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Direct quark-model calculation of weak nonleptonic matrix elements

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It is shown how wave-packet techniques developed by Donoghue and Johnson may be utilized to calculate parity-violating nonleptonic hyperon decay amplitudes directly—without the use of current algebra and PCAC (partial conservation of axial-vector current).

Remarkable progress has occurred during the past two decades in our understanding of weak nonleptonic phenomena. In the early 1960s it could reasonably be stated that there existed virtually no contact between weak nonleptonic phenomenology and the underlying theory which attempted to explain it. By 1967 the situation had changed dramatically and current algebra with PCAC (partial conservation of axial-vector current) enabled (i) a calculation of $K\to 3\pi$ in terms of experimental $K\to 2\pi$ amplitudes[1] and (ii) relations between various parity-violating hyperon decay amplitudes.[2] However, no absolute theoretical predictions—calculations independent of empirical weak-decay information—were possible. The latter situation changed during the mid-1970s with the availability of quark models, most noticeably the MIT bag model.[3] Baryon-to baryon and meson-to-meson matrix elements of the weak Hamiltonian could then be calculated directly which, together with current algebra and PCAC, enabled predictions to be made for experimental amplitudes.[4]

We wish in this paper to demonstrate that the calculations of nonleptonic decay parameters can be carried out *entirely* within the quark model, without the use of current algebra and PCAC. Use of quark-model techniques together with a few small assumptions enables a direct theoretical calculation of

$$\langle B'\pi \,|\, H_w \,|\, B\rangle \text{ and } \langle \pi\pi \,|\, H_w \,|\, K\rangle$$

amplitudes. The key to this work is the use of the wave-packet method to describe bag-model states.[5] This procedure allows the calculation of matrix elements with different numbers of particles in the initial and final states.

The original application was to $\langle 0 \,|\, A_\mu \,|\, \pi\rangle$, i.e., the calculation of F_π. Recently the technique has been applied to the direct evaluation of $g_{\pi NN}$, $g_{\pi N\Delta}$, and $g_{\rho NN}$ coupling constants, when supplemented with the use of the Bogoliubov transformation.[6] These applications have been reasonably successful, despite the approximation involved, which suggests that similar ideas may be utilized in the calculations of other processes, such as weak nonleptonic matrix elements. Note that we do not mean to imply that the use of PCAC is a problem to be avoided. Indeed PCAC is on a much firmer theoretical footing than the quark-model techniques which we employ in this paper. However, it is satisfying to note that the matrix elements may be understood entirely within a quark-model context.

As a test case consider the parity-violating (PV) baryon decay

$$\langle B'\pi \,|\, H_w^{\rm PV} \,|\, B\rangle \; .$$

One cannot hope to treat the parity-conserving (PC) matrix element in this fashion, since the existence of baryon-pole terms implies the importance of long-distance contributions which cannot be handled by this simple projection technique.[2] On the other hand, the Swift-Lee theorem guarantees that such pole (long-distance) terms will not play an important role in parity-violating decay.[7]

To be definite, consider the transition $\Lambda\to N\pi$. We shall utilize the MIT bag model for our specific calculation, but many of our conclusions are independent of the particular quark model employed. We begin with a bag-model Λ state constructed from these quarks in their lowest-energy $(1S_{1/2})$ levels

$$|\,\Lambda\uparrow\rangle = \frac{\epsilon^{ijk}\epsilon^{tt'}}{\sqrt{12}} b_{ut}^{i\dagger} b_{dt'}^{j\dagger} b_{s\uparrow}^{k\dagger}\,|\,0\rangle \; , \tag{1}$$

where $b_{fs}^{i\dagger}$ is a creation operator for a quark of flavor f, color index i, and spin projection s along some axis of quantization. Now operate on this state with the weak Hamiltonian

$$H_w \sim :\bar\psi_d \gamma_\mu (1+\gamma_5)\psi_u \bar\psi_u \gamma^\mu (1+\gamma_5)\psi_s: \; . \tag{2}$$

Here each field operator ψ contains both a quark destruction operator b_s *and* an antiquark creation operator $d_f^\dagger$. Thus, when H_w operates upon the baryon $|\,\Lambda\uparrow\rangle$ it must produce either a $3q$ state (when both ψ_s *and* ψ_u annihilate the initial u,s quarks) or a $4q\bar q$ state (when either ψ_s *or* ψ_u annihilates an initial quark). Noting that

$$\begin{aligned}
&\langle q' \,|\, V_0 \,|\, q\rangle = uu'+vv', \quad \langle q'\bar q \,|\, V_0 \,|\, 0\rangle = (u'v+v'u)\vec\sigma\cdot\hat r \; , \\
&\langle q' \,|\, \vec{\rm V} \,|\, q\rangle = -(vu'+uv')\vec\sigma\times\hat r, \quad \langle q'\bar q \,|\, \vec{\rm V} \,|\, 0\rangle = i(uu'+vv')\vec\sigma - i2vv'\hat r\vec\sigma\cdot\hat r \; , \\
&\langle q' \,|\, A_0 \,|\, q\rangle = i(u'v-uv')\vec\sigma\cdot\hat r, \quad \langle q'\bar q \,|\, A_0 \,|\, 0\rangle = -i(uu'-vv') \; , \\
&\langle q' \,|\, \vec{\rm A} \,|\, q\rangle = -(uu'-vv')\vec\sigma - 2vv'\hat r\vec\sigma\cdot\hat r, \quad \langle q'\bar q \,|\, \vec{\rm A} \,|\, 0\rangle = -(uv'+u'v)\hat r \; ,
\end{aligned} \tag{3}$$

when the quark and antiquark ground-state wave functions are represented as

$$q(r)=\begin{bmatrix} iu(r)\chi \\ v(r)\vec{\sigma}\cdot\hat{r}\chi \end{bmatrix}, \quad \bar{q}(r)=\begin{bmatrix} iv(r)\vec{\sigma}\cdot\hat{r}\chi \\ u(r)\chi \end{bmatrix}, \tag{4}$$

we see that in the SU(3) limit we have

$$\begin{aligned}
&\langle qq \,|\, H_w^{\rm PC} \,|\, qq \rangle \sim \int d^3r[u^2(r)+v^2(r)]^2(\mathbb{1}\times\mathbb{1}-\vec{\sigma}\cdot\vec{\sigma}) , \\
&\langle qq\bar{q} \,|\, H_w^{\rm PC} \,|\, q \rangle \sim 0 , \\
&\langle qq \,|\, H_w^{\rm PV} \,|\, qq \rangle \sim 0 , \\
&\langle qq\bar{q} \,|\, H_w^{\rm PV} \,|\, q \rangle \sim -i \int d^3r[u^4(r)-v^4(r)](\mathbb{1}\times\mathbb{1}-\vec{\sigma}\cdot\vec{\sigma}) .
\end{aligned} \tag{5}$$

We note then that the parity-conserving weak Hamiltonian $H_w^{\rm PC}$ connects *only* to final $3q$ states while its parity-violating counterpart $H_w^{\rm PV}$ connects *only* to the $qqq+q\bar{q}$ sector in accord with the Swift-Lee theorem.[7] That is (here $|\,\rangle_B$ refers to a bag state),

$${}_B\langle B' \,|\, H_w^{\rm PV} \,|\, B \rangle_B = 0 . \tag{6}$$

Also, it is apparent that matrix elements of

$${}_B\langle B' \,|\, H_w^{\rm PC} \,|\, B \rangle_B \text{ and } {}_B\langle B'\pi \,|\, H_w^{\rm PV} \,|\, B \rangle_B$$

are related by a calculable constant, as given by current algebra and PCAC.

In order to determine the constant we employ the procedure of Donoghue and Johnson,[5] which represents bag states as superpositions of plane-wave eigenstates, with a wave packet $\phi(p)$ for mesons or $\chi(p)$ for baryons:

$$\begin{aligned}
|\,\pi,\vec{x}\,\rangle &= \int \frac{d^3p}{2\omega_p}\phi(p)e^{ip\cdot x}\,|\,\pi(p)\rangle , \\
|\,N,\vec{x}\,\rangle &= \int \frac{d^3p}{E_p} m_N \chi(p)e^{ip\cdot x}\,|\,N(p)\rangle .
\end{aligned} \tag{7}$$

If we write the final $|\,N\pi\,\rangle$ state as a direct product of bag wave functions

$$|\,p\uparrow\pi^-\,\rangle = \frac{1}{\sqrt{18}}\epsilon^{ijk}\epsilon^{tt'}\frac{1}{\sqrt{6}}\delta^{mn}\delta^{rr'}b_{ut}^{i\dagger}b_{dt'}^{j\dagger}b_{u\uparrow}^{k\dagger}b_{dr}^{m\dagger}d_{\bar{u}r'}^{n\dagger}\,|\,0\rangle , \tag{8}$$

then the matrix element factorizes into spin-color and radial pieces

$$\begin{aligned}
{}_B\langle \pi^- p \,|\, H_w^{\rm PV} \,|\, \Lambda \rangle_B &= \int d^3x \int \frac{d^3k}{2\omega_k}\phi(k)e^{-i\vec{k}\cdot\vec{x}} \\
&\quad\times \int d^3p'\frac{m_N}{E'}\chi^*(p')e^{-i\vec{p}\,'\cdot\vec{x}} \int \frac{d^3p}{E_\Lambda}m_\Lambda\chi(p)e^{i\vec{p}\cdot\vec{x}}\langle p(p')\pi^-(k) \,|\, H_w^{\rm PV} \,|\, \Lambda(p)\rangle \\
&= -i \int d^3r[u^2(r)+v^2(r)][u^2(r)-v^2(r)]M_3 ,
\end{aligned} \tag{9}$$

where

$$M_3 = \frac{1}{\sqrt{18\times12\times6}}\epsilon^{ijk}\epsilon^{tt'}\epsilon^{abc}\epsilon^{qq'}\delta^{mn}\delta^{rr'}\langle 0 \,|\, d_{\bar{u}r'}^{n}b_{dr}^{m}b_{u\uparrow}^{c}b_{dq'}^{b}b_{uq}^{a}(\delta^{pp'}\delta^{00'}-\vec{\sigma}^{\,pp'}\cdot\vec{\sigma}^{\,00'}){:}b_{dp}^{f\dagger}d_{\bar{u}p'}^{f}b_{u0}^{g\dagger}b_{s0'}^{g}{:}b_{ut}^{i\dagger}b_{dt'}^{j\dagger}b_{s\uparrow}^{k\dagger}\,|\,0\rangle . \tag{10}$$

On the other hand we note

$$\begin{aligned}
{}_B\langle n\uparrow \,|\, H_w^{\rm PC} \,|\, \Lambda\uparrow \rangle_B &= \int d^3x \int \frac{d^3p'}{E}m_N\chi^*(p)e^{-i\vec{p}\,'\cdot\vec{x}} \int \frac{d^3p}{E}m_\Lambda\chi(p)e^{i\vec{p}\cdot\vec{x}}\langle n(p') \,|\, H_w^{\rm PC} \,|\, \Lambda(p)\rangle \\
&= \int d^3r[u^2(r)+v^2(r)]^2M_2 ,
\end{aligned} \tag{11}$$

where

$$M_2 = \frac{1}{\sqrt{18\times12}}\epsilon^{ijk}\epsilon^{tt'}\epsilon^{abc}\epsilon^{qq'}\langle 0 \,|\, b_{d\uparrow}^{c}b_{dq'}^{b}b_{uq}^{a}(\delta^{pp'}\delta^{00'}-\vec{\sigma}^{\,pp'}\cdot\vec{\sigma}^{\,00'}){:}b_{dp}^{f\dagger}b_{up'}^{f}b_{uq}^{g\dagger}b_{sq'}^{g}{:}b_{ut}^{i\dagger}b_{dt'}^{j\dagger}b_{s\uparrow}^{k\dagger}\,|\,0\rangle . \tag{12}$$

However, it is straightforward to show that

$$M_3 = -\frac{1}{\sqrt{6}}M_2 \tag{13}$$

so we have

$$\begin{aligned}
{}_B\langle p\pi^- \,|\, H_w^{\rm PV} \,|\, \Lambda \rangle_B &= \frac{i}{\sqrt{6}} \int d^3r[u^4(r)-v^4(r)]M_2 \\
&= \int \frac{d^3k}{2\omega_k} \int d^3p \frac{m_\Lambda}{E(p)}\frac{m_N}{E'(p-k)}(2\pi)^3\phi(k)\chi^*(p-k)\chi(p)\langle \pi^-(k)p(p-k) \,|\, H_w^{\rm PV} \,|\, \Lambda(p)\rangle
\end{aligned} \tag{14}$$

and

$$
\begin{aligned}
{}_B\langle n \,|\, H_w^{\mathrm{PC}} \,|\, \Lambda \rangle_B &= \int d^3r[u^2(r)+v^2(r)]^2 M_2 \\
&= \int \frac{d^3p}{E(p)} m_\Lambda \frac{m_N}{E'(p)} (2\pi)^3 \chi^*(p)\chi(p) \langle n(p) \,|\, H_w^{\mathrm{PC}} \,|\, \Lambda(p) \rangle \; .
\end{aligned}
\tag{15}
$$

Thus, if we are willing to neglect momentum dependence for $\langle \pi^-(k)p(p') \,|\, H_w^{\mathrm{PV}} \,|\, \Lambda(p) \rangle$ we find

$$
\begin{aligned}
\langle \pi^-(k)p(p') \,|\, H_w^{\mathrm{PV}} \,|\, \Lambda(p) \rangle &= \frac{i}{\sqrt{6}} \frac{\int d^3r[u^4(r)-v^4(r)]}{\int d^3r[u^2(r)+v^2(r)]^2} {}_B\langle n \,|\, H_w^{\mathrm{PC}} \,|\, \Lambda \rangle_B \\
&\quad \times \left[\frac{1}{\int (d^3k/2\omega_k) \int d^3p[m_N m_\Lambda / E(p)E'(p-k)](2\pi)^3 \phi(k) \chi^*(p-k)\chi(p)} \right]
\end{aligned}
\tag{16}
$$

and

$$
{}_B\langle n \,|\, H_w^{\mathrm{PC}} \,|\, \Lambda \rangle_B = \langle n(p) \,|\, H_w^{\mathrm{PC}} \,|\, \Lambda(p) \rangle \int d^3p \, \frac{m_\Lambda m_N}{E(p)E'(p)} (2\pi)^3 \chi^*(p)\chi(p) \; . \tag{17}
$$

We thus have derived the quark-model equivalent of the PCAC relation

$$
\langle \pi^-(k)p(p') \,|\, H_w^{\mathrm{PV}} \,|\, \Lambda(p) \rangle \cong \frac{i}{\sqrt{2}F_\pi} \langle n(p) \,|\, H_w^{\mathrm{PC}} \,|\, \Lambda(p) \rangle \tag{18}
$$

with $F_\pi = 94$ MeV replaced by

$$
F_{\mathrm{th}} = \sqrt{3} \frac{\int d^3r[u^2(r)+v^2(r)]^2}{\int d^3r[u^4(r)-v^4(r)]} \frac{\int (d^3k/2\omega_k) \int d^3p[m_N m_\Lambda / E(p)E'(p-k)] \phi(k) \chi^*(p-k)\chi(p)(2\pi)^3}{\int d^3p[m_N m_\Lambda / E(p)E'(p)] \chi^*(p)\chi(p)(2\pi)^3} \; . \tag{19}
$$

In order to compute F_{th} we require models for the wave functions. Donoghue and Johnson have suggested specific forms. However, for our purposes it is enough to use approximate expressions

$$
\begin{aligned}
\phi(k) &= C_\pi e^{-\alpha_\pi^2 k^2} \; , \\
\chi_N(p) &\cong \chi_\Lambda(p) = C_N e^{-\alpha_N^2 p^2} \; .
\end{aligned}
\tag{20}
$$

The wave functions are subject to the normalization conditions

$$
\begin{aligned}
1 &= \int d^3p \frac{m_N}{E(p)} (2\pi)^3 \,|\, \chi(p) \,|^2 \\
&= \int d^3k \frac{1}{2\omega_k} (2\pi)^3 \,|\, \phi(k) \,|^2
\end{aligned}
\tag{21}
$$

so that approximating $E(p) \approx m_N$ and $\omega_k \approx k$ we determine

$$
\begin{aligned}
C_N &= \frac{1}{(2\pi)^2} \frac{2\alpha_N^{3/2}}{\pi^{1/4}} 2^{1/4}, \quad \alpha_N = \frac{1}{\pi} R_N (\ln 2)^{1/2} \; , \\
C_\pi &= \frac{\alpha_\pi}{2\pi^2}, \quad \alpha_\pi = \frac{1}{\pi} R_\pi (\ln 2)^{1/2} \; .
\end{aligned}
\tag{22}
$$

The wave-function integrals can now be evaluated, yielding

$$
\begin{aligned}
\int \frac{d^3k}{2\omega_k} \int d^3p \frac{m_N m_\Lambda}{E(p)E'(p-k)} \phi(k) \chi^*(p-k) \chi(p) (2\pi)^3 &\cong \frac{\pi C_\pi}{\alpha_\pi^2 + \frac{1}{2}\alpha_N^2} \; , \\
\int d^3p \frac{m_N m_\Lambda}{E(p)E'(p)} \,|\, \chi(p) \,|^2 (2\pi)^3 = 1 \; , &
\end{aligned}
\tag{23}
$$

and

$$
F_{\mathrm{th}} = \frac{\sqrt{3}}{2R_\pi (\ln 2)^{1/2}} \frac{1}{1 + \frac{1}{2}(R_N^2/R_\pi^2)} \times \frac{\int d^3r[u^2(r)+v^2(r)]^2}{\int d^3r[u^4(r)-v^4(r)]} \; . \tag{24}
$$

Use of the conventional values

$$
R_N = 5.5 \text{ GeV}^{-1}, \quad R_\pi = 3.5 \text{ GeV}^{-1} \tag{25}
$$

yields

$$
F_{\mathrm{th}} = 1.38 F_\pi \frac{\int d^3r[u^2(r)+v^2(r)]^2}{\int d^3r[u^4(r)-v^4(r)]} \; . \tag{26}
$$

In the nonrelativistic limit we have $v(r) = 0$ and

$$
F_{\mathrm{th}} = 1.38 F_\pi \; , \tag{27}
$$

while using bag-model wave functions

$$
u(r) = N j_0(\omega r), \quad v(r) = -N j_1(\omega r) \; , \quad \omega = 2.04/R \tag{28}
$$

with N a normalization constant defined in Ref. 4, we find

$$
F_{\mathrm{th}} = 2.5 F_\pi \; . \tag{29}
$$

Agreement is reasonable, given the approximations involved.

In order to understand the reliability of our result, we note the existence of at least two factors which could modify the calculated value of F_{th}. The procedure does not include any measure of bag fissioning. If this introduced a suppression in the amplitude, then the value of F_{th} would be larger. We have not been able to obtain any estimate of the effect, although previous results do not suggest this is a large effect.[6] Also neglected was any momentum dependence for the $B \to B'\pi$ vertex. One would expect that this vertex should have a dependence on k^2, the pion momentum, that could be parametrized by a form factor $F(k^2)$. Inclusion of this term would lower the calculated value of $F_{\rm th}$. To estimate the size of this decrease we can choose a simple form for $F(k^2)$, namely,

$$F(k^2)=e^{-\beta^2 k^2}\,, \tag{30}$$

where β can be fixed by requiring that the form factor have the same expansion about $k^2=0$ as does the form factor of the axial-vector current,[8] i.e.,

$$\beta^2=\tfrac{1}{6}\langle r^2\rangle_{\rm axial}=0.09R_N{}^2\ . \tag{31}$$

The integral in Eq. (23) now becomes

$$\int \frac{d^3k}{2\omega_k}\int d^3p \frac{m_N m_\Lambda}{E(p)E'(p-k)}\phi(k)\chi^*(p-k)\chi(p)F(k^2)(2\pi)^3=\frac{\pi C_\pi}{\alpha_\pi{}^2+\frac{1}{2}\alpha_N{}^2+\beta^2} \tag{32}$$

and the final value [Eq. (29)] now reads

$$F_{\rm th}=0.97F_\pi\ . \tag{33}$$

Agreement is obviously excellent. However, we have introduced this discussion not to obtain the right answer but rather to emphasize some of the inherent uncertainties in our result.

The direct calculation of the weak-decay matrix elements can now proceed in a manner similar to evaluations which use PCAC.[4] The main features have already been pointed out, in a somewhat different context, by Desplanques and the authors.[9] A complete correspondence can be seen between the diagrams of $B\to B'\pi$ and those obtained via PCAC and $B\to B'$. In particular, the Clebsch-Gordan coefficients are the same, so the relative weights of the diagrams are unchanged. The Pati-Woo theorem, which states that $B\to B'$ is purely $\Delta I=\frac{1}{2}$, also carries over to the calculation of $B\to B'\pi$, so that a $\Delta I=\frac{1}{2}$ rule is obtained.[10]

We have thus utilized the wave-packet technique developed in the bag model to derive a relation between $B\to B'\pi$ matrix elements and $B\to B'$ amplitudes which is similar in form to that of PCAC, and numerically agrees to within a factor of about 2. This then provides a *direct* quark-model calculation of the hyperon decay amplitudes. However, despite the interest in such a procedure, the uncertainties are sizable, and the most reliable method remains the standard techniques involving PCAC.

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