Spring May 22, 2021

1 principles.pdf

Lester G Telser

5/22/21

Principles for a Circuit Economy*

L . G . Telser

Introduction

A circuit economy is particularly useful to study the distribution of the returns to the participants of an economy. My circuit model of the economy uses fair polygons whose vertexes are linked by certain one-way arrows. “Fair” comes from tournaments where all players have equal chance of winning and may form teams. Odd m-polygons are fair but even m-polygons are unfair. In an even m-polygon an odd vertex has one more winner than an even vertex. In an odd m-polygon all vertexes have the same number of winners so odd m-polygons are fair. Only an odd m-polygon has partitions. An even m-polygon has no partitions. A partition of arrows has special importance in my circuit model of an economy.

The decisive argument in favor of a partition follows from examination of any alternative arrangement. Each arrow appears only once In a partition. This is also true for a simple circuit. However, the distribution of arrows across circuits is not uniform. Also, the more nearly alike the productivity of the circuit members, the more valuable is the circuit. Among the rejected solutions in the core is the one where some circuit members get the whole value of the circuit and all the other members get nothing. The Noether algebra applied to a simple circuit says in this case the value of the simple circuit becomes zero so it would not happen.

A model where coalitions of participants are simple circuits has a remarkable property. Only odd fair m - polygons can have partitions of arrows made of simple circuits. This is because each vertex of an odd m - polygon satisfies the Euler condition that every vertex has the same number of incoming and outgoing arrows and every row has the same number of incoming arrows.

The Principles

1. Each polygon in a defined set of of m-polygons has m(m-1)/2 arrows. A fair polygon in this set has the most simple [Hamiltonian] circuits. An m-polygon can have a hierarchy without simple circuits. A hierarchy is not a tree. A tree is a set of nodes and edges without any circuits. The set can be an m-polygon and its selected arrows. A super circuit is a set of non overlapping simple circuits such that each has at least one junction with another circuit. A partition is a super circuit but a super circuit need not be a partition. An Eulerian circuit is a super circuit but a super circuit need not be an Eulerian circuit

2. The adjacency matrix for a fair odd m-polygon has m(m-1)/2 terms equal to 1. The remaining terms, $m^2 - m(m-1)/2 = m(m-1)/2 = 0$. The sum of the terms in row i and column i = (m-1)/2 for all i = 1, 2, ..., m. The diagonal terms = 0.

3. Only a fair odd m-polygon has at least one partition of its arrows arranged in non overlapping simple circuits. A partition in a fair odd m-polygon is an Eulerian circuit and conversely. The Mathematica procedure EulerianCycleQ decides whether an m-polygon and its selected arrows is a partition.

4. The s arrows in a simple s-circuit do not overlap. Let v[i] denote vertex i in an m-polygon. Let a[i,j] denote a one way arrow from v[i] to v[j]. A pair of adjacent arrows in an s-circuit is {a[i,j], a[j,k]}, i = 1, 2, ... , s. The arrow whose source vertex is s is a[s, 1]. Details for the hexagon and septagon appear later.

5. Every vertex in a fair odd m-polygon has m-1 arrows, half incoming and half outgoing. An even m-polygon is different. Thus no arrow goes from vertex m to vertex 1.

6. A circuit economy has a non empty core if all values are in logs.

7. The smallest simple circuit has 3 one-way arrows. The biggest has m

arrows.

Arrows and Simple Circuits for the Pentagon

```
{1, 2} {1, 4}   (1, 2, 3) (1, 4, 2, 3)
{2, 3} {2, 5}   (2, 3, 4) (2, 5, 3, 4)
{3, 4} {3, 1}   (3, 4, 5) (3, 1, 4, 5)
{4, 5} {4, 2}   (4, 5, 1) (4, 2, 5, 1)
{5, 1} {5, 3}   (5, 1, 2) (5, 3, 1, 2)

 (1, 2, 3, 4, 5) (1, 4, 2, 5, 3)
```

Partitions in the Pentagon
Concise notation shows only the source vertex for each arrow.

```
(1,2,3),(3,4,5),(4,2,5,1)

(2,3,4),(5,1,2),(3,1,4,5)

(1,4,2,5,3),(1,2,3,4,5)
```

The next procedures use the non zero terms from the bands parallel to the principal diagonal of the adjacency matrix. Bands have arrows arranged as nonoverlapping circuits forming a partition. Fair odd m-polygons also have partitions with the most simple circuits by using as many non overlapping simple 3-circuits as feasible. This poses two problems, first, find the best partition, second, devise how to attain it.

Hexagon

In[•]:=
```
MatrixForm[makeY[6]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & \{1,2\} & 0 & \{1,4\} & 0 & \{1,6\} \\ 0 & 0 & \{2,3\} & 0 & \{2,5\} & 0 \\ \{3,1\} & 0 & 0 & \{3,4\} & 0 & \{3,6\} \\ 0 & \{4,2\} & 0 & 0 & \{4,5\} & 0 \\ \{5,1\} & 0 & \{5,3\} & 0 & 0 & \{5,6\} \\ 0 & \{6,2\} & 0 & \{6,4\} & 0 & 0 \end{pmatrix}$$

The theoretical bands are {b1, b2, b3} (mod 6). Denote them {mb1, mb2, mb3}. A hexagon has 15 arrows. Only 9 of its theoretical bands are correct.

```
In[∘]:= b1 := {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}}

In[∘]:= b2 := {{1, 4}, {2, 5}, {3, 6}, {4, 7}, {5, 8}, {6, 9}}

In[∘]:= b3 :=  {{1, 6}, {2, 7}, {3, 8}, {4, 9}, {5, 10}, {6, 11}}
```

The true bands are {ab1, ab2, ab3}.

```
In[∘]:= ab1 := {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}}

In[∘]:= ab3 := {{1, 6}, {3, 1}, {4, 2}, {5, 3}, {6, 4}}

        ab2 := {{1, 4}, {2, 5}, {3, 6}, {6, 2}, {5, 1}}
```

Next are the bands {b1, b2, b3} (mod 6).

```
In[∘]:= Mod[b1, 6]

        mb1 := {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 1}}

        Mod[b2, 6]

In[∘]:= mb2 := {{1, 4}, {2, 5}, {3, 6}, {4, 1}, {5, 2}, {6, 3}}

In[∘]:= Mod[b3, 6]

In[∘]:= mb3 := {{1, 6}, {2, 1}, {3, 2}, {4, 3}, {5, 4}, {6, 5}}
```

Septagon

```
In[∘]:= makeA[7]
```

In[∘]:=
```
MatrixForm[makeY[7]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & \{1,2\} & 0 & \{1,4\} & 0 & \{1,6\} & 0 \\ 0 & 0 & \{2,3\} & 0 & \{2,5\} & 0 & \{2,7\} \\ \{3,1\} & 0 & 0 & \{3,4\} & 0 & \{3,6\} & 0 \\ 0 & \{4,2\} & 0 & 0 & \{4,5\} & 0 & \{4,7\} \\ \{5,1\} & 0 & \{5,3\} & 0 & 0 & \{5,6\} & 0 \\ 0 & \{6,2\} & 0 & \{6,4\} & 0 & 0 & \{6,7\} \\ \{7,1\} & 0 & \{7,3\} & 0 & \{7,5\} & 0 & 0 \end{pmatrix}$$

Initial Bands

In[∘]:=
```
b71 := {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6},
  {6, 7}, {7, 8}}
```

In[∘]:=
```
b72 := {{1, 4}, {2, 5}, {3, 6}, {4, 7}, {5, 8},
  {6, 9}, {7, 10}}
```

In[∘]:=
```
b73 := {{1, 6}, {2, 7}, {3, 8}, {4, 9}, {5, 10},
  {6, 11}, {7, 12}}
```

Initial Bands (Mod 7)

In[∘]:=
```
Mod[b71, 7]
```

In[∘]:=
```
mb71 := {{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6},
  {6, 7}, {7, 1}}
```

In[∘]:=
```
Mod[b72, 7]
```

In[∘]:=
```
mb72 := {{1, 4}, {2, 5}, {3, 6}, {4, 7}, {5, 1},
  {6, 2}, {7, 3}}
```

In[∘]:=
```
Mod[b73, 7]
```

In[∘]:=
```
mb73 := {{1, 6}, {2, 7}, {3, 1}, {4, 2}, {5, 3},
  {6, 4}, {7, 5}}
```

The formulae for the three bands in the septagon are neat unlike those for the hexagon.

```
a[1, j], j = 2, 3, …, 7 (Mod 7)

a[1, j], j = 4, 5, …, 9 (Mod 7)

a[1, j], j = 6, 7, … , 11 (Mod 7)
```

For the general odd m polygon the formulae for its bands are a[1, j] , j = { 2, 4, 6, … ,m-1} (Mod m).

Program