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1 semi-private.pdf

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Semi-Private Goods in a Circuit Core Model

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1 Introduction

Semi-private goods are financed and owned by a circuit. Its value increases with the homogeneity of its members. This feature is an implication of Noether algebra.The value of some semi-private goods to an individual increases ith the number of users. Computer software is the leading case. It means the demand for software is unusual because this implies its demand function is upward sloping, not downward sloping as is typical of private goods. It follows that a circuit model is a better tool for analysing most semi-private goods than is the an economic model of demand that ignores this fact. Indeed, it is not all that new. The theory of conspicuous consumption, more than a century old, implies an upward sloping demand for fashion goods.

Both a country club and computer software are semi-private goods. Only members and their guests can use the facilities of their country club. Only buyers of a software package can use it. Most users of the software did not create it. Those who bore the cost of creating software had estimated its potential market, recognizing that it depends how many will use it unlike private non fashion goods. Unlike software, users of country clubs may value it more, the more it restricts its membership to certain classes.

Semi-private goods fall into two different classes, unrestricted and restricted. Software belongs to the unrestricted class because its value to a user rises with the number who use it. Country clubs, elite colleges and fashion goods belong to the restricted class because its value increases, the more restricted the acceptable users.

2 Restricted Semi-Private Goods

Services from a semi-private good are available only to its owners. Circuits in a fair m-polygon can own semi-private goods. Production of a semi-private good typically incurs an outlay depending on whether there are enough users to cover the cost. Let w[i] denote the amount user i would contribute to the total cost of creating the semi-private good.

(1) $\left(\sum_{i=1}^{n} w[i] \right) \begin{pmatrix} < \\ \geq \end{pmatrix} K \Longrightarrow \begin{pmatrix} \text{do not} \\ \text{do} \end{pmatrix}$ make the semi-private good.

Arrange w[i] by size from the biggest to the smallest so that the number of users is an increasing concave step function. This sets a lower bound on the value of the semi-private good.

The standard model typically ignores relations among users of a commodity. It measures the value of a product by the sum of its value to its users. It says the total value is $\sum_{i=1}^{n} v[i]$ where v[i] is the value to user i. Figure 1 illustrates a continuous, increasing concave function that approximates the discrete function, $\sum_{i=1}^{n} v[i]$.

How to Measure Gain

In[•]:=
```
gan[bot_, top_] :=
 Plot[bot + Log[(x - bot)], {x, bot, top},
  PlotStyle → {Red, Thick}, AxesOrigin → {0, 0},
  AxesLabel → {Subscribers, Gain}]
```

In[•]:=
```
cst[bot_, K_] :=
 Graphics[{Blue, Thick, Line[{{0, K}, {bot, K}}]}]
```

In[•]:=
```
psub[a_, bot_, top_] :=
 Plot[a x, {x, bot, top}, PlotStyle → {Black, Dashed},
  AxesOrigin → {0, 0}]
```

```
In[∘]:= Show[gan[3., 25], cst[24, 5.6], cst[12, 3.5],
 psub[.81, 0, 11], psub[.36, 3, 30], PlotRange → All]
```

Out[∘]=

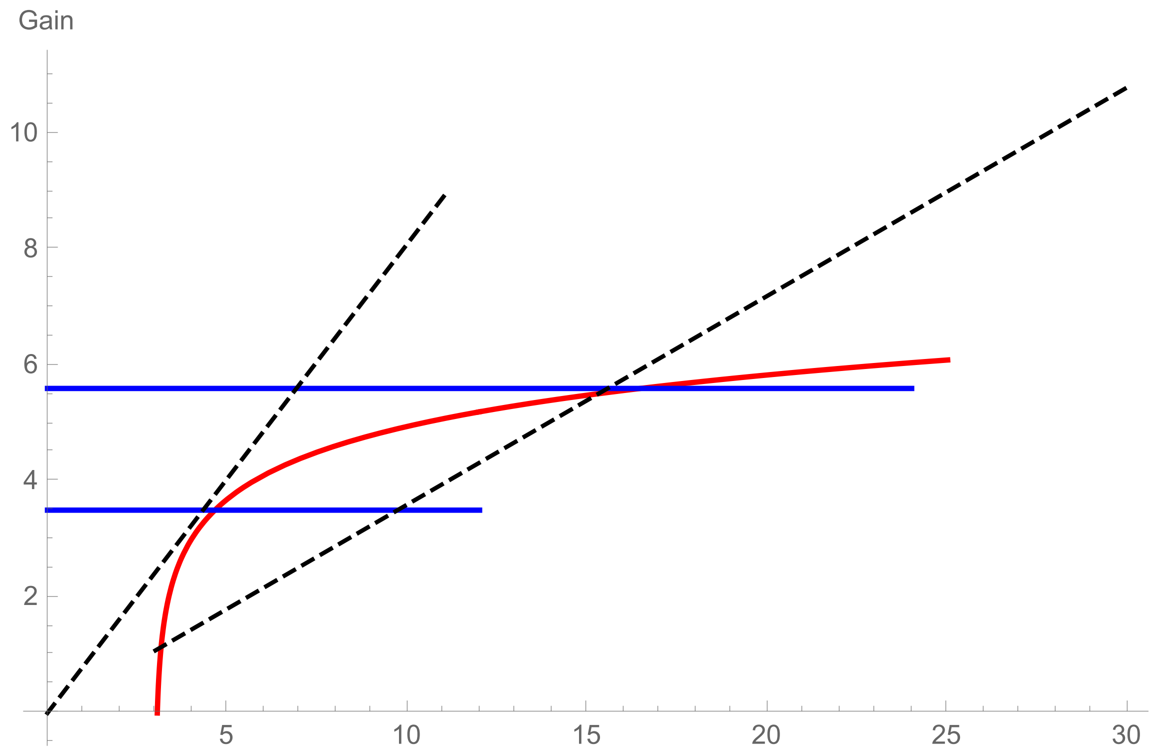


Figure 1

According to Figure 1, there would be no gain from the semi-private good at the price line tangent to the value function.

3 How a Circuit Model Values an Unrestricted Semi-Private Good

The circuit model assumes the members derive benefit from the semi-private good that increases at a decreasing pace with circuit size. It differs from a private good because anyone in the circuit decides and obtains as much as he wants without affecting the amount available for fellow members. While It is equally available to all members, it is unavailable to outsiders. Formula (1) says the value of a circuit of size k, Π_k, is multiplicative. It comes from the Nöther algebra for a simple circuit explained in Circuit Valuation and Core Status. The value of a semi-private good to member i of the circuit is x[i]. The value of the circuit is Π_k.

(1) $\Pi_k = \prod_{i=1}^{k} x[i]$, x[i]≥1 for i = 1, 2 ,..., k.

Next is an important property of the circuit model.

If the individual value were the same in every simple circuit in a fair odd m-polygon, then

(2) $\Pi_{k+1} - \Pi_k = \Pi_k (x[k+1]-1) > 0 \Longleftrightarrow x[k+1] - 1 \geq 0$ for all k ≥ 3 and k ≤ m.

$(\Pi_{k+2} - \Pi_{k+1}) - (\Pi_{k+1} - \Pi_k) = \Pi_k * (x[k+2]\, x[k+1] - (x[k+1) - x[k+1] - 1) =$

$\Pi_k * (x[k+2]\, x[k+1] - 2\, x[k+1] + 1).$

Therefore, Π_k is concave in k if and only if

(3) $(x[k+2]\, x[k+1] - 2\, x[k+1] + 1) \leq 0.$

Inequality (3) is equivalent to

(4) $x[k+1] \leq 2 - 1/x[k]$, k = 3, 4, … ,m.

Figure 2 shows the x-region where Π_k would be concave in x[k].

```
In[•]:= ccl[top_] := Plot[2 - 1 / x, {x, 1, top},
          PlotStyle → {Red, Thick}, AxesLabel → {X[n], Gain}]

In[•]:= Show[ccl[10], cst[11, 2.], AxesOrigin → {0, 1},
         PlotRange → All]
```

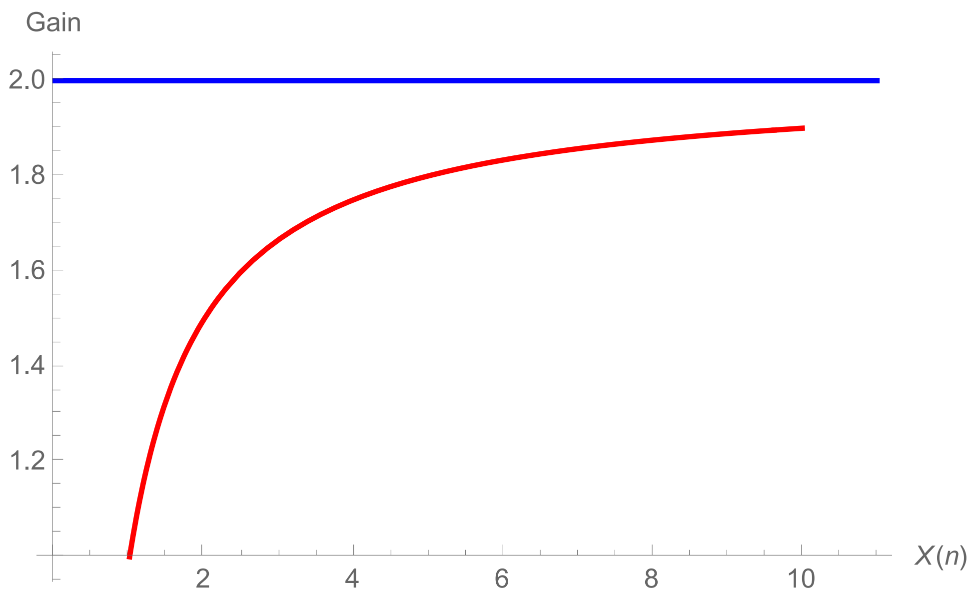
Out[•]=


Figure 2

4 A Social Network Is an Unrestricted Semi-Private Good

If the value of the semi-private good to individual i in the k-circuit depends on the circuit size, k, then the value becomes x[i, k]. Each individual derives a benefit from the semi-private good that is bigger, the bigger the membership of the circuit. This applies to computer software.

Let p[m] denote an odd fair m-polygon. It has n arrows in (1)
(1) $n = m(m-1)/2$.
The next polygon in the sequence of these polygons is p[m+2]. It has (m+2)(m+1)/2 arrows. The number of arrows increases from p[m] to p[m+2] by
(2) $\Delta n = 2m+1$.
Each circuit in p[m] is also a valid circuit in p[m+2], although it is not a sub circuit of any circuit in p[m+2]. Perimeters of the m-polygons, band[1,5] and band[1,7], show this.Triplet {5⟷6,6⟷7,7⟷1} in band[1,7] replaces arrow 5⟷1 in band[1,5] and band[1,7] gives the perimeter of the septagon.

```
band[1, 5] := {1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 4, 4 ⟷ 5, 5 ⟷ 1}
```

```
band[1, 7] := {1 ⟷ 2, 2 ⟷ 3, 3 ⟷ 4, 4 ⟷ 5, 5 ⟷ 6,
   6 ⟷ 7, 7 ⟷ 1}
```

That the value of membership to individual i can depend on circuit size is shown by the presence of k in the valuation x[i,k] of Individual i who derives more value from membership in a bigger circuit. Individual i in p[m+2] has 2m+1 more contacts than in p[m] since the latter has a smaller network.
(3) $\Pi_{m+2} - \Pi_m = \Pi_m \left(\Sigma_{i\in p[m+2]}\, \Delta x[i, m+2]-1\right) > 0$
    if $\left(\Sigma_{i\in p[m+2]}\, \Delta x[i, m+2]-1\right) > 1$.
Calculation of the second differences shows the shape of Π_m as a function of m. If Π_m is convex and increasing, then it is unbounded and yields increasing returns to the semi-private good. Figure 3 shows continuous approximations to the pertinent discrete step functions. The upper curve shows the benefit as the envelope derived from the benefits to the partitions is an increasing function of m. The lower curve shows the benefit for the partition in p[m]. Let G[m] denote the benefit to the partition in p[m]. Let F[m] denote the envelope of these functions. There is a sequence {x[i]} of tangent points such that
(4) F(x[i]) = G(x[i]) and $\partial_x F(x[i]) = \partial_x G(x[i])$.
Figure 3, shows one tangent point for convex F(x) and concave G(x). Figure 4 shows two tangent points. The unit benefit for each p[m] is a maximum at the tangent point. The bigger is m, the bigger the benefit per unit.

An increasing concave function is unbounded on the positive orthant. A diagram similar to Figure 4 would show where the benefit Π_m, approximated by

F[x], is concave and increasing. Such a concave F[m] would be unbounded and yield increasing returns.

In general, F[x] is always increasing, but not necessarily at the same rate, more rapidly for smaller x then less rapidly for bigger x. In the latter case there is an inflection point where the sign of the second derivative changes from positive, convexity, to negative, concavity. Figure 3 shows that F[x] and G[x] do not intersect. They touch only at tangent points.

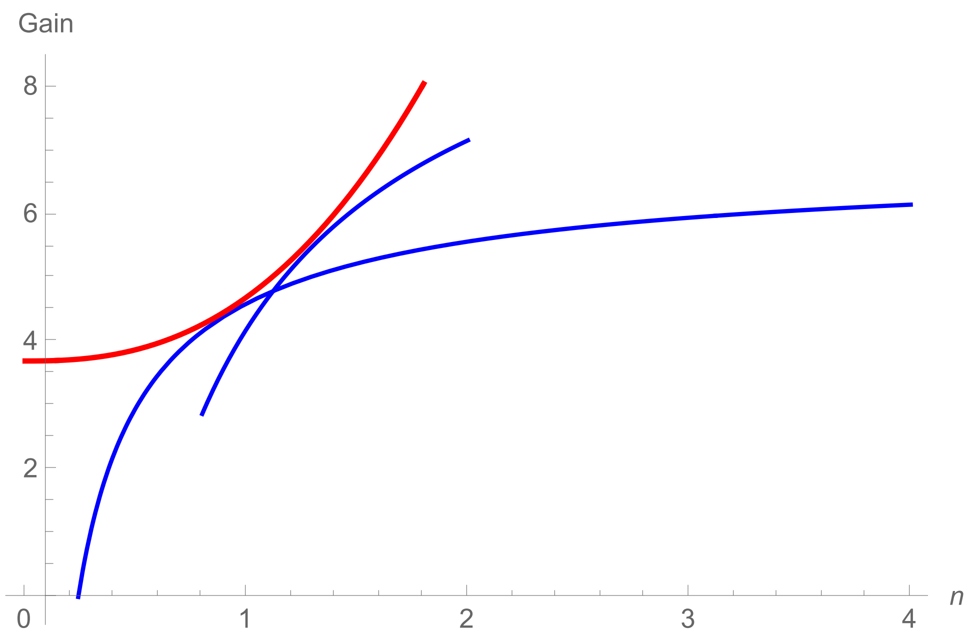


Figure 3

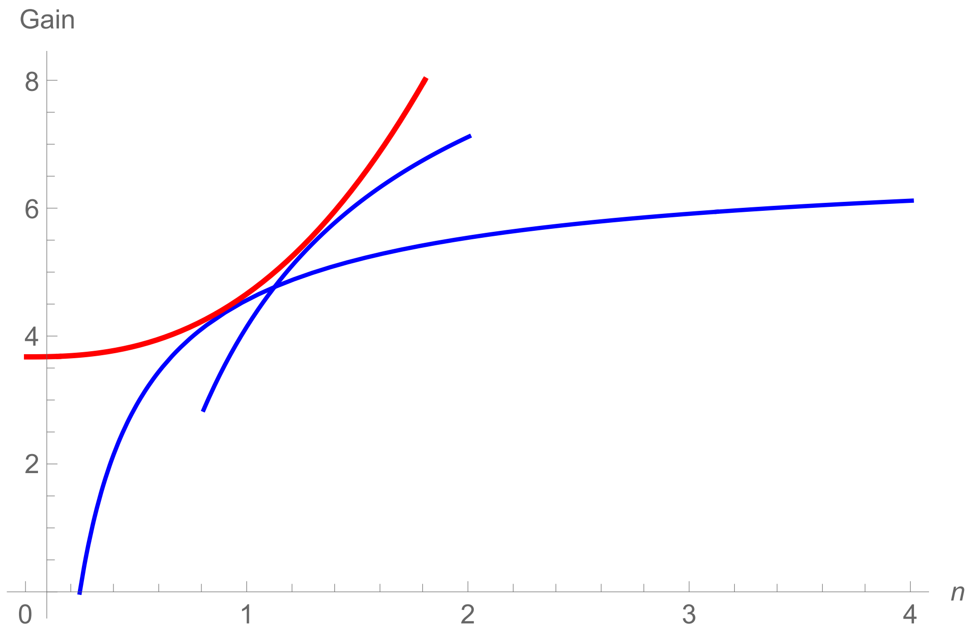


Figure 4

The implication is clear. A social network business has increasing returns.

5 The Value of a Partition

The value of a partition in a fair m-polygon with n = m(m-1)/2 arrows is
(1) $\prod_{i=1}^{n} x[i]$.
In logs this becomes
(2) $\text{Log}[\prod_{i=1}^{n} x[i]] = \sum_{i=1}^{n} \text{Log}\, x[i]$.
Let L[n] denote this function. It is nonnegative if x[i]≥1.
(3) L[n+1] - L[n] = Log x[n+1] + L[n] = L[n] = Log[x[n+1] ≥ 0
Hence L[n] is an increasing function of n. For the second differences
(4) (L[n+2]-L[n+1]) - (L[n+1] - L[n]) = L[n+2]++ L[n] =
= Log[x[n+2] + Log[x[n+1] + 2 L[n] > 0.
Therefore, L[n] is an increasing, convex, non negative, unbounded sequence, for n ≥ 3.

A continuous approximation to L[n] is y = $x^{2+\theta}$

Some Formal Details

Gain per Unit

Programs