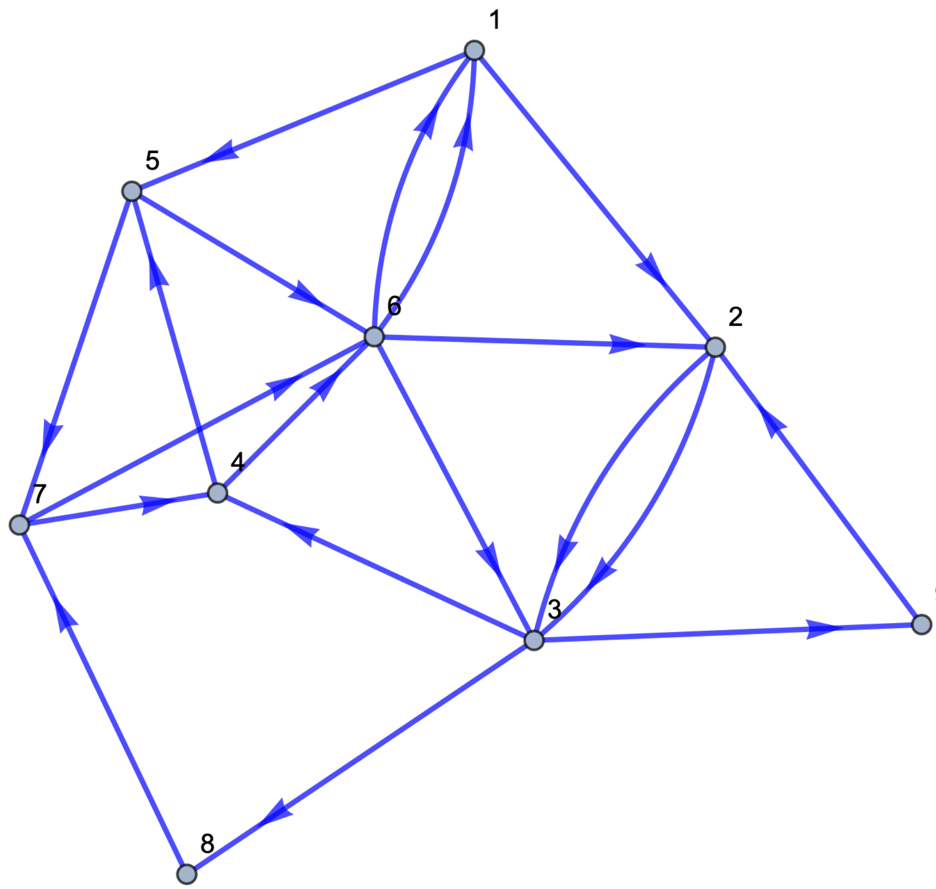
Out[•]=



Graph of Θ using Join

In[•]:=
```
fig[Join[Θ⟦1⟧, Θ⟦2⟧, Θ⟦3⟧, Θ⟦4⟧], 2]
```

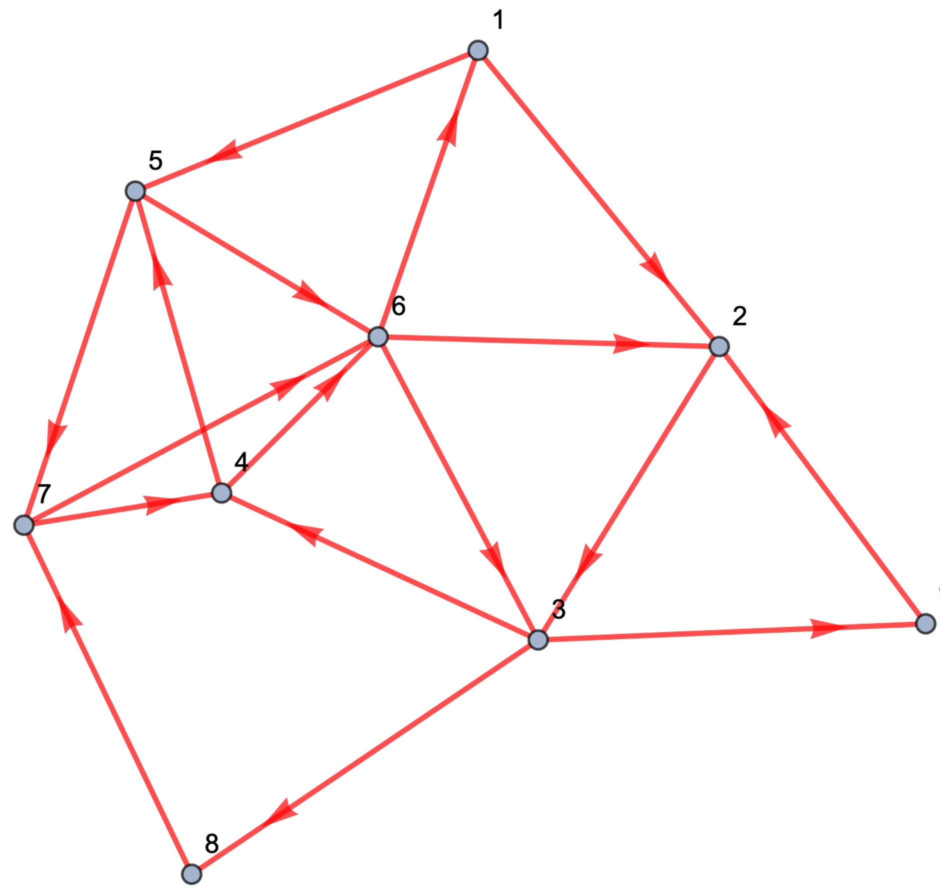
Out[•]=



This graph show merger of circuits. It correctly shows that 6→1 and 2→ 3 are each used twice in layers. The next graph uses Union to remove duplicate arrows.

In[∘]:=
```
fig[Θ⟦1⟧ ⋃ Θ⟦2⟧ ⋃ Θ⟦3⟧ ⋃ Θ⟦4⟧, 3]
```

Out[∘]=



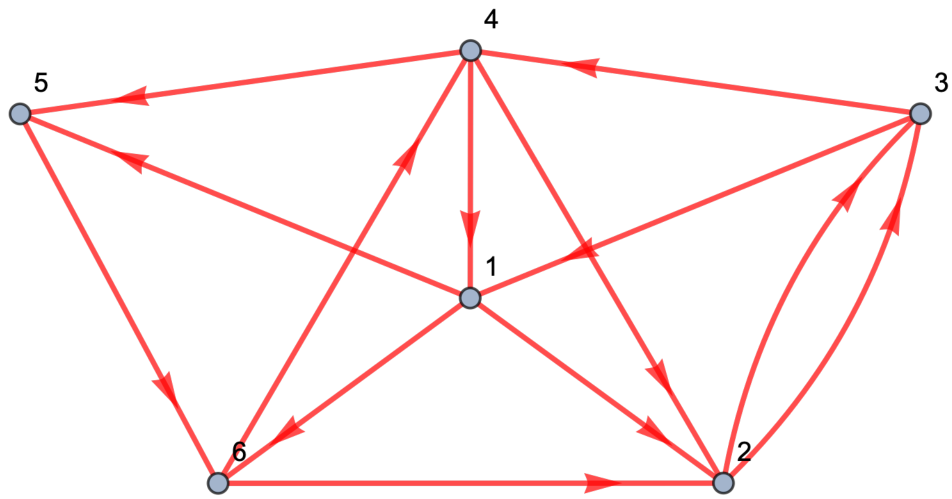
Graph of layered pentagon Θ = Θ⟦1⟧ ⋃ Θ⟦2⟧ ⋃ Θ⟦3⟧ ⋃ Θ⟦4⟧, center at vertex 6.
{3 → 4 → 5 → 7 → 8 → 3} uses two sides of the pentagon.

The next set f has different layered structures.

In[∘]:=
```
f := {{2 → 3, 3 → 4, 4 → 1, 1 → 5, 5 → 6, 6 → 2},
   {6 → 4, 4 → 5, 1 → 6}, {1 → 2, 4 → 2, 2 → 3, 3 → 1}}
```

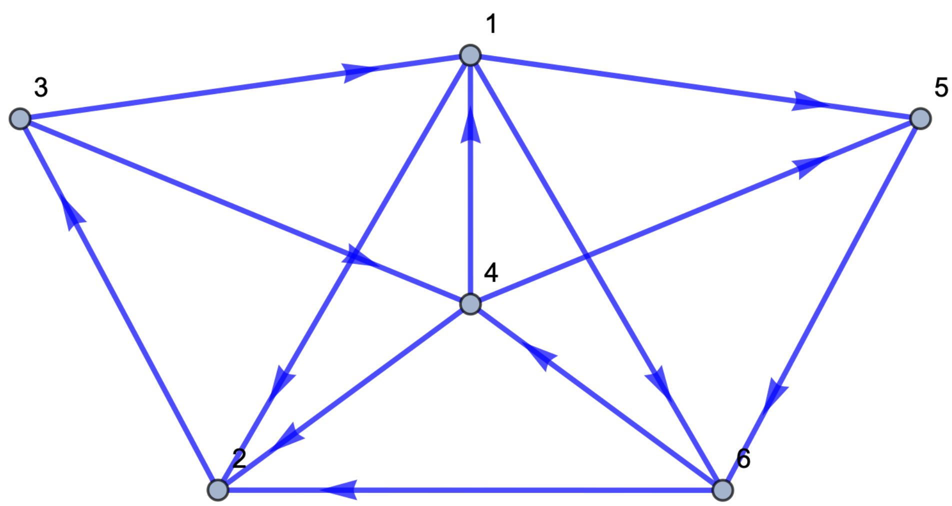
In[•]:=
```
fig[Join[f⟦1⟧, f⟦2⟧, f⟦3⟧], 3]
```

Out[•]=



In[•]:=
```
fig[f⟦1⟧ ⋃ f⟦2⟧ ⋃ f⟦3⟧, 2]
```

Out[•]=



Union removes 2→3 from Join.

Layers in a Hexagon

This hexagon has 15 arrows.

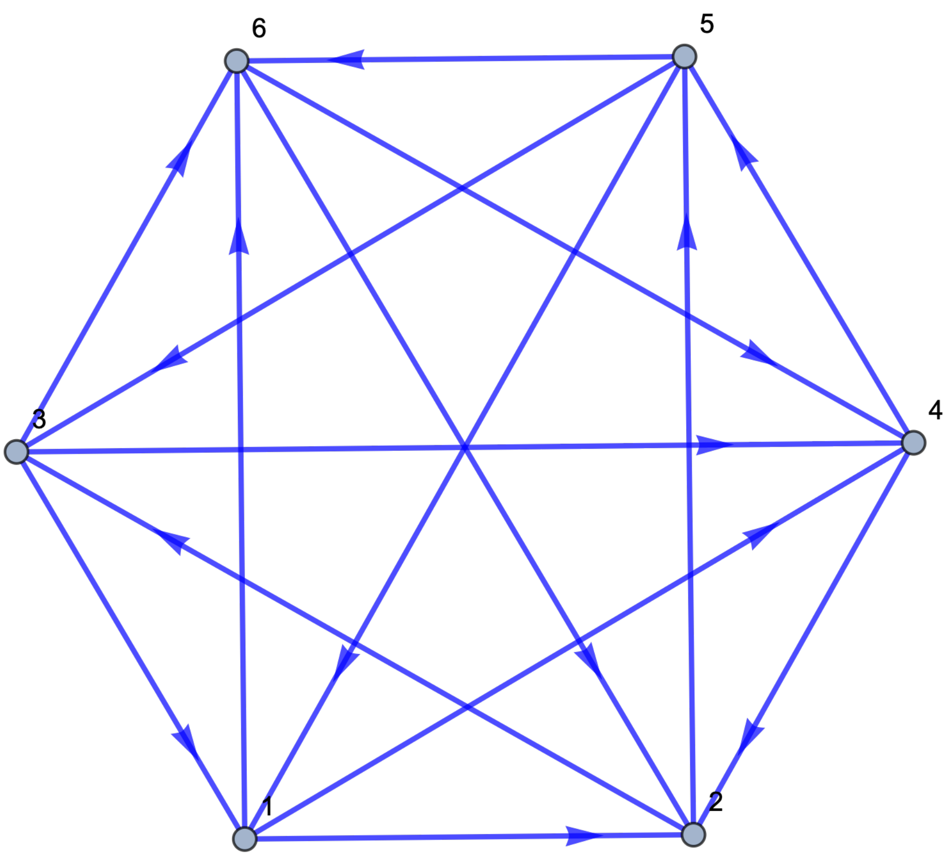
In[•]:=
```
makeA[6]
```

In[∘]:=
```
Apply[DirectedEdge, arrw, 1]
```

The next set of arrows in the hexagon are called six.

In[∘]:=
```
six := {1 ⧴ 2, 1 ⧴ 4, 1 ⧴ 6, 2 ⧴ 3, 2 ⧴ 5, 3 ⧴ 1,
  3 ⧴ 4, 3 ⧴ 6, 4 ⧴ 2, 4 ⧴ 5, 5 ⧴ 1, 5 ⧴ 3, 5 ⧴ 6,
  6 ⧴ 2, 6 ⧴ 4}
```

In[∘]:=
```
fig[six, 2]
```

Out[∘]=


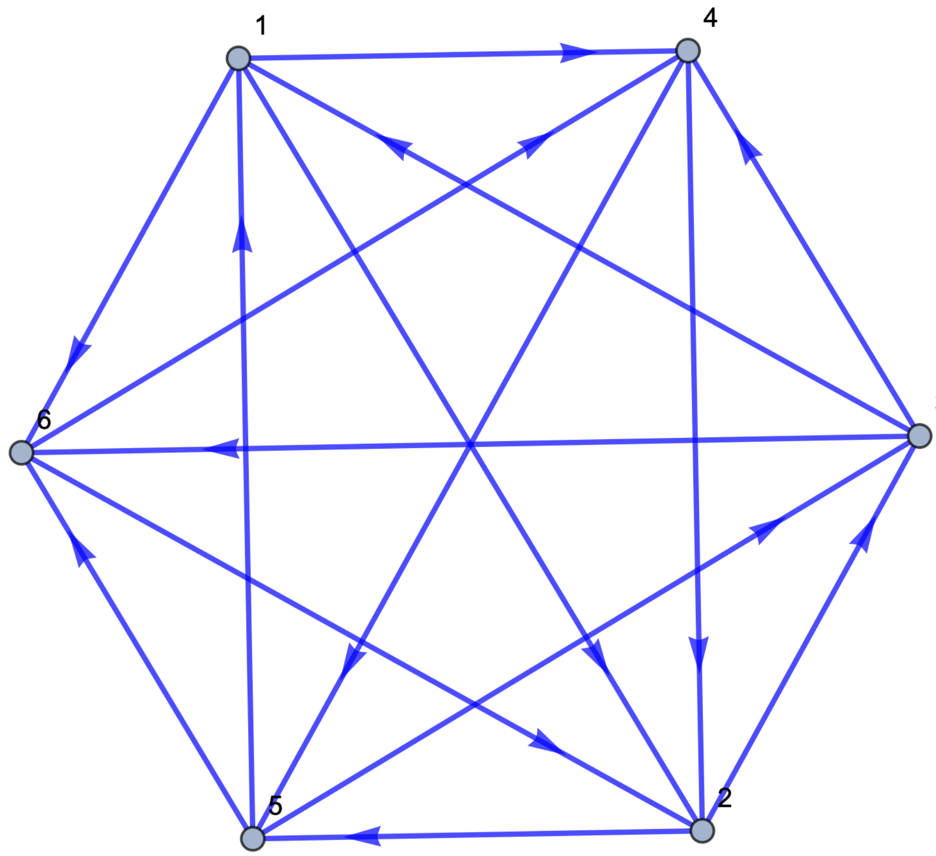
Graph of six showing its 15 arrows

In[∘]:=
```
Six := {{1 ⧴ 2, 2 ⧴ 3, 3 ⧴ 4, 4 ⧴ 5, 5 ⧴ 1},
   {1 ⧴ 4, 4 ⧴ 2, 2 ⧴ 5, 5 ⧴ 3, 3 ⧴ 1},
   {1 ⧴ 6, 3 ⧴ 6, 5 ⧴ 6, 6 ⧴ 2, 6 ⧴ 4}}
```

The circuits Six[[1]] and Six[[2]] share its edges with the hexagon.

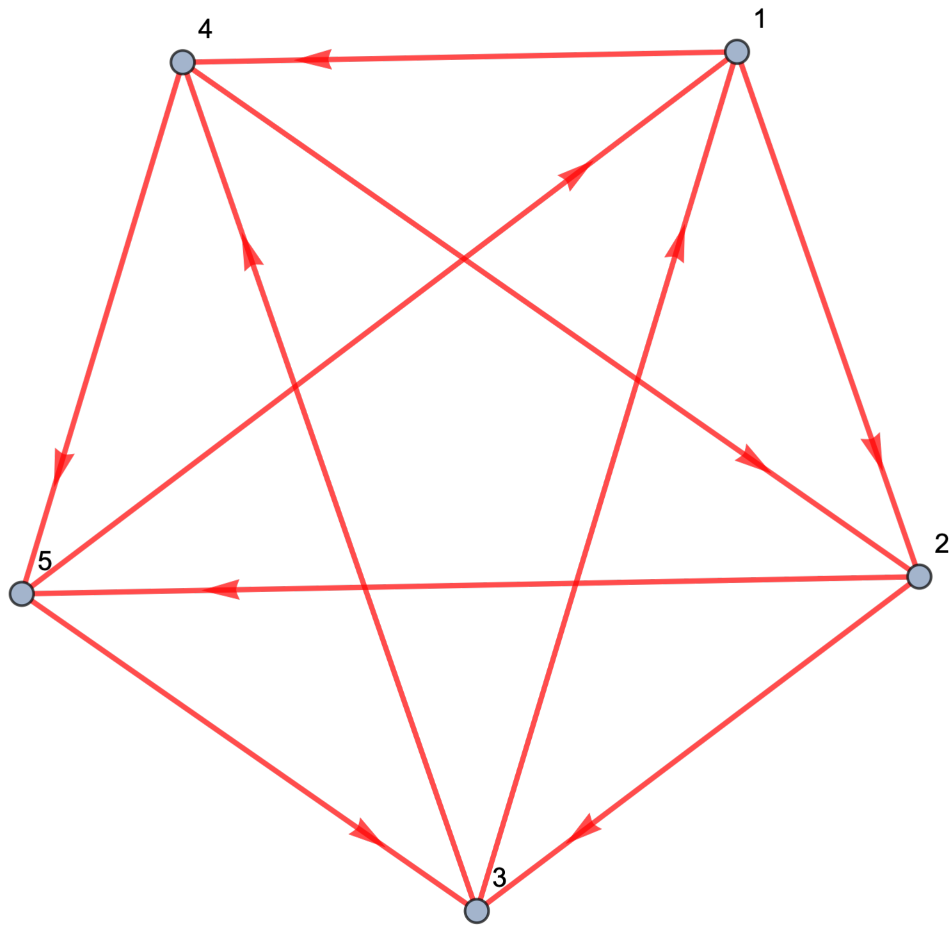
In[•]:=
```
fig[Join[Six⟦1⟧, Six⟦2⟧, Six⟦3⟧], 2]
```

Out[•]=



In[•]:=
```
fig[Six⟦1⟧ ⋃ Six⟦2⟧, 3]
```

Out[•]=



Graph of Six⟦1⟧ ⋃ Six⟦2⟧

In[•]:=
```
s12 := {{0, 1, 0, 1, 0}, {0, 0, 1, 0, 1}, {1, 0, 0, 1, 0},
   {0, 1, 0, 0, 1}, {1, 0, 1, 0, 0}}
```

This graph of Six[[1]] ∪ Six[[2]] is the pentagon inside the hexagon. It has five 3-circuits, five 4-circuits and two 6-circuits. These layers are inside s12. The pentagon inside the hexagon shares three edges with the perimeter edges. The pentagon belongs to a layer inside the hexagon.

In[•]:=

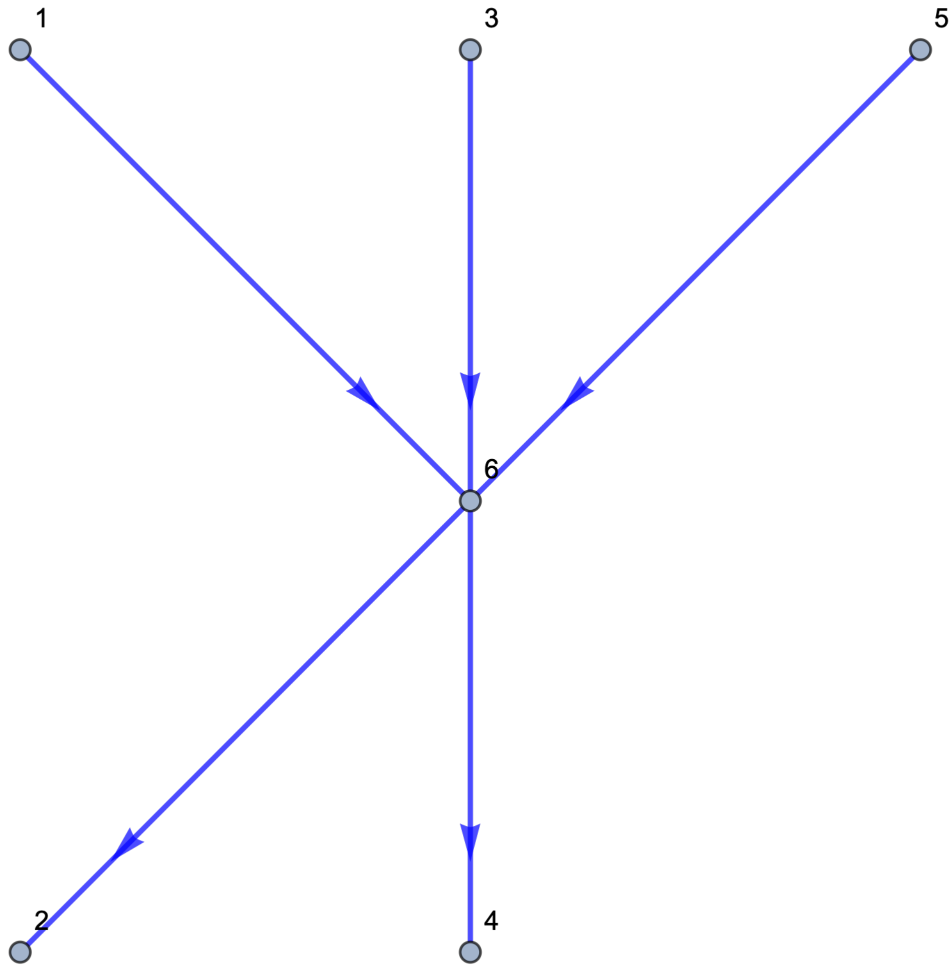
```
MatrixForm[findAllCycles[s12]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \{3 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 3\} \\ \{2 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 2\} \\ \{1 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \\ \{1 \leftrightarrow 2,\ 2 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \\ \{1 \leftrightarrow 2,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 1\} \\ \{2 \leftrightarrow 5,\ 5 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 2\} \\ \{1 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 3,\ 3 \leftrightarrow 1\} \\ \{1 \leftrightarrow 4,\ 4 \leftrightarrow 2,\ 2 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \\ \{1 \leftrightarrow 4,\ 4 \leftrightarrow 2,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 1\} \\ \{1 \leftrightarrow 2,\ 2 \leftrightarrow 5,\ 5 \leftrightarrow 3,\ 3 \leftrightarrow 1\} \\ \{1 \leftrightarrow 4,\ 4 \leftrightarrow 2,\ 2 \leftrightarrow 5,\ 5 \leftrightarrow 3,\ 3 \leftrightarrow 1\} \\ \{1 \leftrightarrow 2,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \end{pmatrix}$$

In[•]:=
```
fig[Six⟦3⟧, 2]
```

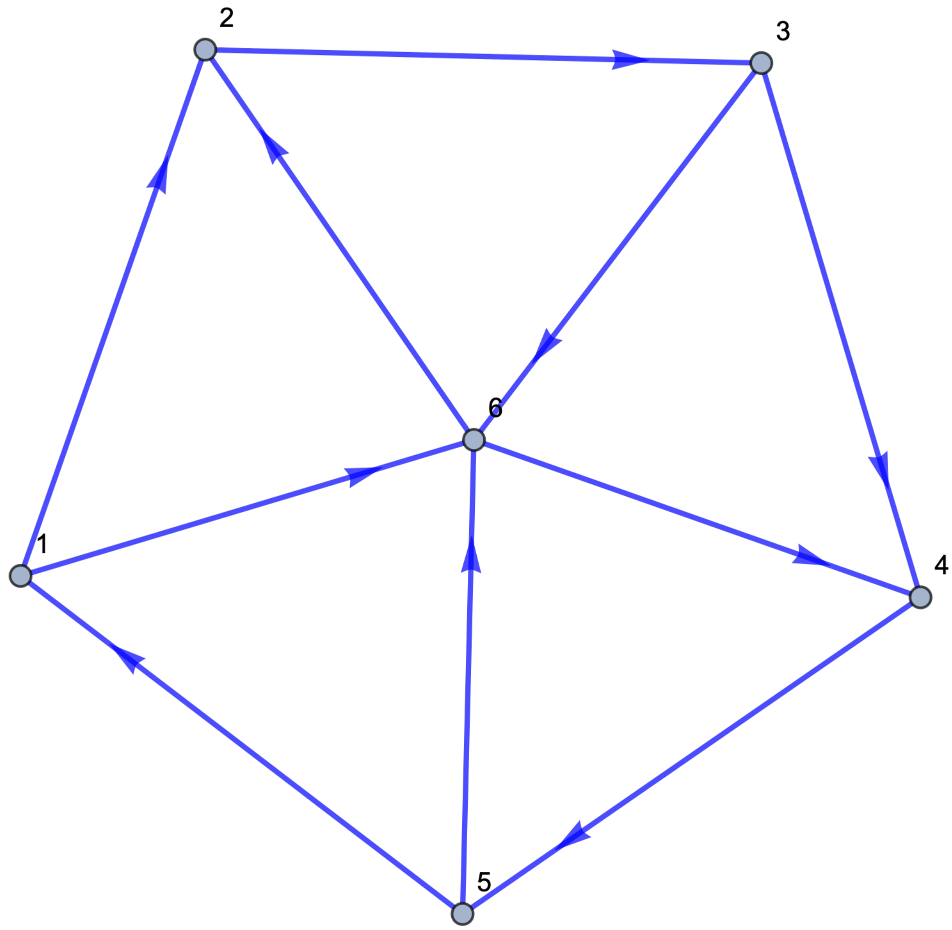
Out[•]=



This is the graph of the 5 arrows connected to vertex 6 in the heptagon.

In[•]:=
```
fig[Join[Six⟦1⟧, Six⟦3⟧], 2]
```

Out[•]=



In[•]:=
```
s13 := {{0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0},
   {0, 0, 0, 1, 0, 1}, {0, 0, 0, 0, 1, 0},
   {1, 0, 0, 0, 0, 1}, {0, 1, 0, 1, 0, 0}}
```

In[∘]:=
```
MatrixForm[findAllCycles[s13]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \{4 \leftrightarrow 5,\ 5 \leftrightarrow 6,\ 6 \leftrightarrow 4\} \\ \{2 \leftrightarrow 3,\ 3 \leftrightarrow 6,\ 6 \leftrightarrow 2\} \\ \{1 \leftrightarrow 6,\ 6 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \\ \{2 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 6,\ 6 \leftrightarrow 2\} \\ \{1 \leftrightarrow 2,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \\ \{1 \leftrightarrow 6,\ 6 \leftrightarrow 2,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \\ \{1 \leftrightarrow 2,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 6,\ 6 \leftrightarrow 4,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 1\} \end{pmatrix}$$

The circuits formed by merger of Six⟦1⟧ and Six⟦3⟧ share their edges with the hexagon. They are layers of the hexagon.

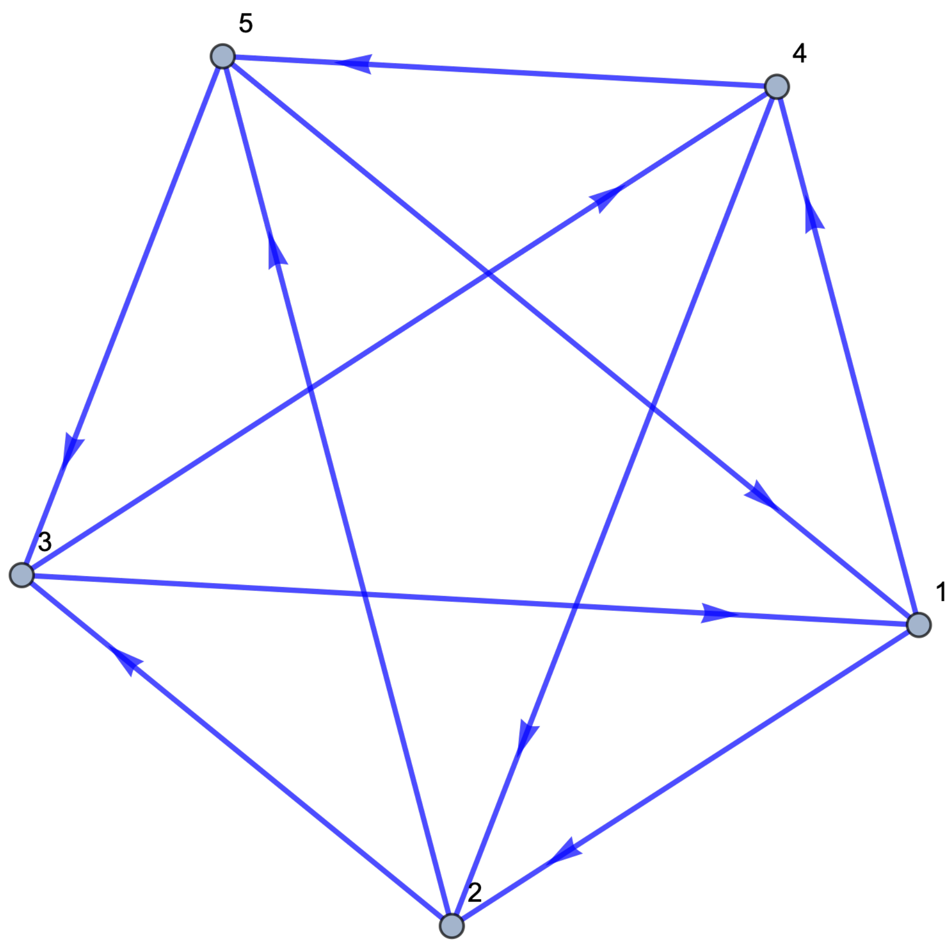
In[∘]:=
```
EulerianGraphQ[fig[Join[Six[[1]], Six[[3]]], 2]]
```

Out[∘]= False

Since all the vertexes of s13 are odd, it is not an Eulerian graph.

In[•]:=
```
fig[Join[Six⟦1⟧, Six⟦2⟧], 2]
```

Out[•]=

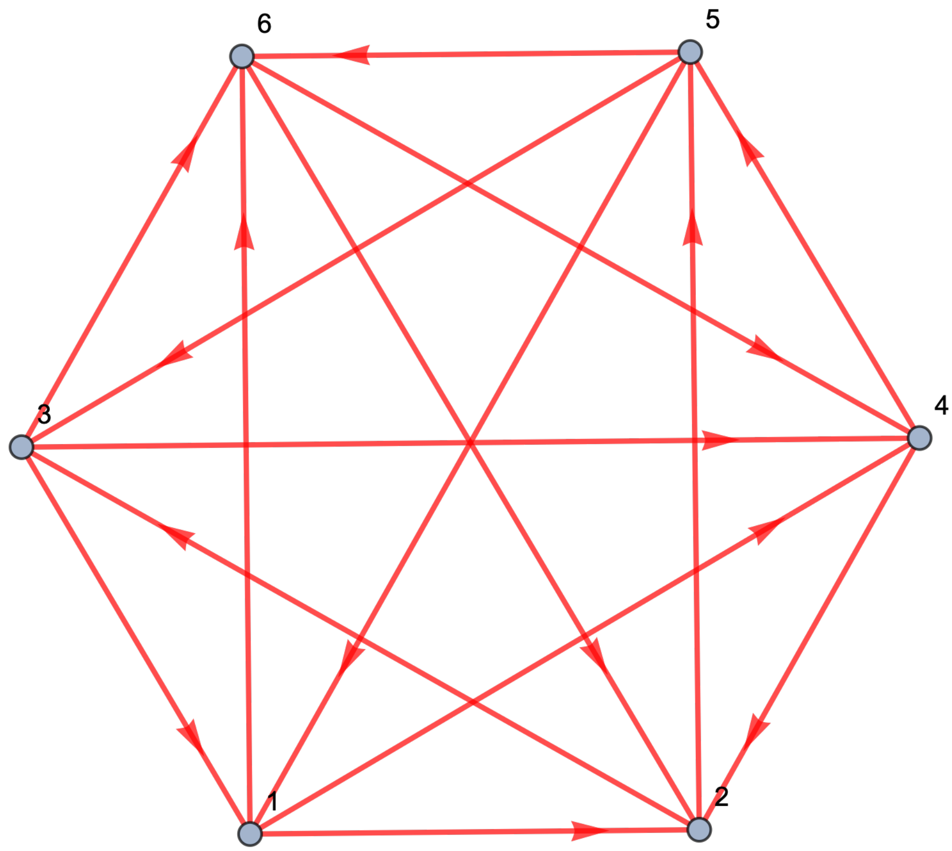


In[•]:=
```
EulerianGraphQ[fig[Join[Six⟦1⟧, Six⟦2⟧], 2]]
```

Out[•]=
```
True
```

All the vertexes of six are even. No edge appears more than once. Therefore, six is an Eulerian cycle.

In[•]:=
```
fig[six, 3]
```

Out[•]=


Money in the Circuit Economy

A model with money as its numeraire applies to circuits. Value and cost are monetary variables. Buyers pay with money. Sellers are paid in money. Money need not be a physical entity such as coins or currency. An economy includes banks that offer their clients demand deposits. In the model bank clients are circuits and owners of arrows. More than one bank needs a clearing house. Demand accounts are the money in this model. Money in this form does not need government. Circuits and owners of arrows have bank accounts on which they write checks for making and receiving payment. These payments and receipts of a circuit are not continuously equal. Funds in a circuit's bank accounts move up or down in step with receipts and outlays. There is no need to impose the rule that firms must balance budgets. A circuit cannot survive continuing losses nor can it accumulate gains indefinitely. The present form of the layered circuit model is static that avoids these complications.

Money as a numeraire of value differs from the other publicly accepted numerical measures of length, volume and weight because there are two types of value, objective and subjective. Price is an objective, not a subjective, measure of value. Yet two-part prices are a complication as an objective measure of value. Price per unit may be available for many commodities and services. The maximum price a buyer is willing to pay for a commodity may be its subjective value for that buyer. A seller may have a subjective measure of value given by the minimally acceptable price. Circuit models use money as an objective measure of value.

A Note on Mathematica

Mathematica uses Join of sets to indicate merger of sets. This usage clarifies the role of Union.

$$\mathbf{Join[x[i]]} - \bigcap \mathbf{x[i]} = \bigcup \mathbf{[x[i]]}$$

$$\text{If } \mathbf{Join}\left[\mathbf{x[i]}\right. = \bigcup \mathbf{[x[i]}\text{ , then } \bigcap \mathbf{x[i]} = \emptyset.$$

Mathematica distinguishes between join [merger] and union.

References

Telser, L . G . 2021. Circuit Economy. Berkeley Electronic Press.

_____ . 2022. Making Nöther Matrics. Berkeley Electronic Press.

_____ . 2022 Hamiltonian Circuits as Coalitions and their Core Status.

Programs