Summer July 14, 2021

The Rise and Fall of Business Enterprise

Lester G Telser

7/14/21

The Rise and Fall of Business Enterprise

L . G . Telser

A successful innovation leads to an enterprise that develops it. Growing an enterprise usually takes longer than the decline often starting with a minor set back. Autos illustrate a fall, computers, a rise.

The triangle, pentagon, septagon and nonagon illustrate the model. First is the triangle. Next comes the pentagon. It inherits a simple 3-circuit from the triangle and adds its 7 new arrows in two simple circuits, one 4-circuit and one 3-circuit. The septagon inherits the circuits from the pentagon. It adds 11 new arrows in two 4-circuits and one 3-circuit. Next comes the nonagon. It inherits all the circuits from septagon and adds 15 new arrows in three 4-circuits and one 3-circuit.

As success continues, the new enterprise rises step by step. New arrows go into new non overlapping simple circuits. Growth is incremental owing to continuing success. The end often starts with a small misstep. The enterprise goes down more rapidly than the time it took to reach its peak.

This resembles the rise and fall of families. A family is a sequence of generations each descending from its fore bares. New members combine in various ways. Every generation is the ancestor of generations to come. Eventually every family disappears.

Triangle

```
In[•]:= makeA[3]
```

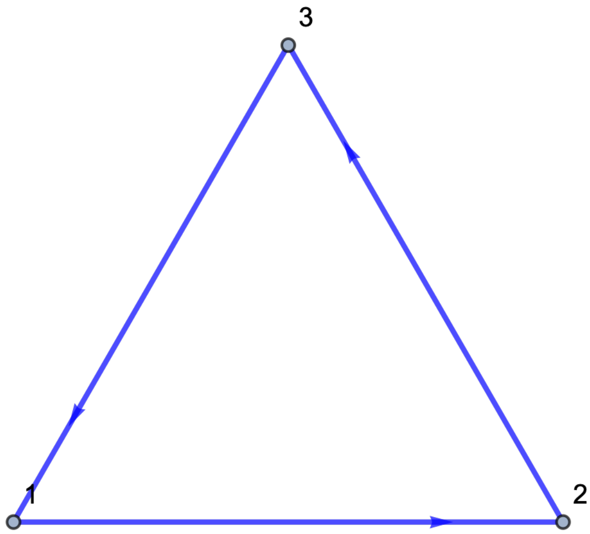
```
In[•]:= Apply[DirectedEdge, arrw, 1]
```

```
In[•]:= ar3 := {1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 1}
```

Nöther algebra applies to simple circuits. Let v[i⟶j] denote the value of arrow i⟶j, $\prod_{i,j\epsilon\gamma} v[i \longrightarrow j]$ is the value of circuit ar3.

In[•]:=
```
fig[ar3, 2]
```

Out[•]=



Graph of ar3

Pentagon

In[•]:=
```
makeA[5]
```

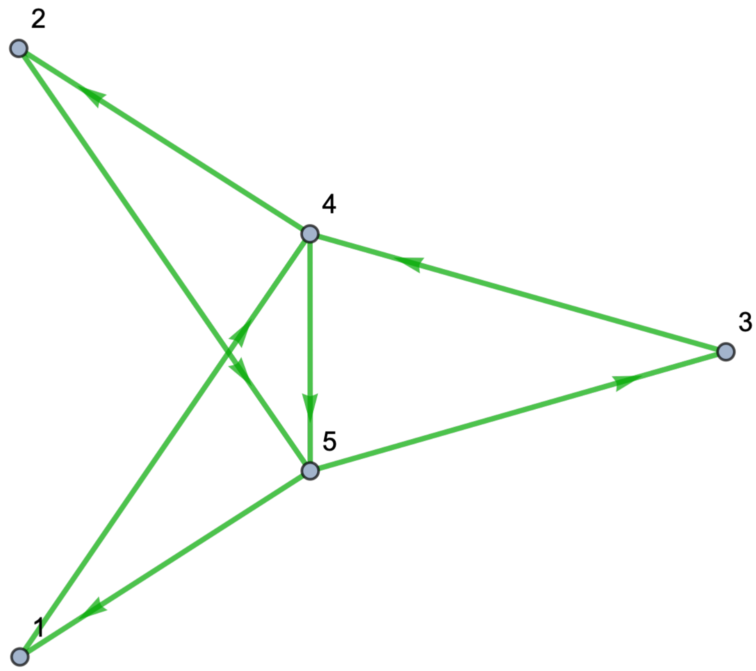
In[•]:=
```
Apply[DirectedEdge, arrw, 1]
```

The new arrows for the pentagon, ar5, are next. They are put into two non overlapping simple circuits, a 3-circuit and a 4-circuit.

In[•]:=
```
ar5 := {{1 ⥅ 4, 4 ⥅ 5, 5 ⥅ 1},
   {2 ⥅ 5, 5 ⥅ 3, 3 ⥅ 4, 4 ⥅ 2}}
```

In[•]:=
```
fig[Join[ar5⟦1⟧, ar5⟦2⟧], 4]
```

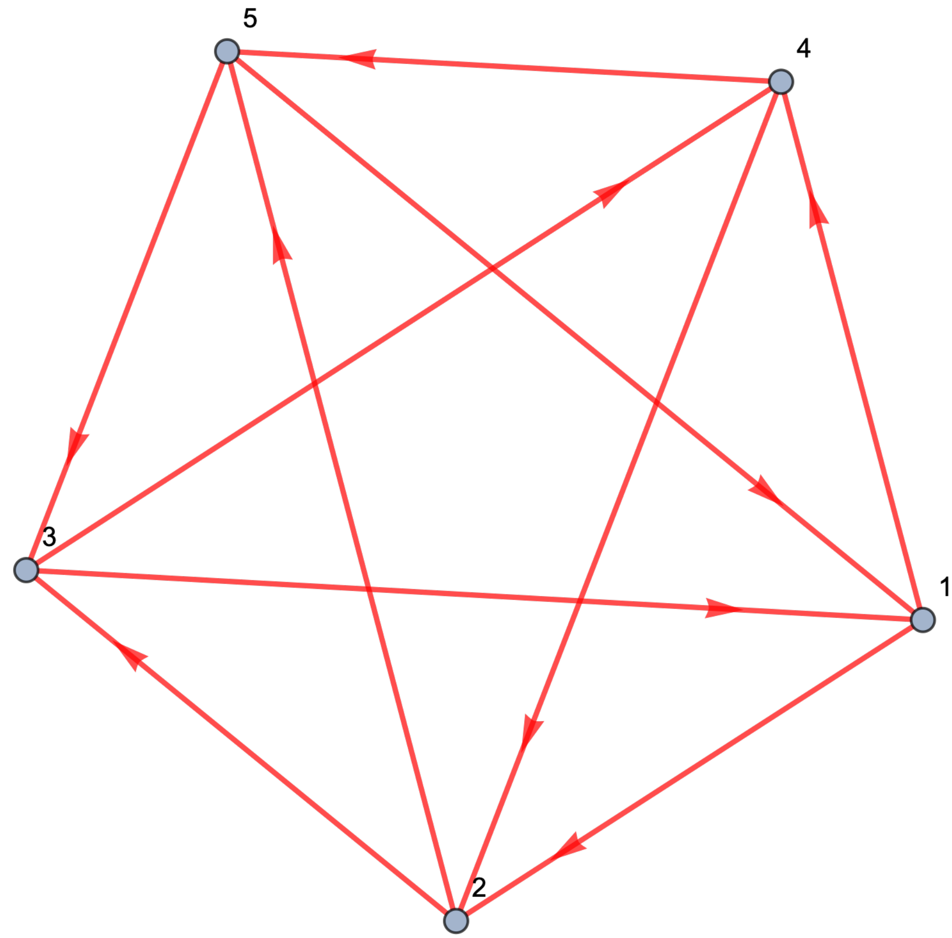
Out[•]=



Graph of New arrows in Pentagon

In[•]:=
```
fig[Join[ar3, ar5⟦1⟧, ar5⟦2⟧], 3]
```

Out[•]=



Graph of ar3 ∪ ar5⟦1⟧ ∪ ar5⟦2⟧: The Pentagon

This graph of the pentagon shows a union of three non overlapping circuits. The union forms a partition of the arrows of the pentagon. It is a significant finding that will be verified by the following analyses of the septagon and nonagon. Without changing the relative order of the arrows in these circuits, it is not difficult to prove that the value of the partition equals the product of the values of the non overlapping simple circuits. This result uses Nöther algebra explained (Weyl, 1953, chaps III and IX).

Septagon

The septagon has 21 arrows, 11 are new, 11 =21-10. These are placed into 3 non overlapping simple circuits.

```
In[•]:= makeA[7]
```

```
In[•]:= Apply[DirectedEdge, arrw, 1]
```

```
{1 ↦ 6, 2 ↦ 7, 3 ↦ 6, 4 ↦ 7, 5 ↦ 6, 6 ↦ 2, 6 ↦ 4,
 6 ↦ 7, 7 ↦ 1, 7 ↦ 3, 7 ↦ 5}
```

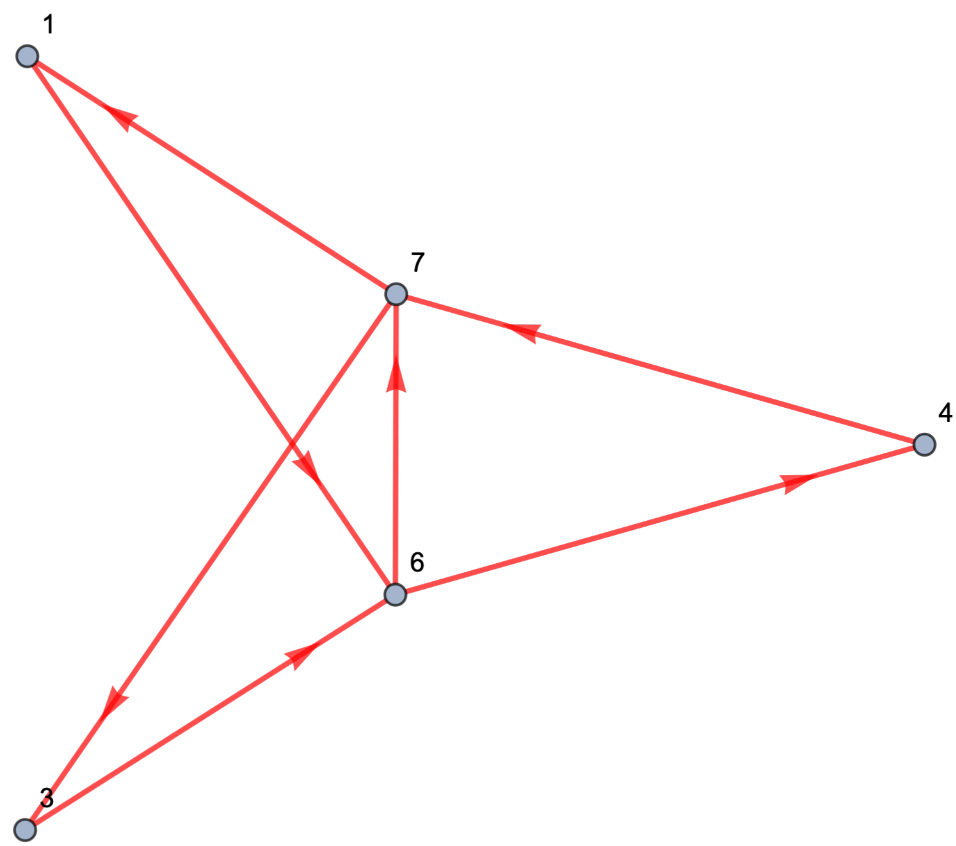
```
In[•]:= Apply[DirectedEdge, arrw, 1]
```

ar7 has the three new non overlapping simple circuits; two 4-circuits and one 3-circuit.

```
In[•]:= ar7 := {{1 ↦ 6, 6 ↦ 4, 4 ↦ 7, 7 ↦ 1},
   {2 ↦ 7, 7 ↦ 5, 5 ↦ 6, 6 ↦ 2}, {3 ↦ 6, 6 ↦ 7, 7 ↦ 3}}
```

```
In[•]:= fig[Join[ar7⟦1⟧, ar7⟦3⟧], 3]
```

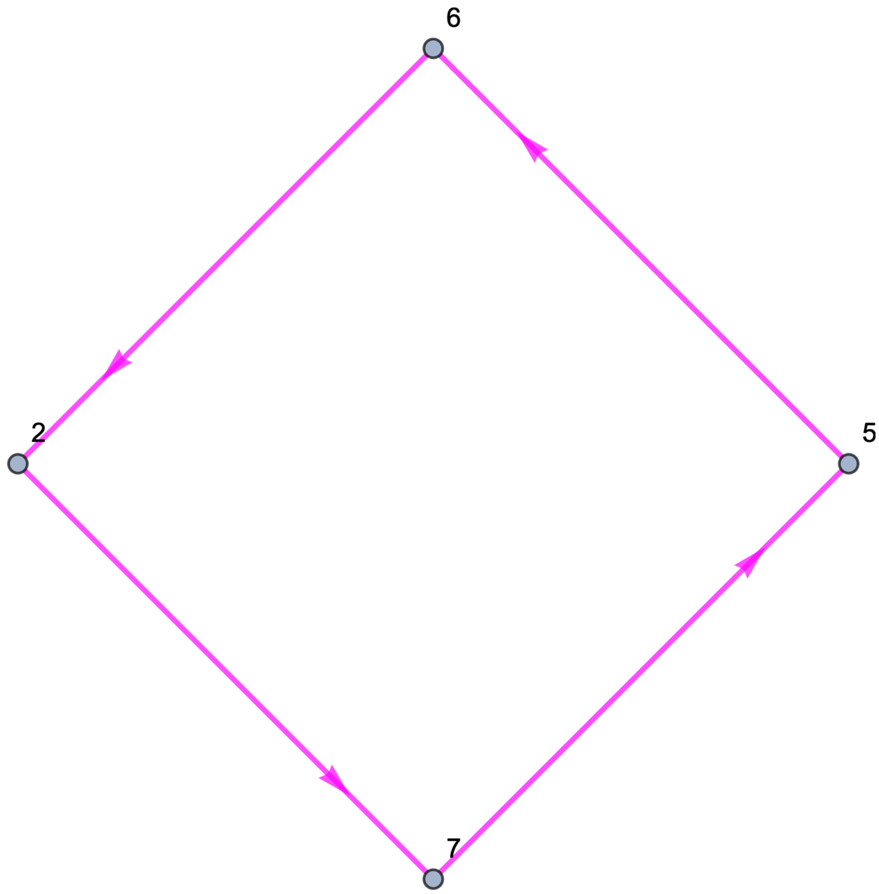
Out[•]=



Graph of ar7⟦1⟧ ∪ ar7⟦3⟧

In[•]:=
```
fig[ar7⟦2⟧, 8]
```

Out[•]=



Graph of ar7⟦2⟧

Because ar3 ∩ ar5 ∩ ar7 = ∅ and ar3 ∪ ar5 ∪ ar7 = arrw all the arrows of the septagon, the union is a partition of the arrows of the septagon.

Nonagon

The nonagon is the first composite m-polygon. It has 36 arrows. It has 15 new arrows, 36 - 21 =15. The 15 new arrows go into 4 non overlapping simple circuits, ar9; three 4-circuits and one 3-circuit

In[•]:=
```
makeA[9]
```

In[•]:=
```
Apply[DirectedEdge, arrw, 1]
```

```
{1 → 2, 1 → 4, 1 → 6, 1 → 8, 2 → 3, 2 → 5, 2 → 7,
 2 → 9, 3 → 1, 3 → 4, 3 → 6, 3 → 8, 4 → 2, 4 → 5,
 4 → 7, 4 → 9, 5 → 1, 5 → 3, 5 → 6, 5 → 8, 6 → 2,
 6 → 4, 6 → 7, 6 → 9, 7 → 1, 7 → 3, 7 → 5, 8 → 2,
 8 → 4, 8 → 6, 8 → 9, 9 → 1, 9 → 3, 9 → 5, 9 → 7}
```

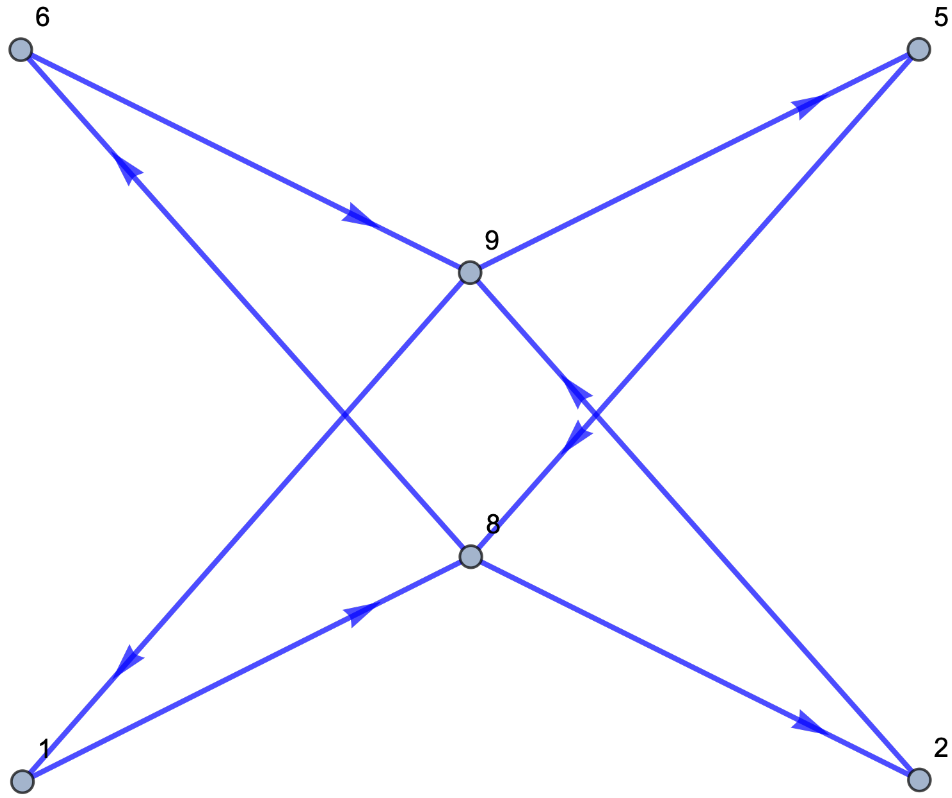
In[•]:=
```
ar := {1 → 8, 2 → 9, 3 → 8, 4 → 9, 5 → 8, 6 → 9,
  7 → 8, 8 → 2, 8 → 4, 8 → 6, 8 → 9, 9 → 1, 9 → 3,
  9 → 5, 9 → 7}
```

Here are the 4 non overlapping simple circuits made of the 15 new arrows in the nonagon, ar9.

In[•]:=
```
ar9 := {{1 → 8, 8 → 2, 2 → 9, 9 → 1},
  {3 → 8, 8 → 4, 4 → 9, 9 → 3},
  {5 → 8, 8 → 6, 6 → 9, 9 → 5}, {7 → 8, 8 → 9, 9 → 7}}
```

In[∘]:= fig[Join[ar9⟦1⟧, ar9⟦3⟧], 2]

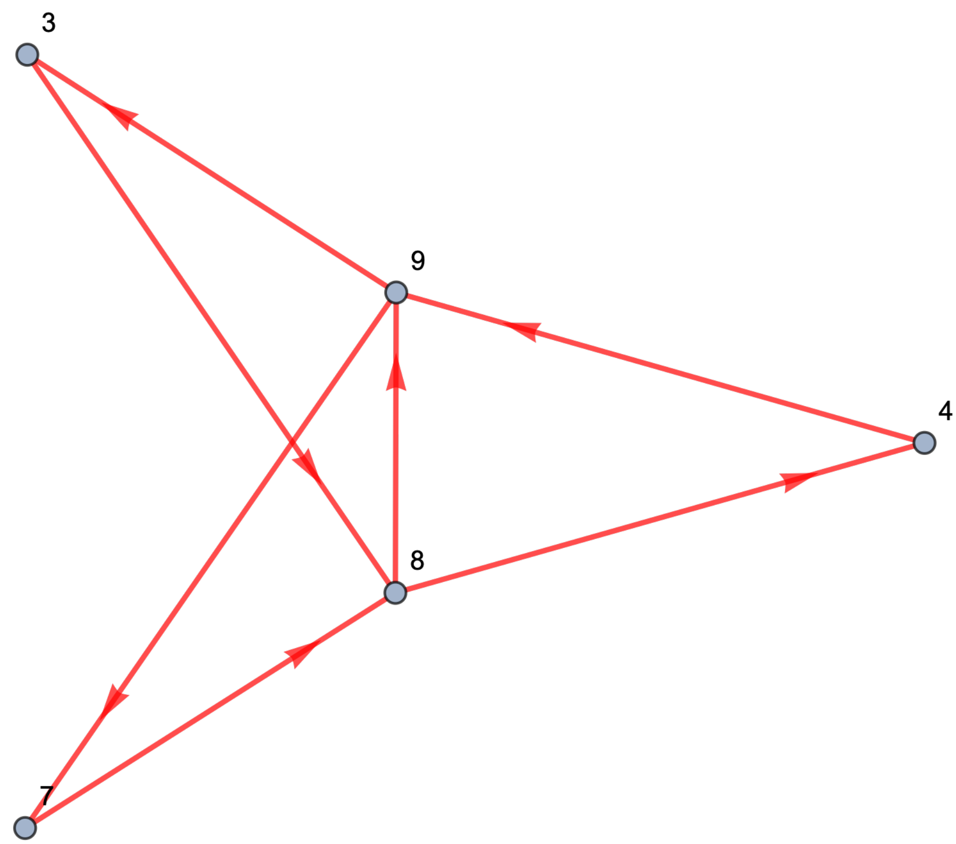
Out[∘]=



Graph of ar9⟦1⟧ ∪ ar9⟦3⟧

In[∘]:=
```
fig[Join[ar9⟦2⟧, ar9⟦4⟧], 3]
```

Out[∘]=



Graph of ar9⟦2⟧ ∪ ar9⟦4⟧

$\bigcap_{3}^{m} \text{ari} = \emptyset$, $\bigcup_{3}^{m} \text{ari} = \text{nonagon arrows}$,

shows that the union of the increments are a partition of the nonagon arrows.

11 - polygon

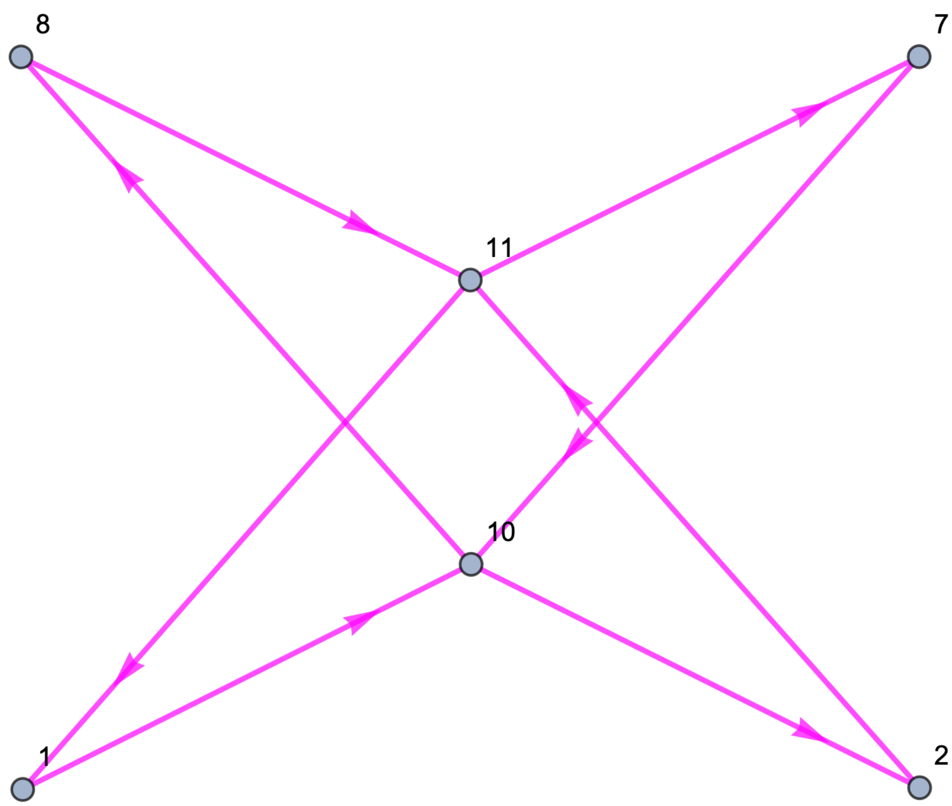
In[∘]:=
```
makeA[11]
```

In[∘]:=
```
Apply[DirectedEdge, arrw, 1]
```

```
In[•]:= ar11 := {{1 ⟷ 10, 10 ⟷ 2, 2 ⟷ 11, 11 ⟷ 1},
    {3 ⟷ 10, 10 ⟷ 4, 4 ⟷ 11, 11 ⟷ 3},
    {5 ⟷ 10, 10 ⟷ 6, 6 ⟷ 11, 11 ⟷ 5},
    {7 ⟷ 10, 10 ⟷ 8, 8 ⟷ 11, 11 ⟷ 7},
    {11 ⟷ 9, 9 ⟷ 10, 10 ⟷ 11}}

In[•]:= fig[Join[ar11[[1]], ar11[[4]]], 8]
```

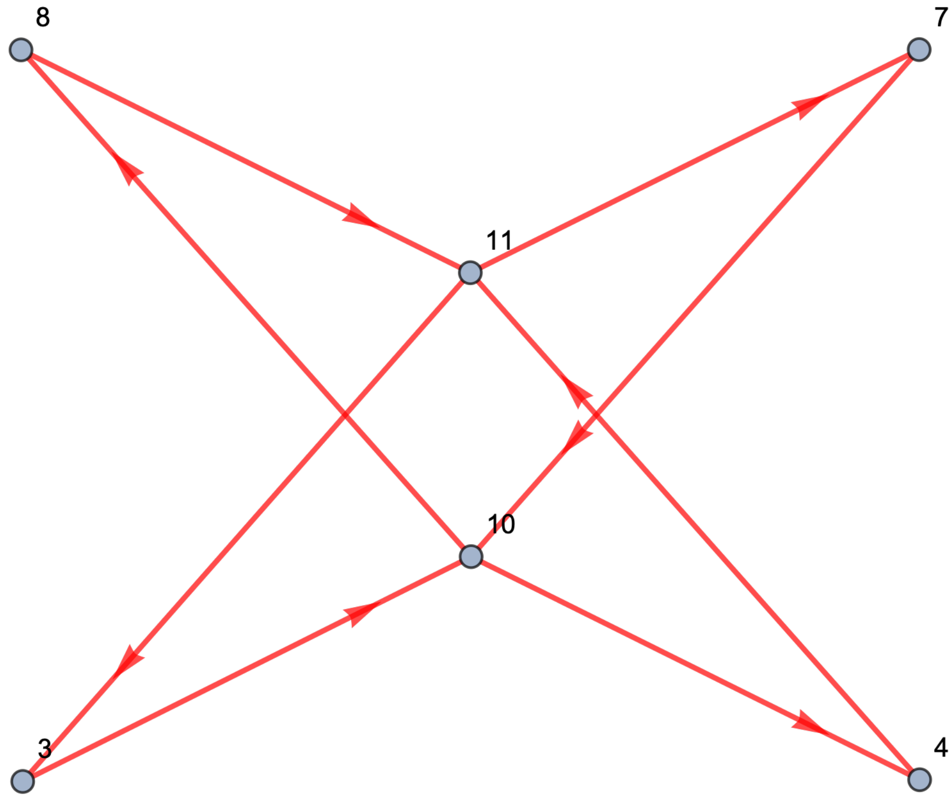
Out[•]=



Graph of ar11⟦1⟧ ∪ a11⟦4⟧

In[•]:=
```
fig[Join[ar11⟦2⟧, ar11⟦4⟧], 10]
```

Out[•]=



Graph of ar11⟦2⟧ ∪ ar11⟦4⟧

Summary

m	name	circuit composition	Δm	size	total composition
3	ar3	one 3 – circuit	3	3	1, 3 – circuit
5	ar5	one 4 – circuit, one 3 – circuit	7	10	1, 4 – circuit, 2, 3 – circuits
7	ar7	two 4 – circuits, one 3 – circuit	11	21	3, 4 – circuits, 3, 3 – circuits
9	ar9	three 4 – circuits, one 3 – circuit	15	36	6, 4 – circuits, 4, 3 – circuits
11	ar11	four 4 – circuits, one 3 – circuit	19	55	10, 4 – circuits, 5, 3 – circuits
13	ar13	five 4 – circuits, one 3 – circuit	23	78	15, 4 – circuits, 6, 3 – circuits
15	ar15	six 4 – circuits, one 3 – circuit	27	105	21, 4 – circuits, 7, 3 – circuits

ari ≡ 3 (Mod 4) , i = 1, 2, … ,
The newer generations are bigger than the older ones.

The Structure of Simple 4 - Circuits

The structure of the 4 - circuits in a 5 - polygon must reckon with the fact that the integers are Mod 5. Row 1 in Table 1 shows the simple 4-

circuit. Row 2 adds 1 to each term in the pair of row 1. Row 3 adds 1 to each term in the pair of row 2. And so on. However, starting from 1 instead of 0, requires treating 5 (Mod 5) congruent to 1 instead of 0. Keeping this in mind, the results are next. All terms in Table 2 correspond to the terms in Table 1 congruent to 5. For example, the term in row 3, column 4 of Table 1, (6,3)(Mod 5) ≡ (1,3) the term in row 3, column 4, Table 2.

Table 2 shows the 4-circuits from Table 1 (Mod 5). Each row shows the 4 arrows in the simple 4-circuits of a 5-polygon. The 4-circuit in row 5 repeats the circuit in row 1 (Mod 5). The relative order of the arrows is the same in all 4 rows (Mod 5). There are 2 principal diagonals orthogonal to each other, one runs northeast, the other southwest. The terms parallel to the southwest diagonal are equal. The terms parallel to the northeast diagonal are not equal but are in the same relative order as the 4-circuits.

(1, 2)	(2, 3)	(3, 4)	(4, 1)
(2, 3)	(3, 4)	(4, 5)	(5, 2)
(3, 4)	(4, 5)	(5, 6)	(6, 3)
(4, 5)	(5, 6)	(6, 7)	(7, 4)
(5, 6)	(6, 7)	(7, 8)	(8, 5)

Table 1

(1, 2)	(2, 3)	(3, 4)	(4, 1)
(2, 3)	(3, 4)	(4, 5)	(5, 2)
(3, 4)	(4, 5)	(5, 1)	(1, 3)
(4, 5)	(5, 1)	(1, 2)	(2, 4)
(5, 1)	(1, 2)	(2, 3)	(3, 5)

Table 2

Nöther Matrics

A simple 4-circuit has a sequence of 4 Nöther matrics. Each arrow has a 5X5 matric whose terms are 0 except the term that represents the arrow in the matric. This term is 1. The first simple 4-circuit in the next table shows its matric.

In[•]:= n12 := $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[•]:= n23 := $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[•]:= n34 := $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[•]:= n41 := $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

The next product, n1, has 1 in row 1, column 3.

```
In[•]:= MatrixForm[n12.n23]
```

In[•]:= n1 := $\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

The next product, n2, has 1 in row 3, column 1.

```
In[•]:= MatrixForm[n34.n41]
```

$$\text{In[•]:= } n2 := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The following matric has 1 in row 1, column 1. This diagonal term is the product of the 4 arrows in the 4-circuit, row 1 of Table 2. If arrow a[i,j] were replaced by its value, v[i,j] then the diagonal term in row 1, column 1 would be v[1,2] v[2,3] v[3,4] v[4,1, the product of the 4 matrics is in the same relative order as the arrows in the 4-circuit.

```
In[•]:= MatrixForm[n1.n2]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This example illustrates a very important feature of simple circuits. The value of a circuit is the product of the values of its arrows. The value of arrow a[i,j] is v[a[i,j]]. Let γ(n) denote a simple circuit made of n arrows a[i,j]. Let V[γ(n)] denote the value of the circuit.

$$V[\gamma(n)] = \prod_{a[i,j]\in\gamma} v[a[i,j]] \tag{1}$$

This formula for the value of a circuit has two important implications. First, the value of a circuit is a maximum when the values of its arrows are equal.

$$V[\gamma(n)] \leq \text{Max}\{v[a[i,j]\}^{n} \tag{2}$$

Second, the value of a circuit is least when all but one of the arrows has the least value and one arrow has the most value. The distribution of the values is maximally heterogeneous. To say it plainly, the distribution of the values would be as skewed as possible. If the minimum value were zero, then the value of the circuit would be zero.

Useful Formulae

The increment of the total number of arrows from the m-polygon to the (m+2)-polygon, is.

$$\begin{aligned}\Delta m &= [(m+2)(m+1) - m(m-1)]/2\\ &= \left(m^2 + m + 2m + 2 - m^2 + m\right)/2\\ &= (4m + 2)/2 = 2m + 1\end{aligned}$$

The change in Δm is

$$\Delta^2 m = 2(m+2) + 1 - (2m+1) = 4$$

The sum of the integers from 1 to n is S[n]

$$S[n] = \sum_{i=1}^{n} i;$$

$$n^2 = S[n] + S[n-1];$$

$$S[n] = S[n-1] + n;$$

$$n^2 = 2S[n] - n;$$

$$S[n] = n(n-1)/2.$$

The ari do not overlap. A partition of the arrows in an odd m-polygon is given by the union of the ari from 3 to m.

Reference

Weyl, Hermann, 1953. The Classical Groups. 2nd ed. Princeton: Princeton University Press. Chaps. III & IX, Matric Algebra

Program