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Long distance chiral contribution to the KLKS mass difference

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LONG DISTANCE CHIRAL CONTRIBUTIONS TO THE $K_L - K_S$ MASS DIFFERENCE

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We use chiral symmetry to calculate the very long distance contributions to the $K_L - K_S$ mass difference. We determine that the dominant effect arises from the two-pion dispersive contribution, which by itself produces a large result comparable to or greater than the experimental mass difference (with the correct sign). Some contributions from intermediate distance are also estimated and could be significant. These conclusions reinforce the need, first pointed out by Wolfenstein, to include an additive parameter to account for the long distance contributions in phenomenological studies of the $K_L - K_S$ mass difference. This parameter may be determined from the data if the B-meson lifetime is as large as recent experiments indicate.

The $K_L - K_S$ mass difference ($\Delta m \equiv m_L - m_S$) is one of the most sensitive measures of $\Delta S = 2$ weak interaction processes. It has been frequently been used as an important constraint on the underlying parameter space of electroweak gauge theories [‡1].

In attempting to gain a theoretical understanding of Δm, it is useful to characterize its contributing amplitudes as either short-distance ($r \lesssim m_c^{-1}$), long-distance (say $r > m_\epsilon^{-1}$ where ϵ is the $I = J = 0$ $\pi\pi$ enhancement), or of intermediate range ($m_c^{-1} \lesssim r \lesssim m_\epsilon^{-1}$). The short-distance contribution, calculated from the box diagrams of fig. 1, is quite often the only one employed in phenomenological applications [2]. However as emphasized by Wolfenstein [3] and by Hill [4] there may also be significant effects arising from long-range ("dispersive") contributions associated with low-lying intermediate states. Such long-distance contributions can modify in an important way the results of analyses which omit them. Indeed Wolfenstein has advocated that an additive constant be included in calculations involving Δm in order to at least represent the possibility of dispersive components.

It is our purpose in this paper to calculate the longest distance contributions using the techniques of chiral and SU(3) symmetries. These contributions (which turn out to be large) are not optional; they *must* be accounted for. We will at the end of the paper also comment on how such dispersive contributions modify attempts to constrain the t-quark mass.

Let us first point out that recent measurements of the B-meson lifetime, if correct, allow one to extract the size of the dispersive contribution from Δm. Following refs. [3,4], we introduce a quantity D to parameterize non-box amplitudes,

$$\Delta m = \Delta m_{\text{box}} + D\Delta m \,, \qquad (1)$$

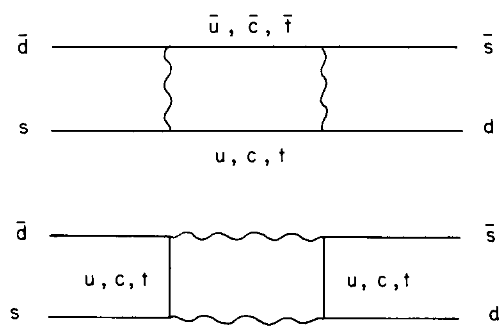

Fig. 1. Short distance contribution to K_L–K_S mixing. Solid (wavy) lines depict the propagation of quark (W-boson) degrees of freedom.

‡1 References to phenomenological uses of the $K_L K_S$ mass difference may be found in ref. [1].

with

$$\Delta m_{\rm box} = \tfrac{1}{3}(G_{\rm F} M_{\rm W} f_{\rm K} m_{\rm K}/\pi)^2 B$$

$$\times \sum_{i,j={\rm u,c,t}} V_{is} V^*_{id} V_{js} V^*_{jd} A_{ij} , \qquad (2)$$

where V_{ij} are the KM matrix elements, A_{ij} are known functions of quark masses [1], and [with $\Gamma^\mu_{\rm L} = \gamma^\mu(1+\gamma^5)$], and

$$B = \tfrac{3}{8}(m_{\rm K} f_{\rm K})^{-2} \langle \bar{\rm K}^0 | \bar{\rm s}\Gamma^\mu_{\rm L}\, {\rm d}\bar{\rm s}\Gamma_{{\rm L}\mu}{\rm d}|{\rm K}^0\rangle . \qquad (3)$$

The parameter B has recently been extracted from $\Delta I = 3/2$ $K\pi\pi$ data by using PCAC and SU(3) symmetry (i.e., the same assumptions employed in this paper) with the result [5] $|B| = 0.33$. There may be some uncertainty in this value (perhaps 50%) due to possible errors induced by the use of PCAC and SU(3), but this is a small effect compared to the uncertainties contained in the dispersive contribution. Now if recent measurements of the B meson lifetime [‡2] are correct we can use eqs. (1)–(2) to constrain the parameter D. This occurs because the B lifetime is evidently so large that it requires the KM angles $\theta_{2,3}$ to be very small. For example, in the limit $\theta_2 \simeq \theta_3 \simeq 0$ (and with $m_{\rm c} = 1.5$ GeV), the charm-quark intermediate state dominates the box diagram and we find

$$1 = \Delta m_{\rm box}/\Delta m + D$$

$$= 0.51\, m_{\rm c}^2 B + D$$

$$= \pm 0.33\ (\pm 50\%) + D , \qquad (4)$$

where the sign ambiguity is associated with the quantity B. Eq. (4) translates into

$$D = +0.67\ (\pm 0.17) , \quad {\rm if}\ B > 0 ,$$

$$= +1.33\ (\pm 0.17) , \quad {\rm if}\ B < 0 . \qquad (5)$$

There is some theoretical evidence that the $B > 0$ case is to be preferred [5]. However, we include the possibility of having the opposite sign for completeness. The inclusion of a t-quark can affect these numbers somewhat, but as long as $\theta_2, \theta_3 < 0.1$ the value of D is not changed by more than 20% [7].

Now that we have seen existing data to be consistent with a nonzero (and even large!) value for the "non-box" component, we turn to the problem of estimating the magnitude of the longest-range contributions to Δm. Clearly, at short distances the appropriate degrees of freedom are quarks and gauge bosons; these are used in the evaluation of the box diagrams. At the longest distances (low energies) however, the theory is best described in terms of the near Goldstone bosons of chiral symmetry, viz., the pions [and if one uses SU(3), K and η]. At intermediate scales phenomenological quark models are useful. Since we wish to study the contributions to Δm which come from very low energy physics, we shall concentrate on the constraints of chiral symmetry. Of course, in the study of Δm, intermediate scales must also be included. Our reason for singling out the low energy contribution is that it can unambiguously signal the importance of long distance effects, since these are relatively model independent. At this point we wish to acknowledge previous studies [8] which even go back to pre-gauge-theory days. Our analysis updates such works and by including the constraint of chiral symmetry, reinforces the belief that dispersive effects are inescapably required. Unfortunately the *overall* value of D does not yet appear to be reliably calculable and presumably must at present be left as a free parameter in phenomenological studies.

The $K_L - K_S$ mass difference is experimentally

$$(m_{\rm L} - m_{\rm S})/\Gamma_{\rm S} \equiv \Delta m/\Gamma_{\rm S} = 0.48 \pm 0.02 . \qquad (6)$$

The sign of the various long distance contributions can easily be understood. Theoretically they are described by

$$\Delta m = \sum_I \left(\frac{|\langle {\rm K_L}|H_{\rm w}|{\rm I}\rangle|^2}{E_{\rm K} - E_{\rm I}} - \frac{|\langle {\rm K_S}|H_{\rm w}|{\rm I}\rangle|^2}{E_{\rm K} - E_{\rm I}} \right) . \qquad (7)$$

If one neglects *CP*-violation, the *CP* quantum numbers of the intermediate states dictate the sign of their contribution. (We remind the reader that K_S has $CP = +1$ and K_L has $CP = -1$.) For example the vacuum intermediate state will increase the K_S mass, but not K_L, making a negative contribution to Δm. The one pion state on the other hand, makes a positive contribution by raising the K_L mass. Other single particle intermediate states, more massive than the kaon, produce $\Delta m < 0$ if they have $CP = -1$ and favor $\Delta m > 0$ if they have $CP = +1$. The two-pion contribution can have either sign, with a negative contribution when $m_{\pi\pi}$

‡2 As cited by Reay in ref. [6].

$< m_K$ and positive when $m_{\pi\pi} > m_K$. However in such circumstances the dispersive part above the pole is almost always stronger, as found below, leading to an overall positive $\pi\pi$ contribution to Δm.

The weak amplitudes which we need involve vertices where a kaon couples to two, one, and zero pseudoscalars. These are all related to each other through the use of the soft meson theorems and PCAC. For pseudoscalars other than the pions one also needs to use SU(3). The momentum dependence of the vertices may be calculated [9] by requiring consistency of all the soft meson limits and imposing a well-known constraint [10] on the vanishing of the matrix element in the SU(3) limit. These observations are neatly summarized in the effective lagrangian which emerges [11] from chiral SU(3)

$$L_{\mathrm{int}} = g\ \mathrm{tr}(\partial_\mu M^+ \partial^\mu M \lambda_6)\ , \tag{8}$$

where g is a constant and

$$M = \exp(\mathrm{i}\phi^{\mathrm{A}}\cdot\lambda^{\mathrm{A}}/\sqrt{2} f_\pi)\ , \tag{9}$$

with ϕ^{A} being the octet of pseudoscalar mesons and f_π is the pion decay constant ($f_\pi \simeq 94$ MeV). This provides a simple, unified description of the $\mathrm{K} \to 3\pi$, $\mathrm{K} \to 2\pi$, $\mathrm{K} \to \pi$, $\mathrm{K} \to$ vacuum amplitudes, and others related to these by SU(3). The full description of kaon amplitudes could also involve effective lagrangians with *more* derivatives, corresponding to higher-order momentum dependence in the weak matrix elements. By dimensional considerations, these will be proportional to inverse powers of the mass scale of chiral symmetry breaking. At energies well below this scale, eq. (8) therefore gives the dominant contribution, and we are justified in taking it as the full effective lagrangian. However, at higher energies, this description will fail, and one either needs to include the higher derivative lagrangian, or modify the description in some other way. We will employ eq. (8) up to a scale Λ, where these corrections are expected to become important, in order to estimate the effect of dispersive contributions below this energy.

The chiral lagrangian contains derivative coupling, corresponding to momentum dependent vertices. In treating this perturbatively the presence of derivative coupling implies modifications of the usual Feynman rules, as pointed out by Gerstein et al. [12] [‡3]. If one divides an arbitrary lagrangian into a free and interaction part, with L_{int} being of order g in some coupling constant, the interaction picture hamiltonian can be found by canonical quantization. The canonical momentum for field ϕ_i is

$$\Pi_i = \dot{\phi}_i + \partial L_{\mathrm{int}}/\partial\dot{\phi}_i \tag{10}$$

and the hamiltonian is formed as usual

$$H = \sum_j \Pi_j \dot{\phi}_j - (L_0 + L_{\mathrm{int}})\ . \tag{11}$$

However $\dot{\phi}_j$ and L must be both expressed in terms of Π_j. Expanding the interaction terms

$$L_{\mathrm{int}}(\dot{\phi}, \phi) = L_{\mathrm{int}}(\Pi, \phi)$$

$$+ \sum_j (\dot{\phi}_j - \Pi_j)(\partial L_{\mathrm{int}}/\partial\dot{\phi}_j)(\Pi, \phi) + \ldots \tag{12}$$

one finds, after some algebra, a simple result at $\mathrm{O}(g^2)$,

$$H_{\mathrm{int}} = -L_{\mathrm{int}}(\Pi, \phi) + \sum_j \tfrac{1}{2} |\partial L_{\mathrm{int}}/\partial\dot{\phi}_j|^2 + \mathrm{O}(g^3)\ . \tag{13}$$

Simultaneously, propagators involving derivative interactions are modified [11], i.e.

$$\Delta(k) = \int \mathrm{d}^4x\ \mathrm{e}^{ikx} \langle 0|T(\phi(x)\,\phi(0))|0\rangle$$

$$= \mathrm{i}/(k^2 + \mathrm{i}\epsilon)\ , \tag{14a}$$

$$\Delta_\mu(k) = \int \mathrm{d}^4x\ \mathrm{e}^{ikx} \langle 0|T(\partial_\mu\phi(x)\phi(0))|0\rangle$$

$$= k_\mu/(k^2 + \mathrm{i}\epsilon)\ , \tag{14b}$$

but

$$\Delta_{\mu\nu}(k) = \int \mathrm{d}^4x\ \mathrm{e}^{ikx} \langle 0|T(\partial_\mu\phi(x)\ \partial_\nu\phi(0))|0\rangle$$

$$= \mathrm{i}[k_\mu k_\nu/(k^2 + \mathrm{i}\epsilon) - g_{\mu 0} g_{\nu 0}]\ . \tag{14c}$$

Both H_{int} and the propagator are not covariant. However the noncovariance will cancel, yielding a covariant physical answer.

The lowest energy intermediate state is the vacuum. However the $\mathrm{K} \to$ vacuum amplitude vanishes in chiral SU(3), as can be seen from the effective lagrangian eq. (8). (The same result is obtained trivially in the valence quark model.) The next lowest energy contribution comes from π^0, η_8 poles (fig. 2a). In the case of single

[‡3] We thank H. Georgi [13] for reminding us of these important facts.

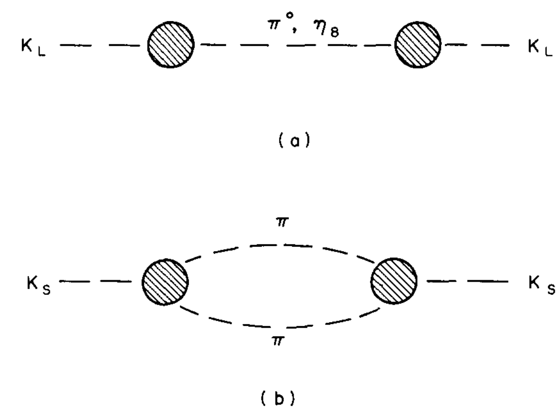

Fig. 2. Long distance contribution to K_L, K_S self-energies. The shaded circles represent the action of the weak hamiltonian.

meson poles, the noncovariant terms cancel exactly, with the result being the same as naive perturbation theory. Let us look at the π^0 pole to see this. We have

$$L_{\rm int} = g'\partial_\mu K_L \partial^\mu \pi^0 \,, \tag{15}$$

$$H_{\rm int} = -g'\partial_\mu K_L \partial^\mu \pi^0 + \tfrac{1}{2} g'^2 (|\dot{K}_L|^2 + |\dot{\pi}^0|^2) \,. \tag{16}$$

To order g'^2 the mass shift is then (k_μ is the momentum of the K_L)

$$2m_K \Delta m = g'^2 [k^4/(k^2 - m_\pi^2) - k_0^2] + g'^2 k_0^2$$

$$= g'^2 k^4/(k^2 - m_\pi^2) \,. \tag{17}$$

In the above, the first terms, in brackets, are the modified propagator while the last piece is the "contact" term in $H_{\rm int}$.

For the present, we employ η_8, the 8th member of the octet, rather than the physical $\eta(549)$ which contains a component of the SU(3) singlet and as such lies partially outside of our framework. The contribution of the single meson poles is

$$2m_K \Delta m = |\langle K_L | H_w | \pi^0 \rangle|^2 / (m_K^2 - m_\pi^2)$$

$$+ |\langle K_L | H_w | \eta_8 \rangle|^2 / (m_K^2 - m_8^2) \,. \tag{18}$$

Use of SU(3) implies

$$\langle K_L | H_w | \pi^0 \rangle = \sqrt{3} \langle K_L | H_w | \eta_8 \rangle \,, \tag{19}$$

so that

$$2m_K \Delta m = [(4m_K^2 - 3m_8^2 - m_\pi^2)/3(m_K^2 - m_\pi^2)(m_K^2 - m_8^2)]$$

$$\times |\langle K_L | H_w | \pi^0 \rangle|^2 \,. \tag{20}$$

However the numerator vanishes by the Gell-Mann–Okubo formula for the mass of the octet isosinglet η_8. Thus we find *no* overall contribution from the single octet particle states. It is this form which was given by Itzykson et al. in 1967, who obtained a nonvanishing value by using the physical η mass in place of m_8. This is not justified if eq. (11) is also used, as η–η' mixing, which shifts the mass of η_8, can also modify the weak amplitude. We return to this point when we discuss intermediate range contributions such as the η'.

The above analysis points to the dominant low energy contribution being the 2π continuum. The 3π effects are weaker, yielding a mass shift proportional to Γ_L. The chiral lagrangian for the $K_S \to 2\pi^0$ amplitude is

$$L_{\rm int} = g_{2\pi}(K_S \partial_\mu \pi \partial^\mu \pi - \pi \partial_\mu K_S \partial^\mu \pi) \,, \tag{21}$$

which yields an interaction hamiltonian

$$H_{\rm int} = -L_{\rm int} + \tfrac{1}{2} g_{2\pi}^2 (2K_S \dot{\pi} - \dot{K} \pi)^2 \,, \tag{22}$$

The coupling constant $g_{2\pi}$ is related to the total $K_S \pi\pi$ decay rate by

$$\Gamma_S = (3/8\pi m_K) g_{2\pi}^2 (m_K^2 - m_\pi^2)^2 (1 - 4m_\pi^2/m_K^2)^{1/2} \,. \tag{23}$$

We calculate the mass shift by studying the K_S self energy $\Sigma(s)$ employing two different methods, (i) use of a Feynman diagram, and (ii) use of a dispersion relation. In both we neglect terms of $O(m_\pi^2/m_K^2)$ in order to simplify the resulting algebra.

In the Feynman diagram approach the process in fig. 2b is easily computed,

$$\Sigma(s) = \frac{g^2}{2} \int \frac{d^4 l}{(2\pi)^4} \{(2l^2 - k^2 - 2k\cdot l)^2$$

$$\times (l^2 - m_\pi^2)^{-1} [(k - l)^2 - m_\pi^2]^{-1} - 4\} \,, \tag{24}$$

where $s = k^2$. The first term above is the result of naive perturbation theory, and the last (i.e. -4) is the summation of the propagator modification and the contact piece in $H_{\rm int}$. The loop integral is divergent, reflecting the fact that the chiral form of the vertex is valid only at very low energies, and fails above the scale of chiral symmetry breaking ‡4. We proceed by cutting off the loop integral at a maximum value $l^2 = \Lambda^2$. If this is done, the form of the self energy is

(adding in also the $\pi^+\pi^-$ contribution)

$$\Sigma(s) = -(3g^2s^2/8\pi^2)[\tfrac{19}{12}\Lambda^2/s + \ln(\Lambda^2/s) + \tfrac{121}{120} - \tfrac{93}{280}s/\Lambda^2 + i\pi] . \quad (25)$$

We note that the effect of the modifications due to derivative couplings is to cancel what would have been a quartic divergence in $\Sigma(s)$ and reduce the answer to a quadratic divergence. This enforces the requirement, demanded by chiral symmetry [‡3], that $\Sigma(s = 0) = 0$. The above calculation leads to a mass shift

$$\Delta m/\Gamma_S = (1/2\pi)[\tfrac{19}{12}(\Lambda/m_K)^2 + \ln(\Lambda/m_K)^2 + \tfrac{121}{120} - \tfrac{93}{140}(m_K/\Lambda)^2] \approx 0.72 , \quad (26)$$

where we have used a rather conservative value of $\Lambda = 700$ MeV (as would be expected if form factor effects damp the loop integral). If we had used $\Lambda = 1$ GeV, the result would have been $\Delta m/\Gamma_S \approx 1.4$.

Our second approach involves the use of the dispersion intergral for the self energy

$$\Sigma(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\Sigma(s')}{s' - s - i\epsilon} .$$

However chiral symmetry requires that $\Sigma(0) = 0$, which implies that we must make one subtraction to satisfy this constraint

$$\Sigma(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds'\, \mathrm{Im}\,\Sigma(s')[(s' - s - i\epsilon)^{-1} - s'^{-1}] . \quad (27)$$

In this case we insert a cutoff in the center of mass integration variable s' at $s' = \Lambda^2$. Again dropping terms $O(m_\pi^2/m_K^2)$, we find

$$\Sigma(s) = -(g^2s^2/8\pi^2)[\Lambda^2/s + \ln(\Lambda^2/s - 1) + i\pi] \quad (28)$$

and

$$\Delta m/\Gamma_S = (1/2\pi)\{(\Lambda/m_K)^2 + \ln[(\Lambda/m_K)^2 - 1]\} . \quad (29)$$

The fact that the two expressions for Δm do not agree in form is to be expected, as the meaning of the cutoffs is different in the two cases. In the latter approach the choice of Λ is perhaps somewhat clearer. It indicates a CM scale at which corrections to our chiral symmetry representation begin to become significant. It cannot be chosen too low, as the strength of the vertex at $s' = m_K^2$ is determined by the experimental decay rate. We feel that it is conservative to choose the cutoff not too far above m_K, say at the mass of the $I = J = 0$ $\pi\pi$ enhancement, $\Lambda \simeq m_\epsilon \simeq 0.7$ GeV, for which we find $\Delta m/\Gamma_s \simeq 0.32$. For $\Lambda = 1$ GeV, $\Delta m/\Gamma_S = 0.71$. The cutoff at which $\Delta m = 0$ is so low ($\Lambda \simeq 0.56$ GeV) that any reasonable choice of Λ does not affect our qualitative conclusion, viz., the long-range dispersive effect is both large and of the correct sign.

Incidentally, use of the full pseudoscalar octet would require the inclusion of $\pi^0\eta$, K^+K^- and $K^0\bar{K}^0$ intermediate states in addition. However in the dispersive analysis, with our rather low value of the cutoff, these contributions would all be absent as their threshold is at or above the cutoff. In the Feynman diagram approach this would increase $\Delta m/\Gamma_S$ somewhat. The results are more complex and we do not feel the need to quote them.

It would not be surprising that intermediate-range contributions are also important. We wish to provide one estimate of such effects by refining the estimate of the π^0–η_8 contributions to include the η. The SU(3) octet η_8 and singlet η_0 mix to produce the physical η and η' states. We can model this by considering the mass matrix

$$m^2 = \begin{pmatrix} m_8^2 & m_{08}^2 \\ m_{08}^2 & m_0^2 \end{pmatrix},$$

with $m_8^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2$, and m_0^2 is the (unknown) bare mass of the SU(3) singlet and m_{08}^2 is a mixing parameter. Fitting the physical η and η' masses we find $m_0^2 = 0.903\ \mathrm{GeV}^2$. $|m_{08}^2| = 0.106\ \mathrm{GeV}^2$. This mixing also modifies the η_8 contribution to Δm. Generalizing the mixing with the K_L one has

$$m^2 = \begin{pmatrix} m_K^2 & a & b \\ a^* & m_8^2 & m_{08}^2 \\ b^* & m_{08}^2 & m_0^2 \end{pmatrix}, \quad (30)$$

‡4 The quadratic divergence is due to the momentum dependence of the chiral coupling. However sizable results are obtained even with a coupling constant independent of momentum. In this case we would obtain instead of eq. (16) $\Delta m/\Gamma = (1/2\pi)[\ln \Lambda^2/m^2 + 1 - m^2/3\Lambda^2] = 0.24$, for $\Lambda = 700$ MeV.

where

$$a \equiv \langle K_L | H_w | \eta_8 \rangle = (1/\sqrt{3}) \langle K_L | H_w | \pi^0 \rangle ,$$

$$b \equiv \langle K_L | H_w | \eta_0 \rangle \equiv -(2\sqrt{2}\,\rho/\sqrt{3}) \langle K_L | H_w | \pi^0 \rangle . \quad (31)$$

The parameter b (or ρ) is unfortunately not fixed by any symmetry argument. However penguin dominance of H_w plus the quark model suggests $\rho = 1$ which is why we have normalized b as above. In any case, diagonalizing the mass matrix and adding the π^0 term one finds

$$2m_K \Delta m = \{(m_K^2 - m_\pi^2)^{-1} - [\tfrac{1}{3}(m_0^2 - m_K^2) + \tfrac{8}{3}\rho^2 (m_8^2 - m_K^2)] \times [(m_8^2 - m_K^2)(m_0^2 - m_K^2) - m_{08}^4]^{-1}\} \, |\langle K_L | H_w | \pi^0 \rangle|^2 . \quad (32)$$

Using the fit values of m_0^2 and m_{08}^2 and the weak K $\to \pi$ matrix element

$$|\langle K_L | H_w | \pi^0 \rangle|^2 = \tfrac{32}{3} \pi F_\pi^2 m_K \Gamma_S , \quad (33)$$

one obtains

$$\Delta m/\Gamma_S = -(0.20 + 0.78\,\rho^2) = -0.98 \; (\rho^2 = 1) . \quad (34)$$

We have been expanding $\Sigma(s)$ in a loop expansion, including up to one loop order. The constraint $\Sigma(0) = 0$ is maintained at each order. Our result unfortunately depends strongly on the parameter Λ which represents the scale where higher order corrections to the low energy effective lagrangian become important. If properly included these corrections would lead to a finite total answer. Our calculation does provide an indication of the overall size of long distance effects. We have found large dispersive contributions of both signs, individual terms ranging from $D = -2$ to $D = 2.8$. It is certainly unrealistic to expect that these contributions would exactly cancel and there are clearly other intermediate range effects which we have not considered. However, our results do support a large value for the dispersive contribution, as also seems to be indicated by the data. The inclusion of these low-energy effects modifies many phenomenological analyses. The additive constant D *cannot* be simply modeled by an effective B parameter, because the B parameter multiplies *all* contributions, including that of the top quark, while D is independent of heavy quark parameters. The presence of D implies that most of the uses of Δm in attempting to bound the top quark mass are invalid. We shall report elsewhere on the ways that dispersive contributions modify the analysis of *CP* violation.

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