May, 2019

Strategic Ignorance in Sequential Procurement

Silvana Krasteva, *Texas A & M University - College Station*
Huseyin Yildirim, *Duke University*

Strategic Ignorance in Sequential Procurement

Silvana Krasteva and Huseyin Yildirim*

August 20, 2018

Abstract

Should a buyer approach sellers of complementary goods informed or uninformed of her private valuations, and if informed, in which sequence? In this paper, we show that an informed buyer would start with the high-value seller to minimize future holdup. Informed (or careful) sequencing may, however, hurt the buyer as sellers "read" into it. The buyer may, therefore, commit to ignorance, perhaps, by: overloading herself with unrelated tasks; delegating the sequencing decision; or letting sellers self-schedule. Absent such commitment, we show that ignorance is not time-consistent for the buyer but it increases trade. Evidence on land assembly supports our findings.

JEL Classifications: C70, D80, L23.

Keywords: informed sequencing, uninformed sequencing, procurement, complements

1 Introduction

The procurement of complementary goods and services often entails dealing with independent sellers. Examples include: a real estate developer buying up adjacent parcels from different landowners; a lobbyist securing bipartisan support; and a vaccine manufacturer obtaining required antigens from patent holders. In many cases, the buyer needs to deal with the sellers bilaterally – perhaps, convening multiple sellers is infeasible, or the sellers fear leaking business plans to the rivals. Given the complementarity between

*Krasteva: Texas A&M University, Allen 3106, College Station, TX 77843, email: ssk8@tamu.edu. Yildirim: Duke University, Box 90097, Durham, NC 27708, email: hy12@duke.edu email2. This paper was previously entitled "Information Acquisition and Strategic Sequencing in Bilateral Trading: Is Ignorance Bliss?" We thank Johannes Hörner (Editor), two anonymous referees, and seminar participants at the Duke Theory Lunch, NYU-Stern, Texas Theory Camp, UC-Berkeley, UC-San Diego, and University of Toronto as well as those at various conferences for helpful comments. All remaining errors are ours.

them, careful sequencing of the sellers should, therefore, be an important bargaining tool for the buyer.[1] Complicating the buyer's strategy, however, is her potential uncertainty about each deal's individual worth in case she ends up purchasing only one object. In this paper, we explore the buyer's incentive to resolve such uncertainty and its welfare implications. Our main finding is that when approaching the sellers, ignorance may be bliss for the buyer but it may also be time-inconsistent: the buyer would want the sellers not to "read" into her sequence (claiming it is random), but given their prices, she would approach them informed.

To make our point, we construct a simple model that features one buyer and two sellers of complementary goods. The buyer's joint valuation is commonly known while her stand-alone valuations are private and initially unknown.[2] The buyer can, however, discover all her valuations at a cost prior to meeting with the sellers. In each meeting, the seller offers a confidential price, which the buyer pays upon acceptance. The buyer's meeting sequence as well as purchase history are public – perhaps, due to the visibility of such transactions.

Our analysis reveals that equilibrium prices trend upward: the first seller charges no higher and, ignoring the previous payment and targeting the extra surplus from the complementarity, the second seller charges no lower than the stand-alone value. To counter the price surge and improve her bargaining position against future holdup, an informed buyer is likely to begin with the high value seller.[3] Together, the buyer's sequencing and the sellers' price response to it determine the value of information for the buyer. For moderate complements, we show that the value of information is *negative*; in particular, the price increase by the leading (high value) seller outweighs the benefit of informed sequencing. Hence, even with *no* cost of acquiring information, the buyer would publicly commit to being uninformed or ignorant so the sellers would not read into her sequence. In practice, she might achieve such commitment to ignorance by: (1) overloading herself with other – unrelated – tasks (Aghion and Tirole, 1997); (2) delegating the sequencing

[1] For an interesting discussion and further applications of sequencing in bilateral trading, see Sebenius (1996) and Wheeler (2005).

[2] In particular, the buyer's stand-alone valuations are assumed to be more uncertain than her joint valuation. For instance, a developer may be less sure about the success of a smaller shopping mall built on a single parcel; a lobbyist may be more worried about passage of the legislation through only one-party endorsement; or a vaccine manufacturer may be more uncertain about the effectiveness of the vaccine that uses only a subset of the required antigens.

[3] In fact, an informed buyer's sequencing may be strict. The reason is that the leading seller sometimes offers a price discount to entice the low value buyer who is more exposed to future holdup.

decision to an uninformed third party; or (3) letting the sellers self-select into the meeting schedule. For strong complements, the value of information is positive since the pricing of the leading seller is now favorable to an informed buyer, implying that the buyer would commit to becoming informed even though she is unlikely to acquire a single item in this case.[4]

In many applications, the buyer may fail to commit to her information acquisition strategy because it is unobservable to the sellers.[5] In particular, under unobservability, the buyer is unable to influence sellers' prices; thus, she acquires information too much for moderate complements (when the pricing effect is negative) and too little for strong complements (when the pricing effect is positive). The unobservability of her information acquisition clearly hurts the buyer, but it may improve social welfare. Note that for complements, efficiency requires a joint purchase of complementary objects, which exposes the buyer to holdup. By strategic sequencing, an informed buyer is able to mitigate this problem and in turn, is more likely to purchase the bundle than the uninformed, implying social value to informed sequencing.

We consider several other extensions pertaining to the bargaining protocol and information structure. Most notably, we show that the buyer may prefer sequential procurements to an auction, in which the sellers make simultaneous price offers. The reason is that while eliminating the holdup problem through ex post purchasing decisions, the auction encourages each seller (not just the last one in sequence) to target the buyer's extra surplus from complementarity. We also show that strategic sequencing substitutes other sources of bargaining power: it is less valuable to a buyer who is more likely to set the prices. Last but not least, comparing various disclosure regimes, we find that while socially most desirable, price disclosure is the least preferred by the buyer. This is because being a continuous variable, price is a more precise signal of buyer's valuation than trade, enabling the sellers to better coordinate their offers.

There is some evidence in favor of our findings for strategic sequencing. In land assembly, Fu et al. (2002), Cunningham (2013), and Brooks and Lutz (2016) all estimate a significant premium to assembled parcels, indicating strong (but imperfect) complementarity among them. In particular, Cunningham (2013) finds that "parcels toward the center

[4] The value of information is trivially zero for weak complements (as would be the case for unrelated goods) in our model and thus not the focus of our discussion here.

[5] It is conceivable that a developer can privately research alternative uses of land parcels; or a lobbyist can privately investigate the long-term political significance of a democratic versus republican support for its proposal.

of the development may command a larger premium than those at the edge, suggesting that developers retain or are perceived to retain some design flexibility."[6] Similarly, as in our investigation, Fu et al. (2002) "identify patterns in the sequencing of acquisition among heterogeneous owners that reflect the trade-off of the opportunity cost of not assembling the preferred set of sites vs. exposure to greater hold-out risk." Given strong complementarity in land assembly, our model also predicts the buyer to be informed and carefully sequence purchases from a high to low value parcel.[7]

Aside from papers mentioned above, our work relates to a burgeoning literature on one-to-many bargaining, where one central player bargains with several others. This literature has mostly assumed complete information, so information acquisition is a non-issue and in many settings, especially those involving Nash bargaining, the buyer turns out to be indifferent to the order of bilateral negotiations despite the sellers' heterogeneity; see, e.g., Horn and Wolinsky (1988), Cai (2000), Marx and Shaffer (2007), Moresi et al. (2008), Krasteva and Yildirim (2012a), Göller and Hewer (2015) and Xiao (2018).

Our work also relates to a large literature studying Coasian bargaining with one-sided private information, e.g., Fudenberg and Tirole (1983), Evans (1987), Gul and Sonnenschein (1988), Vincent (1989), Hörner and Vieille (2009), and Hwang and Li (2017).[8] As in our model, this literature commonly assumes that offers are made by the uninformed party. Most of this literature, however, considers bargaining over a single good and focuses on inefficiencies stemming from delay in reaching an agreement. In contrast, we focus on multiple complementary deals, in which not only the buyer's trade history but also her sequencing can signal her private valuations. Moreover, we endogenize information decision for the buyer. Interestingly, we show that the buyer might choose to remain uninformed even with no cost.[9]

The strategic value of being uninformed has also been indicated in other contexts. For instance, Carrillo and Mariotti (2000) argue that a decision-maker with time-inconsistent

[6] Notable architectural re-designs due to holdouts include Macy's and Rockefeller Center in New York; see `http://untappedcities.com/2014/09/02/10`.

[7] Applied to land assembly, our model assumes that whether a parcel is high or low value is private to the buyer through her architectural design. When the design is sufficiently public, however, careful sequencing may also be due to estimated values of parcels; see Section 5.7 for a formal analysis.

[8] For an overview of the literature, see Ausubel et al. (2002).

[9] Krasteva and Yildirim (2012b) consider a similar setting to the present one but rule out ex ante information acquisition (hence the signaling and strategic ignorance issues) and explore, instead, the optimal sequencing of the sellers with ex ante heterogenous bargaining powers – i.e., the probability of making the offer.

preferences may choose to remain ignorant of the state to control future consumption. In a principal-agent framework, Riordan (1990), Cremer (1995), Dewatripont and Maskin (1995) and Taylor and Yildirim (2011), among others, show that an uninformed principal may better motivate an agent while Kessler (1998) makes a similar point for the agent who may stay ignorant to obtain a more favorable contract. Perhaps, in this vein, papers closest in spirit to ours are those that incorporate signaling. Among them, Kaya (2010) examines a repeated contracting model without commitment and finds that the principal may delay information acquisition to avoid costly signaling through contracts. In a duopoly setting with role choice, Mailath (1993) and Daughety and Reinganum (1994) show that the choice of production period (as well as production level) may have signaling value and dampen incentives to acquire information. The issue of signaling in our setting is very different from these models, and the value of information critically depends on the prior belief in a non-monotonic way.

The rest of the paper is organized as follows. The next section sets up the base model, followed by the equilibrium characterization with exogenous information in Section 3. Section 4 endogenizes information. We explore several extensions and variations in Section 5 and the case of substitutes in Section 6. Section 7 concludes. The proofs of formal results are relegated to an appendix.

2 Base model

A risk-neutral buyer (b) aims to purchase two complementary goods such as adjacent land parcels from two risk-neutral sellers (s_i, $i = 1, 2$). It is commonly known that the buyer's joint value is 1, while her stand-alone value for good i, v_i, is an independent draw from a nondegenerate Bernoulli distribution:[10] $\Pr\{v_i = 0\} = q \in (0, 1)$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q$. We say that as q increases, goods become stronger complements for the buyer. In particular, with probability q^2 goods are believed to be perfect complements. The outside option of each player is normalized to 0.

The buyer meets with the sellers only once and in the sequence of her choice: $s_1 \to s_2$ or $s_2 \to s_1$. Refer to Figure 1. Prior to the meetings, the buyer publicly decides whether

[10]For expositional purposes, we take $v_i \in \{0, \frac{1}{2}\}$ throughout, but our results, especially that on the negative value of information, would generalize to $v_i \in \{v_L, v_H\}$ where $0 \leq v_L < v_H \leq \frac{1}{2}$; see Appendix B.

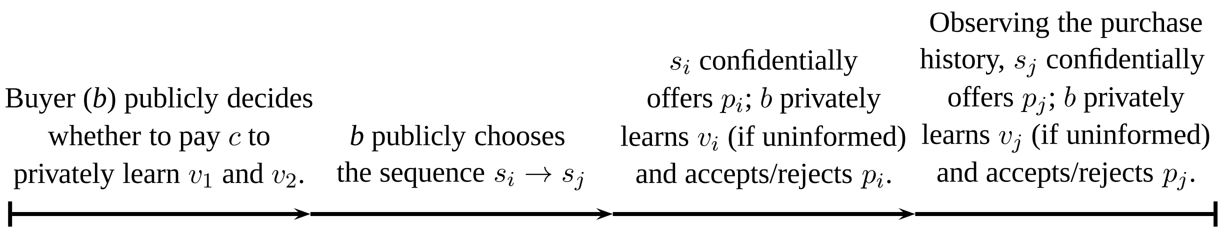

Figure 1: Timing and Information Structure

or not to discover her private values v_1 and v_2 by paying a fixed cost $c > 0$.[11] In each meeting, the buyer receives a confidential price offer p_i and if previously uninformed, privately learns v_i at no extra cost during this meeting – perhaps, through free consultation with the seller. The offer is "exploding" in that it compels a purchasing decision without visiting the next seller. Exploding offers, also known as binding-cash offers in the literature, are ubiquitous in labor and real estate markets (e.g., Niederle and Roth, 2009; Lippman and Mamer, 2012). We assume that the buyer's sequence as well as her purchase history are public. Our solution concept is "symmetric" perfect Bayesian equilibrium, which we discuss in the next section.

More on the model. Our base model is designed to identify sequencing as the unique source of signaling and bargaining power for the buyer. To this end, the buyer is assumed to make informed purchasing decisions, so her ex ante information acquisition matters only for sequencing of the sellers.[12] The buyer is also assumed to commit to information acquisition. This helps focus the baseline analysis and establish a benchmark for an extension to noncommitment in Section 5.1. It is also assumed that the sellers are ex ante identical. This renders sequencing inconsequential for an uninformed buyer, which we briefly relax in Section 5.7. Finally, we restrict attention to one-time bilateral interactions; see Hörner and Vieille (2009) for a similar restriction. This greatly simplifies the analysis with multiple sellers and is reasonable if the buyer has a limited time to undertake the project. In addition to those mentioned, we consider several other extensions pertaining to information structure and bargaining protocol in Section 5.

We begin our analysis with an exogenous information structure and then determine the value of being informed for the buyer. Without loss of generality, we re-label the sellers

[11] Perhaps, the buyer retains a renowned expert. We rule out $c = 0$ in the analysis to avoid trivial equilibrium multiplicity when the value of information is exactly zero, though some of our key results will hold even for $c = 0$.

[12] We elaborate on the possibility of uninformed purchases in the next section. It is, however, readily verified that with commonly known stand-alone values, v_1 and v_2, the buyer would receive a payoff of 0 regardless of the sellers' heterogeneity or sequence (Krasteva and Yildirim, 2012a).

so that seller 1 refers to the first or leading seller in the sequence unless stated otherwise.

3 Informed vs. uninformed sequencing

Suppose that it is commonly known whether the buyer sequences informed (I) or uninformed (U). As alluded to above, given ex ante identical sellers, sequencing is inconsequential for an uninformed buyer. For an informed buyer, let $\theta_1(v_i, v_{-i})$ be the probability that the first (-place) seller has stand-alone value v_i. To ease our analysis, we restrict attention to "symmetric" equilibria: sellers' strategies depend only on the sequence, and the buyer treats equal sellers equally, i.e., $\theta_1(v, v) = \frac{1}{2}$, which reduces her sequencing decision to choosing $\theta_1(\frac{1}{2}, 0)$.[13] Moreover, note that under complements, a joint purchase is (socially) efficient. Thus, we break indifferences in favor of efficiency (i.e., buying and selling more units) unless it is uniquely pinned down in equilibrium. Let $h \in \{0, 1\}$ indicate the buyer's trade history and $(p_1^z, p_2^z(h))$ denote the corresponding pair of prices where $z = I, U$. Our first result shows that under weak complements, equilibrium prices do not respond to informed sequencing.

Lemma 1 *Suppose $q \leq \frac{1}{2}$. In equilibrium, **(a)** $(p_1^z, p_2^z(h)) = (\frac{1}{2}, \frac{1}{2})$ for all z and h, and **(b)** the buyer purchases the bundle with certainty.*

If goods were independent, i.e., $q = 0$, each seller would post his monopoly price of $\frac{1}{2}$, inducing a joint purchase irrespective of the buyer's information. Lemma 1 implies that the same applies to weak complements, $q \leq \frac{1}{2}$. Lemma 1 is, however, uninteresting for our purposes as it trivially rules out information acquisition. Proposition 1 characterizes the equilibrium for $q > \frac{1}{2}$, which is also the focus of our ensuing analysis.[14]

Proposition 1 *Suppose $q > \frac{1}{2}$. In the unique equilibrium, $p_2^U(h = 0) = p_2^I(h = 0) = \frac{1}{2}$. Moreover, we have the following*

[13] As with most signaling games, there is a trivial equilibrium in which the buyer ignores her private information when sequencing and has no incentive to acquire information as a result. Such equilibrium involves picking a "favorite" seller to visit first and the sellers' offering their uninformed prices, with an off-equilibrium belief that switching the sequence implies a high value for the first good, which, in turn, engenders a price pair $(\frac{1}{2}, \frac{1}{2})$ and no deviation surplus to the buyer. By having the buyer treat equal sellers equally, we eliminate such trivial equilibria, and focus, instead, on those that are responsive to information.

[14] The uniqueness obtains after we break the equilibrium price multiplicity at $q = \frac{1}{\sqrt{2}}$ in favor of an efficient trade.

(a) prices for an uninformed buyer:

$$p_1^U = \begin{cases} \frac{1-q}{2} & \textit{with prob.} \quad \frac{1-q}{q} \\ \\ \frac{1}{2} & \textit{with prob.} \quad \frac{2q-1}{q} \end{cases} \quad \textit{and } p_2^U(h=1) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad 1-q \\ \\ 1 & \textit{with prob.} \quad q, \end{cases}$$

(b) prices for an informed buyer: *$p_1^I = p_2^I(h=1) = \frac{1}{2}$ and $\theta_1(\frac{1}{2}, 0) > \frac{1}{2}$ for $q \leq \frac{1}{\sqrt{2}}$; and*

$$p_1^I = \begin{cases} \frac{1-q^2}{2} & \textit{with prob.} \quad \frac{1-q^2}{q^2} \\ \\ \frac{1}{2} & \textit{with prob.} \quad \frac{2q^2-1}{q^2} \end{cases} \quad \textit{and } p_2^I(h=1) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad 1-q^2 \\ \\ 1 & \textit{with prob.} \quad q^2, \end{cases}$$

and $\theta_1(\frac{1}{2}, 0) = 1$ for $q > \frac{1}{\sqrt{2}}$, and

(c) demand: *A buyer with $v_1 = 0$ accepts only the low p_1^z but all $p_2^z(h=1)$ whereas a buyer with $v_1 = \frac{1}{2}$ accepts all p_1^z but only the low $p_2^z(h=1)$.*

To understand the equilibrium characterization in Proposition 1, consider first an uninformed buyer. Working backwards, notice that upon observing a prior purchase (and ignoring its payment), the second seller optimally charges the buyer's marginal value from the bundle, $1 - v_1$, which is either $\frac{1}{2}$ or 1.[15] In equilibrium, he must mix between these two prices and hold a posterior belief such that $\widehat{q}_1 \equiv \Pr\{v_1 = 0|h = 1\} = \frac{1}{2}$. Otherwise, a sure price of 1 would strictly discourage a low value buyer ($v_1 = 0$) from acquiring the first good, inducing $\widehat{q}_1 = 0$ and leading the second seller to reduce his price to $\frac{1}{2}$, whereas a sure price of $\frac{1}{2}$ would guarantee the sale of the first good, leaving $\widehat{q}_1 = q$ and encouraging the second seller to raise his price to 1 given that $q > \frac{1}{2}$. Let $\beta \in (0, 1)$ be the probability that the second seller charges $\frac{1}{2}$, implying an expected payment of $1 - \frac{\beta}{2}$ for the second object. Anticipating this, the first seller will either set a price of $\frac{\beta}{2}$ and secure a sale or set a price of $\frac{1}{2}$ and target only the high value buyer ($v_1 = \frac{1}{2}$). Note that the first seller must also mix between his two options in order to engender $\widehat{q}_1 = \frac{1}{2}$.[16] Let $\gamma \in (0, 1)$ be the probability that the first seller offers $\frac{\beta}{2}$. Together, we must have that

[15] Since $1 - v_1 \geq v_2$ by complementarity, conditional on a prior purchase, the second seller's pricing depends only on $1 - v_1$ and in turn, on his posterior belief about v_1.

[16] Otherwise, a sure price of $\frac{\beta}{2}$ or $\frac{1}{2}$ by the first seller would induce a posterior belief $\widehat{q}_1 = q > \frac{1}{2}$ or $\widehat{q}_1 = 0$, respectively, yielding a contradiction.

$\frac{\beta}{2} = (1-q)\left(\frac{1}{2}\right)$ by the first seller's mixing, and $\widehat{q}_1 = \frac{\gamma q}{\gamma q + 1 - q}$ by Bayesian updating, which result in $\beta = 1 - q$ and $\gamma = \frac{1-q}{q}$, establishing part (a).

Inspecting part (a), it is intuitive that seller 2 mixes according to the prior belief about the first good and stochastically increases his price with the probability of a low value buyer, q. Clearly, a low value buyer demands the first good in the hope of paying less than the full surplus for the second. In equilibrium, such a buyer expects to pay $\frac{1+q}{2}$ for the second good and is therefore willing to pay $\frac{1-q}{2}$ for the first, which is exactly what seller 1 might offer. As q increases, seller 1 drops this discount price to (partially) subsidize a low value buyer for the future holdup, but interestingly he also drops the frequency, $\frac{1-q}{q}$, of this enticing offer so that his subsidy is not captured by seller 2.[17] The uninformed prices in part (a) also explain the equilibrium demand in part (c): a low value buyer purchases the first good only at the discount price, upon which she proceeds to purchase the second with certainty, while the opposite is true for a high value buyer.

Note that uninformed prices trend upward: the first seller charges no higher and the second seller charges no lower than the stand-alone value. Hence, to generate surplus, an informed buyer is more likely to sequence the sellers from high to low value. If this sequencing is strict, namely $\theta_1(\frac{1}{2}, 0) = 1$, then the informed buyer has low value for the first good only in the case of perfect complements, occurring with probability q^2. Substituting this posterior for the prior q in the uninformed prices yields informed prices in part (b) so long as $q > \frac{1}{\sqrt{2}}$. That is, an informed buyer begins with the high value seller if goods are "strong" complements.[18] For "moderate" complements, $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, the informed buyer might mix over the sequence although due to rising prices, she is still strictly more likely to begin with the high value seller, $\theta_1(\frac{1}{2}, 0) \in (\frac{1}{2}, 1]$. Such mixing over the sequence requires equal prices, which can only be at $\frac{1}{2}$.[19]

From Proposition 1, we can determine the buyer's payoff and identify the two key

[17]Indeed, with probability $\frac{1-q}{q}q = 1 - q$, a low value buyer ($v_1 = 0$) acquires both goods but ends up with a loss of $\frac{1-q}{2}$, illustrating the holdup problem.

[18]To emphasize, the source of buyer's strict sequencing from high to low value in our model is the discount price offered by the first seller. As explained above, the first seller offers the discount because, upon the first purchase, the buyer's marginal value for the second good is $1 - v_1$, which is negatively correlated with v_1. That is, a lower value buyer of the first good has a higher stake in obtaining the second. Since this creates the potential for the future holdup, the first seller offers a partial subsidy to entice a low value buyer.

[19]In Proposition 1, the reader may notice discontinuities in the expected equilibrium prices at $q = \frac{1}{2}$ and $\frac{1}{\sqrt{2}}$ for uninformed and informed buyers, respectively. As should be clear from Proposition A1(b) and the proof of Proposition 1, however, those discontinuities are due solely to our equilibrium selection in favor of social efficiency when there is a trivial equilibrium multiplicity.

effects of being informed: sequencing and pricing. Recall that the first seller offers the discount price to entice a low value buyer, leaving her with no expected surplus. This means that despite a joint purchase, a low value buyer incurs a loss if she receives a high price from the second seller. Such holdup does not apply to a high value buyer because she can opt to purchase only the first good. Corollary 1 records this useful observation about the payoffs.

Corollary 1 *A low value buyer of the first good ($v_1 = 0$) obtains an expected payoff of* 0 *while a high value buyer ($v_1 = \frac{1}{2}$) obtains a positive expected payoff equal to her expected payoff from the first purchase.*

From Proposition 1 and Corollary 1, the expected payoff of an uninformed buyer is found to be

$$\begin{aligned} B^U(q) &= (1-q)\frac{1-q}{q}\left(\frac{1}{2} - \frac{1-q}{2}\right) \\ &= \frac{(1-q)^2}{2} \text{ if } q > \frac{1}{2}, \end{aligned} \tag{1}$$

where $1-q$ is the probability that $v_1 = \frac{1}{2}$ and $\frac{1-q}{q}$ is the probability of the discount price, $\frac{1-q}{2}$, by the first seller. For strong complements, the expected payoff of an informed buyer is analogously found by replacing $1-q$ in (1) with $1-q^2$ – the probability that $v_1 = \frac{1}{2}$ under strategic sequencing. For moderate complements, the expected informed payoff is zero since the first seller targets the high value buyer; hence,

$$B^I(q) = \begin{cases} \frac{(1-q^2)^2}{2} & \text{if} \quad q > \frac{1}{\sqrt{2}} \\ \\ 0 & \text{if} \quad \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \tag{2}$$

To identify the two effects of being informed, we also compute a counterfactual payoff for the buyer in which she sequences informed but the sellers are "nonstrategic" in that they keep their uninformed prices. Substituting the probability $1-q^2$ for $1-q$ in the first term of (1), we find the expected informed payoff with nonstrategic sellers:

$$\overline{B}^I(q) = \frac{(1-q^2)(1-q)}{2} \text{ if } q > \frac{1}{2}. \tag{3}$$

Evidently, $\overline{B}^I(q) > B^U(q)$, implying a *positive sequencing effect* of being informed:

given uninformed prices, the buyer strictly benefits from the ability to match a high value good with a low price seller. Moreover, $B^I(q) < \overline{B}^I(q)$ for $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, and $B^I(q) > \overline{B}^I(q)$ for $q > \frac{1}{\sqrt{2}}$; so the *pricing effect* of being informed is *negative* for moderate complements and *positive* for strong complements. As indicated by Corollary 1, the direction of the pricing effect depends on the first seller. Note from Proposition 1 that the first seller offers an expected price of $\frac{q}{2}$ to an uninformed buyer while he offers a higher price of $\frac{1}{2}$ for moderate complements and a lower expected price of $\frac{q^2}{2}$ for strong complements to an informed buyer. Intuitively, informed sequencing increases the probability that the first seller faces a high value buyer. For moderate complements, this probability is significant enough that the second seller chooses a low price, ruling out the holdup and in turn, inducing aggressive pricing by the first seller. For strong complements, the probability of a high value buyer is less significant and thus the second seller also puts weight on the full – surplus extracting – price of 1, leading the first seller to decrease his average price for a low value buyer. An interesting implication of the pricing effect is that for strong complements, an informed buyer prefers *strategic* sellers who read into her sequencing to those who do not while for moderate complements, she prefers nonstrategic sellers.[20]

4 Information acquisition

Equilibrium information acquisition. By definition, the buyer's value of information is the difference between her informed and uninformed payoffs: $\Delta(q) \equiv B^I(q) - B^U(q)$. Using (1) and (2), we have

$$\Delta(q) = \begin{cases} \frac{(1-q)^2(q^2+2q)}{2} & \text{if} \quad q > \frac{1}{\sqrt{2}} \\ \\ -\frac{(1-q)^2}{2} & \text{if} \quad \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \tag{4}$$

Eq.(4) implies that for moderate complements, the buyer is strictly worse off being

[20]It is also worth noting that to isolate the value of informed sequencing from the value of informed trade, we have assumed that even an uninformed buyer privately learns v_i before purchasing good i, perhaps through free consultation. If, on the contrary, an uninformed buyer also made uninformed trades, then the first seller would extract all her (expected) surplus, i.e., $B^{UU} = 0$. Clearly, $B^U - B^{UU} > 0$, so the value of informed trade is positive. The effect of informed seqeuencing, $B^I - B^U$, would, however, remain since $B^I - B^{UU} = 0$ for moderate complements, and $B^I - B^{UU} > B^U - B^{UU}$ for strong complements.

informed! As discussed above, informed sequencing causes the first seller to set the high price in this case, leaving no surplus to the buyer. Put differently, for moderate complements, the negative pricing effect of being informed dominates the positive sequencing effect. For strong complements, both effects are positive and so is the value of information, which the buyer weighs against the cost of information, c.

Proposition 2 *If goods are strong complements, $q > \frac{1}{\sqrt{2}}$, and the information cost is low enough, $c < \Delta(q)$, then the buyer optimally acquires information. If, on the other hand, goods are moderate complements, $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, she optimally stays uninformed.*

Hence, the buyer prefers informed sequencing if and only if goods are strong complements and the information cost is low. Otherwise, even with no information cost, the buyer prefers to sequence uninformed. The buyer can credibly remain uninformed by: (1) significantly raising her own cost, perhaps through overloading with multiple tasks (Aghion and Tirole, 1997); (2) delegating her sequencing decision to an uninformed third party; or (3) letting the sellers self-sequence.

Note that if the sellers were nonstrategic, the value of information would be positive for *all* $q > \frac{1}{2}$. To see this, we subtract (1) from (3):

$$\overline{\Delta}(q) \equiv \frac{(1-q)^2 q}{2}. \tag{5}$$

Interestingly, $\overline{\Delta}(q) < \Delta(q)$ for $q > \frac{1}{\sqrt{2}}$. That is, for strong complements, the buyer has a greater incentive to be informed when the sellers are strategic and read into her sequence, which simply follows from the positive pricing effect identified above.

The buyer's equilibrium information strategy, however, is unlikely to be (socially) efficient, which we demonstrate next.

Efficient information acquisition. Suppose that a social planner who maximizes the expected welfare can publicly instruct the buyer whether or not to acquire information. Consider an uninformed buyer. From Proposition 1, ignoring the cost of information, the expected welfare defined as the expected total surplus is computed to be

$$\begin{aligned} W^U(q) &= q\left[\frac{1-q}{q}(1) + \frac{2q-1}{q}(1-q)(\frac{1}{2})\right] + (1-q)\left[q(\frac{1}{2}) + (1-q)(1)\right] \\ &= \frac{1}{2}(1-q)(3+q). \end{aligned}$$

Similarly, the expected welfare under an informed buyer is $W^I(q) = \frac{1}{2}(1-q^2)(4-q^2)$ if $q > \frac{1}{\sqrt{2}}$, and $W^I(q) = 1$ if $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$ since in the latter case, the bundle is purchased with certainty. Hence, the social value of information is $\Delta^W(q) \equiv W^I(q) - W^U(q)$ or

$$\Delta^W(q) = \begin{cases} \Delta(q) + \frac{1-q^2}{2} & \text{if} \quad q > \frac{1}{\sqrt{2}} \\ \\ \frac{q^2+2q-1}{2} & \text{if} \quad \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \tag{6}$$

Comparing (6) with (4), we readily conclude:

Proposition 3 *The social value of information is positive and exceeds its private value to the buyer; i.e.,* $\Delta^W(q) > 0$ *and* $\Delta^W(q) > \Delta(q)$*. Hence, the buyer's optimal information acquisition is less than efficient if* $c \in (\Delta(q), \Delta^W(q))$.

Given the complementarity, welfare is maximized by a joint sale and informed sequencing helps with this objective[21] – either by raising the first seller's price for moderate complements or by reducing the risk of holdup for strong complements. Since the buyer does not internalize the positive effect of her information decision on total surplus, however, she acquires it less often than is efficient.

Armed with the baseline analysis, we now consider several extensions and variations related to the information structure and bargaining protocol. In doing so, we show the robustness of our key results, though new insights also emerge.

5 Extensions and Variations

We begin with highlighting a commitment issue in acquiring information and then examine the buyer's choice between sequential procurement and an auction.

[21] From Proposition 1, it can be verified that whereas an uninformed buyer purchases the bundle with probability:

$$q\frac{1-q}{q} + (1-q)(1-q) = (1-q)(2-q),$$

an informed buyer purchases the bundle with probability 1 for moderate complements, and with probability $(1-q^2)(2-q^2)$ for strong complements.

5.1 Unobservable information acquisition

In the base model, the buyer can commit to visiting the sellers informed or uninformed. This is reasonable if the buyer can publicly hire an expert to ascertain the values of the objects. It is, however, possible that her hiring decision is unobservable to the sellers, creating a potential commitment problem. To fix ideas, consider the case of moderate complements for which the buyer would commit to sequencing uninformed. If the sellers believed this to be the buyer's strategy, they would offer their uninformed prices, yielding a positive value of information, $\overline{\Delta}(q)$ found in (5). That is, while being optimal under commitment, remaining ignorant is not time-consistent for the buyer. In the case of strong complements, the value of information, $\Delta(q)$, is positive so information acquisition is likely when unobservable, too. It is, however, less likely than under commitment as Proposition 4 shows. In its statement, let ϕ^* be the buyer's equilibrium probability of being informed before meeting with the sellers.

Proposition 4 *When unobservable to the sellers, the buyer acquires information more (resp. less) frequently than she would under commitment for moderate (resp. strong) complements. Formally,*

(a) *if* $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$ *and* $c < \overline{\Delta}(q)$, *then in the unique equilibrium, the buyer acquires information with probability* $\phi^* = \frac{2q-1}{2q(1-q)} \in (0,1)$, *and the sellers set prices:*

$$p_1^* = \frac{\beta^*}{2} \text{ and } p_2^*(h=1) = \begin{cases} \frac{1}{2} & \text{with prob.} \quad \beta^* \\ \\ 1 & \text{with prob.} \quad 1-\beta^* \end{cases}$$

where $\beta^* = 1 - \frac{2c}{q(1-q)}$;

(b) *if* $q > \frac{1}{\sqrt{2}}$ *and* $c \in ((1+q)\overline{\Delta}(q), \Delta(q))$, *then in the unique equilibrium, the buyer remains uninformed,* $\phi^* = 0$, *and the sellers set their uninformed prices in Proposition 1(a).*

Proposition 4 follows because when information acquisition is unobservable, the buyer does not internalize the pricing effect of being informed, which, as identified in Section 3, is negative for moderate complements and positive for strong complements.[22] Therefore,

[22] In Proposition 4, we present only those formal results that sharply contrast with the case of commitment above. The conclusions, however, hold more generally since it is readily verified that ϕ^* is nonincreasing in c.

compared to the case of commitment, the buyer attaches a higher value to being informed for moderate complements and a lower value to being informed for strong complements. As mentioned above, the buyer would want to sequence the sellers of moderate complements uninformed but this is not credible given $c < \overline{\Delta}(q)$. She would not sequence them informed either, because the (commitment) value of information, $\Delta(q)$, is negative in this region, explaining the strict mixing in information acquisition in part (a). For strong complements, the buyer's incentive is reversed: she would want to sequence the sellers informed when $c < \Delta(q)$, but she also wants to save on the information cost, resulting in equilibrium ignorance when information is sufficiently costly.

It is intuitive that by restricting her ability to commit, the unobservability of information acquisition cannot make the buyer better off than her commitment strategy. In fact, by strictly deviating from her commitment strategy, the buyer is strictly worse off under the parameter conditions in parts (a) and (b). Such a suboptimal behavior by the buyer, however, improves the welfare for moderate complements by encouraging informed sequencing. To see this, note that given the equilibrium prices in part (a), a low value buyer for the first good will always make a joint purchase whereas a high value buyer will not purchase the second good if its price turns out to be high. Therefore, the expected total surplus for moderate complements is:

$$\begin{aligned} W^*(q) &= q(1) + (1-q)[\beta^*(1) + (1-\beta^*)(\frac{1}{2})] \\ &= 1 - \frac{c}{q}. \end{aligned}$$

and given $c < \overline{\Delta}(q)$ and $W^I(q) = 1$, it is readily verified that $W^*(q) \in (W^U(q), W^I(q))$. For strong complements, the unobservability of information acquisition hurts the welfare since private and social incentives for informed sequencing diverge further in this case.

5.2 Sequential procurement vs. auction

In the base model, the buyer also visits sellers sequentially. This is natural if the buyer has a capacity or privacy concern to deal with both sellers. Absent such concerns, the buyer could alternatively hold an auction in which she receives simultaneous price offers from the sellers and decides on her purchases after being informed of all prices and valuations. The obvious advantage of an auction over sequential procurement is that the buyer avoids the holdup problem and will incur no ex post loss. Its potential disadvantage is that having no sequence, both sellers are likely to target the buyer's extra surplus from complementarity.[23] Therefore, in the auction, the sellers essentially play a simultaneous game of price coordination. Focusing on symmetric equilibria of this game, Lemma 2 offers a characterization.[24]

Lemma 2 *In the auction, there is a unique symmetric-price equilibrium,* $p^A = \frac{1}{2}$ *for* $q \in (\frac{1}{2}, 2(\sqrt{2}-1))$. *For* $q \geq 2(\sqrt{2}-1)$, *there is also a continuum of symmetric mixed-strategy equilibria, where*

$$p^A = \begin{cases} \frac{\eta q}{1+\eta q} & \textit{with prob.} \quad \eta \\ \\ \frac{1}{1+\eta q} & \textit{with prob.} \quad 1-\eta \end{cases}$$

and $\eta \in \left[\frac{1}{2} - \frac{\sqrt{(q+2)^2-8}}{2q}, \frac{1}{2} + \frac{\sqrt{(q+2)^2-8}}{2q}\right]$.

The multiplicity of equilibria in the auction is not surprising given the coordination game. In equilibrium, low and high prices sum to the joint valuation of 1, so the buyer obtains a positive surplus by purchasing the bundle or one high value object at a low price in a mixed strategy equilibrium. In particular, the buyer enjoys no surplus in a pure strategy equilibrium but receives the following expected payoff in a mixed-strategy equilibrium:

$$\begin{aligned} B^A(q,\eta) &= \eta^2\left(1 - 2\frac{\eta q}{1+\eta q}\right) + 2\eta(1-\eta)(1-q)\left(\frac{1}{2} - \frac{\eta q}{1+\eta q}\right) \\ &= \eta\frac{1-\eta q}{1+\eta q}[1-(1-\eta)q]. \end{aligned} \tag{7}$$

To understand the buyer's choice between sequential procurement and auction, consider first an uninformed buyer. Recall from (1) that an uninformed buyer has a strictly positive expected payoff, $B^U(q)$, under sequential procurement, which clearly dominates the auction if $p_i^A = \frac{1}{2}$ is anticipated in equilibrium. If, however, a mixed strategy equilibrium with $\eta = \frac{1}{2}$ is anticipated, the buyer is strictly better off conducting an auction since $B^U(q) < B^A(q, \frac{1}{2})$. The next proposition collects these observations.

[23] That disadvantage is absent for substitute goods, which we briefly consider in Section 6.

[24] The focus on symmetric-price equilibria here is not only for tractability but also for removing additional signaling due to asymmetric prices in the auction.

Proposition 5 ***(Uninformed buyer)*** *For all $q > \frac{1}{2}$, there is an equilibrium in which an uninformed buyer chooses sequential procurement over an auction. This equilibrium is unique if $q \in (\frac{1}{2}, 2(\sqrt{2}-1))$; otherwise, there is also an equilibrium in which she holds an auction.*

For an informed buyer, the choice between sequential procurement and auction becomes an additional source of signaling. Depending on the off-equilibrium beliefs, such signaling may leak too much information and leave the buyer with no surplus. There are, however, equilibria in which the buyer receives a positive payoff when goods are strong complements, as the following result shows.

Proposition 6 ***(Informed buyer) (a)*** *For all $q > \frac{1}{2}$, there is an equilibrium in which an informed buyer receives 0 payoff. One such equilibrium is that the buyer conducts sequential procurement unless goods are perfect complements, i.e., $v_1 = v_2 = 0$;* ***(b)*** *an equilibrium with a positive payoff for an informed buyer exists if and only if $q > \frac{1}{\sqrt{2}}$. In such an equilibrium, the buyer strictly chooses sequential procurement for $q \in (\frac{1}{\sqrt{2}}, 2(\sqrt{2}-1))$ unless goods are perfect complements. For $q \geq 2(\sqrt{2}-1)$, the auction can also be supported as an equilibrium choice.*

Intuitively, if an informed buyer is expected to perform sequential procurement only when she has a high value for at least one object, given her incentive to begin with a high value seller, both sellers would charge a price of $\frac{1}{2}$, leaving no surplus to the buyer while inducing a joint purchase. The informed buyer also receives no surplus in the auction because anticipating a bid for perfect complements, the sellers again charge $\frac{1}{2}$ in the symmetric equilibrium. Being indifferent between the two mechanisms, the buyer has no strict incentive to deviate from her strategy, explaining part (a). This clearly demonstrates how an informed buyer can be hurt by strategically choosing the mechanism since she could guarantee herself a positive payoff, $B^I(q)$, for $q > \frac{1}{\sqrt{2}}$ by committing to sequential procurement at the outset. That is, with the strategic choice of the mechanism, the value of information can be negative for all $q > \frac{1}{2}$ – not just for $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$ as in the base model. Nevertheless, part (b) says that there is also an equilibrium with a positive payoff for an informed buyer. Despite the equilibrium multiplicity, Propositions 5 and 6 indicate that sequential procurement can emerge as an equilibrium mechanism, as assumed in the base model.

Remark 1 *Proposition 5 implies that sequential procurement can also emerge in equilibrium if the buyer commits to a procurement mechanism uninformed but has the option to acquire information before approaching the sellers. The reason is that the buyer can choose to stay uninformed in sequential procurement.*

5.3 Seller's vs. buyer's market

To identify strategic sequencing as a source of bargaining power for the buyer, we have also assumed in the base model that sellers make the price offers – i.e., each operates in a seller's market. We predict that the buyer will value sequencing less if she expects a buyer's market. To confirm, let $m_i \in \{s_i, b\}$ denote the state of market i, which favors either seller i or the buyer as the price-setter. We assume that sellers already know their respective market conditions but the buyer needs to find out.[25] Specifically, the buyer is assumed to learn m_1 and m_2 at an interim stage between information acquisition and meeting with the sellers.[26] Letting $\Pr(m_1, m_2)$ be the joint probability distribution over the states of the markets, the following proposition shows that the buyer discounts the value of information by the likelihood of facing a seller's market in each meeting.

Proposition 7 *In the setting just described, the value of information to the buyer is*

$$\overline{\Delta}^m(q) = \Pr(s_1, s_2)\Delta(q).$$

Intuitively, if both markets turn in the buyer's favor, there is no value to informed sequencing because the buyer, informed or uninformed, offers a price of 0 to each seller and secures the highest payoff of 1. Interestingly, the buyer's informed and uninformed payoffs are also equal, though not 1, even if only one market turns in her favor. With a mix of markets, the fact that a purchase is always made in the buyer's market (at the price of 0) implies that the buyer would face the same holdup problem in the seller's market regardless of her sequencing. Given this, it is optimal for an informed buyer to ignore her private information and always begin with the buyer's market in order to mitigate

[25] In the real estate market, the buyer can discover the market condition from the stock of listings or expert opinions.

[26] Though convenient, this timing of events is not crucial. Our conclusion in Proposition 7 would not change if the buyer learned m_1 and m_2 before her information decision.

future holdup.[27] Hence, the buyer cares about informed sequencing to the extent that she anticipates *all* sellers' markets, as assumed in the base model. Put differently, the buyer views strategic sequencing as a substitute to other sources of bargaining power – i.e., strategic sequencing is most valuable to the buyer with the least bargaining power.

5.4 Transparency

Our base model is also well-suited to address the issue of transparency about trade and price histories. Recall that the second seller observes the trade but not the price history, which can serve as an additional signal of buyer's valuation and in turn, impact her payoff. To this end, we consider three other disclosure regimes where both price and trade, only the price, or none can be disclosed.[28] The next proposition determines the buyer's payoff in each, leaving equilibrium details to the appendix.

Proposition 8 *Suppose $q > \frac{1}{2}$. If price history is disclosed, then both informed and uninformed buyers receive an equilibrium payoff of 0, irrespective of trade disclosure. If neither price nor trade history is disclosed, then the informed and uninformed buyers receive equilibrium payoffs $B^I(q)$ and $\frac{1+q}{2-q}B^U(q)$, respectively.*

Proposition 8 is best understood in conjunction with the base model. Consider an uninformed buyer. From Proposition 1, suppose that the first seller ends up offering his discount price, $p_1^U < \frac{1}{2}$, which is accepted by both high and low value buyers. When the second seller observes p_1^U, however, this is no longer an equilibrium: given the prior $q > \frac{1}{2}$ and ignoring the past payment, the second seller would then set the full price of 1, discouraging a low value buyer from purchasing the first item and in turn, encouraging the first seller to target only the high value buyer. Indeed, as shown in the proof, both sellers offer $\frac{1}{2}$ under price disclosure, leaving no surplus to the buyer.[29] The same is also true for an informed buyer because of her incentive to begin with a high value item.

When neither price nor trade history is disclosed, the uninformed buyer fares better than the base model. The reason is that unable to observe the trade history, the second seller is more reliant on his prior, $q > \frac{1}{2}$, and therefore, more likely to offer the holdup

[27] That is, sequential procurement would essentially turn into a one-market problem where the seller makes the offer to a buyer with a privately known outside option, 0 or $\frac{1}{2}$.

[28] Given our focus on informed sequencing, we do not consider nondisclosure of the sequence in this paper; see Krasteva and Yildirim (2012a) for such a consideration in a complete information framework.

[29] Sequencing is also immaterial in this case.

price of 1, which induces the first seller to increase the frequency of his discount price – the only source of buyer's surplus.[30] An informed buyer does not, however, benefit from nondisclosure of trade since her sequencing reveals enough about it in equilibrium.

Comparing the four disclosure regimes regarding trade and price histories, it follows that irrespective of trade disclosure, price disclosure is weakly socially optimal as it eliminates the holdup problem.[31] It is, however, the least preferred by the buyer: an uninformed buyer strictly prefers no disclosure whereas an informed buyer weakly prefers trade only (as in the base model) or no disclosure regimes. Roughly speaking, being a continuous variable, price is a more precise signal of buyer's valuation than trade, allowing the sellers to better coordinate their offers to extract her surplus.

5.5 Correlated values

Our base model can also be easily extended to capture (positive) correlation between the stand-alone values.[32] For instance, a developer who is unable to acquire the desired land parcels for a shopping mall may appraise each similarly for a smaller project. Here we show that correlation reduces the incentive for informed sequencing. To this end, consider the following joint distribution of stand-alone values:

$\Pr(v_1, v_2)$	0	$\frac{1}{2}$
0	$q^2 + rq(1-q)$	$(1-r)q(1-q)$
$\frac{1}{2}$	$(1-r)q(1-q)$	$(1-q)^2 + rq(1-q)$

where $r \in [0, 1]$ denotes the correlation coefficient, with $r = 0$ and 1 referring to the base model and ex post homogenous goods, respectively.

The equilibrium characterization with correlation closely mimics Proposition 1 (see Proposition A3). In particular, the expected uninformed payoff in (1) remains intact since, as in the base model, the buyer's equilibrium payoff depends on the first deal. The expected informed payoff in (2) is, however, slightly modified by replacing the posterior q^2

[30] From Proposition 1(a), it is clear that in the base model, the second seller offers an uninformed price of 1 with an ex ante probability of $2(1-q)q$ whereas, as shown in the proof of Proposition 8, this probability remains q under a no disclosure regime.

[31] As mentioned above, under price disclosure, both sellers offer $\frac{1}{2}$. This maximizes the trade, but interestingly, it does not ensure a joint purchase in equilibrium. If it did, given the first seller's price and his prior $q > \frac{1}{2}$, the second seller would charge the full price of 1, inducing no purchase of the first item by a low value buyer.

[32] The argument for negatively correlated goods is analogous.

with $\Pr(0,0)$:

$$B^{C,I}(q;r) = \begin{cases} \frac{[1-\Pr(0,0)]^2}{2} & \text{if} \quad q > \overline{q}(r) \\ \\ 0 & \text{if} \quad \frac{1}{2} < q \leq \overline{q}(r) \end{cases} \tag{8}$$

where $\overline{q}(r) \geq \frac{1}{2}$ uniquely solves $\Pr(0,0) = \frac{1}{2}$ such that $\overline{q}'(r) < 0$, $\overline{q}(0) = \frac{1}{\sqrt{2}}$, and $\overline{q}(1) = \frac{1}{2}$. By subtracting (1) from (8), we obtain the value of information under correlation:

$$\Delta^C(q;r) = \begin{cases} \frac{[1-\Pr(0,0)]^2-(1-q)^2}{2} & \text{if} \quad q > \overline{q}(r) \\ \\ -\frac{(1-q)^2}{2} & \text{if} \quad \frac{1}{2} < q \leq \overline{q}(r). \end{cases} \tag{9}$$

As expected, $\Delta^C(q;0) = \Delta(q)$. Moreover, $\Delta^C(q;1) = 0$. This makes sense because when goods are ex post homogeneous, the buyer's ability to match a high value good with a low price seller under informed sequencing is inconsequential. More generally, informed sequencing becomes less consequential when goods are more correlated and thus less heterogeneous: formally, $\Delta^C(q;r)$ is strictly decreasing in r for $q > \overline{q}(r)$. It is, however, worth noting that since $\Pr(0,0)$ is increasing in r, $\overline{q}'(r) < 0$; that is, correlation reduces the incentive to remain uninformed by increasing the likelihood of perfect complements.

5.6 Partial information

Another restriction in the base model is that information is all-or-nothing: the buyer can discover both valuations ex ante by paying a fixed cost. If the marginal cost of information is significant, however, the buyer may choose to learn only one valuation. We argue that the buyer is unlikely to gain from such "partial" information. Suppose that prior to meeting with the sellers, the buyer privately discovers only v_i. If she approaches seller i second, then she engenders the uninformed equilibrium described in Proposition 1 and obtains a positive expected payoff for $q > \frac{1}{2}$. If, instead, the buyer approaches seller i first, she receives an expected payoff of 0 irrespective of v_i. For a low value buyer, this follows from Corollary 1. For a high value buyer, this follows because seller i would infer the buyer's valuation from sequencing and charge a sure price of $\frac{1}{2}$, leaving no surplus to the buyer. We therefore obtain Proposition 9.

Proposition 9 *Suppose $q > \frac{1}{2}$ and that the buyer is privately informed of v_i only. Then,* *she optimally sequences seller i second and receives her uninformed payoff in (1).*

Proposition 9 justifies our focus on all-or-nothing information. Intuitively, the buyer cannot exploit partial information as it leaks through her sequencing; to avoid this, the buyer begins with the seller of the uncertain good, effectively committing to behaving uninformed. This contrasts with a fully informed buyer whose sequencing leaves a significant probability that the first seller has a low value item.

5.7 Ex ante heterogenous goods and uninformed sequencing

Assuming ex ante homogenous goods, our baseline analysis has focused exclusively on the value of informed sequencing. Sequencing may, however, matter also for an uninformed buyer if goods are ex ante heterogenous.[33] To see this, let $q_k = \Pr\{v_k = 0\}$ be the common prior on good k. Setting $\phi = 0$ and $q_k(0) = q_k$, Proposition A1 in the appendix readily reveals that by sequencing the sellers $s_i \to s_j$, an uninformed buyer receives the expected payoff:[34]

$$B_{ij}^U = \begin{cases} 0 & \text{if} \quad q_i \leq \frac{1}{2} \\ \\ \frac{(1-q_i)^2}{2} & \text{if} \quad q_i > \frac{1}{2}. \end{cases}$$

As with the informed, an uninformed buyer is indifferent in the sequence for $q_1 < q_2 \leq \frac{1}{2}$ because the sellers charge the monopoly price of $\frac{1}{2}$ regardless. For $\frac{1}{2} < q_1 < q_2$, she optimally begins with the higher value seller – seller 1 – in order to reduce the future holdup. Finally, trading off between receiving a monopoly price from the first seller and being held up by the second, for $q_1 \leq \frac{1}{2} < q_2$, she optimally begins with the lower value seller – seller 2.

5.8 Uncertain joint value

In the base model, we have also maintained that the buyer's joint value is commonly known. This fits well applications where the buyer has a large winning project: e.g., a vaccine company acquiring all the necessary antigens to guarantee an effective vaccine

[33] As alluded to in the introduction, in land assembly, it is possible that the developer's plan is sufficiently public and so are her likely appraisals of the parcels.

[34] For $q_i = \frac{1}{2}$, we again select the efficient equilibrium here, which corresponds to $\beta = 1$ in Proposition A1.

or a lobbyist seeking bipartisan support that secures the favorable legislation. In other applications, the buyer's joint value may be uncertain – at least initially. For instance, a developer may be initially unsure of the profitability of a large shopping mall. To understand buyer's incentive to acquire information in such cases, we examine a setting in which the joint value is uncertain: $V = 1$ or $\overline{V} > 1$ where $\Pr\{V = \overline{V}\} = \alpha \in (0, 1)$. Stand-alone values are, however, equal and commonly known, $v_1 = v_2 = v \in [0, \frac{1}{2}]$, so sequencing is immaterial. As in the base model, the buyer can privately discover V at a cost prior to meeting with the sellers or wait until she meets with both (so the second purchase is always informed). Proposition 10 characterizes the value of information in this setting.

Proposition 10 *Consider the setting with an uncertain joint value as described above. In equilibrium, the buyer's uninformed payoff is $B^{J,U}(\alpha) = 0$ whereas her informed payoff and thus her value of information is*

$$\Delta^J(\alpha) = B^{J,I}(\alpha) = \begin{cases} \alpha(\overline{V} - 1) & \textit{if} \quad \alpha \leq \frac{v}{v+\overline{V}-1} \\ \\ \frac{v}{1-v}\alpha(\overline{V} - 1) & \textit{if} \quad \frac{v}{v+\overline{V}-1} < \alpha \leq \frac{1-v}{\overline{V}-v} \\ \\ 0 & \textit{if} \quad \alpha > \frac{1-v}{\overline{V}-v}. \end{cases}$$

An uninformed buyer receives no expected surplus because it is taxed away by the first seller. This means that an uninformed buyer may realize a loss after the second purchase if her joint value turns out to be low. To minimize such holdup, the buyer therefore has an incentive to approach the sellers informed. An informed purchase from the first seller, however, leads the second seller to be more optimistic about a high joint value and raise price, fully extracting the buyer's surplus when a high joint value is sufficiently likely, i.e., $\alpha > \frac{1-v}{\overline{V}-v}$. Note that this parameter condition is more likely to be satisfied as goods become stronger complements, i.e., a higher $\overline{V}$ or a lower v. Hence, contrasting with the base model, we conclude that whereas informed sequencing with uncertain stand-alone values reduces the potential holdup, an informed purchase with an uncertain joint value exacerbates it. Moreover, $\Delta^J(\alpha)$ and $\Delta(q)$ together imply that the buyer is likely to discover her stand-alone values for strong complements and her joint value for moderate complements.

6 Substitutes

Sequential procurement and thus the issues of information acquisition and sequencing can also be pertinent to substitutes – e.g., parcels at rival locations. We, however, argue that with substitutes, sequential procurement is undesirable for the buyer as it forecloses competition between the sellers; instead, the buyer is likely to hold an auction with simultaneous price offers. To make the point, let the buyer's joint value be 1 (as in the base model) but her stand-alone values be independently distributed such that $\Pr\{v_i = 1\} = q_u$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q_u$. Clearly, with probability q_u^2, goods are perfect substitutes whereas with probability $(1 - q_u)^2$, they are independent. We assume $q_u > \frac{1}{2}$ so that perfect substitutes are more likely.[35]

Proposition 11 *Consider substitute goods with $q_u > \frac{1}{2}$. Then, an uninformed buyer strictly prefers auction to sequential procurement.*

Proposition 11 is easily understood for (almost) perfect substitutes, $q_u \approx 1$. Unsurprisingly, the auction engenders the most competitive prices of 0 and in turn, the highest expected payoff of 1 for the buyer. In contrast, sequential procurement results in the *monopoly* prices of 1 and yields the lowest payoff of 0. The latter follows because with no previous purchase, the last seller sets his monopoly price and anticipating this, so does the first seller, leaving no surplus to the buyer. The buyer continues to receive monopoly prices under sequential procurement for imperfect substitutes, $q_u > \frac{1}{2}$, but due to competition, lower prices are likely in the auction. In particular, the proof of Proposition 11 establishes that there is no pure strategy equilibrium in the auction: the sellers trade off pricing for perfect substitutes and pricing for independent units.

7 Conclusion

In this paper, we have analyzed in some detail the value of information in sequencing complementary negotiations. Our baseline analysis has produced three main observations. First, an informed buyer optimally begins with the high value seller to mitigate the future holdup. Second, because of the sellers' pricing response to the sequence, the buyer

[35]The comparison is for an uninformed buyer because the auction is strategically equivalent to uninformed sequencing except that price offers are "nonexploding," i.e., they do not require an immediate purchasing decision.

may be strictly worse off being informed; that is, ignorance may be bliss. And third, the buyer underinvests in information acquisition from a social standpoint.

Extending the baseline analysis, we have also shown, among other things, that the inefficiency in information acquisition may be lower when it is unobservable to the sellers, and that despite creating a holdup problem, the buyer may prefer sequential procurements of complementary objects to an auction mechanism. The reason for the latter is that without any sequence, the auction motivates each seller (not just the last one in sequence) to bid for the buyer's extra surplus from complementarity. We have further shown that strategic sequencing is indeed a source of bargaining power for the buyer and substitutes others: it is less valuable to a buyer who is more likely to set the prices. Last but not least, we find that being a more precise signal of buyer's willingness to pay than trade, price disclosure facilitates an efficient purchase but it is the least preferred by the buyer. Empirically testing our predictions would be an important next step.

Appendix A

In this appendix, we prove the formal results. As in the text, we re-label the sellers so that the sequence is $s_1 \to s_2$ unless stated otherwise. For future reference, Proposition A1 characterizes the equilibrium with the following information structure: the buyer privately knows $z \in \{I, U\}$ but the sellers commonly believe that $\Pr\{z = I\} = \phi \in [0, 1]$. Conditional on this information structure, let $q_1(\phi) = \Pr\{v_1 = 0|\phi\}$ be the posterior belief that s_1 is of low value.[36]

Proposition A1. *In equilibrium,* $p_2(h = 0) = \frac{1}{2}$ *and*

(a) *if* $q_1(\phi) < \frac{1}{2}$*, then* $p_1 = p_2(h = 1) = \frac{1}{2}$ *and the buyer purchases the bundle with certainty;*

(b) *if* $q_1(\phi) = \frac{1}{2}$*, then* $p_1 = \frac{\beta}{2}$ *and* $p_2(h = 1) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad \beta \\ 1 & \textit{with prob.} \quad 1 - \beta \end{cases}$

where $\beta \geq \frac{1}{2}$*. The buyer purchases from* s_1 *with certainty and* s_2 *only if* $v_1 = 0$ *or* $p_2(h = 1) = \frac{1}{2}$*;*

(c) *if* $q_1(\phi) > \frac{1}{2}$*, then*

$$p_1 = \begin{cases} \frac{1-q_1(\phi)}{2} & \textit{with prob.} \quad \frac{1-q_1(\phi)}{q_1(\phi)} \\ \frac{1}{2} & \textit{with prob.} \quad \frac{2q_1(\phi)-1}{q_1(\phi)} \end{cases} \quad \textit{and } p_2(h = 1) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad 1 - q_1(\phi) \\ 1 & \textit{with prob.} \quad q_1(\phi). \end{cases}$$

Moreover, a buyer with $v_1 = 0$ *accepts only low* p_1 *but all* $p_2(h = 1)$ *whereas a buyer with* $v_1 = \frac{1}{2}$ *accepts all* p_1 *but only the low* $p_2(h = 1)$*.*

Proof. Consider pricing by s_2. Clearly $p_2(h = 0) = \frac{1}{2}$ since s_2 realizes a positive payoff only if $v_2 = \frac{1}{2}$. Let $h = 1$. Then a buyer with $v_1 = 0$ accepts any offer $p_2(h = 1) \leq 1$ whereas a buyer with $v_1 = \frac{1}{2}$ accepts only $p_2(h = 1) \leq \frac{1}{2}$. Thus, s_2's optimal price is

[36] In Proposition A1, we do not yet select among multiple equilibria in favor of efficiency as it may be uniquely pinned down when we endogenize the information structure later.

$$p_2(h=1) = \begin{cases} \frac{1}{2} & \text{if} \quad \widehat{q}_1(\phi,1) \leq \frac{1}{2} \\ \\ 1 & \text{if} \quad \widehat{q}_1(\phi,1) \geq \frac{1}{2}, \end{cases} \tag{A1}$$

where $\widehat{q}_1(\phi,h) = \Pr\{v_1 = 0|\phi,h\}$ is the posterior conditional on the buyer's information and purchase history.

Anticipating $p_2(h=1)$, a buyer with v_1 is willing to pay s_1 up to $\overline{p}_1(v_1)$ such that

$$\max\{1 - \overline{p}_1(v_1) - p_2(h=1), v_1 - \overline{p}_1(v_1)\} = 0,$$

or simplifying,

$$\overline{p}_1(v_1) = \max\{1 - p_2(h=1), v_1\}. \tag{A2}$$

Next we show that $\widehat{q}_1(\phi,1) \leq \frac{1}{2}$ in equilibrium. Suppose, to the contrary, that $\widehat{q}_1(\phi,1) > \frac{1}{2}$. Then $p_2(h=1) = 1$, $\overline{p}_1(v_1 = 0) = 0$, and $\overline{p}_1(v_1 = \frac{1}{2}) = \frac{1}{2}$. But this would imply $p_1 = \frac{1}{2}$ and in turn $\widehat{q}_1(\phi,1) = 0$ – a contradiction. We exhaust two remaining possibilities for $\widehat{q}_1(\phi,1)$.

$\widehat{q}_1(\phi,1) < \frac{1}{2}$: Then $p_2(h=1) = \frac{1}{2}$ from (A-1), and $\overline{p}_1(v_1 = 0) = \overline{p}_1(v_1 = \frac{1}{2}) = \frac{1}{2}$ from (A-2). This implies $\widehat{q}_1(\phi,h) = q_1(\phi)$ and thus $q_1(\phi) < \frac{1}{2}$, which reveals that the buyer purchases the bundle with certainty, proving part (a).

$\widehat{q}_1(\phi,1) = \frac{1}{2}$: By (A-1), s_2 is indifferent between the prices $\frac{1}{2}$ and 1. Suppose s_2 offers $\frac{1}{2}$ with probability β. Then, by (A-2), $\overline{p}_1(v_1 = \frac{1}{2}) = \frac{1}{2}$ and $\overline{p}_1(v_1 = 0) = \frac{\beta}{2}$. Let s_1 mix between the prices $\frac{1}{2}$ and $\frac{\beta}{2}$ by offering the latter with probability $\gamma \in [0,1]$. Evidently, the buyer always accepts $\frac{\beta}{2}$ whereas only the buyer with $v_1 = \frac{1}{2}$ accepts $\frac{1}{2}$. Using Bayes' rule, we therefore have $\widehat{q}_1(\phi,1) = \frac{\gamma q_1(\phi)}{\gamma q_1(\phi) + 1 - q_1(\phi)}$, which, given $\widehat{q}_1(\phi,1) = \frac{1}{2}$, implies $\gamma = \frac{1-q_1(\phi)}{q_1(\phi)}$. For $q_1(\phi) = \frac{1}{2}$, $\gamma = 1$ or $p_1 = \frac{\beta}{2}$. By the buyer's optimal purchasing decision, this means $\frac{\beta}{2}(1) \geq \frac{1}{2}(1 - q_1(\phi))$ or equivalently $\beta \geq \frac{1}{2}$, resulting in the equilibrium multiplicity in part (b). Finally, for $q_1(\phi) > \frac{1}{2}$, $\gamma \in (0,1)$. Such strict mixing by s_1 requires $\frac{\beta}{2} = \frac{1-q_1(\phi)}{2}$ or $\beta = 1 - q_1(\phi)$, proving part (c). ■

Proof of Lemma 1. Suppose $q \leq \frac{1}{2}$. For $z = U$, $\phi = 0$ and $q_1(0) = q$. For $z = I$ or $\phi = 1$, it must be that $q_1(1) \leq \frac{1}{2}$; otherwise, if $q_1(1) > \frac{1}{2}$, equilibrium prices in Proposition A1 would imply an informed sequence strictly from high to low value – i.e., $\theta_1(\frac{1}{2}, 0) = 1$, and in turn, $q_1(1) = q^2 < \frac{1}{2}$ – a contradiction. Given that $q_1(0) \leq \frac{1}{2}$ and

$q_1(1) \leq \frac{1}{2}$, Proposition A1 further reveals that $(p_1^z, p_2^z(h)) = (\frac{1}{2}, \frac{1}{2})$ for $z = I, U$ and $h = 0, 1$, inducing a joint purchase, where the sellers' indifference at $q_1(\phi) = \frac{1}{2}$ is broken in favor of efficient pricing. ■

Proof of Proposition 1. Suppose $q > \frac{1}{2}$. For $z = U$, parts (a) and (c) are immediate from Proposition A1 since $q_1(0) = q$. Next, consider $z = I$. If $q > \frac{1}{\sqrt{2}}$, the proof of Lemma 1 has established that $\theta_1(\frac{1}{2}, 0) = 1$ and $q_1(1) = q^2 > \frac{1}{2}$. Therefore, $q_1(1) \leq \frac{1}{2}$ if $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, where we break the sellers' indifference in favor of efficient pricing at $q_1(1) = \frac{1}{2}$. This implies $\theta_1(\frac{1}{2}, 0) > \frac{1}{2}$ given that by Bayesian updating,

$$\begin{aligned} q_1(1) &= \frac{q^2\frac{1}{2} + q(1-q)[1-\theta_1(\frac{1}{2},0)]}{\frac{1}{2}} \\ &= q^2 + 2q(1-q)[1-\theta_1(\frac{1}{2},0)]. \end{aligned} \tag{A3}$$

Applying Proposition A1, we obtain parts (b) and (c) for $z = I$. ■

Proof of Corollary 1. Follows directly from Proposition 1. ■

Proof of Proposition 2. Follows directly from (4). ■

Proof of Proposition 3. Follows directly from (4) and (6). ■

Proof of Proposition 4. With unobservable information acquisition, let ϕ^* denote the equilibrium probability that the buyer is informed. Then, the equilibrium probability that the buyer has a low value for the first item is given by

$$q_1(\phi^*) = \phi^* q_1(1) + (1-\phi^*)q. \tag{A4}$$

Moreover, from Corollary 1 and Proposition A1, the informed and uninformed buyer's receive expected payoffs:

$$B^I(\phi^*) = [1 - q_1(1)]\left[\frac{1}{2} - E[p_1(\phi^*)]\right] \text{ and } B^U(\phi^*) = (1-q)\left[\frac{1}{2} - E[p_1(\phi^*)]\right] \tag{A5}$$

where $E[p_1(\phi^*)]$ denotes the expected equilibrium price by s_1. Using (A-5), we find the equilibrium value of information:

$$\hat{\Delta}(\phi^*) \equiv B^I(\phi^*) - B^U(\phi^*) = [q - q_1(1)]\left[\frac{1}{2} - E[p_1(\phi^*)]\right]. \tag{A6}$$

To prove part (a), suppose $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$ and $c < \overline{\Delta}(q)$. We first claim that $q_1(\phi^*) = \frac{1}{2}$.

Suppose, instead, $q_1(\phi^*) < \frac{1}{2}$. Then, $p_1 = p_2(h) = \frac{1}{2}$ by Proposition A1, and thus $\hat{\Delta}(\phi^*) = 0 < c$ by (A-6), implying $\phi^* = 0$ and $q_1(\phi^*) = q > \frac{1}{2}$, a contradiction. Next, consider $q_1(\phi^*) > \frac{1}{2}$. From Proposition A1, the corresponding equilibrium prices would result in strict sequencing from a high to low value seller, i.e. $\theta_1(\frac{1}{2}, 0) = 1$, implying that $q_1(\phi^*) = \phi^* q^2 + (1-\phi^*)q > \frac{1}{2}$ and in turn, $\phi^* < 1$. In this case, the value of information would become $\hat{\Delta}(\phi^*) = q(1-q)\frac{1-q_1(\phi^*)}{2}$, which is strictly increasing in ϕ^*. But then, $\hat{\Delta}(\phi^*) \geq \hat{\Delta}(0) = \overline{\Delta}(q)(> c)$, implying $\phi^* = 1$, a contradiction. Hence, $q_1(\phi^*) = \frac{1}{2}$, as claimed. By (A-4), this requires $q_1(1) < \frac{1}{2}$ and $\phi^* > 0$. By Proposition A1 and (A-6), the corresponding value of information is: $\hat{\Delta}(\phi^*) = (q - q_1(1))\frac{1-\beta^*}{2}$. Clearly, $\beta^* < 1$; otherwise, $\hat{\Delta}(\phi^*) = 0 < c$, implying $\phi^* = 0$ and $q_1(\phi^*) = q > \frac{1}{2}$, a contradiction. From Proposition A1, $\beta^* < 1$ means a strict sequencing preference: $\theta_1(\frac{1}{2}, 0) = 1$ and $q_1(1) = q^2$. Moreover, by (A-4), we have $q_1(\phi^*) = \frac{1}{2}$, implying $\phi^* = \frac{2q-1}{2q(1-q)} \in (0,1)$ and in turn, the indifference condition: $\hat{\Delta}(\phi^*) = c$. Solving it, we find $\beta^* = 1 - \frac{2c}{q(1-q)}$ $(\geq \frac{1}{2})$, as stated.

To prove part (b), suppose that $q > \frac{1}{\sqrt{2}}$. Since $q_1(1) \geq q^2$, we have $q_1(\phi^*) > \frac{1}{2}$, which, as before, implies $q_1(1) = q^2$ and $\hat{\Delta}(\phi^*) = q(1-q)\frac{1-q_1(\phi^*)}{2}$. Since $\hat{\Delta}(\phi^*)$ is strictly increasing in ϕ^*, and $\hat{\Delta}(1) = (1+q)\overline{\Delta}(q) < \Delta(q)$, the result follows. ■

Proof of Lemma 2. Consider an auction. Note that the buyer (weakly) accepts a price offer p_i by s_i if and only if

$$\max\{1 - p_i - p_{-i}, v_i - p_i\} \geq \max\{v_{-i} - p_{-i}, 0\}. \tag{A7}$$

In a symmetric pure strategy equilibrium, $p_i = p_{-i} = p^A$. By (A-7), $p^A > \frac{1}{2}$ is rejected for sure whereas $p^A < \frac{1}{2}$ leads to a profitable deviation to $p_i' \in (p^A, \frac{1}{2}]$ since any offer $p^A < \frac{1}{2}$ is accepted with certainty. This leaves $p^A = \frac{1}{2}$ as the only price candidate for a symmetric equilibrium. The buyer's acceptance of $p^A = \frac{1}{2}$ with certainty prevents a unilateral deviation to $p_i^A < \frac{1}{2}$ while a deviation to $p_i^A > \frac{1}{2}$ is not profitable as it violates (A-7) and thus is rejected for sure. This establishes that $p^A = \frac{1}{2}$ is a pure strategy equilibrium for all q.

Next, consider a symmetric mixed strategy equilibrium and let $G(p^A)$ denote its cumulative price distribution with the support $[\underline{p}^A, \overline{p}^A]$. We now establish three properties of $G(.)$.

Property 1. $\overline{p}^A > \frac{1}{2}$ *and* $\underline{p}^A \in (0, \frac{1}{2})$.

Proof. Suppose, to the contrary, that $\overline{p}^A \leq \frac{1}{2}$. Then, by (A-7), any $p_i < \frac{1}{2}$ would be accepted with certainty, implying a profitable deviation to $p_i' \in (p_i, \frac{1}{2}]$. Thus, $\overline{p}^A > \frac{1}{2}$. To establish $\underline{p}^A \in (0, \frac{1}{2})$, note that by (A-7), any $p_i \in \left(0, \frac{1}{2}\right)$ must be accepted with a positive probability, ruling out $p_i = 0$ in equilibrium. Furthermore, if $\underline{p}^A \geq \frac{1}{2}$, (A-7) would imply that any price $p_i > \frac{1}{2}$ would be rejected with certainty. ■

Property 2. $\overline{p}^A = 1 - \underline{p}^A$.

Proof. If $\overline{p}^A > 1 - \underline{p}^A$, then since $1 - \underline{p}^A > \frac{1}{2}$ by Property 1, $\overline{p}^A$ would be rejected with certainty, implying a profitable deviation from $\overline{p}^A$ to $p_i^A \leq 1 - \underline{p}^A$. Conversely, if $\overline{p}^A < 1 - \underline{p}^A$, then any $p_i \in (\underline{p}^A, 1 - \overline{p}^A)$ is accepted with certainty, implying a profitable deviation from $\underline{p}^A$ to p_i. Hence, $\overline{p}^A = 1 - \underline{p}^A$. ■

Property 3. *Let $G(\underline{p}^A) = \eta$. Then, $\eta > 0$, $\underline{p}^A = \frac{\eta q}{1+\eta q}$, and $\overline{p}^A = \frac{1}{1+\eta q}$.*

Proof. Since $\underline{p}^A < \frac{1}{2}$ by Property 1, (A-7) implies that $\underline{p}^A$ is accepted with certainty. Thus, in a symmetric mixed strategy equilibrium, the seller's profit $\pi(p_i = \underline{p}^A) = \underline{p}^A$, and

$$\pi(p_i = \overline{p}^A) = \eta q \overline{p}^A$$

because $p_i = \overline{p}^A$ is accepted if and only if $v_{-i} = 0$ and $p_{-i} \leq 1 - \overline{p}^A = \underline{p}^A$. Given that $\pi(p_i = \underline{p}^A) = \pi(p_i = \overline{p}^A)$ by the equilibrium indifference, we have $\eta > 0$. Finally, using Property 2 and solving for $\underline{p}^A$, we find $\underline{p}^A = \frac{\eta q}{1+\eta q}$ and in turn, $\overline{p}^A = \frac{1}{1+\eta q}$. ■

Employing Properties 1-3, we now establish that $q \geq 2(\sqrt{2} - 1)$ is a necessary condition for the existence of a mixed strategy equilibrium. Consider $p_i < \frac{1}{2}$, which is accepted with certainty if $p_{-i} \geq 1 - p_i$, whereas it is accepted only if $v_i = \frac{1}{2}$ whenever $p_{-i} > 1 - p_i$. Then, no profitable deviation from $G(\cdot)$ requires

$$\pi(p_i) = [G(1 - p_i) + (1 - G(1 - p_i))(1 - q)]p_i \leq \frac{\eta q}{1 + \eta q}.$$

Let $\xi = G(\frac{1}{2}) - \eta$. Then, $\lim_{p_i \to \frac{1}{2}} \pi(p_i) = [\eta + \xi + (1 - \eta - \xi)(1 - q)]\frac{1}{2}$. Thus, a necessary condition for a mixed strategy equilibrium is

$$\frac{\eta q}{1 + \eta q} \geq [\eta + \xi + (1 - \eta - \xi)(1 - q)]\frac{1}{2}$$

or equivalently,

$$\eta \in \left[\frac{1 - \xi}{2} - \frac{\sqrt{[q(1 - \xi) + 2]^2 - 8}}{2q}, \frac{1 - \xi}{2} + \frac{\sqrt{[q(1 - \xi) + 2]^2 - 8}}{2q}\right] \equiv \Gamma$$

Straightforward algebra reveals that $\Gamma \neq \emptyset$ if and only if $q \geq \frac{2(\sqrt{2}-1)}{1-\xi}$, which is increasing in ξ. Thus, setting $\xi = 0$, the resulting necessary condition for a mixed strategy equilibrium is $q \geq 2(\sqrt{2}-1)$. Next, we complete the proof by establishing the existence of a mixed strategy equilibrium for $q \geq 2(\sqrt{2}-1)$. Consider an equilibrium in which the sellers mix between two prices, namely $\underline{p}^A = \frac{\eta q}{1+\eta q}$ with probability η and $\overline{p}^A = \frac{1}{1+\eta q}$ with probability $1-\eta$, as recorded in the text. Note that $\pi(p_i^A < \underline{p}^A) = p_i^A < \underline{p}^A$, $\pi(p_i^A > \overline{p}^A) = 0$, $\pi(p_i^A \in (\frac{1}{2}, \overline{p}^A)) = \eta q p_i^A < \eta q \overline{p}^A$, and $\pi(p_i^A \in (\underline{p}^A, \frac{1}{2}]) = [\eta + (1-\eta)(1-q)]p_i^A$, which is maximized at $\frac{1}{2}$. Thus, no profitable deviation exists if

$$\frac{\eta q}{1+\eta q} \geq [\eta + (1-\eta)(1-q)]\frac{1}{2}$$

or simplifying, if $\eta \in \left[\frac{1}{2} - \frac{\sqrt{(q+2)^2-8}}{2q}, \frac{1}{2} + \frac{\sqrt{(q+2)^2-8}}{2q}\right]$, which is nonempty for $q \geq 2(\sqrt{2}-1)$. ■

Proof of Proposition 5. For $q \in \left(\frac{1}{2}, 2(\sqrt{2}-1)\right)$, the result is immediate from Lemma 2 and the fact that $B^U(q) > 0$. For $q \geq 2(\sqrt{2}-1)$, it is straightforward to verify that $B^U(q) < B^A(q, \frac{1}{2})$, revealing that there exists an equilibrium, in which the buyer chooses to hold an auction. ■

Proof of Proposition 6. Let $\mu^A(v_i, v_{-i})$ denote the probability that conditional on her valuations, the buyer chooses an auction over sequential procurement. In a symmetric equilibrium, $\mu^A(0, \frac{1}{2}) = \mu^A(\frac{1}{2}, 0)$. Then, conditional on the buyer's choice of the protocol, the ex ante probability that she has a low value for each item under the auction and sequential procurement is, respectively,

$$q^A = \frac{q^2\mu^A(0,0) + q(1-q)\mu^A(0,\frac{1}{2})}{q^2\mu^A(0,0) + 2q(1-q)\mu^A(0,\frac{1}{2}) + (1-q)^2\mu^A(\frac{1}{2},\frac{1}{2})} \tag{A8}$$

and

$$q^S = \frac{q - q^2\mu^A(0,0) - q(1-q)\mu^A(0,\frac{1}{2})}{1 - q^2\mu^A(0,0) - 2q(1-q)\mu^A(0,\frac{1}{2}) - (1-q)^2\mu^A(\frac{1}{2},\frac{1}{2})}. \tag{A9}$$

To establish part (a), it suffices to show that $\mu^A(0,0) = 1$ and $\mu^A(v_i, v_{-i}) = 0$ for $(v_i, v_{-i}) \neq (0,0)$ constitute an equilibrium, yielding a payoff of 0 to the buyer. From Lemma 2, recall that $p^A = \frac{1}{2}$ is an equilibrium for any q^A, resulting in 0 payoff for the buyer under the auction. Moreover, given $\mu^A(.)$, $q^S = \frac{q}{1+q} < \frac{1}{2}$, which, by Proposition

1(b), implies that $B^I(q^S) = 0$. Therefore, for all $q > \frac{1}{2}$, an equilibrium with a payoff of 0 for the buyer exists.

To establish part (b), consider first $q \leq \frac{1}{\sqrt{2}}$. To show that in equilibrium the buyer realizes a payoff 0, suppose, to the contrary, that at least one trading mechanism yields a positive payoff. Recall from (2) that a positive payoff for an informed buyer under sequential procurement requires $q^S > \frac{1}{\sqrt{2}}$ and in turn, $q^S > q$. By (A-8) and (A-9), this implies $q^A < \frac{1}{\sqrt{2}}$, which, by Lemma 2, means a payoff 0 for the buyer in the auction. Consequently, $\mu^A(0, \frac{1}{2}) = \mu^A(\frac{1}{2}, 0) = \mu^A(\frac{1}{2}, \frac{1}{2}) = 0$, which, by (A-9), results in $q^S = q\frac{1-q\mu^A(0,0)}{1-q^2\mu^A(0,0)} < q$, a contradiction. Thus, $q^S \leq \frac{1}{\sqrt{2}}$, and by Proposition 1(b), $B^I(q^S) = 0$. Analogously, a positive payoff under the auction requires $q^A \geq 2(\sqrt{2}-1)(> \frac{1}{\sqrt{2}})$, which in turn implies $q^S < q \leq \frac{1}{\sqrt{2}}$ and $B^I(q^S) = 0$. Moreover, since a positive payoff for the buyer in the auction requires that each seller put a positive probability on a price $p^A < \frac{1}{2}$ (see the proof of Lemma 2), the buyer will have a strict preference for conducting an auction, i.e. $\mu^A(v_i, v_{-i}) = 1$ for all (v_i, v_{-i}), which, by (A-9), implies that $q^A = q < 2(\sqrt{2}-1)$, yielding a contradiction. Thus, the buyer realizes a payoff 0 for $q \leq \frac{1}{\sqrt{2}}$. Second, consider $q > \frac{1}{\sqrt{2}}$. An equilibrium with a positive payoff for the buyer always exists in this region since $\mu^A(v_i, v_{-i}) = 0$ for all (v_i, v_{-i}) with $q^S = q$ results in $B^I(q^S) > 0$ and it can be supported as an equilibrium with an off-equilibrium belief that $p^A = \frac{1}{2}$ under the auction.

Next, consider an equilibrium with a positive payoff for $q \in \left(\frac{1}{\sqrt{2}}, 2(\sqrt{2}-1)\right)$. In such an equilibrium, $p^A = \frac{1}{2}$. Otherwise, if $p^A < \frac{1}{2}$ with a positive probability, then $\mu^A(0,0) = 1$ by Proposition 1, which, by (A-9), would yield $q^S \leq \frac{1}{2}$ and in turn, $B^I(q^S) = 0$ by (2). Therefore, the buyer must have a strict preference for the auction for all (v_i, v_{-i}), i.e. $\mu^A(v_i, v_{-i}) = 1$ for all (v_i, v_{-i}). Then, from (A-8), $q^A = q < 2(\sqrt{2}-1)$, which, by Lemma 2, leads to a contradiction since $p^A = \frac{1}{2}$ is the unique symmetric equilibrium pricing for $q^A < 2(\sqrt{2}-1)$. Thus, in any equilibrium with a positive payoff for $q \in \left(\frac{1}{\sqrt{2}}, 2(\sqrt{2}-1)\right)$, we would have $p^A = \frac{1}{2}$ and the auction would produce a payoff 0. This implies that $B^I(q^A) > 0$, and by Proposition 1, $\mu^A(v_i, v_{-i}) = 1$ for all $(v_i, v_{-i}) \neq (0,0)$.

Finally, to show that for $q^A > 2(\sqrt{2}-1)$, the auction can be supported as an equilibrium choice, note that $\mu^A(v_i, v_{-i}) = 1$ for all (v_i, v_{-i}) results in $q^A = q$ by (A-8). Furthermore, it is straightforward to verify that $B^A(q, \frac{1}{2}) > B^I(q)$, proving no profitable deviation to sequential procurement. ■

Proof of Proposition 7. Let $m = (m_1, m_2)$. By definition, the buyer's expected value of information is

$$\overline{\Delta}^m(q) = \Pr(b,b)\Delta^{(b,b)}(q) + \sum_{i=1}^{2} \Pr(s_i,b)\Delta^{(s_i,b)}(q) + \Pr(s_1,s_2)\Delta^{(s_1,s_2)}(q).$$

If $m_1 = m_2 = b$, the buyer optimally offers 0 to each seller, implying $B^I(q) = B^U(q) = 1$ and in turn, $\Delta^{(b,b)}(q) = 0$. If, on the other hand, $m_i = s_i$ for $i = 1, 2$, the setting reduces to our base model, implying $\Delta^{(s_1,s_2)}(q) = \Delta(q)$ where $\Delta(q)$ is as stated in (4). It therefore remains to prove that if $m_i = s_i$ and $m_{-i} = b$, then $\Delta^{(s_i,b)}(q) = 0$. Suppose $m = (s_i, b)$. We consider uninformed and informed buyers in turn.

Uninformed buyer: If the sequence is $s_{-i} \to s_i$, the buyer always purchases from s_{-i} (at price 0) and thus the optimal price by s_i is given by (A-1) where $\phi = 0$ and $\widehat{q}_1(0,1) = q$. This means that the buyer's uninformed payoff is: $B^U(s_{-i} \to s_i) = \begin{cases} \frac{1}{2} & if \quad q \leq \frac{1}{2} \\ \frac{1-q}{2} & if \quad q > \frac{1}{2} \end{cases}$. If, however, the sequence is $s_i \to s_{-i}$, the buyer's expected payoff from rejecting s_i's offer is $\frac{1-q}{2}$, which is simply the expected payoff from acquiring good i only. Therefore, the highest acceptable price by s_i in the first period satisfies $\max\{1 - \overline{p}_1, v_{-i} - \overline{p}_1\} = \frac{1-q}{2}$, revealing $p_1 = \frac{1+q}{2}$ and an expected payoff: $B^U(s_i \to s_{-i}) = \frac{1-q}{2}$. Comparing the two payoffs, $B^U = B^U(s_{-i} \to s_i)$.

Informed buyer: If the sequence is $s_{-i} \to s_i$, the optimal price by s_i is given by (A-1) where $\phi = 1$ and $\widehat{q}_1(1,1) = q_1(1)$ since the buyer always purchases from s_{-i}. If the sequence is $s_i \to s_{-i}$, the highest price acceptable to the buyer in the first meeting satisfies $\max\{1 - \overline{p}_1, v_{-i} - \overline{p}_1\} = v_i$. Therefore, the optimal price by s_i is

$$p_1 = \begin{cases} \frac{1}{2} & if \quad q_2(1) \leq \frac{1}{2} \\ \\ 1 & if \quad q_2(1) \geq \frac{1}{2} \end{cases} \tag{A10}$$

where $q_2(1) = \Pr\{v_{-i} = 0 | s_{-i} \text{ is second}\}$. The buyer with $v_{-i} = 0$ accepts p_1 for sure whereas a buyer with $v_{-i} = \frac{1}{2}$ accepts only the low p_1. Let $\widehat{\theta}_k(v_{-i}) = \Pr\{s_{-i} \text{ is } k\text{th } | v_{-i}\}$ and $q_k(1) = \Pr\{v_{-i} = 0 | s_{-i} \text{ is } k\text{th}\}$. Then, by Bayes' rule,

$$q_k(1) = \frac{q\widehat{\theta}_k(0)}{q\widehat{\theta}_k(0) + (1-q)\widehat{\theta}_k(\frac{1}{2})}. \tag{A11}$$

We show that there is no equilibrium in which $B^I \neq B^U$. If, in equilibrium, $\widehat{\theta}_k(0) = \widehat{\theta}_k(\frac{1}{2}) = 1$ for some $k = 1, 2$, then $q_k(1) = q$ and, by (A-1) and (A-10), $B^I = B^U$. Suppose $\widehat{\theta}_k(0) \in (0, 1)$. Then $q_k(1)$ is uniquely pinned down for $k = 1, 2$ using (A-11). We consider three cases for $q_k(1)$.

- $q_k(1) \leq \frac{1}{2}$ for $k = 1, 2$: Since $\widehat{\theta}_{-k}(v_{-i}) = 1 - \widehat{\theta}_k(v_{-i})$, by (A-11), such an equilibrium belief requires $2q - 1 \leq q\widehat{\theta}_k(0) - (1-q)\widehat{\theta}_k(\frac{1}{2}) \leq 0$ and in turn, $q \leq \frac{1}{2}$. From (A-1) and (A-10), we therefore have that the informed and uninformed prices by s_i is $\frac{1}{2}$, resulting in $B^I = B^U$.

- $q_k(1) > \frac{1}{2}$ for $k = 1, 2$: By (A-11), this requires $q > \frac{1}{2}$, which, by (A-1) and (A-10), induces the informed and uninformed prices of 1 by s_i. Therefore, $B^I = B^U$.

- $q_k(1) \leq \frac{1}{2} < q_{-k}(1)$: By (A-1) and (A-10), approaching s_{-i} in kth place results in a price of $\frac{1}{2}$ by s_i, while approaching s_{-i} in $-k$th place results in a price of 1 by s_i. Therefore, a buyer with $v_{-i} = 0$ has a strict preference to approach s_{-i} kth, i.e. $\widehat{\theta}_k(0) = 1$. By (A-11), however, this implies that $q_{-k}(1) = 0 < q_k(1)$, contradicting the existence of an equilibrium with $q_k(1) \leq \frac{1}{2} < q_{-k}(1)$.

Consequently, there is no equilibrium with $B^I \neq B^U$. An equilibrium with $B^I = B^U$ obtains by setting $\widehat{\theta}_1(0) = \widehat{\theta}_1(\frac{1}{2}) \in (0, 1)$, resulting in $q_k(1) = q$ for $k = 1, 2$, which, by (A-1) and (A-10), yields identical informed and uninformed pricing by s_i. ■

Proof of Proposition 8. Let $x \in \{(h, p_1), h, p_1, \emptyset\}$ denote the history observed by the second seller, where (h, p_1) refers to observable price and trade histories, h to trade history only, p_1 to price history only, and $\emptyset$ to nondisclosure. Since the base model captures $x = h$, we consider the remaining possibilities for x here.

Let $\widetilde{v}_2(x) \in \{0, \frac{1}{2}, 1\}$ denote the buyer's value for the second item. Also let $\widetilde{q}_2(\phi, x) = \Pr(\widetilde{v}_2(x) = 1 | \widetilde{v}_2(x) \geq \frac{1}{2})$. Then, analogous to the base model, s_2's optimal price is

$$p_2(x) = \begin{cases} \frac{1}{2} & \text{if} \quad \widetilde{q}_2(\phi, x) \leq \frac{1}{2} \\ \\ 1 & \text{if} \quad \widetilde{q}_2(\phi, x) \geq \frac{1}{2}. \end{cases} \tag{A12}$$

Given (A-12), $p_1(x) > \frac{1}{2}$ will be rejected for sure, implying that in equilibrium $p_1(x) \leq \frac{1}{2}$. Moreover, analogous to the base model, a buyer with v_1 is willing to pay s_1 up to $\overline{p}_1(v_1|x)$ that satisfies

$$\max\{1 - \overline{p}_1(v_1|x) - p_2(x), v_1 - \overline{p}_1(v_1|x)\} = 0$$

where $p_2(x) \geq \frac{1}{2}$ implies that $\overline{p}_1(\frac{1}{2}|x) = \frac{1}{2} \geq \overline{p}_1(0|x)$. Thus, a buyer $v_1 = \frac{1}{2}$ always accepts $p_1(x) \leq \frac{1}{2}$. Let $\gamma(x)$ denote the probability that $p_1 \leq \frac{1}{2}$ is accepted by a buyer with $v_1 = 0$. Then, by Bayes' rule, $\widetilde{q}_2(\phi, x)$ must satisfy

$$\widetilde{q}_2(\phi, x) = \begin{cases} \frac{\gamma(x)q_1(\phi)}{1-(1-\gamma(x))q_1(\phi)} & \text{if} \quad x \in \{(h = 1, p_1)\} \\ \\ \frac{\gamma(x)q_1(\phi)}{1-(1-\gamma(x))q^2} & \text{if} \qquad x \in \{p_1, \emptyset\}. \end{cases} \tag{A13}$$

To understand (A-13), note that $\widetilde{v}_2(x) = 1$ only if $v_1 = 0$ and the first offer is accepted. This event occurs with probability $\gamma(x)q_1(\phi)$, which explains the numerator. The denominator captures $\Pr(\widetilde{v}_2(x) \geq \frac{1}{2})$. With an observable purchase, i.e. $h = 1$, the value for the second item is $1 - v_1 \geq \frac{1}{2}$. Thus, $\Pr(\widetilde{v}_2(x) \geq \frac{1}{2})$ is simply the likelihood of acceptance of the first offer, $1 - (1 - \gamma(x))q_1(\phi)$. With an unobservable trade history, i.e. $x \in \{p_1, \emptyset\}$, $\widetilde{v}_2(x) \geq \frac{1}{2}$ – either because the first offer is accepted or because the first offer is rejected but the second good has a high stand-alone value. Alternatively, $\widetilde{v}_2(x) \geq \frac{1}{2}$ unless both stand-alone values are low and the buyer has rejected the first offer, which is given by $(1 - \gamma(x))q^2$. Thus, $\Pr(\widetilde{v}_2(x) \geq \frac{1}{2}) = 1 - (1 - \gamma(x))q^2$ for $x \in \{p_1, \emptyset\}$.

Next, we show that for $x \in \{(h, p_1), p_1\}$, $p_1(x) = \frac{1}{2}$ is the only possible price in equilibrium. Consider, by way of contradiction, an equilibrium with a price $p_1' \in (0, \frac{1}{2})$ being offered with a positive probability. Let x' denote the history corresponding to p_1'. Analogous to the base model, $\widetilde{q}_2(\phi, x') \leq \frac{1}{2}$ (see proof of Proposition A1). If $\widetilde{q}_2(\phi, x') < \frac{1}{2}$, then $p_2(x') = \frac{1}{2}$, which, in turn, implies that $\overline{p}_1(v_1|x') = \frac{1}{2}$ for all v_1 and $\gamma(x') = 1$. Then, by (A-13), $\widetilde{q}_2(\phi, x') < \frac{1}{2}$ requires $q_1(\phi) < \frac{1}{2}$. Moreover, since $\widetilde{q}_2(\phi, x)$ is increasing in $\gamma(x)$, $\widetilde{q}_2(\phi, x) < \frac{1}{2}$ for all x. This, however, implies that any price $p_1 \leq \frac{1}{2}$ will be accepted for sure, resulting in a profitable deviation to $p_1 = \frac{1}{2}$.

Alternatively, let $\widetilde{q}_2(\phi, x') = \frac{1}{2}$. Then, by (A-13), $\gamma(x') = \frac{1-q_1(\phi)}{q_1(\phi)} \leq 1$ if $x' = (1, p_1')$, and $\gamma(x') = \frac{1-q^2}{2q_1(\phi)-q^2} \leq 1$ if $x' = p_1'$ and the corresponding payoff for s_1 is $[1-(1-\gamma(x'))q_1(\phi)]p_1'$. Consider a deviation to $p_1'' \in (p_1', \frac{1}{2})$. If $\widetilde{q}_2(\phi, x'') < \frac{1}{2}$, then p_1'' will be accepted for sure and thus s_1 will have a strict incentive to deviate. If $\widetilde{q}_2(\phi, x'') = \frac{1}{2}$, from (A-13), $\gamma(x'') = \gamma(x')$, implying that p_1'' yields a profit of $[1 - (1 - \gamma(x'))q_1(\phi)]p_1''$ and thus, s_1 will again have a strict incentive to deviate. This establishes that $p_1(x) = \frac{1}{2}$ is the only possible equilibrium price for $x \in \{(h, p_1), p_1\}$ with a corresponding payoff of 0 for the buyer. Moreover, in equilibrium $p_2(x) = \frac{1}{2}$ since a higher price would result in $\overline{p}_1(0|x) < \frac{1}{2}$, yielding $\gamma(x) = 0$ and $\widetilde{q}_2(\phi, x) = 0$ for $x \in \{(1, \frac{1}{2}), \frac{1}{2}\}$, which by (A-12), implies $p_2(x) = \frac{1}{2}$.

To establish the existence of an equilibrium with $p_1(x) = p_2(x) = \frac{1}{2}$ for $x \in \{(h, p_1), p_1\}$, note that the buyer is indifferent in her sequencing and acceptance decision under these prices. With an uninformed buyer, for all $p_1 \leq \frac{1}{2}$ let $\gamma(x) = \frac{1-q}{q}$ for $x = (h, p_1)$ and $\gamma(x) = \frac{1-q^2}{2q-q^2}$ for $x = p_1$ ($p_1 > \frac{1}{2}$ is always rejected). By eq. (A-13), this gives rise to $\widetilde{q}_2(0, x) = \frac{1}{2}$, thus making s_2 indifferent between $\frac{1}{2}$ and 1 (eq. (A-12)). If $\beta(p_1)$ denotes the probability of a price offer $\frac{1}{2}$ by s_2 upon observing p_1, $\gamma(x)$ is supported as an equilibrium strategy for the buyer as long as $\overline{p}_1(0|x) = \frac{\beta(p_1)}{2} = p_1 \implies \beta(p_1) = 2p_1$. Thus, s_1 will optimally choose $p_1(x) = \frac{1}{2}$, corresponding to $\beta(\frac{1}{2}) = 1$. This establishes the existence of the equilibrium. The resulting total surplus is $W^U(q, x) = q[\gamma(x) + (1 - \gamma(x))(1 - q)\frac{1}{2}] + (1 - q) > W^U(q)$. Analogously, with an informed buyer, it is straightforward to verify that $p_1(x) = p_2(x) = \frac{1}{2}$ for $x \in \{(h, p_1), p_1\}$ can be supported as an equilibrium with the following strategies: $\theta_1(\frac{1}{2}, 0|x) = 1$; for all $p_1 \leq \frac{1}{2}$, $\gamma(x) = 1$ and $\beta(p_1) = 1$ if $q \leq \frac{1}{\sqrt{2}}$, $\gamma(x) = \frac{1-q^2}{q^2}$ and $\beta(p_1) = 2p_1$ if $q > \frac{1}{\sqrt{2}}$. This gives rise to $W^I(q, x) = 1$ if $q \leq \frac{1}{\sqrt{2}}$ and $W^I(q, x) = W^I(q) + q^2(1 - q^2)\frac{1}{2}$ if $q > \frac{1}{\sqrt{2}}$.

Next, consider $x = \emptyset$. Given $\widetilde{q}_2(\phi, \emptyset)$ defined by (A-13), Proposition A2 characterizes the sellers' equilibrium prices as a function of $q_1(\phi)$.

Proposition A2. *Given $q_1(\phi)$, in equilibrium*

a) *if $q_1(\phi) < \frac{1}{2}$, then $p_1(\emptyset) = p_2(\emptyset) = \frac{1}{2}$ and the buyer purchases the bundle with certainty;*

b) *if $q_1(\phi) = \frac{1}{2}$, then $p_1(\emptyset) = \frac{\beta}{2}$ and* $p_2(\emptyset) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad \beta \\ 1 & \textit{with prob.} \quad 1-\beta \end{cases}$

where $\beta \geq \frac{1}{2}$. The buyer purchases from s_1 with certainty and s_2 only if $v_1 = 0$ or $p_2(\emptyset) = \frac{1}{2}$;

(c) *if $q_1(\phi) > \frac{1}{2}$, then*

$$p_1(\emptyset) = \begin{cases} \frac{1-q_1(\phi)}{2} & \textit{with prob.} \quad \frac{1-q^2}{2q_1(\phi)-q^2} \\ \\ \frac{1}{2} & \textit{with prob.} \quad \frac{2q_1(\phi)-1}{2q_1(\phi)-q^2} \end{cases} \quad \textit{and } p_2(\emptyset) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad 1-q_1(\phi) \\ \\ 1 & \textit{with prob.} \quad q_1(\phi). \end{cases}$$

Moreover, a buyer with $v_1 = 0$ *accepts only low* $p_1(\emptyset)$ *whereas she accepts* $p_2(\emptyset)$ *upon purchasing the first item, or upon rejecting* $p_1(\emptyset)$ *whenever* $v_2 = \frac{1}{2}$ *and* $p_2(\emptyset)$ *is low. A buyer with* $v_1 = \frac{1}{2}$ *accepts all* $p_1(\emptyset)$ *but only the low* $p_2(\emptyset)$*.*

Proof. Analogous to Proposition A1 and thus omitted. ■

For an uninformed buyer, $q_1(0) = q$ with a corresponding payoff and total surplus:

$$B^U(q,\emptyset) = (1-q)\frac{1-q^2}{2q-q^2}\left(\frac{1}{2} - \frac{1-q}{2}\right) = \frac{1+q}{2-q}B^U(q).$$

$$\begin{aligned} W^U(q,\emptyset) &= q\left[\frac{1-q^2}{2q-q^2} + \frac{2q-1}{2q-q^2}\frac{(1-q)^2}{2}\right] + (1-q)[q\frac{1}{2} + (1-q)] \\ &= \frac{1}{2(2-q)}[5-4q-2q^2+q^3] \end{aligned}$$

Straightforward algebra reveals that $W^U(q,x) > W^U(q,\emptyset)$ for $x \in \{(h,p_1),p_1\}$.

For an informed buyer, $q_1(1) = q^2$ for $q > \frac{1}{\sqrt{2}}$ due to the buyer's strict sequencing from a high- to low-value seller. Thus, for $q > \frac{1}{\sqrt{2}}$, the equilibrium prices for s_1 under $x = \emptyset$ and $x = h$ coincide, as evident from Propositions A1 and A2. Consequently, $B^I(q,\emptyset) = B^I(q)$ and $W^I(q,\emptyset) = W^I(q) < W^I(q,x)$ for $x \in \{(h,p_1),p_1\}$. For $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, we have $q_1(\phi) \leq \frac{1}{2}$, which, breaking indifference in favor of efficiency at $q_1(\phi) = \frac{1}{2}$, results in $p_1^I(\emptyset) = p_2^I(\emptyset) = \frac{1}{2}$, leaving a payoff of 0 to the buyer. Thus, for $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, $B^I(q,x) = B^I(q) = 0$ and $W^I(q,x) = W^I(q) = 1$ for $x \in \{(h,p_1),p_1,\emptyset\}$. ■

Proposition A3. (Informed prices with correlation) *As defined in Section 5.5, let* $\overline{q}(r)$ *be the unique solution to* $\Pr(0,0) = \frac{1}{2}$*, where* $\Pr(0,0) = q^2 + rq(1-q)$*. In equilibrium,* $p_1^I = p_2^I(h=1) = \frac{1}{2}$ *for* $q \leq \overline{q}(r)$ *with* $\theta_1(\frac{1}{2},0) > \frac{1}{2}$ *for* $q > \frac{1}{2}$*; and*

$$p_1^I = \begin{cases} \frac{1-\Pr(0,0)}{2} & \textit{with prob.} \quad \frac{1-\Pr(0,0)}{\Pr(0,0)} \\ \\ \frac{1}{2} & \textit{with prob.} \quad \frac{2\Pr(0,0)-1}{\Pr(0,0)} \end{cases} \quad \textit{and } p_2^I(h=1) = \begin{cases} \frac{1}{2} & \textit{with prob.} \quad 1-\Pr(0,0) \\ \\ 1 & \textit{with prob.} \quad \Pr(0,0) \end{cases}$$

and $\theta_1(\frac{1}{2},0) = 1$ *for* $q > \overline{q}(r)$.

Proof. Using the joint distribution $\Pr(v_1, v_2)$ in Section 5.5, the posterior belief in (A-3) generalizes to:

$$\begin{aligned} q_1(1) &= \frac{\Pr(0,0) \times \frac{1}{2} + \Pr(\frac{1}{2},0)\left[1-\theta_1(\frac{1}{2},0)\right]}{\frac{1}{2}} \\ &= \Pr(0,0) + 2\Pr(\frac{1}{2},0)\left[1-\theta_1(\frac{1}{2},0)\right]. \end{aligned}$$

By Proposition A1, if $q_1(1) > \frac{1}{2}$, then $\theta_1(\frac{1}{2},0) = 1$ and $q_1(1) = \Pr(0,0)$. Therefore $q_1(1) > \frac{1}{2}$ if and only if $\Pr(0,0) > \frac{1}{2}$, or equivalently $q > \overline{q}(r)$. On the other hand, for $q \leq \overline{q}(r)$, $q_1(1) \leq \frac{1}{2}$, which requires $\theta_1(\frac{1}{2},0) > \frac{1}{2}$. Equilibrium prices follow from Proposition A1. ■

Proof of Proposition 9. Suppose $q > \frac{1}{2}$ and that the buyer is privately informed of v_i only. If s_i is second in the sequence, the buyer receives the uninformed payoff in (1), $B^U(q) > 0$, because she is uninformed of v_{-i} and the second seller's pricing depends only on the prior q in this case. Suppose, instead, that s_i is first and let $\widetilde{q}_1 = \Pr\{v_i = 0 | s_i$ is first$\}$. If $v_i = 0$, the buyer receives an expected payoff of 0 because, by Proposition A1, for $\widetilde{q}_1 \leq \frac{1}{2}$, each seller charges $\frac{1}{2}$ whereas for $\widetilde{q}_1 > \frac{1}{2}$, s_i sets his low price to leave no expected surplus. Hence, a buyer with $v_i = 0$ strictly prefers to sequence s_i second and obtain $B^U(q) > 0$. This implies $\widetilde{q}_1 = 0$ and by Proposition A1, an expected payoff of 0 for the buyer when approaching s_i first. Therefore, in equilibrium, s_i is sequenced second, yielding the buyer her uninformed payoff in (1). ■

Proof of Proposition 10. Suppose that $v_1 = v_2 = v \in [0, \frac{1}{2}]$ and the joint value is $V = 1$ or $\overline{V} > 1$ where $\Pr\{V = \overline{V}\} = \alpha \in (0,1)$. Let $\hat{\alpha}(h) = \Pr(V = \overline{V}|h)$. Then, the optimal price by the second seller upon observing a prior purchase is

$$p_2(h=1) = \begin{cases} 1-v & \text{if} \quad \hat{\alpha}(1) \leq \frac{1-v}{\overline{V}-v} \\ \\ \overline{V}-v & \text{if} \quad \hat{\alpha}(1) \geq \frac{1-v}{\overline{V}-v} \end{cases} . \tag{A14}$$

Without a prior purchase, the second seller trivially offers $p_2^z(h=0) = v$. The pricing by the first seller depends on whether the buyer is informed or uninformed.

Uninformed buyer: Let $\overline{p}_1$ be the first seller's maximum price acceptable to the buyer. Denoting by $E[.]$ the usual expectation operator, $\overline{p}_1$ satisfies: $\max\{E[V] - \overline{p}_1 - E[p_2(h=1)], v - \overline{p}_1\} = 0$, or

$$\overline{p}_1 = \max\{E[V] - E[p_2(h=1)], v\}.$$

Since any higher price is rejected for sure, $p_1^U = \overline{p}_1$ and by Bayes' rule, $\hat{\alpha}(1) = \alpha$. Therefore, for $\alpha \leq \frac{1-v}{\overline{V}-1}$, $p_1^U = \alpha\left(\overline{V}-1\right) + v$ and $p_2^U(h=1) = 1-v$ while for $\alpha > \frac{1-v}{\overline{V}-1}$, $p_1^U = v$ and $p_2^U(h=1) = \overline{V} - v$. The resulting expected payoff for the buyer is $B^{J,U}(v) = 0$.

Informed buyer: In this case, $\overline{p}_1$ satisfies

$$\overline{p}_1 = \max\{V - E[p_2(h=1)], v\}.$$

We consider three possibilities for $\hat{\alpha}(1)$.

- $\hat{\alpha}(1) < \frac{1-v}{\overline{V}-v}$: Then, $p_2(h=1) = 1-v$ by (A-14), implying that $\overline{p}_1^L = v$ is accepted for sure, whereas $\overline{p}_1^H = (\overline{V}-1)+v$ is accepted only if $V = \overline{V}$. Therefore, for $\alpha \leq \frac{v}{v+\overline{V}-1}\left(< \frac{1-v}{\overline{V}-v}\right)$, the first seller optimally sets $p_1 = v$ (breaking the indifference at $\alpha = \frac{v}{v+\overline{V}-1}$ in favor of efficiency), which reveals $\hat{\alpha}(1) = \alpha$. As a result, for $\alpha \leq \frac{v}{v+\overline{V}-1}$, the price pair $p_1^I = v$ and $p_2^I(h=1) = 1-v$ constitute an equilibrium, resulting in the payoff: $B^{J,I}(\alpha) = \alpha(\overline{V}-1)$. For $\alpha \in \left(\frac{v}{v+\overline{V}-1}, \frac{1-v}{\overline{V}-v}\right)$, the first seller sets $p_1 = (\overline{V}-1)+v$, which implies $\hat{\alpha}(1) = 1$ and a profitable deviation for the second seller to $p_2(h=1) = \overline{V}-v$. Hence, $\hat{\alpha}(1) < \frac{1-v}{\overline{V}-v}$ only if $\alpha \leq \frac{v}{v+\overline{V}-1}$ resulting in $\Delta^J(\alpha) = B^{J,I}(\alpha)$.

- $\hat{\alpha}(1) = \frac{1-v}{\overline{V}-v}$: Then, the second seller is indifferent between $\overline{V}-v$ and $1-v$. Suppose that he offers $1-v$ with probability σ. Then, $E[p_2(h=1)] = \overline{V} - v - \sigma(\overline{V}-1)$, implying that $\overline{p}_1^L = v$ is accepted for sure by the buyer while $\overline{p}_1^H = v + \sigma(\overline{V}-1)$ is accepted only if $V = \overline{V}$. The first seller is indifferent between $\overline{p}_1^L$

and $\overline{p}_1^H$ if $\sigma = \frac{(1-\alpha)v}{\alpha(\overline{V}-1)}$, in which case the first seller's mixing $\beta = \Pr(p_1 = v) = \frac{\alpha(\overline{V}-1)}{(1-\alpha)(1-v)} \leq 1$ engenders an equilibrium belief $\hat{\alpha}(1) = \frac{\alpha}{\beta+(1-\beta)\alpha} = \frac{1-v}{\overline{V}-v}$. Then, $\overline{p}_1^H = \frac{v}{\alpha}$. Note that $\sigma \leq 1$ for $\alpha \geq \frac{v}{v+\overline{V}-1}$ and $\beta \leq 1$ for $\alpha \leq \frac{1-v}{\overline{V}-v}$. Therefore, the price pair

$$p_1^I = \begin{cases} v & \text{with prob.} \quad \beta \\ \frac{v}{\alpha} & \text{with prob.} \quad 1-\beta \end{cases} \quad \text{and } p_2^I(h=1) = \begin{cases} 1-v & \text{with prob.} \quad \sigma \\ \overline{V}-v & \text{with prob.} \quad 1-\sigma \end{cases}$$

is an equilibrium for $\alpha \in \left(\frac{v}{v+\overline{V}-1}, \frac{1-v}{\overline{V}-v}\right]$. In such an equilibrium, $\Delta^J(\alpha) = B^{J,I}(v) = \frac{v}{1-v}\alpha(\overline{V}-1)$. Note that for $v=0$, $\sigma = 0$ and in turn, $p_2^I(h=1) = \overline{V}$. Then, $\overline{p}_1 = 0$. Using the efficient tie-breaking rule, the first seller offers $p_1^I = 0$. Let $\eta(V)$ denote the probability that the buyer accepts the first offer given V. Then, by Bayes' rule, $\hat{\alpha}(1) = \frac{\eta(\overline{V})\alpha}{\eta(\overline{V})\alpha+\eta(1)(1-\alpha)}$. Since efficiency is maximized for $\eta(\overline{V}) = 1$, $\hat{\alpha}(1) = \frac{1}{\overline{V}}$, and $\eta(1) = \frac{\alpha(\overline{V}-1)}{(1-\alpha)}$. Therefore, for $v = 0$, the price pair $p_1^I = 0$ and $p_2^I(h=1) = \overline{V}$ is supported by $\eta(1) = \frac{\alpha(\overline{V}-1)}{(1-\alpha)(1-v)}$ and $\hat{\alpha}(1) = \frac{1-v}{\overline{V}-v}$. The buyer's payoff is $B^{J,I}(v=0) = 0 = \Delta^J(\alpha)$.

- $\hat{\alpha}(1) > \frac{1-v}{\overline{V}-v}$: Then, $p_2(h=1) = \overline{V}-v$ and $p_1^I = \overline{p}_1 = v$. The first price is always accepted by the buyer, implying that $\hat{\alpha}(1) = \alpha > \frac{1-v}{\overline{V}-v}$ and $B^{J,I} = 0 = \Delta^J(\alpha)$. ■

Proof of Proposition 11. Let $q_u > \frac{1}{2}$. Consider sequential procurement with uninformed buyer. If s_2 observes no prior purchase, he offers $p_2(h=0) = 1$ since it is accepted with probability q_u, resulting in a payoff of q_u, whereas the alternative price of $\frac{1}{2}$ is accepted with certainty, resulting in a payoff of $\frac{1}{2}$. If s_2 observes a prior purchase, he offers $p_2(h=1) = \frac{1}{2}$ since the buyer's marginal value for his good is $\frac{1}{2}$ or 0. Anticipating such pricing, the highest price, $\overline{p}_1$, acceptable to the buyer in the first meeting satisfies: $\max\{1 - p_2(h=1) - \overline{p}_1, v_1 - \overline{p}_1\} \geq 0$, or simplifying

$$\overline{p}_1 \leq \max\{1 - p_2(h=1), v_1\}.$$

This implies $p_1 = 1$ since $p_1 = 1$ is accepted with probability q_u and $p_1 = \frac{1}{2}$ is accepted for sure. Given the equilibrium prices, sequential procurement yields a payoff of 0 to the buyer. To prove that the auction yields a positive payoff, it suffices to show that in equilibrium, the sellers choose prices lower than 1 with a positive probability. The following two claims make this point.

Claim 1 *In the auction, there is no pure strategy equilibrium.*

Proof. As in the standard Bertrand competition, $p_1 = p_2 = p > 0$ cannot arise in equilibrium because with probability q_u^2, goods are perfect substitutes and a slightly lower price would guarantee a sale in this realization. Without loss of generality, suppose $p_1 < p_2$. If $\frac{1}{2} < p_1 < p_2$, then s_1 receives an expected profit $\pi_1 = \left[q_u(1-q_u) + q_u^2\right] p_1$, implying a profitable deviation to $\tilde{p}_1 = \frac{p_2+p_1}{2}$. The same profitable deviation also exists if $p_1 < p_2 \leq \frac{1}{2}$, because in this case, p_1 is accepted unless $v_2 = 1$ and $v_1 = \frac{1}{2}$, resulting in $\pi_1 = [1 - q_u\,(1-q_u)]p_1$. Finally, if $p_1 \leq \frac{1}{2} < p_2$, the sellers' expected profits are

$$(\pi_1, \pi_2) = \begin{cases} (p_1, 0) & \text{if} \quad p_1 < p_2 + \frac{1}{2} \\ ([1-(1-\sigma_1)q_u(1-q_u)]p_1, q_u(1-q_u)(1-\sigma_1)p_2) & \text{if} \quad p_1 = p_2 + \frac{1}{2} \\ ([1-q_u(1-q_u)]p_1, q_u(1-q_u)p_2) & \text{if} \quad p_1 > p_2 + \frac{1}{2} \end{cases}$$

where $\sigma_1 \in [0,1]$ is an arbitrary tie-breaking rule when the buyer is indifferent. For $p_1 < p_2 + \frac{1}{2}$, s_2 clearly has a strict incentive to lower his price. The same is true for $p_1 = p_2 + \frac{1}{2}$, in which case the buyer is indifferent. For $p_1 > p_2 + \frac{1}{2}$, s_2 would deviate to $\tilde{p}_2 = \frac{p_2+p_1-\frac{1}{2}}{2}$. In sum, there is no pure strategy equilibrium. ■

Claim 2 *In the auction, the following c.d.f. constitutes a symmetric mixed strategy equilibrium:*

$$F(p) = \frac{1}{q_u^2}\left[1 - q_u(1-q_u) - \frac{1-q_u}{2p}\right] \textit{ for } p \in \left[\frac{1-q_u}{2(1-q_u(1-q_u))}, \frac{1}{2}\right].$$

Proof. Consider a symmetric mixed strategy equilibrium with a continuous support $\underline{p} < \overline{p} \leq \frac{1}{2}$ and no mass points. Then,

$$\pi(p) = \left[F(p)(1-q_u) + (1-F(p))(1-q_u(1-q_u))\right] p = \overline{\pi},$$

where $\overline{\pi}$ is the indifference profit across $p \in [\underline{p}, \overline{p}]$. Note that $\pi(\overline{p}) = (1-q_u)\overline{p}$ is increasing in $\overline{p}$, implying a profitable deviation to $p \in (\overline{p}, \frac{1}{2}]$. Therefore, $\overline{p} = \frac{1}{2}$. Re-writing,

$$F(p) = \frac{1}{q_u^2}\left[1 - q_u(1-q_u) - \frac{\overline{\pi}}{p}\right].$$

Since $F(\frac{1}{2}) = 1$, $\overline{\pi} = \frac{1-q_u}{2}$. Given this and the fact that $F(\underline{p}) = 0$, we find that $\underline{p} = \frac{1-q_u}{2(1-q_u(1-q_u))}$. Thus,

$$F(p) = \frac{1}{q_u^2}\left[1 - q_u(1-q_u) - \frac{1-q_u}{2p}\right] \text{ for } p \in \left[\frac{1-q_u}{2(1-q_u(1-q_u))}, \frac{1}{2}\right],$$

as claimed. It remains to show that there is no unilateral deviation incentive to $p \notin \left[\underline{p}, \frac{1}{2}\right]$. Without loss of generality, consider a deviation by s_1. Clearly, $p_1 < \underline{p}$ is not profitable, because $\pi(p_1) = (1-q_u(1-q_u))p_1 < (1-q_u(1-q_u))\underline{p} = \pi(\underline{p})$. Next consider a deviation to $p_1 > \frac{1}{2}$. Since $p_1 > 1$ is rejected with probability 1, we restrict attention to $p_1 \in (\frac{1}{2}, 1]$. In this case, s_1 realizes a sale only if $v_1 = 1$, $v_2 = \frac{1}{2}$, and $1 - p_1 > \frac{1}{2} - p_2$, or equivalently $p_2 > p_1 - \frac{1}{2}$. We exhaust two cases:

- $p_1 - \frac{1}{2} \leq \underline{p}$: Then, $\pi(p_1) = q_u(1-q_u)p_1$. Since this deviation profit is increasing in p_1, the maximum deviation profit in this region is

$$\pi(\underline{p} + \frac{1}{2}) = q_u(1-q_u)\left(\frac{1}{2} + \frac{1-q_u}{2(1-q_u(1-q_u))}\right) < \frac{1-q_u}{2} = \overline{\pi}.$$

  Therefore, there is no incentive to deviate to $p_1 \in (\frac{1}{2}, \frac{1}{2} + \underline{p}]$.

- $\frac{1}{2} + \underline{p} < p_1 \leq 1$: Then, s_1's probability of a sale is $q_u(1-q_u)\Pr(p_2 > p_1 - \frac{1}{2})$ and his deviation profit is

$$\begin{aligned}\pi(p_1) &= q_u(1-q_u)\left[1 - \frac{1}{q_u^2}\left(1 - q_u + q_u^2 - \frac{1-q_u}{2(p_1 - \frac{1}{2})}\right)\right]p_1 \\ &= \frac{(1-q_u)^2}{q_u}\left[\frac{1}{2(\tilde{p}_i - \frac{1}{2})} - 1\right]p_1.\end{aligned}$$

  Simple algebra shows that $\pi(p_1) < \frac{1-q_u}{2} = \overline{\pi}$. ■

Together Claims 1 and 2 prove Proposition 9. ■

Appendix B

Here, we extend our analysis to a more general Bernoulli distribution with valuations $v_i \in \{v_L, v_H\}$ where $0 \leq v_L < v_H \leq \frac{1}{2}$ and $\Pr\{v_i = v_L\} = q \in (0,1)$. The main difference from the special case in the text ($v_L = 0$ and $v_H = \frac{1}{2}$) is that with $v_L > 0$, the second seller may not always charge the high price v_H upon observing no purchase from the first seller – i.e. $h = 0$. In particular, by charging a low price of $v_L > 0$, the second seller now leaves a positive surplus to the buyer who did not acquire the first object.

As in the base model, let $z = I, U$ refer to informed and uninformed sequencing, and $(p_1^z, p_2^z(h))$ be the corresponding pair of prices. Analogous to the base model, for $q \leq \frac{1-v_H}{1-v_L}$, the value of information is 0 since $p_1^z = p_2^z(h=0) = v_H$ and $p_2^z(h=1) = 1 - v_H$ in equilibrium. Thus, our analysis here focuses on $q > \frac{1-v_H}{1-v_L}$. The following proposition characterizes a symmetric equilibrium for the intermediate values of q featuring a negative value of information as in the base model.

Proposition B1. *Let $q \in \left(\frac{1-v_H}{1-v_L}, \min\left\{\sqrt{\frac{1-v_H}{1-v_L}}, 1 - \frac{v_L}{v_H}\right\}\right)$. There is an equilibrium such that $p_2^U(h=0) = p_2^I(h=0) = v_H$. Moreover, for*

(a) an uninformed buyer:

$$p_1^U = \begin{cases} v_H & \textit{with prob.} \quad 1 - \frac{1-q}{q}\frac{1-v_H}{v_H - v_L} \\ & \\ (1-q)v_H & \textit{with prob.} \quad \frac{1-q}{q}\frac{1-v_H}{v_H - v_L} \end{cases}$$

and

$$p_2^U(h=1) = \begin{cases} 1 - v_H & \textit{with prob.} \quad \frac{v_H(1-q) - v_L}{v_H - v_L} \\ & \\ 1 - v_L & \textit{with prob.} \quad 1 - \frac{v_H(1-q) - v_L}{v_H - v_L}; \end{cases}$$

(b) an informed buyer:

$$p_1^I = v_H \text{ and } p_2^I(h=1) = 1 - v_H.$$

(c) Demand: *An informed buyer accepts both sellers' offers, while an uninformed buyer with $v_1 = v_L$ accepts only the low p_1^U but all $p_2^U(h=1)$ whereas an uninformed buyer with $v_1 = v_H$ accepts all p_1^U but only the low $p_2^z(h=1)$.*

Proof. Let $\widehat{q}_1(z,1) = \Pr\{v_1 = 0 | h = 1\}$ and $\widehat{q}_2(z,0) = \Pr\{v_2 = 0 | h = 0\}$ denote the sellers' posterior beliefs given the buyer's information and purchase history. The optimal pricing by s_2 is

$$p_2^z(h=1) = \begin{cases} 1 - v_H & \textit{if} \quad \widehat{q}_1(z\,,1) \leq \frac{1-v_H}{1-v_L} \\ \\ 1 - v_L & \textit{if} \quad \widehat{q}_1(z,1) \geq \frac{1-v_H}{1-v_L} \end{cases}$$

since $1 - v_H$ is accepted for sure and $1 - v_L$ only if $v_1 = v_L$; and

$$p_2^z(h=0) = \begin{cases} v_H & \textit{if} \quad \widehat{q}_2(z\,,0) \leq 1 - \frac{v_L}{v_H} \\ \\ v_L & \textit{if} \quad \widehat{q}_2(z,0) \geq 1 - \frac{v_L}{v_H} \end{cases}$$

since v_L is accepted for sure and v_H only if $v_2 = v_H$.

For $z = U$, we have that $\widehat{q}_2(z\,,0) = q$, since the buyer is uninformed about v_2 when making a purchasing decision about good 1. Therefore, for $q < 1 - \frac{v_L}{v_H}$, $p_2^U(h=0) = v_H$. Then, the equilibrium derivation under $z = U$ is analogous to the proof of Proposition 1 and thus omitted here.

Next, consider $z = I$. Clearly, $p_1 = v_H$ is a best response to $p_2(h=1) = 1 - v_H$. Given this pricing, the buyer accepts both offers with probability 1. Therefore, upon observing $h = 1$, s_2 has no incentives to deviate as long as $\widehat{q}_1(1,1) = q^2 + 2q(1-q)\theta_1(v_L, v_H) \leq \frac{1-v_H}{1-v_L}$. Since the buyer is indifferent in the order, $\theta_1(v_L, v_H) = 0$ and $q < \sqrt{\frac{1-v_L}{1-v_H}}$ ensure that s_2 has no incentive to deviate. Finally, $p_2(h=0) = v_H$ is supported by the following off-equilibrium belief:[37] $\widehat{q}_2(1,0) \leq 1 - \frac{v_L}{v_H}$. $\blacksquare$

Note that for $v_L = 0$ and $v_H = \frac{1}{2}$, the pricing in Proposition B1 reduces to that in Proposition 1. Moreover, it is readily verified that the value of information is

$$\Delta(q) = -(1-q)^2 \frac{(1-v_H)v_H}{v_H - v_L} < 0,$$

which, given a payoff of 0 for the informed buyer, is simply the negative of the buyer's uninformed payoff. Therefore, for moderate complements, i.e., $q \in \left(\frac{1-v_H}{1-v_L}, \min\left\{\sqrt{\frac{1-v_H}{1-v_L}}, 1 - \frac{v_L}{v_H}\right\}\right]$, it is optimal for the buyer to stay uninformed, extending Proposition 2. Analogous to the base model, it can also be shown that the buyer's optimal strategy to remain uninformed is

[37] This belief satisfies the Intuitive Criterion since the buyer's equilibrium payoff is 0, while rejecting the first offer would result in a payoff of at least 0 for the buyer under the most favorable beliefs regarding v_2 (corresponding to $p_2(h=0) = v_L$). Thus, rejecting the first offer is not equilibrium dominated for any realization of v_2.

not credible if it is unobservable to the sellers. In particular, if the cost is not too high, the buyer would acquire information with some positive probability in equilibrium, extending Proposition 4.

References

[1] Ausubel, L. M., Cramton, P., & Deneckere, R. J. (2002). *Bargaining with incomplete information*. Handbook of Game Theory with Economic Applications, 3, 1897-1945.

[2] Aghion, P., and J. Tirole. (1997) "Formal and real authority in organizations." *Journal of Political Economy*, 1-29.

[3] Brooks, L., and B. Lutz (2016). "From today's city to tomorrow's city: An empirical investigation of urban land assembly." *American Economic Journal: Economic Policy*, 8(3), 69-105.

[4] Cai, H. (2000). "Delay in multilateral bargaining under complete information." *Journal of Economic Theory*, 93, 260-276.

[5] Carrillo, J. and T. Mariotti (2000). "Strategic Ignorance as a Self-Disciplining Device", *Review of Economic Studies*, 66, 529–544.

[6] Cremer, J. (1995). "Arm's Length Relationships." *Quarterly Journal of Economics*, 110, 275-95.

[7] Cunningham, C. (2013) "Estimating the Holdout Problem in Land Assembly." FRB Atlanta Working Paper No. 2013-19. Available at SSRN: `http://ssrn.com/abstract=2579904`

[8] Daughety, A., and J. Reinganum. (1994). "Asymmetric Information Acquisition and Behavior in Role Choice Models: An Endogenously Generated Signaling Game." *International Economic Review*, 35 (4), 795-819.

[9] Dewatripont, M., and E. Maskin. (1995). "Contractual Contingencies and Renegotiation." *RAND Journal of Economics*, 26, 704-19.

[10] Evans, R. (1989). "Sequential bargaining with correlated values. " *The Review of Economic Studies*, 56.4, 499-510.

[11] Fu, D., Y. McMillen, and T. Somerville (2002). "Land Assembly: Measuring Holdup." University of British Columbia, CUER Working Paper 02-08. Available at `https://circle.ubc.ca/handle/2429/50535`

[12] Fudenberg, D., and J. Tirole (1983). “Sequential bargaining with incomplete information.” *The Review of Economic Studies*, 50.2, 221-247.

[13] Göller, D., and M. Hewer.“Breakdown in multilateral negotiations.” *Journal of Economic Theory* 157 (2015): 478-484.

[14] Greenstein, M., and K. Sampson. *Handbook for Judicial Nominating Commissioners*. American Judicature Society, 2004.

[15] Gul, F., and H. Sonnenschein (1988). “On delay in bargaining with one-sided uncertainty.”*Econometrica*, 601-611.

[16] Horn, H., and A. Wolinsky (1988). “Worker substitutability and patterns of unionization.” *Economic Journal*, 98, 484-497.

[17] Hörner, J., and N. Vieille (2009). “Public vs. private offers in the market for lemons.” *Econometrica*, 77.1, 29-69.

[18] Hwang, I., and F. Li (2017). “Transparency of outside options in bargaining.“ *Journal of Economic Theory*, 167, 116-147.

[19] Kaya, A. (2010). “When does it pay to get informed?” *International Economic Review*, 51(2), 533-51.

[20] Kessler, A. (1998). “The Value of Ignorance,” *RAND Journal of Economics* 29, 339–54.

[21] Krasteva, S., and Yildirim, H. (2012a). “On the role of confidentiality and deadlines in bilateral negotiations.” *Games and Economic Behavior,* 75(2), 714-730.

[22] Krasteva, S., and Yildirim, H. (2012b). “Payoff uncertainty, bargaining power, and the strategic sequencing of bilateral negotiations.” *RAND Journal of Economics,* 43(3), 514-536.

[23] Lippman, S., and J. Mamer. (2012) “Exploding offers.” *Decision Analysis* 9(1): 6-21.

[24] Mailath, G. (1993) “Endogenous Sequencing of Firm Decisions,” *Journal of Economic Theory,* 59, 169-82.

[25] Marx, L., and G. Shaffer (2007). "Rent Shifting and the Order of Negotiations," *International Journal of Industrial Organization*, 25(5), 1109-25.

[26] Moresi, S., S. Salop, and Y. Sarafidis (2008), "A Model of Ordered Bargaining with Applications." Available at SSRN: `http://ssrn.com/abstract=1287224`

[27] Niederle, M., and A. E. Roth. (2009). "Market culture: How rules governing exploding offers affect market performance." *American Economic Journal: Microeconomics*, 199-219.

[28] Riordan, M. "What Is Vertical Integration?" In M. Aoki, B. Gustafsson, and O.E. Williamson, eds., *The Firm as a Nexus of Treaties*. London: Sage Publications, 1990.

[29] Sebenius, J.K. 1996. Sequencing to build coalitions: With whom should I talk first? In *Wise choices: Decisions, games, and negotiations*, edited by R. J. Zeckhauser, R. L. Keeney, and J. K. Sebenius. Cambridge, MA: Harvard Business School Press.

[30] Taylor, C., and H. Yildirim. (2011) "Subjective Performance and the Value of Blind Evaluation." *Review of Economic Studies*, 78, 762-94.

[31] Vincent, D. (1989) "Bargaining with common values." *Journal of Economic Theory*, 48.1, 47-62.

[32] Wheeler, M. (2005) "Which Comes First? How to Handle Linked Negotiations." *Negotiation* 8, no.1.

[33] Xiao, J. (2018) "Bargaining Order in a Multi-Person Bargaining Game," *Games and Economic Behavior*, 107, 364-79.