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Project management decisions with uncertain targets

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Abstract

Project management decision rules presume that fixed and inflexible targets have been defined for the project. If a project's slack is defined as the difference between actual project performance and these targets, then these decision rules can be characterized as maximizing the probability that slack is non-negative (i.e., maximizing the probability of meeting the targets). These rules rely on z-scores to compare uncertain performance to target levels. Following these decision rules will not always suffice for the project manager to act consistently with customer preferences. In particular, actual requirements may be uncertain or subject to change, and customers may have some flexibility. A decision analytic approach accounting for these factors can allow the project manager to maximize the customer's expected utility. We redefine project slack to reflect the difference between performance and a random target that reflects both the customer's risk tolerance and uncertainty about the actual requirement. The z-score associated with this slack is shown to be proportional to the certainty equivalent for a project. Thus utility maximizing decision rules in the language of decision analysis can be readily translated into z-score maximizing decision rules in the language of project management. From this, we discuss how related decision analytic concepts such as value of information might be applied to families of problems in project management.

1 Introduction

Project management (PM) is a broad discipline encompassing numerous techniques for assuring the successful completion of projects. This discipline, as presented in the Project Management

Body of Knowledge (PMBOK, Project Management Institute, 2013) sets requirements, develops plans for activities, allocates resources to those activities in order to satisfy the requirements, and monitors and controls those activities. Furthermore, the certified project manager, as a professional, is committed to a code of ethics (Project Management Institute, 2014) which references the duty of loyalty, i.e., acting in the best interests of the client, and this paper presumes that the project manager wishes to use decision rules that are consistent with those interests. While decision analysts evaluate alternatives based on the certainty equivalents of the prospects associated with them, PM typically uses simple z-scores to evaluate the prospects of plans in the face of uncertainty. This paper reconciles these two views by defining z-scores and decision rules based on them that are consistent with the principles of decision analysis and dutifully incorporate additional information the customer's about risk tolerances and requirements.

With many moving parts, project management is complex. For this reason, quantitative approaches to project management decision making tend to focus on deterministically optimizing under fixed constraints. An example of this is the Critical Path Method, or CPM (Kelley, 1963; PMBOK p. 176). CPM assumes completion times for a set of interdependent activities. It identifies those critical activities whose acceleration will accelerate the completion of the project. It then allocates additional resources to critical activities to the point that the project will be completed on time.

In practice, much is uncertain in projects. For example, activities might have better or worse results than assumed. Tools have been developed to deal with this kind of performance uncertainty, e.g., project risk management ascribes uncertain completion times to individual activities, and from these calculates uncertain project completion times. To manage project risk, the Program Evaluation and Review Technique, or PERT (Malcom et al, 1959), which includes these uncertainties, is often combined with CPM so that resources can be allocated to activities with the aim of reducing the risk that a project will fail to meet its requirements.

Such formalized approaches still face practical challenges arising from the following two implicit assumptions:

Assumption 1: *Project requirements are inflexible, with no additional benefit for performance that exceeds the requirements rather than just meeting them, and no benefit for performance that almost meets the requirement rather than falling far short of it.* But this may not reflect customers' actual preferences.

Assumption 2: *Project requirements are known and fixed.* But in fact, requirements can change. Since PM recognizes this, PM's qualitative processes for managing changes are well-developed.

While the two assumptions simplify analysis, they can lead to imprecise analysis and to time-consuming follow up efforts that may involve costly course corrections.

Decision analytic techniques can potentially incorporate uncertainties and preferences into PM decision making. In fact, there have been significant efforts to apply decision analysis to PM (e.g., Powell and Buede, 2009, Schuyler, 2001, Virine and Trumper, 2007), and decision trees are known to PM (e.g., PMBOK, p.339). But the use of decision analysis in such cases has been an augment to – and largely distinct from – traditional PM practice.

This paper develops an approach that treats project requirements as changing or uncertain, and treats customer preferences more flexibly than traditional PM. The approach used draws on target oriented utility (TOU), which recognizes a kind of equivalence between utility functions and the cumulative distribution function (cdf) of random variables. Target-oriented utility was originally interpreted in terms of uncertain requirements in Castagnoli and LiCalzi (1996) and Bordley and LiCalzi(2000), as well as in multiattribute extensions by Bordley and Kirkwood (2004), Tsetlin and Winkler (2006), Tsetlin and Winkler (2007). Abbas and Matheson (2005, 2009) emphasized however that the interpretation of utility as a cumulative probability distribution reflected a utility/probability duality which does not require that utility be interpreted in terms of uncertain requirements. Consistent with this research, this paper presents an integrated treatment which includes both the case in which the customer has requirements about which a project manager is uncertain as well as the case in which the customer has a utility function. For example, a continuous utility function for performance on a metric can be replaced by (or can replace) a continuous random variable where there is utility of 1 for performance meeting or exceeding the value of this variable and utility of 0 for performance that falls short of it. We shall see that this idea fits naturally with PM formulations which project managers find comfortable and, presumably, are not eager to change. It allows results that extend standard formulations, especially when we assume suitable Gaussian distributions, and thus may be relatively painless to incorporate into existing practice.

The rest of the paper is structured as follows. In section 2, we develop the simple example of a decision between two alternative project plans. In this setting, we motivate and define key terms and and illustrate the basic ideas with a set of numerical examples. Section 3 presents results involving several types of decision analytic value calculations including certainty equivalents, marginal utility, and value of information. In section 4, we summarize the potential effects on practice of this approach, and point to directions for expanding it.

2 Uncertain target approach

2.1 Basic model

Consider a simple PM decision: choosing a plan with uncertain project performance. Under Assumptions 1 and 2 above, the objective used by the project manager is equivalent to maximizing the probability of meeting the customer's fixed targets. This section assumes that project performance is evaluated on the basis of one dimension, e.g., completion time, with all of the other dimensions (e.g., cost, quality, etc. being presumed known). But this approach will also apply if there is a more general performance measure that is a function of cost, quality, etc. To fix an example in mind, we quantify performance as the negative of project completion time (in order to have a metric for which higher scores are better).

A standard approach, consistent with PMBOK, makes the following assumption:

Assumption 0 is that the project manager's uncertainty about project performance is sufficiently approximated by a Gaussian distribution.

We note that accounting practice also often assumes cost-variances are Gaussian (e.g., Weil and Maher, 2005, p.459), and statistical quality metrics often involve this distribution (e.g., Pyzdek and Keller, 2009, p.278). We introduce the following notation:

X is a random real-valued variable denoting the performance of the project.

$E[X]$ is the expectation of X, and

σ_X is the standard deviation of X.

x denotes a particular level of performance, and

t denotes a particular target level.

PM refers to the disparity between the completion time and the required completion time as the level of *project slack*. We incorporate this notion as follows:

$y = x - t$ denotes project slack for a particular level of performance, while

Y denotes an uncertain level of project slack (e.g., because X is uncertain), where

$E[Y]$ is the expected project slack, and

σ_Y is the standard deviation of Y, and

$z = E[Y]/\sigma_Y$ is the z-score PM uses to evaluate uncertain slack associated with a project plan.

$\Phi(z)$ denotes the standard normal cumulative distribution function evaluated at z.

Since $y = x - t$, and t is known, $E[Y] = E[X] - t$, and $\sigma_Y = \sigma_X$ (and the variance of Y is thus equal to the variance of X). Following Abbas et al (2009), we assign utility function $u(x, t)$ a value of 1 in the case of success ($x \geq t$) and 0 in the case of failure ($x < t$). Since $y = x - t$, $u(y, 0) = u(x, t)$. We shall henceforth suppress the second argument, 0, in writing $u(y)$, and note $u(y) = 1$ if $y \geq 0$, and $u(y) = 0$ if $y < 0$. The customer's expected utility $E[u(Y)]$ is equal to $Pr\{Y \geq 0)\}$. Given the Gaussian assumption (Assumption 0), $Pr\{Y > 0)\} = \Phi(z)$ which is monotonically increasing in z. Thus, to maximize the customer's expected utility, the manager should simply choose the plan that maximizes z. The way we think about this z-score plays a critical role in the rest of the paper.

The primary contribution of this paper is in showing how the z-score (or standardized expected slack metric), with some minor updating, can still be used even if both Assumptions 1 and 2 are generalized. This section will show how assumptions 1 and 2 can be generalized and the z-score metric updated. The remaining sections then show how this updated z-score can be used in project management.

Let us replace Assumption 1 (inflexible requirements) with

Assumption 1*(flexible requirements): *Customer preferences over performance are described by a Gaussian utility function where performance at level x has expected utility equal to the cumulative density of a Gaussian variable with mean t and standard deviation σ_e.*We can do this by introducing a random variable e following a Gaussian distribution with mean 0 and standard deviation σ_e, and giving utility of 1 when $x \geq t + e$, so that $E[u(x, t, e)] = Pr\{x \geq t + e\}$. In working with this assumption, we shall denote particular values of e with the symbol ϵ. As σ_e decreases to zero, Assumption 1* reverts to Assumption 1.

We note several consequences of this assumption.

1. The curve for this Gaussian utility function (Berhold, 1973) has the same S-shape as the cumulative density function for a variable with mean t and standard deviation σ_e. To represent customer preferences, the value t would be selected to have utility halfway between the worst case and the best case.

2. Utility for performance is convex for $x \leq t$, concave for $x \geq t$ and has risk tolerance $\sigma_e^2/(x-t)$.

   For performance metrics such as completion time, this S-shape can be justified by asymptotic behavior that bounds utility, e.g., a project that is too late has utility dropping toward some minimum, and while there may be economic benefit to a project finishing ahead of schedule, we might expect diminishing marginal returns to improved completion time. A similar argument could hold for quality, depending on how it is measured.

3. The Gaussian utility function is risk-seeking with respect to losses and risk averse in terms of gains relative to the threshold. Thus, a decision maker with such a utility function for a dimension of performance – even one for whom this utility is not just a descriptive result of framing effects – would demonstrate some of the behavior described by prospect theory (Kahneman and Tversky, 1979) as observed by Abbas (2006) as well as by Castagnoli and Li Calzi (1996).

4. The Gaussian utility function implies that risk tolerance is increasing with performance on timeliness. Increasing risk tolerance would be a less suitable assumption for the attribute of wealth.

Note, as is often the case with assumptions, the use of a Gaussian utility function simplifies analysis. Before applying this assumption in any particular situation, however, it would be prudent to judge whether such a function reasonably approximates customer preferences.

We might also replace Assumption 2 (known requirements) with

Assumption 2*(uncertain requirements): *To be successful, rather than meeting a known target, the project performance must reach some uncertain threshold level* (Θ) *described by a Gaussian distribution with mean* $E[\Theta]$ *and standard deviation* σ_Θ.A particular value of Θ will be denoted θ. A Gaussian distribution may be reasonable if, for example, deviations from the estimate are the result of numerous unanticipated events or errors in forming the judgement. Note that as σ_Θ shrinks to zero, Assumption 2* reduces to Assumption 2. If we also assume that Θ is independent of X, then the distribution on project slack Y has mean $E[Y] = E[X] - E[\Theta]$, and standard deviation $\sigma_Y = \sqrt{(\sigma_X^2 + \sigma_\Theta^2)}$. As before, to maximize the customer's expected utility, the manager maximizes the probability of success $Pr\{Y \geq 0)\}$ by choosing the plan that maximizes $z = E[Y]/\sigma_Y$. The only difference is that in obtaining the variance of Y, the variance of Θ is now added to the variance of X.

While Assumption 1 and Assumption 1* are statements about the nature of the customer's preferences, Assumption 2 and Assumption 2* can be thought of as statements about the project manager's *knowledge about* the customer's preferences.

Assumption 1* and Assumption 2* can be combined in the following manner. We define a new random variable:

$T = \Theta + e$, which represents a random target centered on the unknown threshold, with standard deviation $\sigma_T = \sqrt{\sigma_\Theta^2 + \sigma_e^2}$. Particular values of T are $t = \theta + \epsilon$. The uncertain project slack with respect to this random target is $Y = X - T$, where $y = x\theta + \epsilon$, so $u(x, t, \epsilon) = u(y, 0, 0)$, and again, we suppress the latter arguments and write $u(y) = 1$ when $y \geq 0$ (i.e., when performance meets or exceeds the random target) and $u(y) = 0$ for $y < 0$. Now, $E[u(x,T)] = Pr\{x \geq T\}$, $E[u(X,T)] = Pr\{X \geq T\}$, and $E[u(Y)] = Pr\{Y \geq 0\}$.

This paper's primary modification to project management is redefining slack so that, instead of being calculated with respect to an actual known target, it is defined with respect to this random target (which will be the same as an actual target when the customer is only concerned with meeting a particular real target.) As this paper will show, this simple adjustment allows for the more general application of decision analytic reasoning to project management.

In later sections, some of the results involve only X and T, without distinguishing whether the distribution on T is associated with the customer's utility function or with uncertainty about the threshold, or both. Furthermore, T and X will be assumed Gaussian and independent unless otherwise stated.

2.2 Numerical examples

The different combinations involving Assumptions 1 and 1* and Assumptions 2 and 2*, their meaning, and the way we represent them in terms of how we define the z-score associated with the uncertain project slack can be summarized as follows:

1. Case 1 - Standard project management (Assumption 1 and Assumption 2): Maximize probability of exceeding a known target. *In this case the z-score is computed with project slack defined relative to an actual known threshold.*

2. Case 2 - Uncertain requirements (Assumption 1 and Assumption 2*): Maximize probability of exceeding the random target: *In this case, the z-score is computed with project slack defined relative to an actual but unknown threshold.*

3. Case 3 - Flexible requirements (Assumption 1* and Assumption 2): Maximize expectation

of a known customer utility function: *In this case, the z-score is computed with project slack defined relative to a random target centered on a known threshold.*

4. Case 4 - General case (Assumption 1* and Assumption 2*): Maximize expectation of the uncertain customer utility function: *In this case, the z-score is computed with project slack defined relative to a random target centered on an unknown threshold.*

We now develop a numerical example across these four cases. By illustrating the calculations for making decisions under each set of assumptions, this example gives context to the concepts introduced here. Beyond that, the different cases of the example demonstrate that, indeed, the assumptions the project manager makes can quite easily affect choices made by the project manager – which means that project managers making the wrong assumptions might be making the wrong decisions.

Case 1 – Standard project management: The required completion time for a project is 100 days. The decision maker must choose between

Plan A: Time to completion is normal with mean 90, standard deviation 10, and

Plan B: Time to completion is normal with mean 85, standard deviation 20.

For consistency later on in this paper, we shall in all cases use performance measures where higher scores are better, so we define the performance metric "*timeliness*" as the negative of completion time (in days in this case). Thus, the threshold for success is $t = -100$. For Plan A, timeliness has expected value $E[X] = -90$, and standard deviation $\sigma_X = 10$. Thus, $Y = X - t$ (the gap between performance and the requirement), has expected value $E[Y] = 10$, standard deviation $\sigma_Y = 10$, and $z = E(Y)/\sigma_Y = 1.0$. The expected utility $E[u(X,t)] = Pr\{X \geq t\}$, which is clearly equal to $E[u(Y,0)] = Pr\{Y \geq 0\}$. This probability is $\Phi(1.0) = 0.84$. For Plan B, we have $E(X) = -85$, $\sigma_X = 20$, and thus $z = 15/20 = 0.75$, and $E[u(X,t)] = \Phi(0.75) = 0.77$. Thus, the plan (Plan A) associated with the higher z-score has higher expected utility and is preferred. Note, although PM commonly assumes Gaussian completion times, a well-known limitation of this assumption is that in reality, completion times are non-negative. When $E[X]/\sigma_X$ is far from 0, however, this has negligible effect on probability calculations.

Case 2 – Uncertain requirements: In Case 1, we can replace t with θ, the particular value of the random variable Θ, which here happens to have standard deviation $\sigma_\Theta = 0$ and mean $E[\Theta] = -100$. Project slack is then the random variable $Y = X - \Theta$, and z remains $E[Y]/\sigma_Y$. Because $\sigma_\Theta = 0$, σ_Y is unchanged, and so are the z-scores and utilities. For Case 2, assume there is uncertainty about the required timeliness, i.e., Θ is a random variable with positive standard deviation. Assume Plan A and Plan B are as in case 1, while Θ follows a normal distribution with mean $E[\Theta] = -100$, but now with standard deviation $\sigma_\Theta = 20$.

Then for Plan A, $E[Y] = 10$, $\sigma_Y = \sqrt{10^2 + 20^2} = 22.36$, so $z = 0.447$ and $E[u(Y)] = 0.673$.

For Plan B, $E[Y] = 15$, $\sigma_Y = \sqrt{20^2 + 20^2} = 28.28$, $z = 0.53$, and $E[u(Y)] = 0.702$.

Thus, with the uncertainty added to the target, the improved mean performance grows in importance compared to the uncertainty of that performance. This results in a higher z-score for Plan B than for Plan A, and thus a higher utility, and Plan B is now preferred.

Case 3 – Flexible requirements: We now define slack $Y = X - t + e$ and let the target oriented utility function be $u(x, t, e) = Pr\{x \geq t + e\}$ so that $E[u(Y)] = Pr\{Y \geq 0\}$ incorporates the variable e which has mean 0 and standard deviation σ_e. Case 1 assumed there is no additional upside to early completion, and no difference between missing the requirement by a small amount or a large amount, which is equivalent to assuming $\sigma_e = 0$, so that the values of σ_Y and thus the expected utilities for Plan A and Plan B are unchanged.

For Case 3, assume that the customer assigns a Gaussian utility function to timeliness, where a project meeting the requirement of 100 days has utility halfway between the worst and best possible cases, and where the utility for a project that is ten days late has utility halfway between the worst case and the case where the project exactly meets its requirement. We represent this with $u(x,t,e)$. Fitting $u(100,t+e)=0.5=Pr\{100\geq t+e\}$ and $u(90,t,e)=0.25=Pr\{90\geq t+e\}$, we find $\sigma_e=14.8$. Thus, for Plan A, $\sigma_Y=17.9$, $z=10/17.9=0.559$, and $E[u(Y)]=0.712$. For Plan B, $\sigma_Y=24.9$, $z=15/24.9=0.603$, and $E[u(Y)]=0.726$. Thus, for the customer with higher risk tolerance in Case 3 vs. Case 1, the riskier but on-average better performance of Plan B becomes preferable. Case 4 – General case: We now assume that, in contrast to Case 1, the customer has a utility function around the threshold, but the project manager is also uncertain about this threshold. We include the random variables added in both Case 2 and Case 3, letting $E[\Theta]=-100$, $\sigma_\Theta=20$, and $\sigma_e=14.8$, and assume e and Θ independent. Defining $Y=X-\Theta+e$, we find that for Plan A, $\sigma_Y=26.8$, $z=0.373$ and $E[u(Y)]=0.645$, while for Plan B, $\sigma_Y=0.319$, $z=0.470$, and $E[u(Y)]=0.681$. Not only is Plan B preferred to Plan A in this case, but the difference in utility is larger than in Case 2 or Case 3.

Thus, the question of whether or not the project manager should even select the project with the shorter completion time if it comes with more uncertainty depends on how consequential that uncertainty is. This is reflected in the calculation of z-scores which correspond to the resulting expected utilities. In the first case, it was consequential enough to make the manager choose Plan A, while in cases 2 (due to the manager's uncertainty about requirements) and 3 (due to the customer's risk tolerance) the risk is less consequential and Plan B is preferred. Finally, in case 4, both factors make the faster but riskier Plan B even more attractive relative to Plan A. The pattern of which plan is preferred in which case could change with different values on any of the parameters.

Figure 2 illustrates the utility functions that may be faced by the project manager. The solid curve consisting of a vertical line at -100 is the utility function for Case 1; Case 2 is not shown, but

	Case 1	Case 2	Case 3	Case 4
Assumptions				
Known requirements	Yes	No	Yes	No
Fixed requirements	Yes	Yes	No	No
Requirements				
E[T] = -100				
σ_θ	NA	20	NA	20
Risk tolerance				
σ_e	NA	NA	14.8	14.8
Performance				
Plan A: E[X]=-90, $\sigma_X = 10$				
E[Y] = 10				
σ_Y	10	22.36	17.9	26.8
z	1	0.447	0.559	0.373
E[u(X)]	0.84	0.673	0.712	0.645
Plan B: E[X]=-85, $\sigma_X = 20$				
E[Y] = 15				
σ_Y	20	28.28	24.9	31.9
z	0.75	0.53	0.603	0.47
E[u(X)]	0.77	0.702	0.726	0.681
Choice				
Plan selected	A	B	B	B
Difference between E[u(x)] for Plan A and Plan B	0.07	-0.029	-0.014	-0.036

Figure 1: Comparison of numerical example cases

would just have different possible positions of the vertical line corresponding to different possible threshold levels; Case 3 is illustrated by the center dashed curve (labelled M) showing the customer's utility when the threshold is known to be 100 and the customer has flexible requirements; in Case 4, the customer's threshold is also uncertain, the dashed curves on the left and right represent possible customer utility functions with low (L) and high (H) thresholds one standard deviation above and below the mean threshold, while the wider solid curve represents the expected utility taking into both the flexibility of the customer's requirements and the uncertainty of the customer's threshold

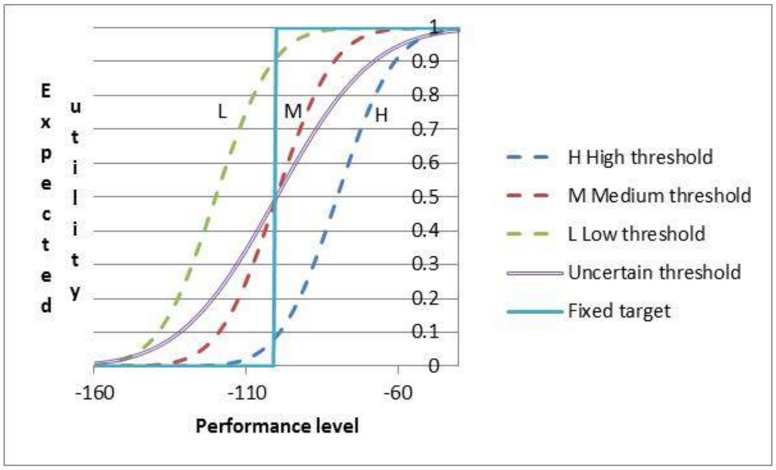

Figure 2: Project utility functions under different assumptions

level.

In Figure 3, we compare Case 1 and Case 4 by inspecting the cdf for the slack associated with the two project plans. The expected utility of each plan is $1 - Pr\{Y < 0\}$. In Case 1, with the fixed target, Plan A has a higher z-score and hence lower chance of having negative slack (failure) than plan B, while in Case 4, Plan A has a higher chance of failure.

3 Results

3.1 The certainty equivalent

Having gone from z-scores to expected utilities, we now circle back to decision analytic certainty equivalents (CEs). We shall let c_X denote the CE for performance, i.e., the particular performance level x which has the same expected utility as the uncertain performance X. Both perspectives can be useful. To show that these two ways of ranking decisions are consistent, we prove:

Proposition 3.1. *In a project with independent Gaussian distributions for performance and target, the certainty equivalent for performance is linear in the product of the risk tolerance and the z-score*

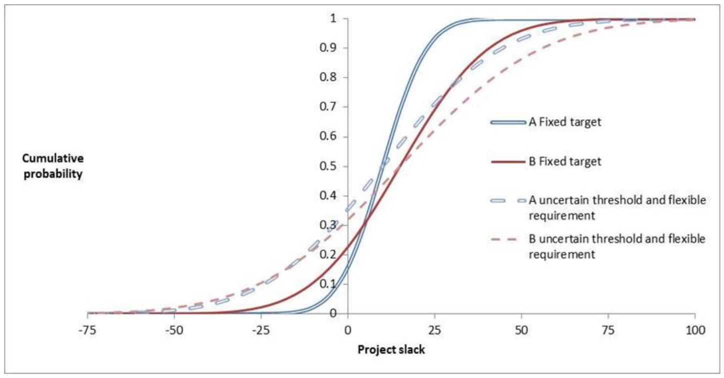

Figure 3: Everything combined into uncertain project slack

for project slack.

Proof: Suppose X and T are Gaussian and independent, and $u(x,t) = 1$ if $x \geq t$ and $u(x,t) = 0$ if $x < t$. Then we can write

$$u(x) = Pr\{x \geq T\} = \Phi(\frac{x - E[T]}{\sigma_T})$$

Since c_X, the certainty equivalent for X, solves $u(c_X) = Pr\{c_X \geq T\} = u(X) = Pr\{X \geq T\}$, $\Phi[\frac{c_X - E[T]}{\sigma_T}] = \Phi[\frac{E[X-T]}{\sigma_{X-T}}]$. Since the z-score is $\frac{E[X-T]}{\sqrt{\sigma_X^2 + \sigma_T^2}}$. this implies

$$c_X = E[T] + \frac{\sigma_T}{\sigma_{X-T}} E[X - T] = E[T] + z\sigma_T$$

Note that $u(x) = \Phi(\frac{x-E[T]}{\sigma_T})$, $u'(x) = \Phi'(z)/\sigma_T$ and $u''(x) = \Phi''/\sigma_T^2$. As a result, the risk tolerance is $-u'/u'' = \sigma_T \Phi'/\Phi''$ where Φ' and Φ'' are independent of σ_T. Thus $c_X = E[T] - z\frac{u'}{u"}\frac{\Phi"}{\Phi'}$ and c_X is linear in the product of z and the risk tolerance $\frac{-u'}{u'}$. QED

Thus, in the PM context, the z-score as developed here can function as a simple proxy for the conventional certainty equivalent for project managers to maximize. Other surrogates or even alternate formulations of the certainty equivalent have been adopted in different applied contexts, e.g., risk and return based formulations in finance. Thus there may be practical value to other statistics that are similar to the CE. This paper's formulation suggests several such possibilities which we consider in the appendix.

Related to the CE of an uncertain outcome is its *risk premium*, which quantifies the undesirability of the risk in a gamble as the difference between its CE and its expected value. If we define the risk-adjusted weighting factor as $\rho = \sigma_T/(\sigma_T^2 + \sigma_X^2)^{1/2}$ then the certainty equivalent c_X can be rewritten as $E[T] + \rho E[Y]$ where $\rho < 1$ implies risk aversion in X. In the limit, as $\sigma_T \to \infty$, $\rho \to 1$, a condition which implies risk-neutrality. It is then meaningful to talk about a performance risk premium, $E[X] - c_X = (1-\rho)E[Y] = (1-\rho)z\sigma_T$ and this is also an affine transformation of the z-score.

In plan A of our previous example, $z = 10/26.8 = 0.373$, $\sigma_T = 24.88$, and $E[T] = -100$, so $c_X = -100 + 0.373 * 24.88 = -90.72$ and the risk premium is 0.72 (which is fairly low because the risk tolerance is somewhat higher than the amount of performance risk).

This can be seen graphically if we modify the parameters to accentuate the effects. If $E[X] = -90$, $E[T] = -100$, so that $E[Y] = 10$ as before, but $\sigma_X = 20$ and $\sigma_T = 15$, so that $\sigma_Y = 25$, $z = 10/25 = 0.4$, $\rho = 15/25 = 0.6$, $z\sigma_T = 6$, and $c_X = E[T] + z\sigma_T = -94$. In Figure 4, we see that the cdf for $X - T$ intersects the cdf for $E[X] - T$ at the 50th percentile, and intersects the cdf for $c_X - T$ at 0, as the curve is shifted left by the risk premium $(1-\rho)\sigma_T = 4$.

3.2 Sensitivity to changes in means and standard deviations

In the standard case with a fixed target, t, if a particular change to the plan adjusts the mean timeliness by some amount $\delta E[X]$ and the standard deviation by some amount $\delta\sigma_X$, then a first-

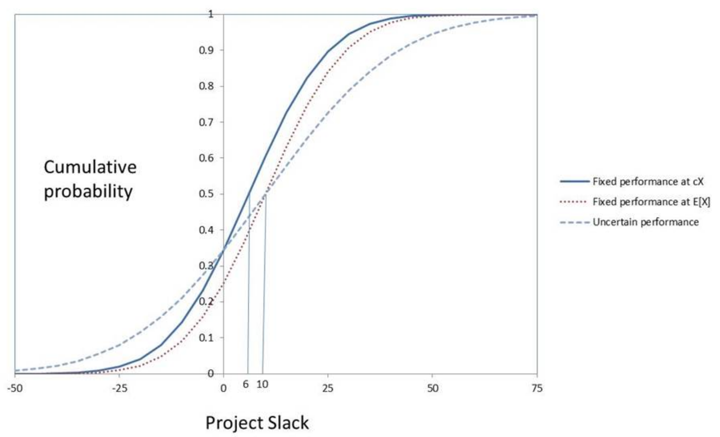

Figure 4: Performance certainty equivalent and the distribution on project slack

order approximation of the change in z-score for project slack is

$$\delta z \approx -\frac{\delta E[X]}{\sigma_X} - \frac{E[X]-t}{\sigma_X^2}\delta\sigma_X = \frac{1}{\sigma_X}\delta E[X] - z\delta\sigma_X$$

Since the project manager should only accept changes which improve the z-score, any change which increases σ_x by $\delta\sigma_X$ will only be acceptable if it improves expected performance by $z\delta\sigma_X$.

We can interpret the right-hand side of the equation above as a decision rule for contemplated changes. It shows whether the change in mean is offset by the change in standard deviation in such a way that the z-score improves and therefore the probability of success improves. Hence the impact of a change on the z-score also defines the amount of added risk $\delta\sigma_X$ which the manager is willing to accept in order to receive an incremental improvement, $\delta E[X]$, in expected performance.

When the target T is uncertain, the first-order change in z-score induced by a change in $E[X]$ and σ_X is now

$$\delta z \propto \frac{\delta E[X]}{\sigma_Y} - \frac{1}{2}\frac{E[X]-E[T]}{(\sigma_X^2+\sigma_T^2)^{3/2}} 2\sigma_X\delta\sigma_X = \frac{1}{\sigma_Y}\delta E[X] - z\frac{\sigma_X}{\sigma_Y}\delta\sigma_X$$

As before, the decision about whether to make the change is guided by whether the change increases the z-score for project slack, which now incorporates target uncertainty. When the target T is uncertain, the coefficient of $\delta\sigma_X$ in the previous expression changes from z to $z\frac{\sigma_x}{\sigma_y}$. As a result, a $\delta\sigma_X$ increase in the plans standard deviation will now be acceptable if it improves performance by $z\frac{\sigma_x}{\sigma_Y}$. As the risk tolerance increases, σ_Y increases and the required improvement in performance decreases.

3.3 Value of information

In this section, we derive an equation describing the increase in expected utility arising arising from acquisition of information. From this, value of information can be readily calculated in particular situations.

During the course of the project, the manager will have the opportunity to obtain information prior to make some decisions. Consider the following two-stage problem. In the first stage, the manager has a choice of whether or not to gather information I. In the second stage, the manager, based upon the information collected in the first stage, can decide between continuing with his current plan A or moving to a different plan B. The manager has three choices

1. Do not acquire information I and choose plan A
2. Do not acquire information I and choose plan B
3. Acquire the information I and choose either plan A or plan B depending upon what I reveals.

Let X_A denote the uncertain performance associated with the execution of plan A while X_B denotes the same for plan B, with $Y_A = X_A - T$, $Y_B = X_B - T$. Assume X_A and X_B are Gaussian with means $E[X_A]$ and $E[X_B]$ and standard deviations σ_{X_A} and σ_{X_B} so that the project slack associated with the plans would be Gaussian with means $E[Y_A]$ and $E[Y_B]$ and standard deviations σ_{Y_A} and σ_{Y_B} implying z-scores $z_A = E[Y_A]/\sigma_{Y_A}$ and $z_B = E[Y_B]/\sigma_{Y_B}$.

After learning I, the manager forms updated probability distributions for $X_{A|I}$, $X_{B|I}$ and T_I, which we shall assume are also Gaussian with standard deviations $\sigma_{X_{A|I}}$, $\sigma_{X_{B|I}}$ and $\sigma_{T|I}$. The manager then calculates distributions for $Y_{A|I}$ and $Y_{B|I}$, with means $E[Y_{A|I}]$ and $E[Y_{B|I}]$ with standard deviations $\sigma_{Y_{A|I}}$ and $\sigma_{Y_{B|I}}$, and calculates the associated z-scores with $z_{A|I}$ and $z_{B|I}$.

The expected utility of choosing plan A is thus $\Phi(z_A)$ prior to collecting the information and $\Phi(z_A|I)$ after, and likewise for the expected utility of choosing plan B.

The manager's decision problem is shown in Figure 5. We see here that some calculation is required to estimate the probability that acquired information will lead to Plan A or Plan B being preferred, but once that information is acquired (or if no information is acquired), the choice between Plan A and Plan B is straightforward. Finally, more calculation is required to determine the chance of success once that plan is selected. Thus, determining the value of information is not trivial.

The project manager should, upon learning I, switch to plan B if $z_{B|I} \geq z_{A|I}$ since this offers the manager a higher utility. Thus if $k = 1$ when $z_A \geq z_B$ and zero otherwise, the project manager's utility upon learning I is

$$\Phi(z_{A|I})k + \Phi(z_{B|I})(1 - k)$$

The distributions associated with the variables before learning I will be called prior distributions and the distributions associated with the variables after learning I will be called posterior distributions. Prior to learning I, the means of the posterior distributions for the variables $E[X_{A|I}]$, $E[X_{B|I}]$, $E[T_I]$ can themselves be considered as (Gaussian) random variables with means equal to the means of the prior distributions and variances equal to the difference between the variances of

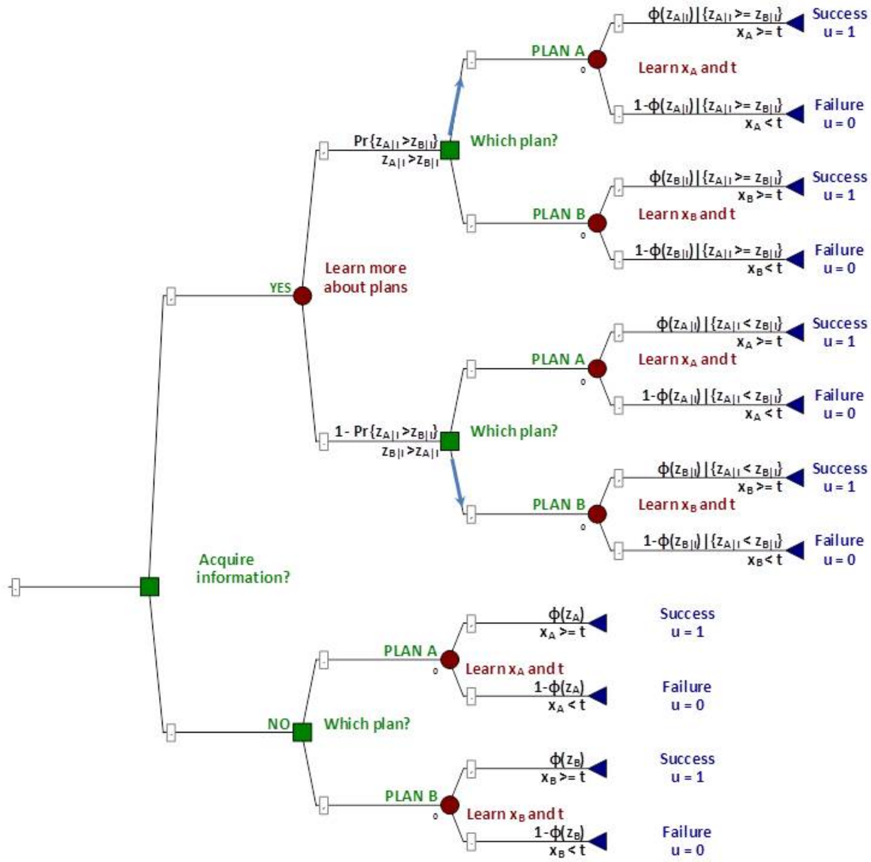

Figure 5: Decision tree for selecting a project plan, with an opportunity to acquire information

their prior and posterior distributions (as in Raiffa and Schlaifer, 1961). Thus, prior to learning I, $E[Y_{A|I}]$, $E[Y_{B|I}]$, $z_{A|I}$ and $z_{B|I}$ can also be treated as random variables whose distributions can be calculated. When treating the last two as random variables, we shall use the notation $\tilde{z}_{A|I}$ and $\tilde{z}_{B|I}$. As $\tilde{z}_{A|I}$ and $\tilde{z}_{B|I}$ are random variables, $\Phi(\tilde{z}_{A|I})$ and $\Phi(\tilde{z}_{B|I})$ are themselves random variables, with expected values $E[\Phi(\tilde{z}_{B|I})]$ and $E[\Phi(\tilde{z}_{B|I})]$.

With this treatment, the manager's new expected utility for the project prior to learning the outcome of I (but after committing to gather the information) is

$$E[\Phi(\tilde{z}_{A|I})|\{\tilde{z}_{A|I} \geq \tilde{z}_{B|I}\}]\Pr\{\tilde{z}_{A|I} \geq \tilde{z}_{B|I}\} + E[\Phi(\tilde{z}_{B|I})|\{\tilde{z}_{B|I} > \tilde{z}_{A|I}\}]\Pr\{\tilde{z}_{B|I} > \tilde{z}_{A|I}\}$$

We can rewrite the original expected utility (where Plan A is selected regardless of what I reveals) as $\Phi(z_A) = E[\Phi(z_A|I)|z_A \geq z_B]\Pr\{z_A \geq z_B\}] + E[\Phi(z_A)|z_A < z_B]\Pr\{z_A < z_B\}$. Taking the difference between the new and original levels shows that information I increases expected utility by

$$\Pr\{\tilde{z}_{A|I} < \tilde{z}_{B|I}\}[E[\Phi(\tilde{z}_{B|I}) - \Phi(\tilde{z}_{A|I})]|\{\tilde{z}_{A|I} < \tilde{z}_{B|I}\}]$$

If S is a standardized Gaussian random variable (with $\Phi(k) = \Pr\{S \leq k\}$), then this change in utility can be written as

$$\Pr\{S \leq \tilde{z}_{B|I}, \tilde{z}_{A|I} < \tilde{z}_{B|I}\} - \Pr\{S \leq \tilde{z}_{A|I}, \tilde{z}_{A|I} < \tilde{z}_{B|I}\} = \Pr\{\tilde{z}_{A|I} < S < \tilde{z}_{B|I}\}$$

This can be rewritten as $\Pr\{(X_{A|I} - T)/\sigma_{Y_{A|I}\}} < S < \Pr\{(X_{B|I} - T)/\sigma_{Y_{B|I}}\}$
$= \Pr\{(E[X_{A|I} - E[T])/\sigma_{Y_{A|I}} < S\,S < (E[X_{B|I} - E[T])/\sigma_{Y_{B|I}}\}$
$= \Pr\{(X_{A|I} < T)\,(T < X_{B|I})\} = \Pr\{X_{A|I} < T < X_{B|I}\}$ i.e., it is the probability of the information indicating that plan B exceeds the threshold while plan A does not. This formula can be computed from the probability density for the maximum of two correlated random variables.

To compute this probability density, let M and s, and μ and σ denote the mean and standard deviation of the two performance scores. Also let r be the correlation between the two completion times. Nadarajah and Kotz (2008) show that, if X_1 and X_2 are correlated Gaussian random variables, with $X = \max(X_1, X_2)$, $f_0(x)$, the probability density function for this maximum, satisfies

$$f_0(x) = f_1(-x) + f_2(-x) \tag{1}$$

where

$$f_1(x) = \frac{1}{\sigma}\phi(\frac{x+\mu}{\sigma})\Phi(\frac{r(x+\mu)}{\sigma\sqrt{1-r^2}} - \frac{x+M}{s\sqrt{1-r^2}})$$
$$f_2(x) = \frac{1}{s}\phi(\frac{x+M}{s})\Phi(\frac{r(x+M)}{s\sqrt{1-r^2}} - \frac{x+\mu}{\sigma\sqrt{1-r^2}})$$

If we define $a = 1/\sqrt{1-r^2}$, $\hat{x}_c = \frac{\mu - x}{\sigma}$, $\hat{x}_n = \frac{M-x}{s}$ then equation (1) becomes

$$f_0(x) = \frac{1}{\sigma}\phi(\hat{x}_c)\Phi(a[r\hat{x}_c - \hat{x}_n]) + \frac{1}{s}\phi(\hat{x}_n)\Phi(a[r\hat{x}_n - \hat{x}_c]).$$

The cumulative distribution of the maximum of these two variables can be computed from the probability density using common numerical integration methods such those based on the trapezoidal rule (Davis and Rabinowitz, 1984). This specifies the change in utility induced by the information as a function of the parameters of the distribution of z_A and z_B.

In this paper, performance is in terms of time rather than in terms of money, and we can calculate the value of information in those units (e.g., the delay the project manager would be willing to accept in order to obtain the information) as follows: Let δp denote increase of utility just obtained, and and define p_0 as $\max[\Phi(z_A), \Phi(z_B)]$. We calculate certainty equivalents c_X s.t. $E[u(c_X, T] = p_0$ and c'_X s.t. $E[u(c'_X, T)] = p_0 + \delta_p$. The value of information is then $c'_X - c_X$.

In our Gaussian example, without information the expected utility is $\Phi(z)$ where $z = E[Y]/\sigma_Y$. Define z' so that $\Phi(z') = \max[\Phi(z_A), \Phi(z_b)]$. Suppose we define $\delta E[X]$ as the amount by which timeliness on the current project would have to be improved in order to get the same utility as the PM could attain by getting new information. Then $\Phi([\delta E[X] + \frac{E[Y]}{\sigma_Y}) = \Phi(z')$ and $\delta E[X] = \sigma_Y z' - E[Y]$.

To translate value of information to monetary units, we would need to have a customer utility function u_m over money, and the value of information would then be $u_m^{-1}(p_0 + \delta p) - u_m^{-1}(p_0)$. If project cost were linear in X, value of information in terms of acceptable delay could be translated to an equivalent increased cost. Another straightforward case is where the monetary value of project success is V and failure has value 0, and u_m is linear over $[0, V]$. Then the monetary value of information is simply $V\delta p$.

3.4 Solutions for Different Kinds of Information

As an example, consider the situation depicted in the influence diagram in Figure 6, where the slack on both plans A and B depends in linear fashion on the timeliness X_i of some common activity

i, and the manager must decide whether or not to obtain information about X_i before choosing a plan. Hence if the expected timeliness of this activity is $E[X_i]$ and the actual timeliness is x_i, the slack on both A and B will change by $x_i - E[X_i]$. Learning the timeliness of this activity will also reduce the variance in the slack of both A and B by the same common amount $\sigma^2_{X_i}$. Letting $\delta_i = x_i - E[X_i]$ implies that

$$z_{A|I} = \frac{E[Y_A] + \delta_i}{\sigma_{Y_{A|I}}} = \frac{z_A \sigma_{Y_A} + \delta_i}{\sigma_{Y_{A|I}}} \text{ and } z_{B|I} = \frac{z_B \sigma_{Y_B} + \delta_i}{\sigma Y_{B|I}}$$

The manager will reach the point of switching to plan B when

$$z_{B|I} - z_{A|I} = \delta_i [\frac{1}{\sigma_{Y_{B|I}}} - \frac{1}{\sigma_{Y_{A|I}}}] + z_B \sqrt{1 + (\frac{\sigma_{X_i}}{\sigma_{Y_{B|I}}})^2} - z_A \sqrt{1 + (\frac{\sigma_i}{\sigma_{Y_{B|I}}})^2}$$

Hence if plan B is riskier than plan A, the coefficient of δ_i is negative and the manager's tendency to switch will be greater when δ_i is smaller. This is consistent with the manager only switching to the riskier plan B when the outcome of activity i falls significantly short of the manager's expectations. In addition, the coefficient of z_B is smaller than the coefficient of z_A since $\sigma_{Y_{B|I}}$ is larger. Thus the difference between $z_{B|I} - z_{A|I}$ places greater weight on z_A than z_B.

If activity i was only present in plan A, then $z_{B|I} = z_B$ and getting perfect information on the performance of activity i will bring the manager to the point of switching to B if

$$z_{B|I} - z_{A|I} = z_B - \frac{z_A \sigma_{Y_A} + \delta_i}{\sigma_{Y_{A|I}}} = -\frac{\delta_i}{\sigma_{Y_{A|I}}} + (z_B - z_A) + z_A \frac{\sigma_{Y_{A|I}} - \sigma_{Y_A}}{\sigma_{Y_{A|I}}}$$

In this case, the greater the residual uncertainty (as measured by $\sigma_{Y_{A|I}}$) in plan A, the less prone the manager will be to switch given adverse information about activity i. In particular, if slack is in part based on the random variable centered around the threshold, this component of uncertainty associated with T does not resolve, hence, for the same difference in expected utility of A and B, the greater the customer's risk tolerance, the lower the value of information.

To generalize this result, suppose that the manager, instead of collecting perfect information about the activity's timeliness, conducts an imperfect experiment about activity i, producing a

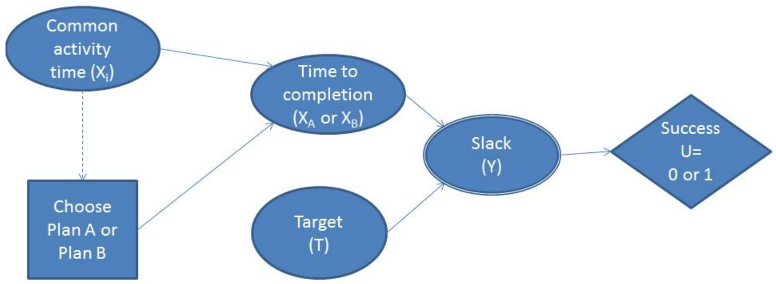

Figure 6: Influence diagram for problem of value of information about an activity common to both plans

signal about X_i. As before, let $\sigma^2_{X_i}$ denote the uncertainty in the manager's beliefs about activity i's timeliness. Let σ^2_D denote the uncertainty in the signal. Suppose that the signal differs from the expected timeliness by δ_i. If the prior estimate of timeliness was $E[X_i]$, then the revised mean timeliness is

$$\frac{(E[X_i]/\sigma^2_{X_i}) + (E[X_i] + \delta_i)/\sigma^2_D)}{(1/\sigma^2_i) + (1/\sigma^2_D)} = E[X_i] + \frac{\delta_i}{1 + (\sigma^2_D/\sigma^2_{X_i})}$$

If we let $\beta_i = 1/[1 + \sigma^2_D/\sigma^2_{X_i}]$, then the posterior mean differs from the prior mean by $\beta_i\delta_i$. The posterior variance can be computed from

$$(1/\sigma_{X_{i|I}})^2 = \frac{1}{\sigma^2_D} + \frac{1}{\sigma^2_{X_i}} = [\frac{\sigma^2_{X_i}}{\sigma^2_D} + 1]\frac{1}{\sigma^2_{X_i}} = \frac{1}{1-\beta_i}\frac{1}{\sigma^2_{X_i}}$$

so that

$$(\sigma_{X_{A|I}})^2 = \sigma^2_{X_A} + \sigma^2_{X_i|I} - \sigma^2_{X_i} = \sigma^2_{X_A} + (1-\beta_i)\sigma^2_{X_i} - \sigma^2_{X_i} = \sigma^2_{X_A} - \beta_i\sigma^2_{X_i}$$

which is equivalent to the previous results apply with δ_i replaced by $\beta\delta_i$ and $\sigma^2_{X_i}$ by $\beta\sigma^2_{X_i}$. As the signal approaches perfection, σ^2_D approaches zero and β_i approaches one.

Hence the increase in expected utility associated with δ_i is the probability that δ_i lies within that interval where plan B meets the threshold but A does not.

4 Conclusion

Project management practice incorporates uncertainty about performance in a way that is correct (utility maximizing) from a decision analytic perspective. We have described how approaches that work for a range of problems using the assumption of fixed requirements can be extended into a target-oriented utility based approach that works when requirements are uncertain and when they are flexible. Common simplifying assumptions to make logical decision rules tractable with fixed targets, e.g., Gaussian distributions, remain tractable with uncertain targets. Standard decision rules function by managing the uncertain slack between performance and requirements; we can allow these requirements to vary while continuing work in terms of project slack. Doing so facilitates integration into formal project management of the information about external uncertainties and customer preferences.

This translation between PM and DA formulations lays the groundwork for future modeling of an array of PM problems as sequential decision problems containing uncertainty about performance and targets, and identify or outline statistics and decision rules to maximize z-scores that project managers may readily calculate, or quantities derived from them.

Future work would move beyond the illustrative examples in this paper to additional formulations capable of capturing more of the considerations that arise in realistic settings. Specific developments might include applications to practical PM problems, such as:

- selecting among alternative project plans affecting separate activities in networks;
- developing tractable decision rules using utility functions that are more flexible than the

Gaussian;

- making tradeoffs in achieving performance on cost, quality and time aiming to meet either a set of targets based on all three of these dimensions;
- designing optimal product testing and validation procedures into project plans based on value of information.

Theoretical developments could also allow for more flexible modeling, e.g. incorporating multi-level uncertain targets, e.g., where there is a value of 1.0 for meeting a stretch goal, 0.5 for meeting a lower threshold, and 0 for failing to meet that lower threshold; or incorporating distributions other than Gaussian.

By explicitly treating targets as both flexible and uncertain, it is possible to formalize many of the practical challenges of PM. This creates interesting theoretical and applied opportunities to enhance PM with powerful decision analytic methods.

A Appendix: Equivalence concepts related to certain equivalence

The concept of certain equivalence in DA is useful in part because it allows the decision maker to remove the dimension of uncertainty in evaluating prospects. In this paper's formulation, where there are several different ways of viewing utility based on several uncertain variables, CE-related devices may be useful when one wishes to remove different aspects of that uncertainty from discussion. The case considered in the main text is similar to the CE common in decision analysis, except that instead of being expressed in monetary units, it is expressed in units of performance (X). As the approach here incorporates targets, it is also possible to calculate an aspiration equivalent as defined in Abbas and Matheson, 2005, which we may think of for present purposes as a CE for the target. We can extend this idea by searching for constant values of other variables or combinations of variables for which similar equations hold, we define several new variations on the CE theme. As with the performance CE, in the Gaussian case these new values are are simply sums of $E[T]$ and multiples of the original z-score (proofs are left as an exercise for the reader).

1. *Performance CE*: c_X such that $E[u(X,T)] = E[u(c_X,T)]$, in the Gaussian case here $c_X = E[T] + z\sigma_T$, finds the certain performance with the same expected utility as the random performance;

2. *Target CE*: c_T such that $E[u(X,T)] = E[u(X,c_T)]$, here $c_X = E[T]+z\sigma_X$,reduces the general case with an unknown and non-binary customer utility function to a fixed target with the same probability of non-negative slack;

3. *Threshold CE*: c_Θ such that $E[u(X,T)] = E[u(X,c_\Theta+e)]$, here $c_X = E[T]+z\sqrt{\sigma_X^2+\sigma_e^2}$,reduces the case with a Gaussian customer utility function with unknown mean to one with a known mean;

4. *Requirement CE*: c_e such that $E[u(X,T)] = E[u(X,\Theta+c_e)]$, here $c_e = E[T]+z\sqrt{\sigma_X^2+\sigma_\Theta^2}$, reduces the general case to one with a binary customer utility function and an unknown threshold shifted to account for the customer's risk tolerance;

5. *Threshold risk adjusted performance CE* $= c_{X,\Theta}$ such that $E[u(X,T)] = E[u(c_{X,\Theta}, E[\Theta]+e)]$, here $c_{X,\Theta} = E[T]+z\sigma_e$, assumes the threshold is as expected and finds the certain performance with the same expected utility as the random performance against an unknown threshold;

6. *Risk sensitivity adjusted performance CE* $= c_{X,e}$ such that $E[u(X,T)] = E[u(c_{X,e},\Theta)]$, here $c_{X,e} = E[T]+z\sigma_\Theta$, allows an uncertain threshold but assumes the customer has a binary utility function and finds a performance CE with the same expected utility as the random

performance against a Gaussian customer utility.

Beyond one-dimensional CE values, we can also identify the following iso-utility curves:

Target uncertainty adjusted iso-utility curve: the set of points $(c1, c2)$ such that $E[u(X,T)] = E[u(c1, c2 + e)]$; and

Risk-tolerance adjusted iso-utility curve: the set of points $(c1, c2)$ such that $E[u(X,T)] = E[u(c1, \Theta + c2)]$, the position of which depends on the customer risk tolerance.

In addition, two more variations on the CE involve situations where some official requirement q is given that has not accounted for customer preferences. In this case, we can find a fixed target for which the uncertain performance has the same expected utility as performance at the requirement as for the uncertain target, i.e., c_q such that $E[u(q,T)] = E[u(X, c_q)]$. Alternatively, we might compute the expected utility of performance using the requirement as a threshold, and find a certain performance level that gives the same expected utility against the uncertain target, $E[u(X,q)] = E[u(c_q, T)]$.

Like the traditional performance CE, the variations described here may be of interest to designers of decision rules, who can use them as markers against which possibilities are compared.

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