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Consensus Commodities

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Introduction

Producers design commodities that appeal to users with different requirements. This poses the problem of finding the best collection of commodities that would maximize their net benefit. Producers decide whether or not a commodity includes a feature desired by some but not by all users. The cost of a feature can differ among producers. Some features may appear in all the commodities but it does not imply different producers offer identical commodities. An upper bound on the quantity of a commodity comes from the number of users, not from the cost conditions. Computer software is the archetype case of these commodities.

Standard economics has little analysis to offer about the market for these commodities. The alternative commodities resemble choice among competing proposals in a legislature. Producers offer their commodities and users choose among them by a process similar to voting in a legislature. Log rolling is a political process. It is present in a legislature that uses majority rule for their decisions. Log rolling creates a market in votes. Different proposals favored by different voters are combined by the legislators into one omnibus bill that passes if at all by nearly a consensus. Conditions sufficient for a non empty core are in Telser (1982).

Each user derives utility from the features in the commodity. If there are n features, then the number of different types of commodities equals 2^n-1. The cost of making a commodity depends on its features not on the number of users. However, the advertising value of a commodity depends on the number of consumers who buy each type. To keep things simple the following analysis assumes there are only 3 features so 7 different types of commodities.

A Realistic Example of Consensus Commodities.

The cost function for a commodity of this type is sub additive in its features. The value function is super additive in its features. Although the advertising value depends on the number of users, the model makes no assumptions about the shape of this function. Ignoring the effect of advertising for now, the net benefit of a commodity, its value minus its cost, is a super additive function of its features. Nothing is said about the relation between the number of users of a given commodity and its value to them. Thus the model does not assume an inverse relation between these two variables.

1. No user pays more for a commodity than its value to him.
2. No commodity is produced at a loss.
3. No user who requires certain features in a commodity is willing to pay more for a commodity that has these additional features.

In the table values are super additive and costs are subadditive. Thus the cost of a commodity with all three features, 100, is less than the sum of the cost of 3 commodities with only one of the 3 features or the sum of the cost of a commodity with 2 of the 3 and one with only 1 of the three features.

type	1	2	3	1, 2	1, 3	2, 3	1, 2, 3
value	7	4	2	12	9	7	13
number of users	d_1	d_2	d_3	$d_{1,2}$	$d_{1,3}$	$d_{2,3}$	$d_{1,2,3}$
all users	D_1	D_2	D_3	$D_{1,2}$	$D_{1,3}$	$D_{2,3}$	$D_{1,2,3}$

The total number of users for each commodity under assumption 3 follows.

$$\mathbf{D_1 = d_1;\ D_2 = d_2;\ D_3 = d_3;}$$
$$\mathbf{D_{1,2} = d_{1,2} + d_1 + d_2;\ D_{1,3} = d_{1,3} + d_1 + d_3;\ D_{2,3} = d_{2,3} + d_2 + d_3;}$$
$$\mathbf{D_{1,2,3} = d_{1,2,3} + d_{1,2} + d_{1,3} + d_{2,3} + d_1 + d_2 + d_3.}$$

commodity type	1	2	3	1, 2	1, 3	2, 3	1, 2, 3
cost	50	40	30	80	70	60	100
cost per user	$50/D_1$	$40/D_2$	$30/D_3$	$80/D_{1,2}$	$70/D_{1,3}$	$60/D_{2,3}$	$100/D_{1,2,3}$
top value	7	4	2	12	9	7	13

Whether cost per user does not exceed the value of the commodity to the user depends how many use that version of the commodity. A producer of a commodity with all three features incurs a total cost of 100.

(1) $D_{1,2,3} = \sum_i^3 d_i + \sum_{i,j=1}^3 d_{i,j} + d_{1,2,3}$.

It becomes a consensus commodity if

(2) $100/D_{1,2,3} \leq \text{Min}\ \{7, 4, 2, 12, 9, 7, 13\}$.

Similar rules apply for a commodity with only 1, 2 or 3 requirements. The amount paid by a user for a consensus commodity does not exceed the least valuable requirement it can satisfy. This assumes no requirement is constrained by a legal barrier to any producer such as a patent or a copyright. The solution depends on the numbers of users of each type given their values of these commodities

$50/D_1 \leq 7$	$40/D_2 \leq 4$	$30/D_3 \leq 2$	$80/D_{1,2} \leq 4$
$7 D_1 \geq 50$	$4 D_2 \geq 40$	$2 D_3 \geq 30$	$4 D_{1,2} \geq 80$

$70/D_{1,3} \leq 2$	$60/D_{2,3} \leq 2$	$100/D_{1,2,3} \leq 2$
$2 D_{1,3} \geq 70$	$2 D_{2,3} \geq 60$	$2 D_{1,2,3} \geq 100$

A commodity attains consensus by offering it cheaply enough to make it acceptable to all potential users.

Enter Advertising

This section is a more abstract description of the economics of a consensus commodity called a package for brevity.

A producer of one of these packages can obtain revenue by selling information about its users to interested firms. Let U(.) denote utility to users, T_i, $T_{i,j}$ $T_{i,j,k}$ denote cost to producers of one of these packages that combine some of these features and S(.) advertising revenue to selected firms. The net cost of package (1,2) becomes $T_{i,j}$ - S($D_{1,2}$) because the producer of this package lowers cost by the revenue received from firm that obtains information about the users of his product.

1. The price per package, π, cannot exceed the least valuable requirement of its users.
2. Hence price per package is nonincreasing in the requirements it can satisfy.
3. There are no side payments among users of the package.

4. All users of the package pay the same amount to its producer, π_i, $\pi_{i,j}$, $\pi_{i,j,k}$, according to the requirements in the package.
5. The total cost of producing the package depends on the requirements it can satisfy but not on the number of users.
6. The total receipts of producers is $\pi_i D_i$, $\pi_{i,j} D_{i,j}$, $\pi_{i,j,k} D_{i,j,k}$, for producer of package with one, two or three requirement.
7. The payments to producers of these packages by firms that obtain information about the users is $S(D_i)$, $S(D_{i,j})$ $S(D_{i,j,k})$. The notation shows these payments increas with the number of users.
8. There are three classes of users, those who want only one requirement i, the who want only two i and j and those who want all three, 1,2,and 3. The utilities for these users are u_i, $u_{i,j}$ and $u_{1,2,3}$. The number of users in each class is d_i, $d_{i,j}$ and $d_{1,2,3}$, The preceding section has the total indicated by upper case D.

We are now ready for some simple algebra. The net revue of producers of these packages are

$$R_i = \pi_i D_i - T_i + S(D_i)$$
$$R_{i,j} = \pi_{i,j} D_{i,j} - T_{i,j} + S(D_{i,j})$$
$$R_{1,2,3} = \pi_{1,2,3} D_{1,2,3} - T_{1,2,3} + S(D_{1,2,3})$$

The net cost of a package decreases with the number of requirements. It follows that the consensus package yields the maximum net revenue if the gross revenue increases with the number of requirement. Nor is this all. Since the payment per user is a non increasing function of the number of requirements a package can satisfy, the net benefit per user is maximum at the consensus package. A package too costly to obtain a consensus without the revenue producers obtain from advertising sold to selected firms may be unable to attain a consensus without this revenue. The presence of the advertising revenue only reenforces the lessons from the theory without advertising.

References

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