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1 Circuit Core Graph

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Circuit Core Graph

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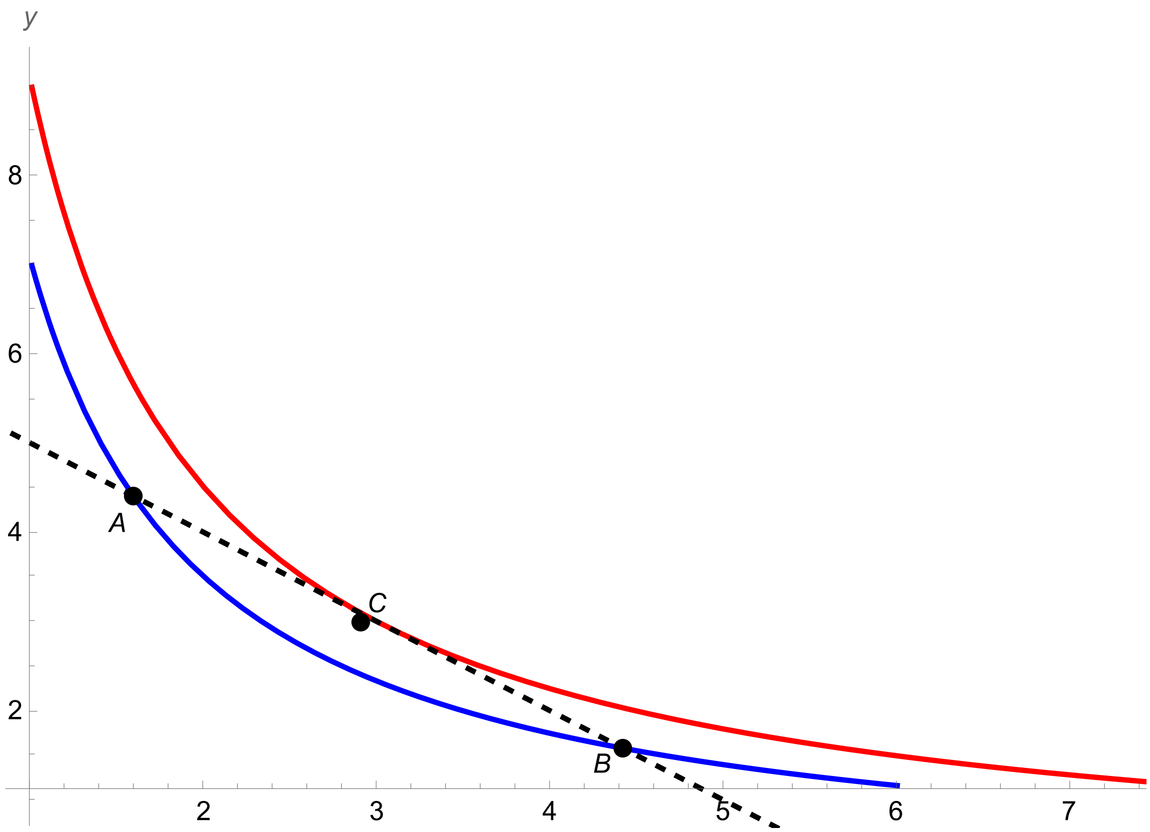
In this essay a graph illustrates core status by showing a non empty and an empty core. The upper hyperbola is the value of the circuit to its members who are represented by the two parties, a pair of numbers (x,y). The upper hyperbola in the graph is $x\ y = 9$. The resources available to the circuit are all the positive points northwest of the line going through A, B and C. This line intersects the lower hyperbola $x\ y = 7$ at A and B. This line is tangent to the upper hyperbola at C where the total value available to (x, y) would be maximal. However, C does not dominate B because the x-payoff is bigger at B than at C. Also C does not dominate A because the payoff to y is bigger at A than at C. Therefore, while the total value is maximal at C, it is not in the core because it is dominated by the feasible alternatives, A and B. Only points less than both coordinates of C are dominated by C. A non empty core exists only with respect to the allocations it dominates, the points the rectangle northwest of C.

The situation for three parties (x, y z) is the same but needs a graph in 3-dimensions. The hyperbola becomes $x\ y\ z = k$. The resource constraint is $x + y + z = l$. In addition to the 3-party circuit there are three two-party circuits for the three pairs; x y, x z, y z. The status of the core depends on these three two party circuits and the three party circuit. The core is the set of undominated payoffs to the three parties. For n parties algebra replaces graphs. There are more constraints imposed by he circuits of various sizes. The core consists of the set of undominated payoffs. If they exist the core is not empty.

In[•]:=
```
Show[p1, p2, l, ptl, ptr, ptc, ta, tb, tc]
```

Out[•]=



Procedures

In[•]:=
```
p1 := Plot[9 / x, {x, 1, 8}, PlotStyle → {Red, Thick},
   AxesLabel → {x, y}]
```

In[•]:=
```
p2 := Plot[7 / x, {x, 1, 6}, PlotStyle → {Blue, Thick},
   AxesLabel → {x, y}]
```

In[•]:=
```
l :=
  Graphics[{Black, Dashed, Thick,
    Line[{{0, 6}, {6, 0}}]}]
```

Solve the two equations x + y = 6 and x y = 7 in order to find the intersection of the lower hyperbola x y = 7 at A and B on the line. Eliminate y and obtain x + 7/x = 6. The roots of x^2- 6 x + 7= 0 are the x-coordinates of A and B as follows.

In[∘]:= Solve$[x^2 - 6x + 7 == 0, \{x\}]$

Out[∘]= $\{\{x \to 3 - \sqrt{2}\}, \{x \to 3 + \sqrt{2}\}\}$

In[∘]:=
```
ptl :=
 Graphics[{Black, PointSize[Large],
   Point[{3 - √2, 7/(3 - √2)}]}]
```

gives the coordinates of A.

In[∘]:=
```
ptr :=
 Graphics[{Black, PointSize[Large],
   Point[{3 + √2, 7/(3 + √2)}]}]
```

gives the coordinates of B.

In[∘]:=
```
ptc :=
 Graphics[{Black, PointSize[Large], Point[{2.9, 3}]}]
```

gives the coordinates of C.

In[∘]:=
```
ta := Graphics[Text[A, {1.5, 4.1}]]
```

In[∘]:=
```
tb := Graphics[Text[B, {4.3, 1.4}]]
```

In[∘]:=
```
tc := Graphics[Text[C, {3., 3.2}]]
```