2018

FURTHER toward a neoclassical theory of sustainable consumption 3-14-18 (rev1).docx

Harry D. Saunders

Further toward a neoclassical theory of sustainable consumption: The Eco-Economy

Harry D. Saunders[1,2]

Abstract

While not known as a factual matter, it is easy to speculate that somewhere in the universe, and quite likely in our own galaxy, there exists a planetary civilization that has achieved indefinite sustainability.

By giving ourselves intellectual leave to contemplate this picture, we can more cleanly consider key questions: How does its planetary eco-economy function, and how do its human economy and natural economy components work together? Can this eco-economy function if its human component is a free-market, private ownership economy, or is that impossible?

A neoclassical framework that formally and rigorously encompasses both the human economy and the natural economy provides some answers.

First, economic growth is not a necessary condition for functioning of a free-market, private ownership economy. Further, it can in principle deliver a sustainable eco-economy that displays the following: indefinite sustainability of the planet's natural capital; ongoing household satisfaction from consumption, leisure time, retirement savings, and directly from natural capital itself; persistent income equity between labor and capital; poverty elimination; and long-term intergenerational equity.

Provided, that is – and this is shown to be a crucial proviso – the human economy can honor certain fundamental biophysical limits imposed by its planet's natural economy.

Keywords: Sustainable consumption; neoclassical theory; eco-economy; ecological economics; natural capital; zero growth; collapse

1. Introduction

The motivation for this article is to ask if a truly sustainable civilization on a spherical planet can have a human economy resembling our present one – in particular, whether its eco-economy can function indefinitely if embodying a human economy that is a free-market, private ownership-type economy. Is the capitalist paradigm embedded in neoclassical economics even consistent with a sustainable civilization? In particular, is it sustainable when dependence of the human economy on the natural economy (which clearly must be in operation and functioning) is explicitly considered?

[1] Corresponding author.
[2] Managing Director, Decision Processes Incorporated; Senior Fellow, The Breakthrough Institute; Advisor, Carnegie Energy Innovation, Carnegie Institution for Science, Department of Global Ecology, Stanford, CA.

A technical ancillary fortuitously arises from exploring this question. A framework is required that may help advance a theoretical unification of ecological economics with neoclassical economics. It is argued that a unification embodying both is needed to imagine what a sustainable eco-economy can look like, including on our own planet earth.

Ecological economists insist that sustainability requires an eco-economy to be sustainable on a physical throughput basis. As Herman Daly describes the steady-state sustainability requirement, "…the entropic physical flow from nature's sources through the economy and back to nature's sinks, is to be non-declining." (Daly, 2007).

In contrast, neoclassical economists typically exclude the natural capital side of this dynamic, or include it only weakly, or indirectly, or incompletely. Yet neoclassical economics can provide a good physical picture of the *human* side of the full eco-economy – that portion complementary to natural capital in the physical flow cycle Daly describes. The working dynamics of the human economy piece of the cycle are fairly well understood in neoclassical terms. Here the natural economy is included.

The theoretical development of what follows is quite mathematical, and its rough description is best first communicated via a visual description of its logical foundation (full mathematics in the Appendices). This visual description steals heavily from the "doughnut economy" visualization provided by Hayworth (2017).

The theoretical framework is deeply grounded in the work of neoclassicists Robert Solow (Solow, 1956, 1988), Kenneth Arrow and Gerard Debreu (Arrow and Debreu, 1954), Franco Modigliani and Richard Brumberg (Modigliani and Brumberg, 1954), Edmund Phelps (Phelps, 1961, 1965) Ronald Shephard (Shephard 1953, 1970), Friedrich Hayek (Hayek, 1945, 1960), and Joseph Schumpeter (Schumpeter, 1947a,b). And it is likewise heavily grounded in the foundational ecological economics work of Nicholas Georgescu-Roegen (Georgescu-Roegen, 1971), Herman Daly (Daly, 1973, 1978, 1980, 1996, 1997, 2000, 2005, 2007, 2008), Robert Costanza (Costanza, 1980, 1984/1987, 2008), and Robert Ayres (Ayres, 1997, 2005, 2009) and their colleagues.

In fact it easily could be said that the framework presented here is little more than a routine assembly job of all these great thinker's ideas, methods and mathematics.

[Author Aside: The present article is the outgrowth of an earlier article in these pages (Saunders, 2014), with extensions made possible by two gratefully-received suggestions from Herman Daly to include the role of natural capital in directly producing human satisfaction, and to incorporate extensions that cast the human productive economy in a form consistent with the production structure developed by Nicolas Georgescu-Roegen (1971.)]

To correctly include the natural economy means honoring Herman Daly's Three Rules:

1. Sustainable use of renewable resources means that the pace should not be faster than the rate at which they regenerate.

2. Sustainable use of non-renewable resources means that the pace should not be faster than the rate at which their renewable substitutes can be put in place.
3. Sustainable rate of emission for pollution and wastes means that it should not be faster than the pace at which natural systems can absorb them, recycle them, or render them harmless.

The dynamic elements we see here are resource use, regeneration, waste infliction, and natural capital absorption capacity. These elements are included in the neoclassical framework described here.

The resulting theoretical framework purports to be complete, formal, closed, and rigorous. But the generality it embodies obscures its functional workings. To help remedy this, the framework is then instantiated in a series of interlocking functional forms that are standard yet specific choices used by neoclassicists and ecological economists. These forms are not entirely arbitrary: they honor commonsense conditions these practitioners favor and have a long and pedigreed history in economics.

The instantiation takes the form of an "emulation" model (distinct from a predictive, or simulation model). This model is designed for exploration of how these standard functional forms interact with each other under different conditions – purely and parsimoniously interact, without obfuscation introduced by arbitrarily added or hidden mechanisms or modeling "tricks." The model is freely available and posted alongside this article.

With this construct in hand, the emulation model allows a rich exploration of some basic ecological economics questions.

The picture revealed is one of promise. According to this framework, indefinite sustainability is possible under a broad set of conditions, provided population stabilizes and markets function well. As with the previous paper's model, we see with this model that it reveals forces pushing toward intergenerational equity, equity between of capital owners and the workers providing labor, and forces pushing to eliminate poverty. Augmenting this, technology improvements in a steady-state zero-growth economy work to increase the amount of leisure time households can enjoy but without surrendering their pre-technology (e.g., automation) levels of income, consumption and saving. At the very same time, the productive human economy reduces its draw on natural capital, meaning the natural economy can expand as humans retreat and allow it to better heal itself from past assaults.

However, this happy picture comes only within binding limits on how high the consumption can be, and how high the population can be for a given level of individual consumption, without causing catastrophic collapse of the natural and human economy (the eco-economy).

Disclaimer: Knowledgeable readers will recognize strong similarities between the emulation model and the Integrated Assessment Models (IAMs) following in the famous DICE/RICE-tradition of modeling (Nordhaus, 1977a,b, 1992a,b, 2008, 2010, 2012), having many of the same features, components and interactive elements. These IAMs are more detailed and richer across several dimensions, including more explicit links between energy use and climate, for instance. But their purpose is different (though certain elements of the present framework may point to useful modifications of existing IAMs).

2. Methods - Formal Framework

The framework is mathematical and may therefore be unnecessarily obscure for certain readers. Honoring the principles of neoclassical economics in the human economy, especially, involves a mathematical rigor that is elegant, but demanding.

So it is useful to begin with a pictoral description of the framework, which then can inform evaluation of the mathematical description for those with the inclination. The following sub-Section 2.1 does this. Those not so inclined can thereafter simply skip the mathematics of Section 2.2 and proceed directly to the Results Sections 4 and 5 where the device of positing different hypothetical planetary civilizations is used to explore eco-economy sustainability drivers quantitatively. Mathematical masochists should read Section 2.2 and proceed for further detail to Appendices A and E.

2.1 The Framework in Pictures

It is easiest to describe the framework by appeal to a picture very much like the picture put forward by Raworth (Raworth, 2017)[3]. We begin by imagining a world without a human economy or civilization, but one in which there is a functioning natural economy. Such a world might look like this (Figure 1):

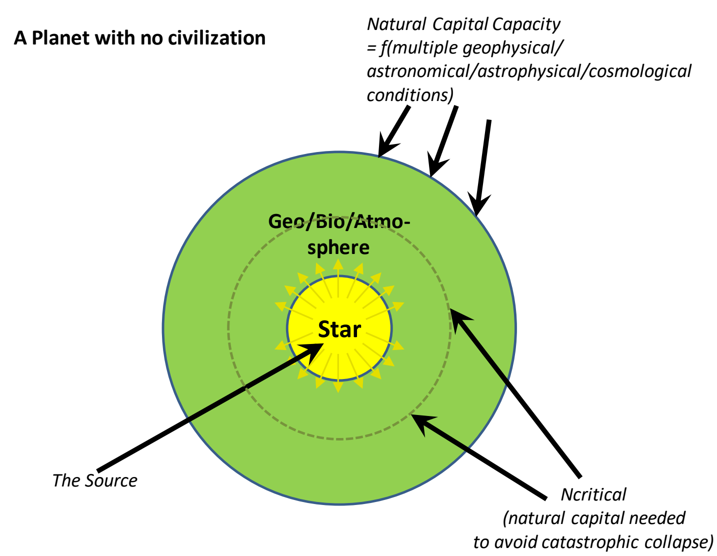

Figure 1: An Eco-economy without a human economy

What we see here is a world with a geo/bio/atmosphere, but no human activity. The natural economy (green) has evolved to fill its natural capital capacity – a capacity determined by its luck of the draw in the cosmological lottery: its size and surface area; distance from its host star; volume of liquid water,

[3] I hereby plainly acknowledge that Kate Raworth and her associates may view this arguably larcenous conscription of her conceptual framework as a poor, presumptuous, possibly deceptive, and dangerously misleading bastardization of the central thesis she offers. Nonetheless, I offer my sincere gratitude for the digestible and powerful imagery she has provided the field.

available chemical elements, etc. In defiance of astronomical reality, the star in this world's solar system is showing at the center of the image.

The star is placed at the center to depict the fact that that the local star is the all-provider of the natural economy's health and functioning.   All energy for biophysical growth and replenishment comes from this star. On earth we see all energy for human use likewise sourced, be it fossil fuels (decayed bio-organisms that long ago captured energy via photosynthesis), hydro power, wind power, solar power, biofuels or tidal power. The only exceptions to this direct provision are geothermal and nuclear power, but these came from long-ago stellar processes fueling supernovas and possibly neutron star mergers or other cosmic cataclysms that made other stars the ancestors of our own sun and solar system. Likewise, the geosphere itself and all its minerals is the progeny of long ago stars that gave birth to our solar system. It is little wonder the ancients worshipped the sun, even if they knew nothing about her functioning or her ancient forebears.

Also showing is a lower limit to the natural capital supply, below which it cannot function and will collapse (perhaps a limit that could be tested by a future civilization that reduces, by its too-great activity, the natural capital supply). We can think of the "doughnut" between this critical level and the natural capital capacity level as the "safe zone" (to again steal a phrase from Raworth, 2017) within which natural capital supply must perpetually reside for healthy functioning of the natural economy.

Next we introduce a human-like civilization. The civilization supports itself by drawing on resources provided by natural capital. It also inflicts waste on the biosphere. The natural economy can respond, to some degree, by replenishing resources (think food, timber, biomaterials) and by absorbing waste (think natural filtering, the carbon cycle, and conversion of biological materials to fertilizing agents via decay). The picture is then this (Figure 2):

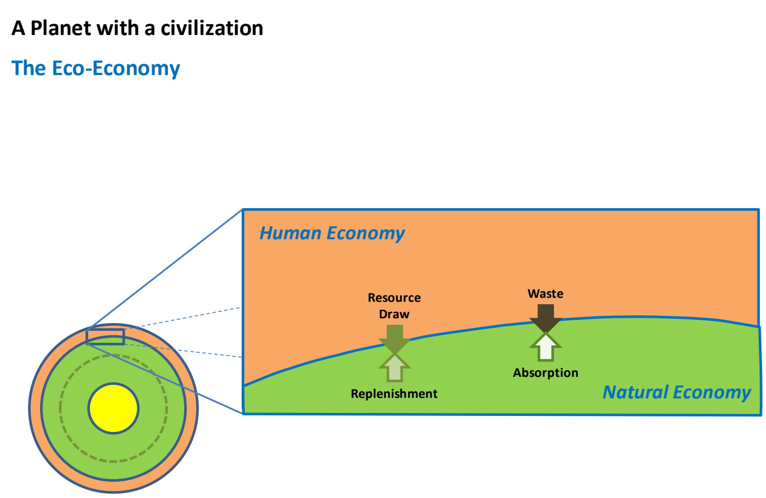

Figure 2: An Eco-economy *with* a human economy

There will be occasion to explore the conditions under which such interaction is sustainable in a later section, but we can see here the dynamic described by Daly as "…The limits to growth, in today's usage, refers to the limits of the ecosystem to absorb wastes and replenish raw materials in order to sustain the economy" (Daly, 2007).

This is where the Georgescu-Roegen production theory comes into play on the human economy side, and consistency requires its inclusion in the framework (see Appendix F). Note that the resource draws from natural capital are technically complementary to physical capital in the human economy rather than being substitutes for each other – both are needed to make the system work.[4] In the words of Daly, "… manmade and natural capital are complements, not substitutes (except over a very small margin). When factors are complements then the one in short supply is limiting." (Daly, 2007). This physical way of looking at things is also embodied concretely in the "embedded energy" concept introduced by Costanza (Costanza, 1980), and the thermodynamic "exergy" formulation put forward by Ayres (Ayres, 1977) and Ayres and Warr (Ayres and Warr, 2005, 2009).

Now we turn to the nature of the human economy itself. The human economy comprises two components: the activity of households and the activity of producers that supply them goods and services (Figure 3):

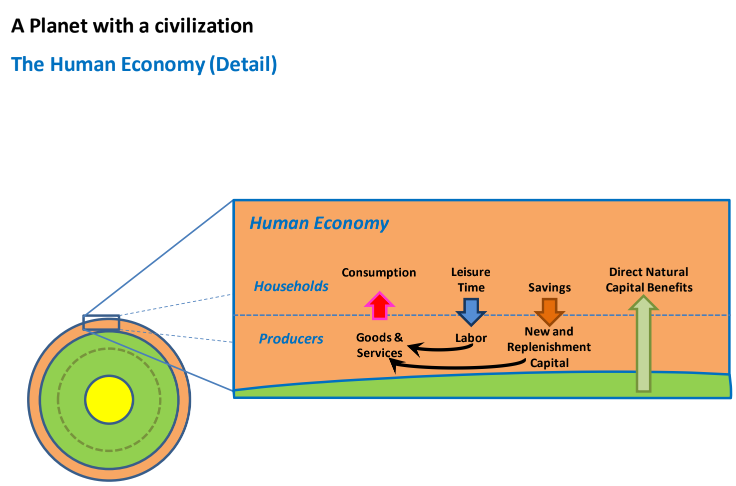

Figure 3: The human economy

Here we see several interactions. First, households supply producers labor at the expense of leisure time. They also supply producers with new capital via their savings (foregone consumption of goods and services that can be directed to the creation of new or replenishment physical capital). Producers use these inputs from households (along with the resources extracted from the natural economy as in Figure 2) to produce the goods and services consumed by households. Households buy these from proceeds extracted from producers in the form of wages and investment returns, which set the household budgets.

Note also that households are depicted as receiving benefits directly from the natural economy (think clean water and air, biodiversity, favorable climate conditions, majestic grandeur, esthetic gratification,

[4] The framework developed here allows for a certain degree of substitutability between the two, but they are fundamentally complements.

etc.)[5]. The case for this direct benefit is beautifully articulated by Emma Marris (Marris, 2017) who argues that households receive "intrinsic benefits" from the natural economy: "…nature is valuable *because people value it for what it is*, independently of any concrete economic or practical value it provides." Of course, the latter "practical" benefits (sometimes called "natural capital services"), must be and are comprehended in this framework as well, as illustrated in Figure 2. Benefits to households are twofold: those related to the intrinsic value delivered them by the natural economy; and benefits that flow indirectly through the productive economy (resource draw for producers to create goods and services for households; replenishment and waste absorption services provided for the same end). Both functions of natural capital need to be comprehended in any general framework of the eco-economy.

A complete picture would take account of goods and services provided collectively by households for social benefit. (While not included in the framework presented here, such an extension would be relatively straightforward.) Social enterprises require physical quantities of goods and services from households:

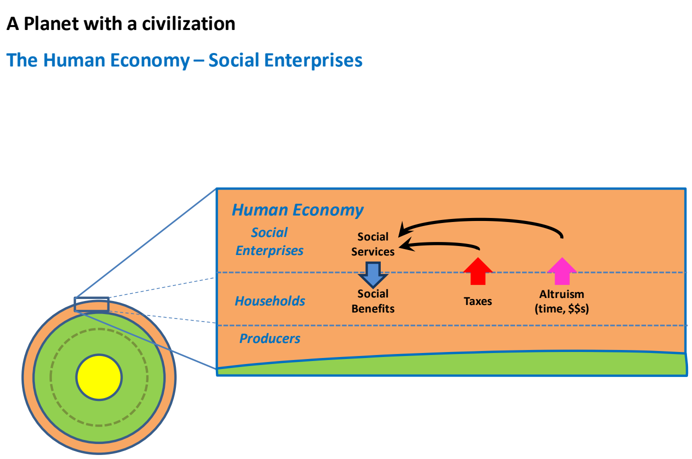

Figure 4: The social economy

Here we see that households surrender claims on goods and services for the collective benefit of the whole. This is done either via taxes collected by governments for social services or via pure altruism, both of which deliver social benefits to households (and arguably, satisfaction to the households providing them).

With these images in mind, the mathematical development following should appear more intuitive. [*But a humbly-offered reminder: The time-constrained reader (who isn't?) may be best served by first skipping to the Results sections (Sections 4 and 5) to see what is being claimed there, returning to the following sections if these claims seem to make little sense or suggest hidden faults.*]

[5] Again, my thanks to Herman Daly for suggesting this feature.

2.2 The Framework in Mathematics

For those readers of a neoclassical mathematics bent, a complete description of the theoretical framework mathematics is given in Appendix A. But even so, rough verbal description of its main elements in terms of the above picture is perhaps a useful guide to those mathematics for those wishing to go one step deeper.

Everything is locked together in the (simplest possible?) neoclassical framework. All the quantity flows are endogenously determined, and consistent with neoclassical theory, including general equilibrium theory.

It is important to recognize that it is, first and foremost, a model of physical quantities. Importantly, too, this formulation escapes the obfuscation laid upon the physical mechanisms by financial markets. The existence of money in the economy, with various multiplier effects introduced by banking long with government monetary policy, obscures what is going on in physical reality. Investment in physical capital can only be so much in an economy, and household savings (forgone physical consumption) drives that. Financial markets cannot override the physical constraints imposed by the real physical economy, which is what is pictured here.[6] Keeping things in physical terms (while honoring pricing mechanisms) allows a better picture of the human economy/natural economy interaction, which is at bottom a quantitative interaction.

Human Economy

It is best to begin with the human economy in Figure 3, and think of it as being a single-period picture for the time being. Technically in this framework, households and producers are each aggregated. This means one can think of the household component here treated either as a single, representative household or as an aggregate of all households. Similarly, producers (arguably harder to envision as a single, representative producer) should be thought of as some global aggregate.[7]

Households maximize their satisfaction, subject to their budget constraint, by choosing among: consumption of goods and services C; savings for retirement S; leisure time not surrendered to the workforce $l = \bar{L} - L$ (where L is the amount of labor-hours surrendered to the workforce and $\bar{L}$ is the total hours available to the household); and direct enjoyment of the natural capital available N (Marris, 2017). This tradeoff is embodied in a utility function:

$$u = u(C, S, l, N) \tag{1}$$

Savings S is the amount set aside for investment by households to be used for new physical capital; it can be thought of in quantitative terms as goods and services production, to which they have contributed labor and capital, but instead of being consumed by households that period are instead

[6] Further detail is given in Appendix D: Financial Markets Mechanics and Zero-growth Dynamics in a Physical Economy.

[7] This is a technical limitation of the framework as clearly both households and producers are not so simply aggregated, a result well known in neoclassical economics. See Section 6, Discussion - Cautions and Limitations.

used to create new capital goods used to replenish and build the productive physical economy. (More detail on why this production dynamic matches desired household savings behavior for retirement funding is given in Appendix C.)

The inclusion of leisure time, as the complement to labor supplied, and the direct household utility for natural capital are not much of an extension of common formulations, analytically. The direct inclusion in the household utility function of natural capital was suggested by Herman Daly.[8] Chan and Gillingham (Chan and Gillingham, 2015) have done a similar thing by broadening the utility function to a social utility function comprehending the dis-benefits from damages due to externalities.

On the other side, the *productive part of the economy* takes labor and capital from households and uses these to create goods and services. In neoclassical terms, this is represented as a production function – a function that, in keeping with the physical underpinnings of economic activity put forward by Daly and colleagues, takes physical input quantities and converts them into physical goods and services. The production function is a physical representation of this process. In simplified form, the production function is

$$Y = f\left(K, L\right) \tag{2}$$

The physical production of goods and services in the overall economy is related to the total quantity of labor used, and the total supply of physical capital (plant, equipment, structures) employed to deliver it. To incorporate the physical role of natural capital supply and linkages, we can extend (2) slightly to account for the fact that some of the capital available and some associated labor will be directed to the extraction of natural resources while the rest will use these resources plus its own physical capital and labor to produce final goods and services:

$$Y = f\left[X\left(K_X, L_X\right), R\left(K_R, L_R\right)\right] \tag{3}$$

This human economy can be thought of as comprising two sectors, R is the resource extraction sector, and X is the rest of the productive economy, each using capital and labor supplied them. Together, via the overall function f, they deliver final goods and services (including capital goods). The physical output is physically attached to the physical inputs. Much further detail is provided in Appendices A and E.[9]

However, to enclose the picture and have it be internally consistent, neoclassical theory requires that costs and prices be comprehended as well as physical quantities, or the system will not function correctly.

Fortunately economists in the personages of Ronald Shephard (Shephard, 1953,1970) and Erwin Diewert (Diewert, 1974) have helped us out there. A beautiful theory called duality theory automatically locks

[8] For which I am again extremely grateful.
[9] Appendix E: Functional Forms and Derivations for the Emulation Model, shows how this production function can be generalized to include technology gains.

prices and quantities together so we can know the system is correctly functioning in profit-maximizing mode, the effects of prices and quantities on each other are properly stated, and all markets clear at the calculated prices. In this framework, duality enables us to endogenize prices – wages, capital returns, and the price of output – in the process allowing the household budget constraint to be endogenously determined as well. This complicates the mathematics, but the power it delivers is worth the extra analytic effort.

This theory states that every production function must have a corresponding dual form that delivers the unit cost of production and embodies the prices of the quantity factor inputs to production. For consistency, both functions must work together. So associated with the production function quantity Y is a unit cost function of the form:

$$c = \boldsymbol{D}\ (Y) = c(p_K, p_L) \tag{4}$$

Where $\boldsymbol{D}$ is the so-called dual operator on Y , and p_K and p_L are the unit prices of capital (otherwise denoted as the rate of return on capital r) and the unit price of labor (otherwise denoted as the wage rate w).[10]

With duality functioning and households being owners of the means of production (via their savings), the general equilibrium principles put forward by Kenneth Arrow and Gerard Debreu (Arrow and Debreu, 1954) will hold. By honoring these principles, the framework becomes a so-called Arrow-Debreu private ownership, free market economy, one where markets clear at the calculated prices, virtually the definition of general equilibrium. The system is thereby closed and complete (in classically-stated terms for the human economy side – the natural economy has yet to be included here, but will be below). The emulation model instantiation of this framework validates that this is what happens in an economy like this. So this is how the cycle from households to producers to goods and services supplied back to households showing in Figure 3 works.

Importantly, in this picture households drive the show. The choices made by households determine everything the productive economy does – the labor and capital investment deployed, and the amounts of each in service; the amount of consumption goods and services called for from the productive economy. The productive side in turn constrains the overall levels of household consumption and savings by way of the budget delivered the household from wages earned and investment returns. The consumption/savings tradeoff dynamic is found to emulate the predictions of Modigliani (Modigliani and Brumberg, 1954) who discovered the dynamic.[11] As with the general equilibrium validation, consumption behaviors predicted by Modigliani are borne out by the emulation model, and households run the show.

[10] Appendix E: Functional Forms and Derivations for the Emulation Model shows how this cost function can be generalized to include production-side technology gains.
[11] See Appendix C. How can capital formation occur in a zero-growth economy where net capital returns are zero? [And a CORRIGENDUM]

Household choices also determine the level of wealth generated by the productive economy. By choosing to supply less labor for instance, households would lower their ability to consume, as the household budget would now have less in the way of wage income. Producers would respond to the consumption reduction by producing fewer goods and services to match demand, and would anyway have fewer labor hours to produce them. A similar story applies to household reductions in savings and resulting investment income. As another example, if households place a higher value on natural capital, this would change their choices among consumption, savings, and leisure time and the output produced by the productive economy. The framework accommodates these features of household behavior and all such interactions that result.

As for time dynamics, the work of Solow and the neoclassical growth economists is here slightly modified to account for the framework being able to generate savings endogenously. But it turns out this generates behaviors identical to the ones familiar to neoclassical growth economists.

Reassuringly, Edmund Phelps' "Golden Rule" behavior (Phelps, 1961, 1965), found to operate in the simpler framework described in a previous paper (Saunders, 2014), continues to hold even when the natural economy is included in the picture, which we turn to doing next.

Appendix B gives a verbal description at a slightly deeper level of formal reasoning of why this all works.

Natural Economy

For the natural economy part of the framework, the linkages showing in Figure 3 are those Georgescu-Roegen (1971) showed to be missing in standard neoclassical theory – in particular how the resource draw and waste interactions between the human/natural economy should be considered in manmade capital production function terms, and the proper accounting distinctions betwen stocks (Georgescu-Roegen calls them "funds") and flows. Appendix F lays out a detailed picture of how the framework relies mathematically on these Georgescu-Roegen concepts. This sets up the interactions coming from the human economy side (resource draw, waste production).

On the natural economy side, it turns out neoclassical economics methods can be usefully applied.[12] The natural economy can be considered a "producer" in the classical sense. And it happens duality principles likewise can be applied. The natural capital production function is simple in principle and has the advantage of being theoretically correct by virtue of the fact it is a physical identity (leaving to the side for the moment that its elements can experience complex dynamics). The identity is this:

$$N_t = N_{t-1} + \left(\text{¡}_t - R_t\right) + \left(A_t - W_t\right) \tag{5}$$

This identity states that the current supply of natural capital N_t is what it was last period plus changes coming from two eco-economy dynamics: first, it is augmented by the difference between its natural replenishment capacity (biospheric reproduction and growth) and resources drawn from it – ¡_t and R_t,

[12] Much more detail is provided in the mathematical expositions in Appendix A and Appendix E.

respectively; second, it is diminished by waste inflicted on it by the human economy, moderated by natural capital's ability to absorb it – W_t and A_t, respectively (with restrictions requiring absorption to be no greater than waste inflicted, and absorption capacity itself to be limited by the prevailing natural capital level).

In the absence of a civilization, a planet's natural capital supply will be determined by a variety of factors including luck of the draw in astrophysical terms (mineral composition, water supply, size, distance from the host star, etc.), the geological evolution of the planet, the evolution and constitution of its atmosphere, and the degree to which it can evolve and sustain living organisms. It will change over time, but at any one time natural capital supply is relatively fixed. When a civilization is introduced, that civilization will extract mineral and biological resources and generate waste. Thus we see in identity (5) the terms R_t and W_t as negative detractors from natural capital supply operating in each time period t.

The time dynamic is simple: natural capital supply at time t is its supply in the previous period, plus all the current changes arising from the human economy-natural economy interactions.

This production function (5) has a dual unit cost function. It turns out to be a very simple one, where the unit cost is uniform and fixed across the entire domain of all production function components. Furthermore, the elements ¡$_t$ R_t, A_t and W_t carry identical dual prices across the domain and deliver a unit cost function value that is unity everywhere.[13] If we call this value c_N, we have $c_N = 1$ everywhere. Technically, it would be tempting to use this fixed value as the mechanism to "charge" the human economy for each unit of natural capital services.

But for the natural economy, there is need to account for the problem of scarcity. First, there is the limited nature of non-renewable resources. But even renewable resources can become too heavily exploited if replenishment is not sufficient to offset resource draw and/or waste absorption is insufficient to offset waste inflicted. Either way, if the supply of natural capital falls too low, nature becomes stingier in supplying needed resource services to the productive economy. The unit cost extracted from the natural capital production function above represents a price "charged" to the human economy that is therefore too low. Another neoclassical economist bails us out here. We can characterize this behavior by appeal to the signature work of David Ricardo (Ricardo, 1817) who formulated the neoclassical theory we know today as "Ricardian rent," or "scarcity rent." While Ricardo applied this concept to land and land ownership (land itself clearly being a defining component of natural capital), it is easily extended to natural capital as a whole.

In terms of a general framework, this relationship describing what the natural economy "charges" the human economy for its services is as follows:

$$p_N = c_N + g(N) \tag{6}$$

[13] Further detail in Appendix A.

where the function g ("Ricardian rent component") increases in value as N declines: when the prevailing supply of natural capital is large (considered relative to its critical level $N_{critical}$), the "rent" charged by natural capital is low; when the supply is low, the rent is high.

While this is a highly general formulation, its nature can be intuitively illustrated by appealing to the particular functional form employed by the emulation model presented later:

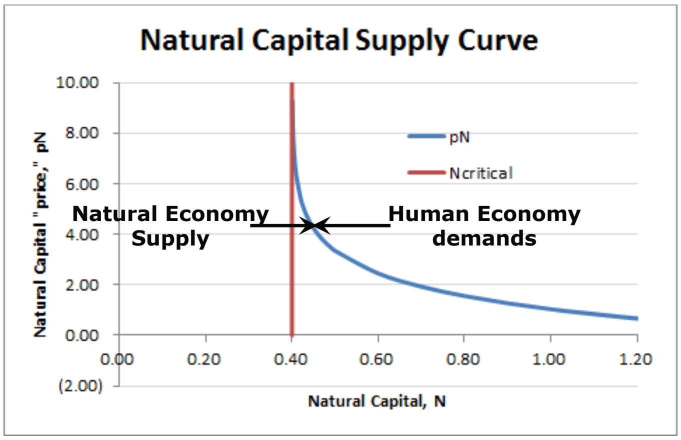

Figure 5: The "push back" from the Natural Economy

In the image of Figure 5, natural capital "charges" the productive economy a "price," showing as p_N. If the supply of natural capital has been reduced by resource needs or waste disposal to levels approaching some critical level ($N_{critical}$), the natural economy approaches catastrophic collapse and charges an asymptotically-increasing price. In this way, the natural economy "pushes back" (toward the right) against over-exploitation (pushing to the left).

In a general framework, the component drivers of natural capital dynamics showing in (5) will need to be specified as well. Consider these one at a time:

Resource draw in each period, R_t, is easily derived from (3) when profit-maximizing conditions are applied. Likewise, waste inflicted in each period, W_t, is clearly related to resource draw (and can be no greater than this), so in general terms:

$$W_t = W\left(R_t\right) \tag{7}$$

Natural replenishment of natural capital in each period, ¡_t, is clearly related to the quantity of natural capital in place (e.g., biological organisms boasting more progenitors can generate more progeny):

$$\text{¡}_t = \text{¡}\left(N_t\right) \tag{8}$$

Natural capital absorptive capacity is likewise dependent on the prevailing supply of natural capital to absorb waste (e.g., CO_2 absorption by the biosphere via the carbon cycle, recycling of animal waste, biodegradability of human economy materials disposed, dilution/natural processing of pollution to non-biotoxic levels, etc.). So this relationship becomes, generally stated:

$$A_t = A(N_t) \tag{9}$$

Notwithstanding this simple-appearing set of relationships, they may mask critical underlying dynamics in the evolution of the eco-economy. The connection between waste and absorption is complex, as it can involve complex atmospheric and biospheric chemistry, to mention only one consideration.

One might worry that past waste has to be taken account of whenever it has exceeded natural capital absorption capacity (think CO_2 emissions) historically. The absorptive capacity of the planet's natural capital has to deal with cumulative unabsorbed waste it has not yet dealt with, not just what is inflicted on it in the current period.

Fortunately from an analytic perspective, the natural capital supply function expressed in (5) is iterative. This means the evolution of natural capital in this framework inherently accounts for all past diminishments of its supply due to waste exceeding natural capital's absorptive capacity.

This completes the theoretical framework. Further detail appears in Appendices A and E.

3. Methods - The Emulation Model

The emulation model is not a model in the usual sense of the word. It is coded in Excel™, yes, but it is a pure, direct instantiation of the framework relationships in the form of specific but standard neoclassical functional forms and relationships, with no hidden modeling tricks or sleight of hand. While this makes the emulation model less general than the framework itself, the functional forms used have a long pedigree in economics and themselves are fairly general.

The model can be interrogated by asking how changes in input conditions affect output conditions of interest when confined to strictly honoring the principles and equations of neoclassical and ecological economics. It is a learning tool as much as anything else. It is used for "expeditionary exploration," to conscript Solow's elegant term to highlight its limits.

It is dubbed an emulation model to distinguish it from models intended to simulate and predict future conditions. It is not a forecast model, nor does it simulate futures in the sense Monte Carlo models do.[14] It is not an econometric model. It is rather intended as an exploration tool, interactive and amenable to tinkering with different parametric and initial conditions assumptions to see the effects (as in, "Why did it do **that** when I did **this**?").

The framework is the anatomy (or "base code," if you like); the emulation model aims at understanding the physiology, how the eco-economy system operates over time. The framework is as parsimonious as

[14] In principle, the model could be used to paint a Monte Carlo picture, but this would require knowing the proper ranges to use for several input variables, and their uncertainty distributions. Empirical data to inform key input choices appear scarce to non-existent at the present time.

appears possible. The emulation model reflects this parsimony, but in a way that is restrictive owing to specific functional forms being chosen. Other more general functional forms could be used, but at least those used here have deep familiarity for neoclassical economists.

[Technical aside: Modeling of this type is an excellent way to validate the mathematics and its correctness. Excel™ is miserly in its tolerance for sloppy reasoning. Using the model to check the math allows benefits to flow both ways: the math is sometimes improved (made correct) by the discipline forced on clear reasoning by the modeling process (e.g., exposing erroneous thinking, or errors in the mathematical derivations); going the other way, the task of proper mathematical implementation often improves the model instantiation itself – results in a better, and more usable model. Along the way, such back-and-forth uncovers new understanding about how the modeled eco-economic system itself must really work, and why.]

I think of the emulation model as a tool to understand how *mechanically* the system works. What affects what, and how, a way to explore relationships and how they work, and specifically what conditions are needed to avoid collapse of an eco-economy.

The Excel model itself is posted alongside this article for users to play with, is user-friendly, interactive, and includes a User Guide. This model is freely available for anyone to use or improve upon (open source collaboration protocol). It can also be used to audit the proper implementation of the equations described herein; or to replicate the results showing in later sections of this article.

The specific functional forms used for the emulation model are given in Appendix E, along with associated mathematical derivations.

4. Results – A Sustainable Civilization

To reinforce the point that the emulation model is not intended as a forecast model, consider a hypothetical planet; call it "Planet A."

The civilization on Planet A has evolved a steady-state zero-growth human economy. The natural economy, which supports this human economy, has likewise reached a steady state where it indefinitely flourishes. The human economy is a private ownership free market economy. The atmosphere and biosphere have stabilized in an aggregate sense, without discernible negative trends in play. Exhaustible resource use has gone away. As to how this civilization has achieved all this is left to the reader's speculation, but this is the picture on Planet A.

Is such a state of affairs technically feasible? Can such an economy, eco-economy, exist, even given these heroic assumptions?

Conditions that appear to be necessary for feasibility are two: one, the population of the planet has to have stabilized in a non-growth mode (some observers say this may come within a few decades on Earth (Robbins, 2016)); and two, that overall physical consumption of goods and services has likewise stabilized on a zero-growth trajectory (though its composition can and no doubt will be changing). This means households on Planet A have realized a "satisficing" level of consumption (Simon, 1956) –

perhaps something similar to what is seen to be beginning to happen in the Nordic countries on our planet.

With these two conditions the model shows surprisingly well-behaved and sustainable operation of the human economy (Figure 6). [15]

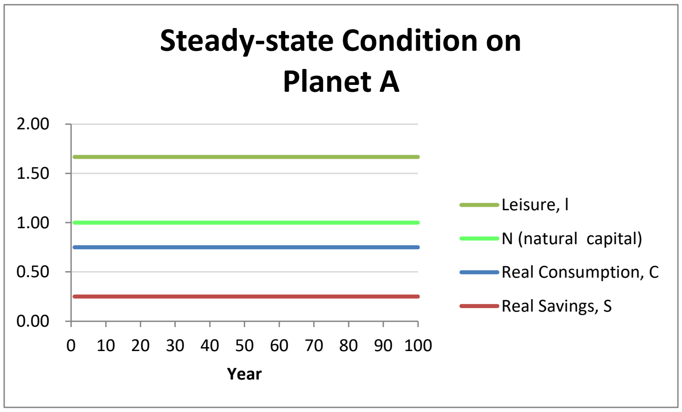

Figure 6: The Eco-economy in Steady-state

This civilization has achieved indefinite sustainability, or at least the preconditions of indefinite sustainability. It appears there is nothing in neoclassical economics that prevents or stands in the way of such state of affairs. The trajectories in Figure 6 show the components of household satisfaction (utility function) over time. But note that also implicitly projected are trajectories of physical labor supply (complement of leisure time) and physical capital invested in the productive capacity of the human economy (real savings, or foregone consumption of goods and services instead used to produce new physical capital, or refurbish old). The physical natural capital is also projected, as this is an element of household utility, but would be subject to waste inflicted and resource draw by the human economy. (But on Planet A, waste inflicted is below absorption capacity, and only renewable resources are used).

In this world, we see a human economy that is behaving in pure neoclassical free market fashion and has achieved fixed and steady level of prosperity, yet leaving the natural economy indefinitely intact. However, for this to happen, another condition we must be able to observe as satisfied on Planet A is that the prevailing level of natural capital is above some critical level, below which it would experience catastrophic collapse. But assume for the moment this, and the other conditions, are seen to prevail.

Under these conditions everything works: household behavior keeps everything in balance – the proper supply of labor is delivered to the productive economy, as is new and replenishment capital supply – just enough to supply the desired level of goods and services to households. But households, and household choices, ultimately run the show and shape the productive capacity of the human economy. Utility households place directly on natural capital is part of this.

[15] Figures throughout show activity levels normalized to a proportional "activity level" index.

As seen in Figure 6, leisure time, natural capital, real consumption and real savings all remain steady. While such results were previously reported for the human economy considered (almost) alone[16], here we see it holding with a more explicit inclusion of the natural economy. Provided, critically, the natural economy collapse avoidance conditions hold.

This zero-growth situation is stable. Forces at work in the neoclassical economy ensure either shortages or surpluses of capital or labor supply experience dynamic corrective forces. [17] A picture somewhat like this was foreseen by William Arthur Lewis (1954), where capital and labor in the course of economic development eventually come into balance quantitatively as labor surplus disappears, wages rise, capital shortage disappears, and capital "rent" extraction diminishes while labor negotiating power strengthens. Some economists[18] refer to these developments as the "Lewis turning point." Long run, behaviors under shortages and surpluses of either production input drive the system to maintain fair balance between the two.

But the technical reasons for this sustainability situation being real are enlightening. General equilibrium theory principles guarantee that this state of affairs will necessarily obtain. At root, the result arises from the fact that, in this framework, all the necessary conditions of general equilibrium theory are met by the emulation model. Primary among these are that the price of output is calculated (endogenously via duality theory), not assumed as an input value; that household utilities satisfy commonsense conditions; and that producer production possibilities (embodied in any production function such as that in the emulation model) are non-increasing returns to scale.[19] Markets must clear at the endogenously-calculated prices. All this occurs in the model. In a beautiful and amazing proof that won Gerard Debreu a Nobel Prize[20] this is precisely what is to be expected theoretically. [See later section 6: "Discussion – Cautions and Limitations" for a more explicit treatment of the required assumptions for this central result.]

An interesting outcome of this dynamic is that new capital supply and labor supply (both provided ultimately by households) come into steady-state balance at a point where there is a fair and equitable distribution of income between capital and labor, just as the "Lewis Model" predicted. Each gets its due and no more. That is, labor receives real wages that match its productivity for the eco-economy as a whole, and capital the same in the returns it receives.[21]

---

[16] Saunders, 2014.

[17] The emulation model, when initiated in a condition of capital shortage and labor surplus, exhibits this "development" dynamic; and capital-labor perturbations of steady state show the system is stable in the dynamic systems sense – forces act to return it to steady-state if one or the other supply is in surplus.

[18] Minami and Ma, 2010; Zhang et al., 2011.

[19] Most prevailing integrated assessment models use a CES production function, which satisfies this condition.

[20] Developed in concert with Kenneth Arrow in Debreu, 1952, 1959, Arrow and Hahn, 1971. [see Luenberger 1995 for cites.)

[21] Saunders, 2014.

On the way to zero growth, capital, if it is in short supply relative to labor, receives added economic rent versus when they come into balance. This $r > g$ "Piketty condition" draws in needed capital during the time before they come into balance. In a zero growth situation, $r = g$ falls to zero, or nearly so.[22]

Even more interesting is the discovery in this model that such an economy automatically delivers what Phelps calls "golden rule" behavior (Phelps, 1961,1965). That is, household savings in the current generation are at the level such that subsequent generations are endowed the same productive economy capacity and natural capital as the current generation enjoys. Phelps dubbed this type of intergenerational equity "golden rule" equity as it honors the directive, "Do unto others [i.e., future generations] what you would have them do unto you." Each generation inherits the capability to recreate the prosperity and natural economy of the generation that preceded it.

As we will see shortly, the introduction of technology advancements only improves this story.

Collapse Conditions

This is the picture of Planet A if its human economy is able to sustain the natural capital level in the "safe zone," residing above the critical level so as to avoid eco-economy collapse. But such catastrophic collapse can occur in this framework in zero growth conditions when household consumption of goods and services exceeds the capacity of the productive economy to deliver it without either excessive draws on resources from the natural economy or waste inflicted on it in excess of its ability to absorb it.

Collapse conditions are also dependent on population levels. Irrespective of the level of a planet's population relative to its natural capital supply, natural drivers of the household consumption of goods and services operate to determine natural capital services needed. A larger population will drive the productive economy to employ all available labor and produce with it goods and services asked for by households so employed, thus raising overall the household budget from new wages. More households also mean more savings, to attach labor supplied to the capital needed for production. But this also means a greater draw on natural capital is needed, whether in the form of more raw resources or greater waste demanding absorption by natural capital.

Consider the first collapse dynamic. Here we have a situation where the productive economy has achieved waste production that is below what natural capital can absorb, and resource draw is such that natural capital replenishes it, but only up to the point population exceeds a certain level. Figure 7 shows that population on Planet A cannot exceed a certain level, waste management and resource draw success notwithstanding, before there is catastrophic collapse of the eco-economy.

[22] See Appendix C. "How can capital formation occur in a zero-growth economy where capital returns are zero? [And a CORRIGENDUM]," for an explanation of how capital formation can still occur when real returns to capital hover near zero, a concern first put forth by Daly (Daly, 2005,2008).

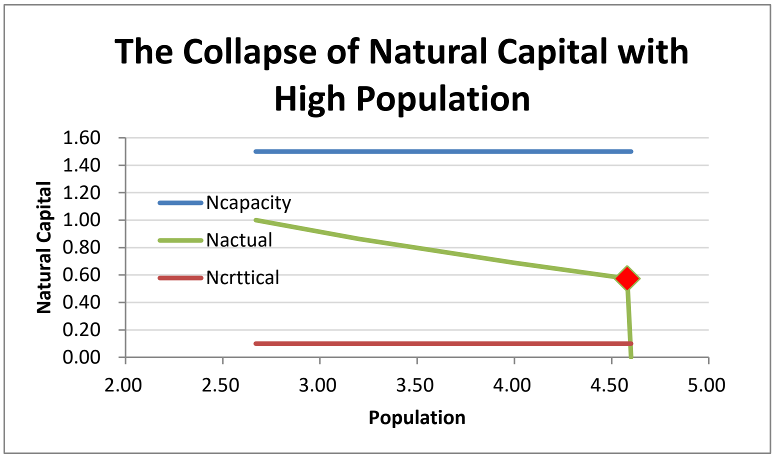

Figure 7: When Population Exceeds Nature's Carrying Capacity

What happens here is that once population gets above a certain threshold, the human economy automatically calls for too much consumption with attendant loss in the natural capital supply arising from resource extraction and waste. Note that collapse occurs even when the level of natural capital is well above the critical level – collapse occurs within about sixty periods in the emulation model, and then occurs swiftly. No countervailing forces are sufficient to prevent it if the human economy behaves in neoclassical free-market fashion.

A similar thing happens when population is fixed, but waste produced exceeds absorptive capacity on an ongoing basis (think greenhouse gas emissions, ocean acidification increases). The relationship between the two is shown in Figure 8.

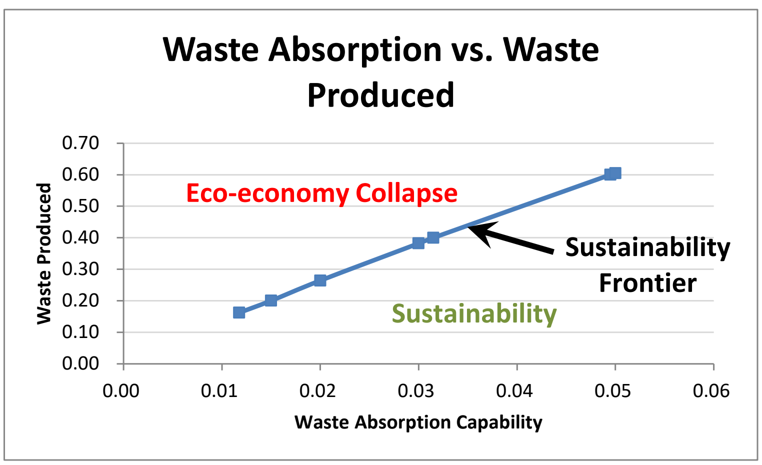

Figure 8: Sustainability Frontier between Ongoing Waste and Absorption

Here we see the quantitatively determined limit of ongoing waste produced by the human economy for mapped against various levels of the natural economy's ability to absorb it. (Note that the natural capital time dynamic accounts for cumulative excess waste). Waste absorption capability must exceed waste production to ensure indefinite sustainability, showing as the region in the lower-right of Figure 8. (Recall that the numerical values are merely indices that relate to a value of natural capital normalized to unity with natural capital capacity arbitrarily chosen to carry a value of 1.50, and its critical value chosen to carry a value of 0.10.[23,24])

If the eco-economy begins in a state wherein there is a legacy waste present (think cumulating greenhouse gases), we again see a collapse condition when its magnitude is too high (Figure 9).

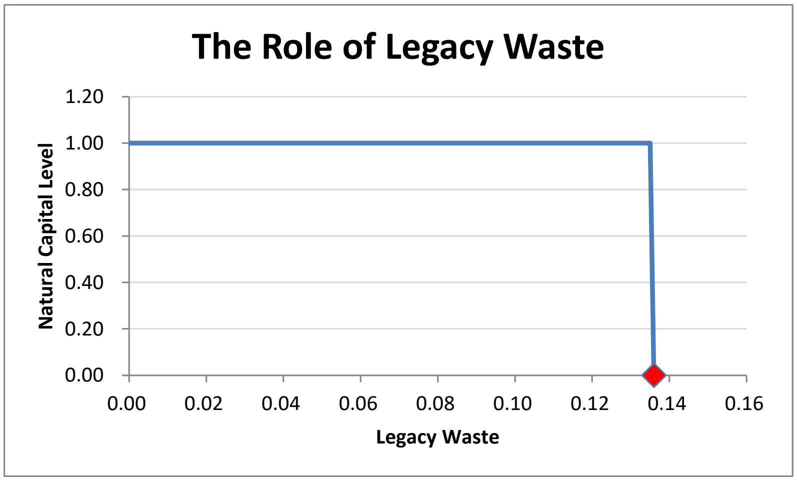

Figure 9: The Limit of Sustainability when Legacy Waste is too High

This shows how there can be a "tipping point" of legacy waste beyond which natural eco-economy dynamics cannot recover. What is happening here is the following: Any quantity of legacy waste reduces natural capital available. Natural capital responds by increasing the Ricardian "rent" it charges the human economy. This reduces production and consumption, having the effect of reducing resource draw and waste inflicted, thus allowing natural capital to recover under certain conditions. If legacy waste is below a certain threshold, natural capital replenishment and waste absorption capacity then become sufficient to "re-grow" natural capital to its steady-state condition. But above this particular threshold of legacy waste, the eco-economic forces in play are insufficient to produce recovery and the system collapses.

[Cautionary note: this is what the emulation model delivers under essentially arbitrary quantitative values chosen for various input parameters. But the intent is to show that the framework comprehends, and the emulation model illustrates, important sustainability dynamics.]

Finally, and importantly, we'd like to understand how the value households place directly on natural capital affects sustainability. Can a larger population be sustained on Planet A if its households place greater value on the benefits they receive directly from natural capital? Figure 10 illustrates.

---

[23] These assumptions can be changed in the emulation model.

[24] Likewise, various human economy indices are normalized to unity in the reference case (see Figure 6): total capital in place, total labor supply, value-added output, consumption, and unit cost (price of output).

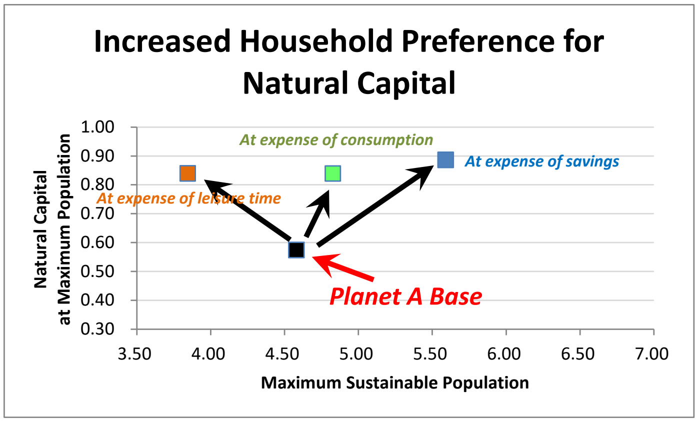

Figure 10: How Changing Household Preferences affect Natural Capital and Maximum Population

If Planet A households place higher value on natural capital than assumed in the reference case, the resulting combination of natural capital level and maximum sustainable population changes, in a way dependent on which particular household value is reduced to accommodate the higher natural capital value. For instance, if household value placed on natural capital increases at the expense of value on consumption, there is a slight increase in the maximum sustainable population (green square). Not surprisingly, the natural capital level increases as well for such a population (green square). The population can be slightly higher because per capita consumption declines. With lower consumption per capita comes less resource draw per capita, meaning a somewhat larger population is sustainable at a given level of resource draw. The result is a slightly higher maximum population alongside a larger magnitude of natural capital.

If the natural capital value increase comes at the expense of value placed on leisure time, the maximum sustainable population goes down (brown square). More labor enters the human economy's workforce, generating more household income and greater per capita consumption. The natural economy can support fewer people at the resulting level of resource draw/waste disposal.

If the natural capital value increase comes at the expense of savings, sustainable population goes up. With fewer savings entering the productive economy, physical capital supply will be lower and its capacity to produce consumption goods and services goes down (blue square). As a result, consumption per capita goes down and the natural economy can sustain a higher population.

This is the picture if Planet A is devoid of technology improvements as time progresses. But the picture changes if the Planet A civilization does improve its technology. In fact, relaxing this condition only brings good news, a discussion we turn to next.

Technology Gains

In comparison to the previously-reported emulation model[25], technology gains behave the same way even when the natural economy is appended to the structure.

When we look more closely at Planet A, we see that technology improvements can come in a few flavors.

First, technology improvement can come in the form of increased labor productivity (e.g., automation). It can also come in the form of increased capital productivity (also automation, engineering improvements). Further, technology gains can come in the form of increased resource use efficiency.

To illustrate, consider increased labor productivity arising from automation. Figure 11 shows what happens when households realize satisficing consumption levels, but automation (in the form of ongoing productive efficiency gains per unit labor) is extant.

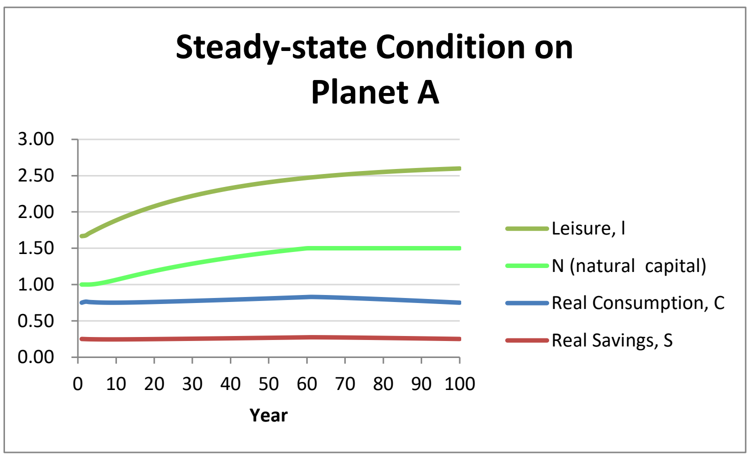

Figure 11: Automation Increases in a Steady-state Consumption Condition

Two things stand out here: First, even while sustaining a fixed level of household consumption, the leisure time of these households increases; and second, the supply of natural capital expands at the same time. Note that this economy still meets the requirement that everything works, with households supplying the exact amount of labor and capital needed to producers, even with their technology gains, and producers supplying the exact quantity of consumption goods and services to households at the same time. What is happening is that technology gains drive down the price (unit cost) of goods and services, and thus increase *real* wages, in a manner that exactly offsets the decline in labor supplied, thereby preserving household budgets while holding consumption steady. Likewise, their household budget continues to support their savings for retirement at its prevailing level, delivering required capital to the productive economy in the process.

---

[25] Saunders, 2014.

Natural capital grows because with more leisure time and less labor supplied to the productive economy (and less needed to provide the same quantity of goods and services owing to the technology improvement), the productive economy likewise needs to extract less resources. This eases the draw and waste inflicted on natural capital and it can grow, even while it extracts less and less "rent" from the human economy, which moderates its growth rate and the resource draw decline.

I was initially astonished to see this result appear in the emulation model, with everything automatically fitting together so perfectly to deliver this result, even with the natural economy considered – not a given that this system of equations would be well-behaved. But there can be no doubt it paints an appealing picture of what life could be like on Planet A if they have found ways to productively displace labor needed while paying workers the same overall wages for less hours worked. And all while growing natural capital supply.[26]

Technology improvements obviously matter to sustainability prospects.

5. Results – Different Sustainable (and Unsustainable) Civilizations

We would like to understand how feasibility of sustainability is affected by natural capital endowment and its eco-economy carrying capacity. For this, we imagine that Planet A has neighboring civilizations differently endowed. Let us say that astronomic/geobiophysical conditions for nearby Planet B have bestowed on it the same overall magnitude of natural capital as Planet A, but its eco-economic carrying capacity is nonetheless lower; for instance, Planet B has larger surface area than Planet A, but is further from its host star than Planet A. It has a larger biosphere, but it is more fragile owing to dimmer insolation. Planet B has the same $N_{capacity}$ as Planet A, but also a higher $N_{critical}$. (Of course this device is also a way of exploring how uncertainty about these magnitudes should be treated on any planet, including Earth.)

In fact, it is useful to introduce another Planet C, which is large but whose biosphere is even more fragile, having a higher level of $N_{critical}$ but the same $N_{capacity}$. What level of sustainable consumption is feasible on each of these planets?

Figure 12 illustrates:

[26] Technical note: The framework accounts for so-called resource "rebound" effects from technology gains, as described by Ayres and Warr (2009), Saunders (1992, 2008, 2014,2015,2017), and others. The results reported here apply even with rebound considered. (See Appendix G).

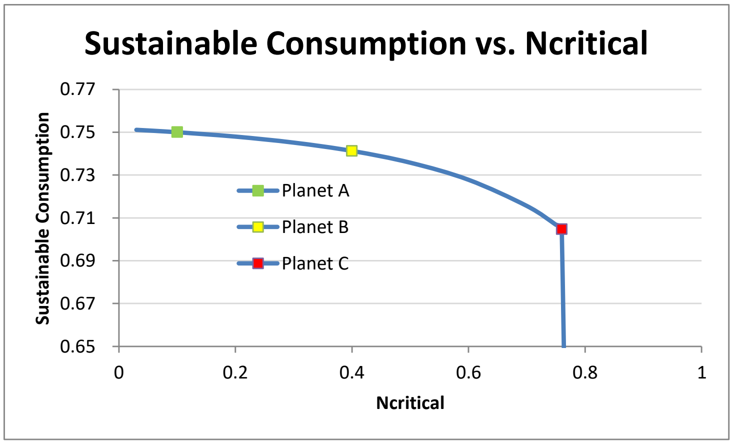

Figure 12: Sustainable Consumption under Different Conditions of Biosphere Fragility

Here we see that Planet B, with a higher threshold of natural capital required to sustain consumption and avoid collapse (higher $N_{critical}$), must have an overall level of consumption lower than Planet A. Planet C, like Planet B, has the same level of natural capital ($N_{capacity}$) as Planet A, but has an even higher level of natural capital needed for the eco-economy to be sustained without collapse ($N_{critical}$). Again, Planet C's level of consumption must be lower than Planet A to keep it in the "safe" zone above the critical level. All of which simply accords with common sense, but now can be validated within a formal neoclassical structure.

Naturally, a larger natural capital endowment means a higher level of feasible sustainable consumption, all else equal. Suppose Planet D resides in a system where its sun is identical to Planet A, it is the same distance from its sun, but it is much larger so its biosphere/geosphere is also larger. Figure 13 illustrates:

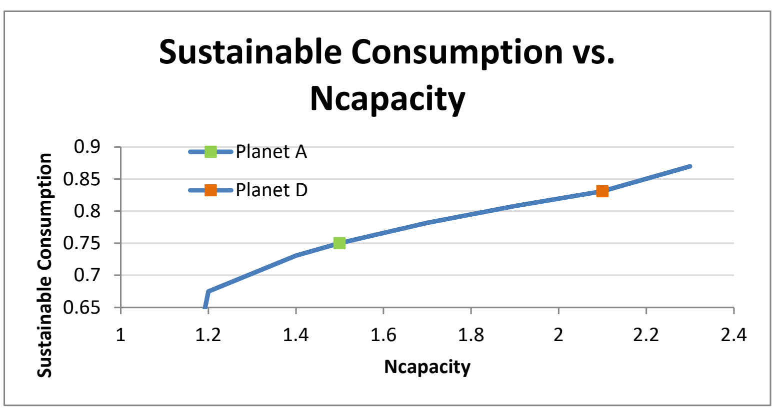

Figure 13: Sustainable Consumption when the Biosphere is Larger

As fits with intuition, a planet with a higher endowment of natural capital (Planet D) can support a human economy with larger overall maximum consumption. Said differently, the natural capital endowment constrains the overall consumption that is sustainable.

To put a finer point on it, Figure 13 compares two planets with identical populations, so these are both the maximum sustainable consumption levels and maximum consumption per person. Accordingly, it is interesting to see how different population levels play into this picture. Consider now a new planet, Planet E, having the same natural capital endowment and carrying capacity as Planet A ($N_{capacity}$ and $N_{critical}$ are the same on both planets). What happens if Planet E population grows beyond that of Planet A before stabilizing at a fixed level? Figure 14 illustrates.

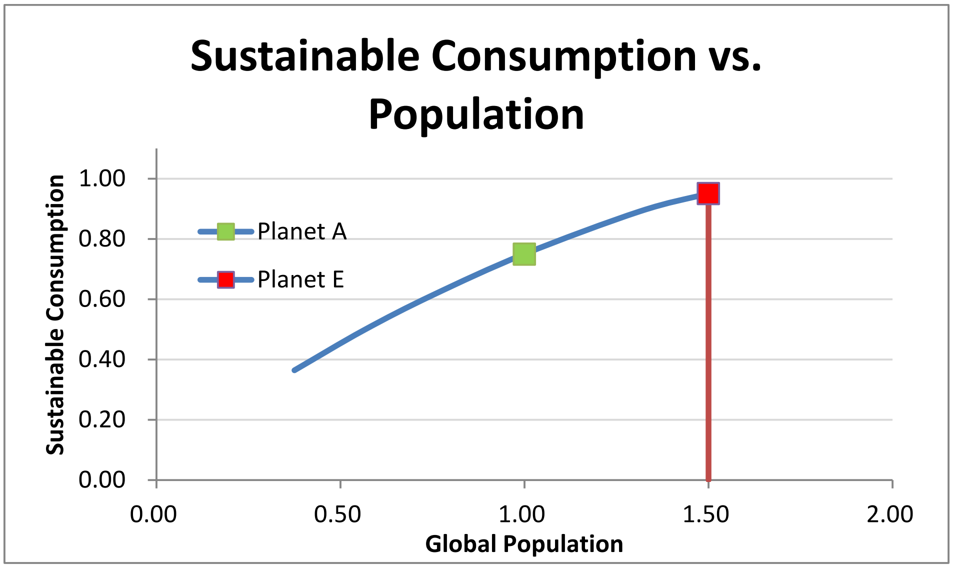

Figure 14: Similar Planets with Different Populations

Planet E can sustain a higher level of population than Planet A, but the civilization of Planet E is right at the precipice of their planet's carrying capacity. Going above this level of population results in catastrophic collapse of the eco-economy. (Note that although overall consumption is higher on Planet E, its per capita consumption is lower.) In the emulation model this collapse happens rapidly (a handful of "years"). Clearly, if the theory is correct, there is a tight link between the sustainable human economy's population and the natural economy, when looked at through the lens of a free-market neoclassical human economy determining the overall consumption level. This accords with common sense. Natural economy carrying capacity and maximum sustainable population are tightly linked because physical consumption level in the human economy (with associated resource draw and waste infliction) is the intermediating determinant of this linkage to the natural economy.

This depiction of the eco-economy dynamics as being on different planets is, of course, fanciful. But this device perhaps adds a degree of clarity to understanding the conditions that could feasibly obtain on our home planet.

6. Discussion – Cautions and Limitations

It is critical to remember that this is an idealized depiction. Readers should bear in mind that it makes certain neoclassical assumptions – assumptions that could be, and sometimes are, contested. A more complete list of these appears in Saunders (Saunders, 2014), but here are a few of the more important of these assumptions in the context of the present eco-economy framework:

1. Competition among producers is robust, and the system therefore mitigates against excess monopolistic rent-taking by capital owners.
2. There is no government – the system operates without centralized macroeconomic control…
    a. … with no taxes and no government spending.
    b. (But the free and fair competition assumption above is arguably consistent with jurisdictional enforcement upon producers by central authorities.)
    c. (Government, taxation, and social enterprises can in principle be incorporated by introducing the social economy as described in Section 2.1 above.)
3. The economy is closed. (Of course, viewed as being the entire global economy, it is closed by definition.)
4. The above two assumptions mean the traditional macroeconomic accounting identity, instead of being $Y = C + I + G + (X - M)$ is instead treated as $Y = C + I$.
5. Aggregation: Each of households and producers is considered as an aggregated whole (alternatively, representative exemplars). Microeconomists have discovered challenging theoretical and technical problems with aggregating both production functions and utility functions.
6. Aggregation: Natural capital is treated as a single, aggregate whole. Of course, it is actually a multi-component system the human economy draws on in a multiplicity of ways.
7. Capital fungibility: A more complete emulation would accommodate the fact that physical capital should be vintaged. That is, as currently formulated, the emulation model considers all capital in place in the economy to be fungible – meaning it is flexible enough to adjust its factor use proportions to match the prevailing market conditions in each period for profit-maximization. An improved emulation model would adopt something resembling a "putty-clay" model (Johansen, 1959, Solow, 1960, Phelps, 1964).
8. Returns to scale: Adherence to general equilibrium principles requires that the production function be non-increasing returns to scale (the CRS assumption used in common practice is fine; decreasing returns to scale is also allowed and consistent[27]).
9. Emulation model: While the framework is general, the emulation model selects particular functional forms for the framework to deliver the reported results and figures.

7. Discussion – Advantages of the Framework and Emulation Model

A few new analytic enhancements are featured here not seen elsewhere:

1. Everything is locked together in the (simplest possible?) neoclassical framework. All the quantity flows are endogenously determined, and are consistent with neoclassical theory, including most unforgivingly, general equilibrium theory:
    a. Households are locked to the productive economy

[27] See Luenberger, 1995, pages 224-225, Condition 7.

b. The productive economy is locked to the natural economy
c. Households are locked directly to the natural economy, as well as indirectly

2. Leisure time and direct benefits from natural capital are included in the household utility function and resulting decision-making.
3. The supply of labor and capital supplied to the productive economy are determined endogenously. This occurs via the explicit link to household preferences and the budget available to households.
4. Household savings is determined endogenously.
5. Endogenization of the budget constraint is automatic by the strict linkages between prices and physical quantities enabled by incorporating duality methods (wages, capital returns, price of output consumed). Households optimize their utility given this budget constraint, delivering savings (capital) and forgone leisure (labor) to the productive economy.
6. Factor prices and dually-linked factor quantities are determined endogenously.
    a. The framework (and emulation model) endogenizes factor prices and the price of output, which assures markets will clear at the calculated prices, and so-called first-order conditions on factor use are correctly represented and accounted for. This may represent an enhancement that prevailing Integrated Assessment Models (IAMs) could take advantage of.
7. The framework (and emulation model) is extremely parsimonious of needed assumptions, both on the human economy side and on the natural economy side.

8. Conclusions

Indefinite sustainability for a planetary eco-economy appears possible when it harbors a neoclassical, free market, private ownership human economy, provided it operates within the "safe zone" accorded it by the natural economy.

The natural economy does, however, impose hard, defining limits. Catastrophic collapse of the eco-economy can result if the civilization's population delivers demand for goods and services consumption that is too large for the planet's natural carrying capacity to support it. If human consumption requires resource extraction beyond the natural economy's capacity to replenish it, and/or generates waste in excess of its capacity to absorb it, natural capital falls below a critical level where its functioning cannot be sustained.

These are arguably not startling conclusions. But they are here offered as being based on deep and formal neoclassical and ecological economics theoretical foundations. And informed by an emulation model that strictly and rigorously adheres to these.

Acknowledgements

All the acknowledgements given in the paper predecessor to this (Saunders, 2014) apply here. Multiple foundational contributions from a host of economics luminaries made this framework even possible. At bottom, the framework presented here (and emulation model) is little more than a routine assembly job of their ideas and methods. Specific to this article, however, I am deeply grateful to Herman Daly for two *key suggestions that allowed integration of the human economy with the natural economy: the idea of including natural capital explicitly in the household utility function; and the idea of incorporating the production function logic introduced by Nicolas Georgescu-Roegen that properly accounts for quantitative raw resource draw and waste inflicted on the natural economy by the human economy. Both these elements are foundational to what is presented here. This research was self-funded and the author declares no conflict of interest. All errors are my own.*

Supplementary Materials: Appendices

Appendix A. Theoretical Framework in Detail

Appendix B. Why it All Works: General Equilibrium Foundations of the Framework (and Emulation Model)

B.1. Why it all Works

B.2. Why does intergenerational equity fall out automatically?

B.3. Why does income inequality between labor and capital disappear automatically?

Appendix C. How can Capital Formation Occur in a Zero-growth Economy where Net Capital Returns are Zero? [And a CORRIGENDUM]

C.1. Corrigendum

C.2 Physical economy mechanics and zero-growth dynamics

Appendix D: Financial Markets Mechanics and Zero-growth Dynamics in a Physical Economy

Appendix E: Functional Forms and Derivations for the Emulation Model

Appendix F: Relation to Georgescu-Roegen Functional Forms, Structure and Functioning

Appendix G: A Note on Resource Rebound Dynamics

Supplementary Materials: Emulation Models

Eco-economy Sustainable Consumption Model 2-11-18.xlsm (Excel®: open-source collaboration protocol; includes a User Guide). This emulation model can be used to audit results reported here, or to explore the dynamics more extensively.

Household Capital Accumulation model 3-14-18.xlxs (Excel®: open-source collaboration protocol). This emulation model can be used to audit results reported here, or to explore the dynamics more extensively.

References

1. Arrow, K. J. and Debreu, G. 1954. Existence of an equilibrium for a competitive economy. *Econometrica* 22 (3): 265–290.
2. Ayres, R., 1997. Comments on Georgescu-Roegen. Ecol. Econ. 22, 285–287.
3. Ayres, R.,Warr, B., 2005. Accounting for growth: the role of physical work. Struct. Chang. Econ. Dyn. 16, 181–209.
4. Ayres, R., Warr, B. 2009. The 'rebound effect' as a driver. In: Herring, H., Sorrell, S. (Eds.), Energy Efficiency and Sustainable Consumption. Palgrave Macmillan, London.
5. Chan, N. and Gillingham, K. 2015. The microeconomic theory of the rebound effect and its welfare implications. Journal of the Association of Environmental and Resource Economists 2(1): 133-159.
6. Costanza, R. 1980. Embodied energy and economic valuation. Science 210:1219-1224. (Also reprinted as pp. 432-444 in: I. A. Sohn, (Ed.), 1986. Readings in Input-Output Analysis, Oxford University Press, and as pp. 103-115 in: Costanza, R., 1997. Frontiers in Ecological Economics: Transdisciplinary Essays of Robert Costanza, Edward Elgar, Cheltenham, 491 pp.).
7. Costanza, R. and R. A. Herendeen. 1984. Embodied energy and economic value in the United States economy: 1963, 1967, and 1972. Resources and Energy 6:129-164. (Also reprinted as pp. 116-150 in: Costanza, R., 1997. Frontiers in Ecological Economics: Transdisciplinary Essays of Robert Costanza, Edward Elgar, Cheltenham, 491 pp.).
8. Costanza, R. 2008. Stewardship for a "full" world. Current History 107:30-35
9. Daly, H. E., ed. 1973. Toward a Steady-state Economy. San Francisco: W.H. Freeman. ISBN 978-0716707936
10. Daly, H.E. 1978. On thinking about energy in the future. Natural Resources Forum. Wiley. 3 (1): 19–26. doi: 10.1111/j.1477-8947.1978.tb00389.x..
11. Daly, H. E., ed. 1980. Economics, Ecology, Ethics: Essays Toward a Steady-State Economy. San Francisco: W.H. Freeman. ISBN 978-0716711797. Revised edition of 1973 anthology.
12. Daly, H. E. 1996. Beyond Growth: The Economics of Sustainable Development. Boston: Beacon Press. ISBN 9780807047095.
13. Daly, H. E. 1997. "Georgescu-Roegen versus Solow/Stiglitz". Ecological Economics. ScienceDirect. 22 (3): 261–266. 10.1016/S0921-8009(97)00080-3.
14. Daly, H. E. 2000. Ecological Economics and the Ecology of Economics: Essays in Criticism. Cheltenham, UK; Northampton, MA: Edward Elgar. ISBN 9781840641097
15. Daly, H.E. 2005. Economics in a full world, Scientific American 293(3)
16. Daly, H.E. 2007. Ecological Economics and Sustainable Development, Selected Essays of Herman Daly. Advances in Ecological Economics series. Edward Elgar, DOI: http://dx.doi.org/10.4337/9781847206947
17. Daly, H.E. 2008. A steady-state economy, Report to the UK Sustainable Development Commission.
18. Diewert, W.E. 1974. Applications of Duality Theory. in *Frontiers of Quantitative Economics*, Vol II; Edited by M.D. Intriligator and D.A. Kendrick, North-Holland.
19. Fisher, I. 1911. The purchasing power of money. The Macmillan Co, New York

20. Georgescu-Roegen, N. 1971. The entropy law and the economic process. Harvard University Press, Cambridge, MA.
21. Hayek, F.A. 1945. The use of knowledge in society. The American Economic Review 35(4): 519:530.
22. Hayek, F.A. 1960. The constitution of liberty. University of Chicago Press. ISBN 0226320847.
23. Intergovernmental Panel on Climate Change. 2007. Fourth Assessment Report, Climate Change 2007: Working Group III: Mitigation of Climate Change, Technical Summary. Available at: http://www.ipcc.ch/publications_and_data/ar4/wg3/en/contents.html
24. Johansen, L. 1959. Substitution vs. fixed production coefficients in the theory of economic growth: a synthesis. Econometrica 27: 157-176.s
25. Leontief, W. 1951a. The structure of the American economy, 1919-1939, (2nd ed). Oxford University Press, New York. (1st ed. 1941).
26. Leontief, W. 1951b. Input-output economics. Scientific American. Reprinted in Input-Output Economics (1966).
27. Lewis, W. A. 1954. Economic Development with Unlimited Supplies of Labor. Manchester School of Economic and Social Studies 22:139–91. Lewis, W. A. 1955. The Theory of Economic Growth. Homewood (IL): Richard D. Irwin.
28. Luenberger, D. 1995. Microeconomic Theory. McGraw-Hill, Inc.
29. Marris, E. 2017. Can we love nature and let it go? Breakthrough Journal 7: 27-38.
30. Minami, R., Ma, X. 2010. The Lewis turning point of Chinese economy: Comparison with Japanese experience. China Economic Journal. 3 (2): 163–179. doi:10.1080/17538963.2010.511912.
31. Modigliani, F. and Brumberg, R. 1954. Utility analysis and the consumption function: an interpretation of cross-section data. Post Keynesian Economics, ed. Kenneth K. Kurihara. Copyright © 1954, by the Trustees of Rutgers College in New Jersey.
32. Nordhaus, T. 2017. The energy rebound battle. Issues in Science and Technology: 51-58
33. Nordhaus, W. D. 1977a. Strategies for the control of carbon dioxide. Cowles Foundation discussion paper no. 443.
34. Nordhaus, W.D. 1977b. Economic growth and climate: the carbon dioxide problem. The American Economic Review 67(1):341-346.
35. Nordhaus, W.D. 1991. To slow or not to slow: the economics of the greenhouse effect. The Economic Journal 101(407):920-937.
36. Nordhaus, W,D. 1992a. The "DICE" model: background and structure of a Dynamic Integrated Climate-Economy model of the economics of global warming. Cowles Foundation discussion paper no. 1009.
37. Nordhaus, W.D. 1992b. Optimal greenhouse-gas reductions and tax policy in the "DICE" model. The American Economic Review 83(2):313-317. Nordhaus WD. 1994a. Managing the Global Commons: The Economics of Climate Change. Cambridge, MA: The MIT Press.
38. Nordhaus, W. D. 2008. A question of balance: weighing the options on global warming policies, Yale University Press, New Haven, CT.
39. Nordhaus, W. D. 2010. Economic aspects of global warming in a post-Copenhagen environment. Proceedings of the U.S. National Academy of Sciences 107 (26): 11721-11726.

40. Nordhaus, W. D. 2012. Integrated Economic and Climate Modeling. Handbook of Computable General Equilibrium Modeling, Peter Dixon and Dale Jorgenson, eds., Elsevier.
41. Phelps, E.S. 1961. The Golden Rule of Accumulation: A fable for growthmen. *The American Economic Review* 51(4): 638-643.
42. Phelps, E.S. 1963. Substitution, fixed proportions, growth and distribution." *International Economic Review* 4: 265–88.
43. Phelps, E.S. 1965. Second essay on Golden Rule of Accumulation. *The American Economic Review* 55(4): 793-814.
44. Piketty, T. 2013. Capital in the 21st century. Paris School of Economics. http://piketty.pse.ens.fr/en/capital21c
45. Raworth, K. 2017. Doughnut Economics: seven ways to think like a 21st-century economist. Penguin Random House, London, UK.
46. Ricardo, D. 1817. On the Principles of Political Economy and Taxation. Piero Sraffa (Ed.) Works and Correspondence of David Ricardo, Volume I, Cambridge University Press, 1951, p. 47.
47. Robbins, P. 2016. After the Baby Bust: The politics and ecology of zero population growth. Breakthrough Journal (6):
48. Saunders, H.D., 1992. The Khazzoom–Brookes postulate and neoclassical growth. Energy J. 13 (4), 131–148.
49. Saunders, H.D., 2005. A Calculator for Energy Consumption Changes Arising from New Technologies. The B.E. Journal of Economic Analysis & Policy/ Topics in Economic Analysis & Policy, 5(1), pp. -. Retrieved 29 Jan. 2018, from doi:10.1515/1538-0653.1467 https://www.degruyter.com/view/j/bejeap.2005.5.issue-1/bejeap.2005.5.1.1467/bejeap.2005.5.1.1467.xml?format=INT
50. Saunders, H.D., 2008. Fuel conserving (and using) production functions. Energy Econ. 30, 2184–2235.
51. Saunders, H.D. 2014. Toward a neoclassical theory of sustainable consumption: eight Golden Age propositions. Ecological Economics 105: 220-232..
52. Saunders, H.D. 2015. Recent evidence for large rebound: Elucidating the drivers and their implications for climate change models. The Energy Journal 36(1): 23-48.
53. Saunders, H.D. 2017: Response to Cullenward and Koomey critique of 'historical evidence for energy efficiency rebound in 30 US sectors. Technological Forecasting and Social Change 119: 184–193.
54. Schumpeter, J. 1947a. The creative response in economic history. The Journal of Economic History 7(2): 149-159. doi:10.1017/S0022050700054279.
55. Schumpeter, J. 1947b. Theoretical problems: theoretical problems of economic growth. The Journal of Economic History 7(Supplement): 1-9. http://www.jstor.org/stable/2113264.
56. Shephard, R.W. 1953. *Cost and Production Functions* Princeton: Princeton University Press.
57. Shephard, R.W. 1970. *Theory of Cost and Production Functions*. Princeton: Princeton University Press (March 1996): 71–89.
58. Sheshinski, E. 1967. Balanced growth and stability in the Johansen vintage model. Review of Economic Studies 34: 239–428.

59. Simon, H. A. 1956. Rational Choice and the Structure of the Environment. Psychological Review. 63 (2): 129–138. doi:10.1037/h0042769.
60. Solow, R. M. 1956. A contribution to the theory of economic Growth. *The Quarterly Journal of Economics* 52: 65-94.
61. Solow, R. M. 1960. Investment and technical progress. In *Mathematical Methods in the Social Sciences, 1959*. [ed. K. Arrow, S. Karlin, and P. Suppes]. Stanford, California: Stanford Univ. Press.
62. Solow, R. M. 1988. Growth theory: An exposition. (includes 1969 Radcliffe Lectures and 1987 Nobel Prize Lecture.) New York. Oxford University Press.
63. Sorrell, S. (Ed.), 2007. The Rebound Effect: An Assessment of the Evidence for Economy-Wide Energy Savings from Improved Energy Efficiency. UK Energy Research Centre (Available at: http://www.ukerc.ac.uk/Downloads/PDF/07/0710ReboundEffect/0710ReboundEffectReport.pdf).
64. Sue Wing, I. 2009. Computable general equilibrium models for the analysis of energy and climate policies. Chapter 14, 322:366 in International Handbook on the Economics of Energy (Eds: Evans, J., Hunt, L.). Edward Elgar, Cheltenham, UK, Northhampton, MA, USA.
65. Zhang, X., Yang, J., Wang, S. 2011. China has reached the Lewis turning point. China Economic Review. 22 (4): 542–554. doi:10.1016/j.chieco.2011.07.002.

A. Appendices for "*Further toward a neoclassical theory of sustainable consumption: The Eco-Economy,*" Harry D. Saunders

Appendix A: Theoretical Framework in Detail

The formal theoretical development of the neoclassical eco-economy is best described with reference to the components and interactions showing in Figure A1.

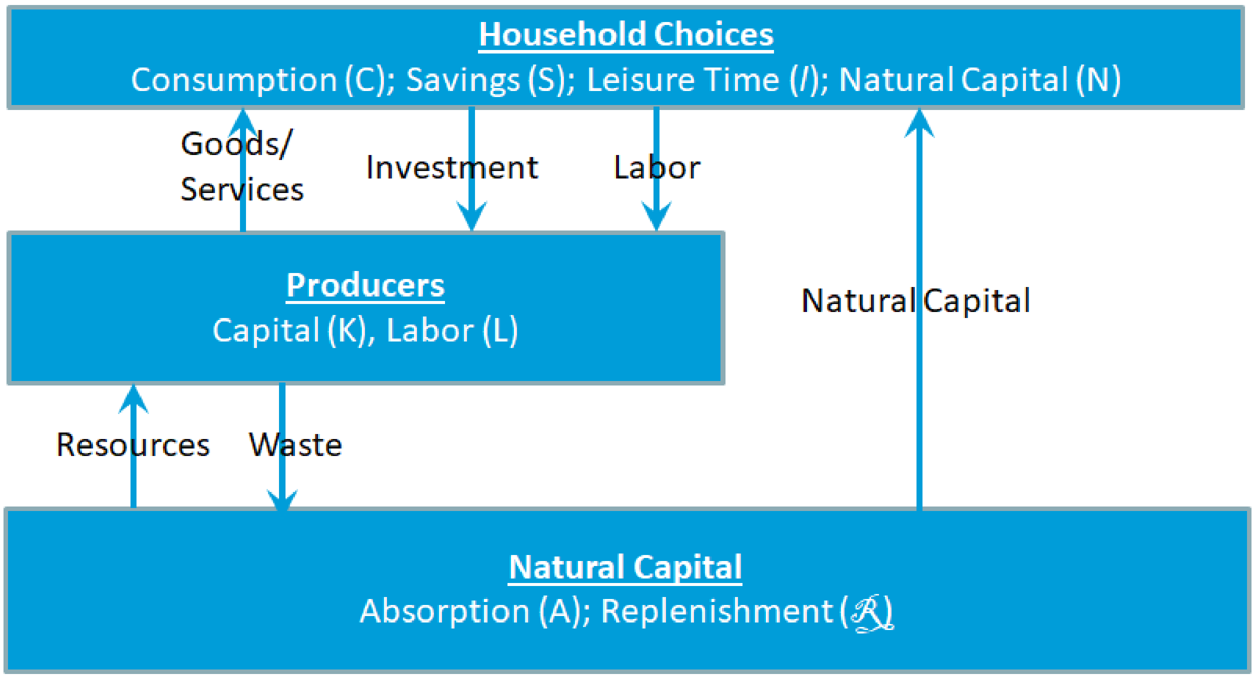

Figure A1. The Eco-Economy

Each of these elements and interactions is described in the following sections.

Households

The Household sector is described by the following utility function, reflecting household preferences (expressed in physical quantity terms):

$$u = u(C, S, l, N) \tag{B.1}$$

Households make tradeoffs among consumption of goods and services available from producers (C), savings to be set aside for future use (S), leisure time (l), and their desire for some level of natural capital from which they draw corresponding direct benefits. The overall levels of these are limited by a household budget constraint (defined below).

By these choices, households determine the levels of production factors (capital and labor) available to producers to produce these goods and services. Labor supplied to producers (L) is the complement of leisure time they choose relative to the total hours available to the household ($\bar{L}$) so that

$$L = \bar{L} - l \tag{B.2}$$

Capital supplied to producers comes from household savings (S), which provides investment (I) to replenish capital stock that has deteriorated (depreciated) out of the system, and/or to add new capital, according to the neoclassical identity

$$I = S \tag{B.3}$$

The picture is this: by not consuming all the output created by the productive economy in any one period, households allow some of that production to be directed instead to the creation of capital goods – goods that replace, refresh, or grow the capital stock in place in that period.

Note that all these components of utility are treated as *physical quantities*. Prices are introduced later.

Producers

Producers in turn take that new capital together with the capital remaining in place (to create total capital in place in the current period, K), and combining this with the labor households supply (L), create the goods and services provided back to households.

This process is captured by means of the following production function:

$$Y = f\left(K, L\right) \tag{B.4}$$

The productive sector takes capital and labor inputs to produce final output (Y) of goods and services, some of which goes to the production of capital goods, with the rest going to the production of consumption goods and services. Again, all these are *physical* quantities.

National accounting practices track this functioning in currency-based terms, not physical terms. The present framework accounts for this by introducing prices based on duality considerations. In particular, the price/cost of output c falls out of the analysis (described and derived below). When combined with the quantity Y , the product cY is equivalent to the "nominal GDP," while Y is equivalent to the "real" GDP.[28]

[28] Cautionary note: It is tempting to think of c as the standard "GDP deflator," but this is not exactly right. In the present framework, c can be altered by shifts in the capital/labor inputs, and importantly, by technology gains in the system that reduce the actual (unit) cost of producing output, making consumption more affordable for households. In contrast, in the world contemplated by national accounting, overall price levels of inputs and outputs can be affected by monetary policy: As a gedanken experiment, consider what happens if money supply were suddenly doubled – all prices of inputs and output will double (and the GDP deflator would double), but

The structure thus embodies the macroeconomic national accounting equation $Y = C + I$, here expressed in physical terms;[29] when dually-derived prices are invoked, the standard currency-based equation is also honored.

Note also that Y is different from the *gross output* of the economy. Of course, the productive economy, in addition to final goods and services, also produces intermediate goods and services, supplied to other producers (and use capital, labor, and natural resources to do it – all of which inputs are embodied in the equation for Y (B.4)); the intermediate goods and services output are recognized in national accounts (in "gross output") but excluded from the GDP accounting measure as not being "value-adding" in the GDP sense.

A good way to visualize how all this works, including intermediates production, is to look at an Input-Output (I-O) formulation such as given in the amazingly clear description provided by Sue Wing (2009, especially Figure 14.2) . There it is seen that GDP is both value-added production (excluding intermediates production), and household earnings from their labor and their capital invested. This is what is captured here as the value cY expressed in currency terms. So Y is best thought of as GDP, though expressed in physical terms instead of monetary terms.

But to accommodate the fact that producers draw on raw resources (R) extracted from natural capital to deliver such production, a more specific functional form is needed:

$$Y = f\left[X\left(K_X, L_X\right), R\left(K_R, L_R\right)\right] \tag{B.5}$$

where K_R and L_R are those portions of capital and labor, respectively, devoted to the extraction of raw resources (think raw foodstuffs, minerals, timber, primary energy resources, water, etc.), while K_X and L_X are those supplied to the production of intermediate and final goods and services, using these raw resources. The individual sectors X and R can be thought of in this way: R is the sector supplying raw resources to the productive economy (those drawn from natural capital); X is the remainder of the productive economy that draws on these resources to produce intermediate and final goods and services (designated "X" to indicate it is actually a crisscross physical flow of products as depicted in I-O models); this combined process generates real final output ("real GDP") Y, which reflects the totality of economic activity required to produce these goods and services, whether they be used for resource extraction, creating new physical capital, or used to provide final goods and services to households.

This does not yet comprehend the role of technology improvements in the production process. Technology gains can enter the production process in several ways. First, improvements can occur in the effectiveness of capital deployed to the resources sector and also the effectiveness of labor thereto

producer profit-maximizing decisions about physical quantities employed/produced (and technology choice) will be unchanged.

[29] Note that the standard national accounting identity is actually $Y = C + I + G + \left(X - M\right)$. But here, exports X and imports M are excluded as a planet-wide economy is by definition closed. Also, in the present framework, government expenditures G (and taxation) are ignored.

deployed, requiring less capital and labor to deliver raw resources to the productive economy. Parameters we can designate τ_{R_K} and τ_{R_L} must be introduced to reflect these gains to resource recovery. Likewise, capital and labor deployed to the intermediates sector can become more effective; we designate the corresponding efficiency gains τ_{X_K} and τ_{X_L}, respectively. On top of this, the intermediates sector can become more efficient in employing these raw resources (using fewer resources to produce the same goods and services), a gain contemplated in the parameter τ_{XR}. Finally, this latter parameter must be understood to account for the important distinction, for energy resources, between raw energy and exergy (Ayers, 1997; Ayers and Warr, 2005,2009) – where exergy is the useful work that can be extracted from a unit of raw energy resources. Exergy is what enables production, not raw energy, requiring a technology parameter we may call $\tau_E < 1$. If E is the raw energy resource extracted and provided to the economy, the productive energy (exergy) actually supplied to producers and production will be $\tau_E E$. Accordingly, to generalize to raw resource use as a whole, the parameter $\tau_{XR} = \tau_{XR}(\tau_E)$, and the generalized production function becomes

$$Y = f\left[X(K_X, L_X), R(K_R, L_R), \tau_{X_K}, \tau_{X_L}, \tau_{R_K}, \tau_{R_L}, \tau_{XR}(\tau_E)\right] \tag{B.6}$$

Prices and the Household Budget

Thus far, the development has been nearly silent on the role of prices. All variables in (B.1) through (B.6) are specified in physical quantity terms. To close this system, prices are needed to assure that solutions to (B.6) honor profit-maximizing conditions, and to specify/determine the household budget constraint, which itself must be honored. This requires knowing the price of output, derivable as the unit cost function c dual to the production function (B.6).

$$c = D\ (Y) \tag{B.7}$$

Where D is the dual operator on Y [30]. If r is the (nominal) cost of capital and w is the (nominal) cost of labor, the profit-maximizing (first-order) conditions for producers are[31]

$$\begin{gathered}\frac{\partial Y}{\partial K_X} = \frac{\partial Y}{\partial K_R} = \frac{r}{c} \\ \frac{\partial Y}{\partial L_X} = \frac{\partial Y}{\partial L_R} = \frac{w}{c}\end{gathered} \tag{B.8}$$

[30] Such a dual exists in principle for every production function, though some are hard to derive analytically.

[31] Note that "profit" in this traditional economics sense is different from what firms typically call "profit." First, "profit" for firms is an accrual measure (not an arguably more accurate cash flow measure upon which they actually make investment decisions); second, "profit" for firms reflects *returns* to capital whereas economists treat profit as revenues less payments to *all* factors of production, *including* payments to capital.

This illustrates the need to have the cost function variable c available if profit-maximizing conditions are to be honored. So-called first-order conditions will not be honored if (potentially changing) c is not used to "deflate" nominal prices. The profit-maximizing employment of inputs K and L will be mis-specified. As K and L are in physical units, their physical quantities will be mis-specified, which are needed to link to the natural economy. Another way to look at this is that factor prices must be expressed in "real" terms, to deliver the *real* physical quantities needed, and which must properly result from the optimization.

While it is technically possible to distinguish conditions (B.8) as different between the two sectors X and R, a useful simplification is to assume labor and capital can move freely between the two sectors. That is, the assumption is that financial markets act to ensure equalization of capital return rates between them; and labor is mobile between the sectors and responsive to wage rate differentials.

Whether or not this simplification is applied, invoking conditions similar to the form (B.8) assures that prices and physical quantities are always locked together when prices and/or quantities change, and are correctly specified in relation to each other for profit maximization and utility maximization.

The dual cost form of (B.6) is needed also for determining the overall household budget, which regulates the overall level of consumption and savings possible, and thus in turn the overall level of economic activity.

Households supply producers with capital and labor. The returns received from these determine the household income budget (returns on savings plus wage earnings). The first of these, capital, is provided via household savings, but this determines only part of the capital supply available to producers – current investment. Capital already in place is available to producers as well (K_{t-1}). But this capital can be thought of as the capital already owned by households owing to past savings. Total capital in place K_t is household total capital assets owned in the current period (and generating current returns).

A further consideration is that some of the capital in place physically deteriorates (depreciates) out of the system in each time period. To account for this, the framework adopts the neoclassical growth capital accumulation model:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{B.9}$$

where K_t is the capital in place in period t, δ is the physical depreciation rate of capital, and $I_t = S_t$ is investment (equals household savings – equals forgone consumption) in period t.

In the present context, capital is split between two sectors, so $K_t = K_X^t + K_R^t$ and there are (in principle) two depreciation rates, δ_X and δ_R. But investment is automatically dispersed across the two sectors to equilibrate returns (method described in Saunders, 2014, Appendix B).

So the household returns on their capital holdings in time period t is $rK_t = r\left(K_X^t + K_R^t\right)$[32]. For household labor, returns are from wages earned in that period, and so the income households realize is $wL_t = wL_X^t + wL_R^t$. This sets the household budget:

$$B_t = rK_t + wL_t \tag{B.10}$$

From this, together with their utility function, households determine their disposition of income between consumption and savings (and in doing so the amount of labor and savings they surrender to producers for current use).

The values of the prices r and w are determined within the framework, with no need to assume arbitrary values. So likewise are the quantities K_X, K_R, L_X, L_R and Y. The power of duality theory is that it allows us to determine both sides of the economic picture at once (physical quantities and prices together).

The Human Economy

This general framework is agnostic as to the functional forms of u, f and c, and is agnostic as to the values of the τ parameters over time. The functional form of the capital accumulation dynamic (B.9) is specified but the depreciation parameters are not pre-specified, and the functional specification is the standard neoclassical one.

With this, the system is highly general and is (nearly) fully closed, with capital supply and labor supply determined without need for arbitrary factor supply functions. Calculated prices are those that clear all markets at the calculated quantities – a requirement of general equilibrium theory. Households own the means of production and choose how much to consume, save, and work, reflective of the private ownership "Arrow-Debreu" economy underlying general equilibrium theory.[33] Capital formation dynamics are likewise internal to the framework, and follow standard neoclassical growth theory.[34]

The Natural Economy

But the system is not yet fully closed. This system resides within a larger system, and on which it draws – natural capital, to coin the terminology of ecological economists. Crucial to the specification of a full eco-economy is the inclusion of the foundational source that fuels and sustains its activity. The above system is still undetermined mathematically inasmuch as the N component of the household utility function (B.1) is so far unaccounted for; but according to this formulation, households place direct value on the planet's supply of natural capital (e.g, clean water and air, biodiversity, favorable climate conditions, majestic grandeur, etc. See Marris, 2017).

---

[32] Note that, since K_t already accounts for capital depreciation each period, households receive capital returns r on that (remaining) invested capital.

[33] Arrow and Debreu, 1954.

[34] Solow, 1956. (Though here the "savings rate" is endogenized.)

Also so far unaccounted for is the draw on natural capital to enable productive economic activity in the human economy as contemplated in the raw resource term R of (B.6).

To correct this, and to fully close the system, requires specification of the functioning of the natural economy. Here, the natural economy is assumed to follow a neoclassical principle that it seeks to maximize, not profit, but natural capital itself. This allows us to characterize natural capital by means of a neoclassical-style production function of the following form:

$$N = N\left(¡\;, R, A, W\right) \tag{B.11}$$

where N is the prevailing stock of natural capital; ¡ is the replenishment provided by the natural capital stock (including agricultural land, water, energy, timber, etc.) ; R is the current natural resource draw by the productive economy (including food, water, energy, minerals, timber, etc.); A is the waste absorbed by prevailing natural capital stock in a given period; and W is the waste delivered to natural capital by the productive economy (and by households directly). In any one period, waste absorption by natural capital cannot be greater than the waste delivered, so $A \leq W$. Nor can absorption be greater than natural capital's capacity to absorb, so $A \leq A_{capacity}$.

Each of these factors of production in turn may be complex functions of natural capital stock, of human economic activity, and of time. For instance, absorptive capacity may diminish over time owing to waste accumulation that has not been absorbed in previous periods; waste generated will be related to the overall production and consumption of goods and services in the human economy, Y , and the resource draw required to produce these; natural replenishment is no doubt a function of the magnitude of extant natural capital; and some resources may be exhaustible, meaning they are only available for a limited time.

Acknowledging all this, we can still formulate a natural capital production function whose validity depends on the fact that it is in fact an identity. With the appropriate time dynamic introduced this is:

$$N_t = N_{t-1} + \left(¡_{\;t} - R_t\right) + \left(A_t - W_t\right) \tag{B.12}$$

In each time period, natural capital is augmented by natural replenishment, but reduced by the human economy's draw on its raw resources. It is further reduced by the difference between waste products generated by the human economy and natural capital's ability to absorb these without degradation. Note that natural capital at time t has already been diminished by any surplus of waste over absorption that occurred in previous periods, thus accounting for its accumulation, if any, over time.

But this is only the quantity side of the natural economy. As with the production function specifying the human economy (B.6), the natural capital production function has a dual cost function, incorporating prices for each of the elements of (5) and delivering a unit cost function c_N . But as shown below, this cost function is simple and the prices are likewise simple, owing to (5) being a linear function.

Formally, natural capital's "profit maximizing" strategy takes the following form (time indices suppressed):

$$\max_{¡,R,A,W} \pi = c_N N - \left(p_{¡}\, ¡ + p_R R + p_A A + p_W W \right) \qquad \text{(B.13)}$$

Solution of this problem shows that c_N is a fixed scalar constant across the entire domain of $\left(¡, R, A, W \right)$. Also, all prices are likewise fixed across this domain and equal c_N.[35]

This means we can normalize c_N to any value we wish. Choosing $c_N = 1$ means $p_{¡} = p_R = p_A = p_W = 1$ everywhere.

Scarcity and Ricardian Rent

In principle, we could think of applying this derived price of raw resources $p_R = c_N$ to characterize the cost of their use in (B.6). However, doing so would ignore the neoclassical principle of Ricardian rent. Ricardian rent arises when an economic agent owns and controls an element of production that is inherently scarce. As demand for it rises, the agent is able to extract a price for that element that exceeds its inherent value to create physical economic activity (e.g., land), a price premium known to economists as "rent." As the supply of natural capital is fundamentally confined by the fact that it occupies a spherical planet, it clearly has the wherewithal to extract rent from the human economy.

Attendant to this is a notion favored by a number of ecological economists that natural capital may have a lower limit, below which it could experience catastrophic collapse. All of which points to the need to incorporate a "natural capital supply curve" showing how the effective price of natural capital delivered to the human economy changes with the quantity of natural capital supplied. The underlying notion is that natural capital charges us rent for her land, raw resources, and waste absorption services. She replenishes elements of herself seasonally, but becomes stingier the lower is her natural capital stock.

If we denote the lower limit of natural capital that avoids catastrophic collapse as $N_{critical}$, the natural capital supply curve takes the form:

$$\begin{aligned} & p_N = p_N \left(N_{current}, N_{critical}, ¡, R, A, W \right) \\ & \textit{subject to} \\ & N_{current} \geq N_{critical} \end{aligned} \qquad \text{(B.14)}$$

As from above we know that $p_R = p_N$, we can say that

[35] The function N is a hyperplane across all four dimensions and the slopes of this hyperplane are likewise fixed across the entire domain, meaning their marginal productivities are constant. The slopes are all (positive or negative) unity and the dual cost function is Leontief-like and delivers a value of positive unity for c_N (proofs available from the author).

$$
\begin{aligned}
&p_R = p_R\left(N_{current}, N_{critical}, \text{¡}\ \ , R, A, W\right) \\
&subject\ to \\
&N_{current} \geq N_{critical}
\end{aligned}
\qquad \text{(B.15)}
$$

This price can be used to penalize the resources sector of (B.6) to reflect the fact that as N declines, more capital and labor must be devoted to the resources sector (K_R, L_R) to deliver the same amount of raw resources to the productive human economy. Accordingly, we can revise the production function of (B.6) by including p_R as follows:

$$
Y = f\left\{X\left(K_X, L_X\right), R\left[K_R\left(p_R\right), L_R\left(p_R\right)\right], \tau_{X_K}, \tau_{X_L}, \tau_{R_K}, \tau_{R_L}, \tau_{XR}\left(\tau_E\right)\right\} \qquad \text{(B.16)}
$$

As before, the exact functional forms of $K_R\left(p_R\right)$ and $L_R\left(p_R\right)$ are left unspecified in the general framework (but specified for the emulation model in Appendix E). But it is at least clear that higher values of p_R will correspond to lowering the effectiveness of K_R and L_R; and the reverse.

The upshot is that households in this framework experience the effects of limitations on natural capital in two ways: first, in the direct value of natural capital to their household satisfaction by way of their utility function; and second in the loss of productive capacity to deliver goods and services they value by way of the rent charged by natural capital with in the form of lower effectiveness of (and returns from) the capital and labor households supply, with resulting limitations on their household budget available for consumption, savings and leisure time. Said another way, natural capital limitations place limitations on household budgets indirectly.

But with this, the resulting claim is we have a complete, closed, neoclassical framework for the entire eco-economy, independent of arbitrary assumptions related to functional forms, parameter values, or obscurely-hidden suppositions. But it is absent a detailed representation on the natural economy side – unlike what is found in many IAM and climate models (e.g., Nordhaus, 1977a,b, 1992a,b, 2008, 2010, 2012; IPCC,2007; others evaluated in Saunders,2015), which elegantly depict many of the more complex interactions among the components of N showing in the identity (5), and certainly better depict reality than the simple functional forms for these components employed in the emulation model, showing below in equations (E.15) of Appendix E.

Appendix B: Why it all Works: General Equilibrium Foundations of the Framework (and Emulation Model)

B.1. Why it All Works

The good behavior of the emulation model reveals itself by delivering long-term Phelps-style "golden rule" conditions under a broad range of initial conditions and parameter values. The time dynamics more generally also move the system in explainable ways.

The good behavior of the emulation model (and framework generally, it could be said, with some exceptions regarding forbidden functional forms) arises from what turns out to be its grounding in general equilibrium theory; and observed behaviors can be formally described in these terms.

The Fundamental Existence Theorem for general equilibrium lays out what must be assumed for general equilibrium to occur in the system (Arrow and Debreu, 1954; Luenberger, 1995).

For the Fundamental Existence Theorem to hold, real (output price-deflated) prices must be used in determining household and producer choices.

The dual unit cost function of the production function used delivers unit cost, which is also the price of output (price of goods and services delivered to households and used for capital formation). While it may seem counterintuitive that the price paid to producers is equal the cost of production, the apparent "zero profit" this implies is only zero profit in the sense of use how neoclassical economists use the term, not what producers would consider zero profit. Neoclassical economists subtract off returns to capital as part of the profit maximization process. To most business ears, "profit" would be precisely that very component economists subtract that is returned to capital owners – revenue generated less the costs of production inputs required to generate it, or profits returned to capital owners.

Prices are also needed to establish the household budget constraint. Workers and capital owners are compensated in nominal dollars, but those nominal dollars go further when the output price drops. As this could induce a change in the physical consumption level, the utility function (mediated by equations (B.8) and (B.10)) will govern the exact household consumption level choice when the budget constraint is loosened. But this will in turn be governed by household responses to what they see as "real" prices, once specified in output price-deflated terms, this being what matters to them in making their consumption choices.

Another way to look at it is through the eyes of Slutzky (Slutzky, 1915): household consumption decisions are determined not just by relative price effects but income effects as well. These in turn depend on the value of C, the nominal price of goods and services. Consider the case where, say, C is going down due to technology gains. In this case households can buy more physical goods and services with lower outflow of (nominal) earnings when their (nominal) price goes down. This increases the household's effective income.

Similarly, producers must ensure their factor-use decisions are profit maximizing. This means they must match physical quantities used and produced to both factor prices and the price of output. To do this, they need to match physical quantities to real prices, and to undertake this profit maximization they need to know at what price they will be able to sell their output to household consumers, namely C (all of which illustrated in equations (B.8)).

Technical theoretical point: Note that good system behavior ensues in the later-described emulation model even when households are assumed to reach a "satisficing" level of consumption. At first glance, this may seem to violate one of the assumptions of the Arrow-Debreu Fundamental Existence Theorem. In particular, the Arrow-Debreu statement of the theorem has a condition commonly understood as, "There is no satiation bundle in any consumer's consumption set."[36] This seems to rule out what we are proposing here, namely that individual households can have a point beyond which they are satisfied with having no additional consumption in any time period, given their budget.

However, further examination of this existence condition shows that "satiation" as applied to the emulation model implementation means something different from what is meant by the Arrow-Debreu condition. The utility function employed here to represent households is a standard Cobb-Douglas function, which inherently has no saturation point: increases in any element of the function increase utility without limit. However, as Arrow-Debreu point out, this does not mean utility is not constrained in normal circumstances – the household budget puts a cap on the "amount" of utility that can be realized at any one time. In the implementation here, household budget is determined endogenously, and so constrains overall consumption level.

To emulate scenarios where overall household consumption is fixed at some zero-growth level (given fixed population), but when technology gains are present, adjustments to the component utility function value attribute elasticities are needed to keep consumption stable. But what is interesting is that this is accompanied by overall utility going up – the technology gains drive increases in leisure time while preserving (or increasing, in the case of labor technology gains) the supply of natural capital. In the emulation model, overall consumption can be set at any level, and while that level may well be unpredictable, households in future might choose to take their newly-increased utility in the form of more leisure time or natural capital expansion, at the expense of increased consumption of goods and services (a trend already visible in certain Scandinavian countries today).

At bottom then, the framework and emulation model structure are – neither one – inconsistent with the Arrow-Debreu satiation condition. And it is very likely that overall conformity with the Arrow-Debreu theorem explains the emulation model being so well-behaved under a variety of sensitivity tests, thus perhaps even helping to endorse the theorem's already broad reach.

B.2. Why does intergenerational equity fall out automatically?

In a steady-state zero-growth human economy that honors the constraints imposed by the natural economy, future generations are endowed with the productive capacity and natural capital of each previous generation. Forces in the eco-economy push in this direction with a private-ownership, free market economy. The system automatically goes to a Phelps "Golden Rule" condition, moving thereafter along a so-called golden rule pathway that maximizes household utility in each generation.

The equations of neoclassical economics guide it this way. Profit-maximization by producers and utility maximization for households (including natural capital utility) are the driving forces of its evolution. The introduction of technological improvements only serves the golden rule more forcefully: each generation endows the next with the technology they have developed, which has served to increase household utility in the current generation, even without growth of the productive economy (a kind of

[36] Condition III.b. page 269 of Arrow and Debreu, 1954. Luenberger uses the language quoted (purloined) here to describe this non-satiation condition (Luenberger, 1995, page 224, condition #2).

"green growth"). Subsequent generations may improve on that, increasing leisure time and reducing assaults on natural capital along the way.

Each generation, in effect, values the welfare of the next as their own. This could be considered to imply a zero social discount rate, though what Phelps calls the "social return" (real returns to capital less depreciation) can be non-zero along the way to a golden-rule pathway.

So why does the system automatically seek out and find a golden-rule pathway? The truthful answer is I was personally surprised to see the emulation model deliver this result, and more surprised to see it be delivered when natural capital is explicitly incorporated in the framework. The weak analytic answer is that the mathematics dictates that this be so; the true answer may lie far deeper than this, and is left for future eco-economists to unravel.

B.3. Why does income inequality between labor and capital disappear automatically?

Thomas Piketty (Piketty, 2013) has presented evidence that, in today's world, capital owners are extracting returns in excess of what would be optimal for households as a whole, relying on concepts developed by Solow (Solow, 1956, 1988) and Phelps (Phelps, 1961, 1965). In particular, the argument is that for a socially fair distribution of economic benefits to labor and capital, the condition $r = g$ must obtain, while Piketty claims currently $r > g$ by a substantial margin.

It is quite easy to argue that this observed reality arises whenever there is insufficient capital in place to accommodate the willing labor supply available. An interesting thing happens when the emulation model is queried on this topic. That is, the emulation model allows the user to reduce the initial supply of capital and let the system adjust. When this is done, the system delivers the Piketty observation of $r > g$ – for a time. But the system acts to draw in more capital via the promise of increased returns – and substitutes toward more labor demanded given the disadvantaged prevailing wage rate owing to the labor surplus. Capital owners in the near term extract more "Ricardian rent," owing to capital scarcity, and this flows through as excess returns. Labor suffers a wage disadvantage for a time, owing to its being in surplus, but demand for labor grows until balance is eventually re-achieved and capitalist rent-taking returns to zero.

An important feature of this dynamic is that, at any one time, each factor of production (capital, labor) is returned its "marginal productivity" – that is, the contribution to economic output (eco-economy output, more exactly) each additional unit of each input delivers. It thus can be argued to be "fair" in this sense, in returning each factor its contribution to the productive process.

This labor disadvantage prevails only for a time. Eventually, the system draws in sufficient capital from households that labor becomes the increasingly scarce input resource. As these two claims on the fruits of production come into balance (and remember this is a balance ultimately chosen by households in their savings/leisure time tradeoff), the marginal productivities of each factor input come into balance. Full (willing) employment matches household decisions on savings and consumption.

Eventually, the system approaches a zero-growth golden-rule pathway/trajectory and the human economy is characterized by $r = g$; and each of capital and labor receive their due in accordance with what each contributes to household economic welfare (including value placed on natural capital).

In steady state, if one of the production inputs of capital and labor should fall short for a time, free-market forces arise to correct the imbalance. Technically speaking, this makes the optimal solution to this dynamic system a stable solution.

This speaks only to income inequality between capital owners and labor. It does not address the distribution of income across households that supply different quantities or "qualities" of labor. This is a more complex question. But an observation that can be drawn from this analysis is that in a condition where labor is fully employed, each household will both decide how much labor it wishes to supply, and will bid for employment at a wage rate that matches its marginal productivity to the economy and to the household. It is easy to speculate that the system will seek to converge to a condition where labor in each household will see its income apportionment to be fair given the leisure/consumption/savings choices it makes, and the value it can rightfully deliver to/extract from the productive economy. No proof of this claim is offered here, but the mathematics seems to indicate this is a formational element of the optimal, while feasible, condition the eco-economy seeks to deliver in the large.

Appendix C. How can Capital Formation Occur in a Zero-growth Economy where Net Capital Returns are Zero? [And a CORRIGENDUM]

C.1. Corrigendum

In the *Ecological Economics* article preceding this one (Saunders, 2014), I claimed to have addressed Herman Daly's worry (Daly, 2005,2008) that in a zero-growth economy real returns to capital would go to zero, with the incentives for capital formation disappearing.

I not only did not actually address it, but the argument I gave there was wrong. My observation that the marginal productivity of capital would always be positive, while actually true, does not mean net returns to capital will be positive. Specifically, for theoretical consistency, a correction must be made to subtract depreciation (degradation of physical capital) from marginal productivity. When this is done, the zero growth economy shows real net return to capital goes to zero, just as Daly predicted. This obviously creates a serious challenge for explaining how such an economy could go on indefinitely, with household savings going to producers only being sufficient to offset deteriorating/depreciating assets in terms of net value creation – zero returns for producers, and zero resulting returns delivered to households. What happened to the incentives for households to save, and producers to produce, if both returns are always zero?

C.2. Physical economy mechanics and zero-growth dynamics

Herbert Simon (Simon, 1956) provided a partial answer to this conundrum with his concept of "satisficing" behavior. He applied this concept to both consumers (individuals, households) and producers. Economic agents act, he said, to make the tradeoff between the cost of effort applied and the value of gains realized from that effort reach an equilibrium – for instance, household consumption vs. leisure time may favor increased leisure time over potential new wages earned from further labor supplied to producers and savings they accrue for retirement. In Simon's picture, households have "satisficed" their consumption needs and, importantly, their savings needs.

This leads us part way. But to complete the theoretical picture we need to call on the insights of other luminaries. This requires separating the household side from the producer side.

Households

Here we call on the work of Franco Modigliani and associates (Modigliani and Brumberg, 1954). The Modigliani picture of household savings/consumption decisions rests on the idea that households want to save for retirement. This will hold true even in a zero-growth economy.

The consequences of this are best illustrated by appeal to another emulation model appended to this article (Household Capital Accumulation model 3-11-18.xlxs).

Figure C.1 shows what savings dynamics look like for a "representative" household over its "lifespan" in a zero-growth economy (assuming no technology gains).

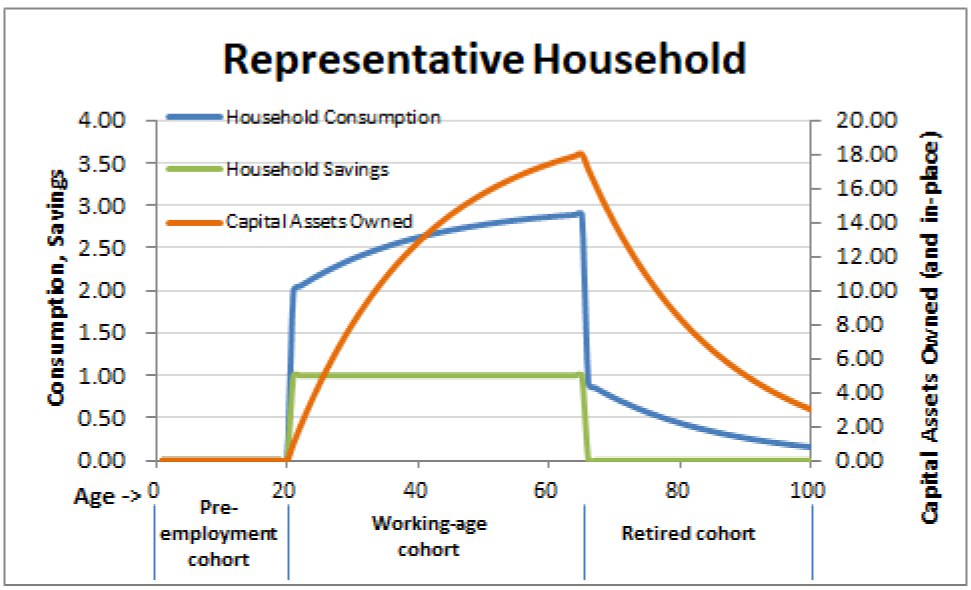

Figure C.1. Capital Accumulation/Savings Dynamics for a Representative Household

Here we see a simplified picture, where the "pre-employment cohort" associated with the household (i.e., children) saves nothing during their first 20 years; also the "retired cohort" saves nothing after age 65. While the "pre-employment cohort" shows as also consuming nothing, the presumption is that the "working-age cohort" supports them during their pre-employment years. In the "retired cohort," households no longer contain any pre-employment individuals, and instead of working rely on drawing down their savings to support their consumption.

Several interesting dynamics become visible here, but first consider what happens when households at different stages of their lifecycle are aggregated together. Figure C.2 illustrates.

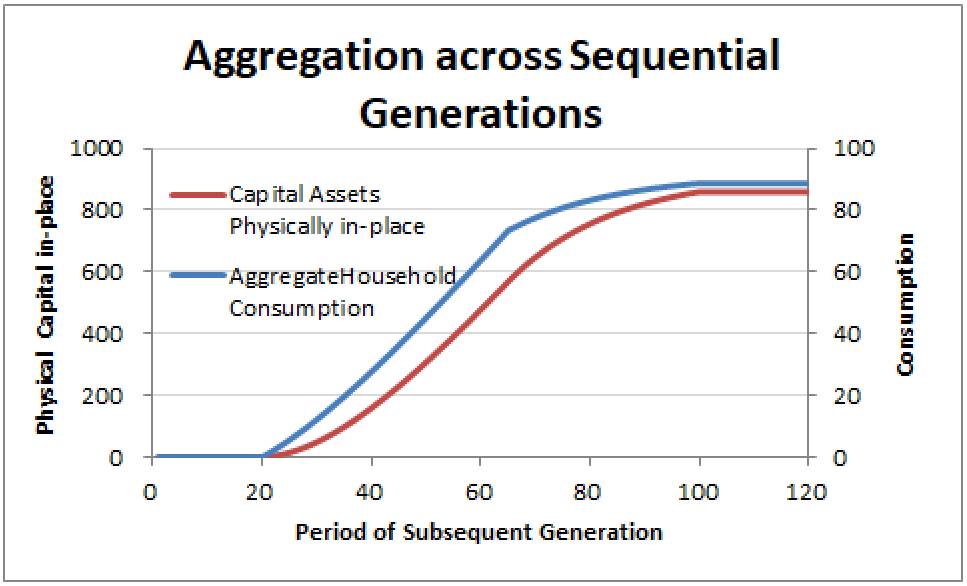

Figure C.2. Capital Accumulation/Consumption Dynamics for Aggregated Households

Here, we assume a "seed" household generation that is followed in each subsequent period by new generations that augment it. Over time, as new generations are added and others die out, we see that the household aggregate of consumption and capital supplied producers from savings approaches a fixed limit and remains steady thereafter. This household savings dynamic creates a zero-growth steady-state economy that sustains itself.

The underlying dynamics involved is best understood by reverting to the "representative" household of Figure C.1. During their working-age careers, households set aside a fixed portion of their budget for savings. In a zero-growth economy, they must reinvest their capital returns ("unearned income") to offset depreciation of the physical assets they have invested in and to maintain these. Failing to do so means the value they lay claim to from these assets diminishes – new and replenishment investment from other sources will dilute the value of their holdings. But doing so generates zero return from these reinvestment investments ($r - d = 0$).

But their savings also come from another source: savings extracted from their "earned income" in excess of the amount needed to offset depreciation (by such savings behavior households forgo some additional consumption of goods and services in principle available from their "earned income" stream). Remember that forgone consumption is what allows producers to produce capital goods instead of consumption goods, so these household savings generate investment capital for the productive economy in this way. As depicted in Figure C.1, household ownership claims to capital increase during the working-age period where savings exceeds reinvestment-level savings. Consumption also goes up.

By age 65, household claims on capital are equal the cumulative savings they have set aside from earned income – they "get back" exactly what they've put in to the system (i.e., again, zero return). In contrast, savings derived from unearned income does not return those savings to households, except inasmuch as they technically pass to the household budget each period as ownership claims destined for reinvestment.

But by this retirement mechanism, households still have incentive to save and invest even a zero-growth economy. And in total, upon retirement, household capital ownership claims exactly match the physical capital they have funded that is still operational.

The emulation model shows the numbers exactly match. The capital value claimed by the household at age 65 is identical to the capital assets physically in place and operating in the productive economy (or more accurately, the individual household's share of such in aggregate household capital ownership). These two numbers are exactly equal the sum total of savings contributed from *earned* income.

Consumption of the working-age cohort rises because over time *unearned* investment income grows as the household accumulates more capital assets, meaning less *earned* income is needed to fund the household's chosen savings level (from their utility function), leaving room for more consumption.

In retirement years, the household ceases saving and lives off unearned income. This income declines, not because marginal returns decline, but because household claims on the capital stock of the productive economy decline as new investment dilutes their ownership claims and holdings. Their household consumption declines as their unearned income to support it declines.

But in aggregate (Figure C.2), household savings maintain the supply of productive capital and household consumption stays steady in a zero-growth economy.

According to the Modigliani model, then, households have incentive to save even a zero-growth economy with zero net return to savings.

Producers

What about producer incentives? If net returns they can deliver using household savings are zero, why invest in capital stock to begin with?

In a stable zero-growth economy, capital and labor supply are in balance, so aggregate corrective forces that arise from imbalance are not in play and don't provide an answer. What profit incentive still exists for producers to maintain production?

One consideration is that households will want to save and invest and, as the ultimate owners of production, could effectively force producers to create capital that replenishes and sustains production and throws off unearned income for retirement. But that sheds little light on the market dynamics.

From the producer side, picture the flow as follows: producers will still want to maximize investment returns. Think of these returns as dividends flowing to households. Producers count on households turning around and reinvesting these dividends to offset depreciation of producer-operated capital assets (at least count on those coming from households in the working-age cohort) and to maintain production levels and profit levels steady. These dividends are available as surplus cash flow from the productive economy (revenues less input costs) because existing capital is throwing off return, even if it's aging. As individual households enter retirement, producers count on new entrants to the system to cover depreciation losses and thereby allow them to continue operation, while sending promised dividends back to those households not contributing capital. One cohort takes up investing to cover the first. The retired cohort pays a price in the form of a declining share of capital ownership as their investment is diluted, but they still receive a return on the declining assets they command and income to support consumption.

Profit-maximizing producers will compete to supply these dividends. If they don't, the market (households) will punish them in the form of reduced market valuation of their assets and operations, and competitors will step in to fill the gap. Even if, on the whole, the productive economy delivers zero net returns when depreciation is accounted for, producers will want to maximize the dividends they can generate and report as investor gains to owners. They can accomplish this by invoking the neoclassical optimization (profit-maximizing) condition, so to speak, which optimizes the output they generate from chosen inputs, and generates surplus cash for dividend payments (or for reinvestment) in the current period.

A deeper look into this picture can be had by considering the work of in Johansen (1959), Solow (1960), and Phelps (1964).[37] These researchers took explicit account of capital vintaging, via a "putty-clay" formulation, which accords with the capital dynamics described here. They envision that capital formation is like a process of adding each period "putty" layers on top of (existing) layers that have hardened into "clay." The new putty layer can be formed to accommodate prevailing conditions of labor and capital supply markets and technology to maximize profits. The older clay (i.e., physical capital) is fixed in its production technology, and slowly deteriorates (depreciates) out of the system.

These researchers show that older vintages still throw off their original return from capital (marginal productivity of capital), but decline in their capacity to produce overall returns (rate times volume). Only by adding new capital can overall production and returns be maintained. This fits exactly with the picture that, as retirees consume returns from their capital invested pre-retirement, they will receive an ever-smaller share of producer returns generated. But they will get back exactly what they put in at the end of the day. It's as if they have forgone funding depreciation from savings and left that to the next generation, while consuming their capital in a way that matches depreciation dynamics (the returns they consume are equal depreciation ($r = d$)).

But production capacity overall remains steady, with producers maximizing profit (returns) along the way.

In summary, physical flows of capital resources between households and producers remain in place in a zero-growth economy, with producers maximizing profits and households maximizing retirement-period utility given them via their savings.

[37] Good expositions are found in Sheshinski (1967) and Gilchrist and Williams (2000).

Bottom-line assertion: Even in a zero-growth eco-economy, capital formation can continue apace to sustain it.

This may provide some comfort to ecological economists, and perhaps help assuage concerns on this foundational question first raised by Herman Daly.

Technology Gains

When technology and innovation come into play, the zero-growth capital formation dynamic is reinforced in an appealing way.

The picture goes back to the insights of Hayek (1945, 1960) and Schumpeter (1947a,b). Here we picture that some producers are able to supply new goods and services to households that are better and cheaper, even while households do not increase overall consumption of these goods and services.

Hayek viewed the system as a complex, evolving one, not one limited to producing only what it has produced before. No product is permanent and economics is about change, not stasis. Competition among producers to better supply the array of goods and services asked for by households, and to grow their share of supplying these, feeds production innovation and new technologies. Even if overall production and consumption of goods and services stays the same, it will be ever-refreshed in its composition. Schumpeter likewise saw the system driven by forces of "creative destruction" that act to generate new goods and services not before seen, eliminating in the process the production of obsolete ones in a complex web of agent interactions. Schumpeter argued that innovators would be able to capture monopolistic "rent" (i.e., higher than average returns to capital) until competition erased it by yet newer innovation.

The emulation model reported here provides supporting evidence that technology improvement mechanisms work in this way even while physical consumption of goods and services remains fixed overall. In this framework, new technology means goods and services are produced with ever-lower cost, effectively requiring less of the household budget to consume – the utility from which can be increased by adjusting the household allocation of utility components. With "satisficing" behavior on the part of households, technology improvements are captured by them in the form of increased leisure time, and/or greater value placed on direct benefits from natural capital, even while producers are enabled to innovate products that are better, cheaper, and moreover require less drain on natural capital assets as well as household labor supply. It reveals itself to be, in other words, still eco-economy sustainability, but enhanced by the Hayek/Schumpeter dynamic and the new technologies that arise therefrom.

Appendix D: Financial markets mechanics and zero-growth dynamics in a physical economy

Financial markets obscure the underlying physical realities. Financial markets are subservient to physical markets, and are ultimately governed by the physical flow dynamics of the real economy. Various dynamics come into play within financial markets regarding banking-based multiplier effects and government "fiat" money (both of which can appear as if from nowhere), that appear to be in violation of the physical capital investment flows described here, but they are not. Financial markets merely supply the "fluid" that directs physical capital supplied by households to its most productive uses, and merely oils the productive machine. But it is a highly valuable lubricant.

For simplicity, first assume that households hold all their savings in the form of *equity securities*. These securities confer ownership to households of the means of production (physical capital in place)[38]. Households also secure ongoing unearned income from capital returns. In such a circumstance, the value of these securities can therefore *not* exceed the productive economy's capacity to produce goods and services from the capital in place.

The physical flow of capital (which, remember, is simply forgone consumption by households directed instead by producers to the creation of capital goods) is intermediated by financial markets as follows:

A new entrant to the working-age cohort decides to set aside some consumption for savings to invest in *equity securities*. In a steady-state economy, these will be purchased from an existing equity owner, (or perhaps from producers issuing new shares to the market if the producer is running short of dividend reinvestments (say from retirees) or sees a profitable investment opportunity). But in any case, the producer in a zero-growth economy cannot provide total returns to owners in excess of the capital's physical capacity to do so. In effect this means either existing equity owners surrender their claims (e.g., retirees surrender their holdings), or if not, the market value of each security declines. Either way, securities markets adjust to keep overall returns to securities owners fixed (even as ownership by non-contributing retirement households diminishes via dilution), and nothing physically is either created or destroyed in the process.

Now suppose households instead decide to keep all their savings in the form of *bank deposits*. Here is where so-called "multiplier effects" obscure the physical dynamics, but do not change them. Fractional-reserve banking enables the system to multiply the supply of money circulating in the economy (banks lend out a large fraction of the deposits they hold, which fund firms who pay suppliers who in turn deposit the proceeds in other banks, who in turn, etc.…, thus apparently "multiplying" household savings deposits available to the economy[39]). This money supply is greater than the redemption claims that could actually be honored should all depositors claim them at once, so the system counts on them not doing so.

But the physical economy ignores all this and the physical capital is what it is and no more, and can produce only what it can produce, irrespective of promised claims residing in financial markets. There can be no multiplier effect on the amount of investment physically put into place as a result. This is restricted by household savings and the capacity of producers to profitably use these funds for new physical capital.

---

[38] Remember that the ownership conveys direct control as well as claims on a firm's profit stream: Equity owners ultimately have absolute control over producers, as they have to power to democratically hire and fire shareholder representatives– Boards of Directors and the management these Boards hire.

[39] The multiplier, m, is formally defined as $m = 1/R$, where R is the fractional reserve requirement, typically imposed by government central banks.

The economy can only produce a limited quantity of goods and services at any one time. For investment to happen, some of these must go to the creation of new or replenishment physical capital – so-called capital goods. These are goods *not* going to households, so households are sacrificing consumption they could otherwise claim from a productive economy that instead used these resources (capital, labor) to create consumption goods and services. This is what is saved by households, in the form of forgone consumption. So both the investment and savings originate in households. And they are equal because the physical economy can only create new capital using goods and services forgone by households. (Thereby honoring a simple identity among total output produced, total consumed and total going to new and replenishment capital via savings.) All this happens notwithstanding anything that is happening in the (more "virtual" than real) financial markets where only promises reside.

Banks are a conduit of household investment capital, lubricating the process, but this cannot exceed the household ability to deliver it in the form of forgone physical consumption (money supply notwithstanding) used instead by producers to produce new physical capital.

But households can invest in other financial instruments. For instance, they could invest in *debt securities*. While debt holdings, like bank accounts, do not exactly confer ownership of production, they do confer a substantial measure of control over producer operations. Corporate bonds issued by firms to raise capital carry with them obligations on producers to perform and deliver returns. This control is not only in the form of bond "covenants" restricting producer operations and financing activities, but producer returns are preferentially directed to bond holders at the expense of equity security holders if necessary – debt holders are "senior" to equity holders in their claims on the profit stream producers generate. Further to that, debt securities most often contain guarantees by producers that "coupons" ("dividend" guarantees) will be issued periodically that debt holders can redeem for cash or reinvest.

Moreover, because debt securities are traded on the open market, debt markets (again, households-driven) are subject to households downgrading/discounting their financial value if producers are not performing to expectations. Bond rating agencies can "downgrade" corporate bonds, influencing their market value and the ability for such producers to issue further debt. Financial markets thus allow households holding debt securities to "punish" under-performing producers. Again, in the end, producers can supply returns to debt holders no more than producers' capital in place physically allows.

Bottom line: financial markets in no way violate the underlying physical dynamics of the human economy, but only make its operation smoother (by comparison, say, to a currency-free barter economy).

Monetary Policy and Government "fiat" Money

Governments can affect the quantity of currency circulating in the economy. Their central banks do this by various mechanisms, including adjusting banking fractional reserve requirements and these central banks' lending rates. Their government fiscal counterparts can do this by issuing government securities, effectively "printing" money. But this does not change the physical realities of economic production in any substantive way.

It is easiest to see this by considering a situation in which the physical productive economy is operating (as in a zero-growth condition) at its capacity to produce goods and services. Suppose the government, for the sake of argument, decides to immediately double the money supply circulating. What would then happen is that the prices of everything would double (roughly speaking, twice as many "dollars" chasing the same amount of goods and services). No more goods can be produced by this economy, but they would cost consumers twice as much (and, because production inputs would also cost twice as much, would cost producers twice as much to produce). But nothing physically would change.

Slightly more formally, this dynamic is described by the identity developed by Irving Fisher (1911): $PY = mv$, where Y is the "real" (i.e., physical) output of the economy, m is the money supply circulating, v is the "velocity" with which the money is circulating, and P is the price of that output (often characterized as the "output deflator"). From this identity we see that if real physical output Y is fixed (zero-growth economy) and the velocity of money remains constant, doubling the money supply m merely serves to double prices. Nothing changes in the physical economy. The physical economy ultimately governs what happens, irrespective of money (currency) supplies or financial market dynamics.

Bottom line for Appendix D: household choices, and physical principles, ultimately rule, not financial markets or government money creation. Money (currency or its equivalent) is merely the lubricant of the physical system that commands the core of sustainability dynamics.

Appendix E: Functional Forms and Derivations for the Emulation Model

The emulation model adopts specific functional forms for use with the general framework presented in Section 2.2 of the main article and Appendix A above.

Households

For the emulation model, households are assumed to exhibit a household utility function of the following form:

$$u(C,S,l) = u_0 C^{\gamma} S^{\nu} l^{\omega} N^{\pi} \tag{F.1}$$

While more sophisticated functional forms are often used, this Cobb-Douglas form has the advantage that the exponents of C, S, l and N denote households' preferred shares of consumption, savings, leisure time, and desire for natural capital respectively, thus informing intuition in the results that follow. [40] The emulations reported in this article assume u is a constant-returns-to-scale (CRS) function, so $\pi = 1 - \gamma - \nu - \omega$.

To reflect the possibility of an eco-economy wherein households limit their consumption of goods and services to some "satisficing" level (Simon, 1956), we introduce a series of parameters to adjust the

[40] As noted in the main text, I am grateful to Herman Daly for suggesting inclusion of natural capital explicitly in the utility function.

exponents of (F.1) over time. Specifically, we introduce parameters r_γ, r_ν, r_π indicating periodic percentage changes in γ, ν, π respectively, to reflect a situation where overall household utility does not change, but its components do. Owing to the CRS assumption, this means ω becomes as follows:

$$\begin{aligned} &\pi = 1-\gamma-\nu-\omega \\ &\Rightarrow \omega = 1-\gamma-\nu-\pi \\ &\Rightarrow \omega = 1-\gamma_0 e^{r_\gamma t} - \nu_0 e^{r_\nu t} - \omega_0 e^{r_\pi t} \end{aligned} \tag{F.2}$$

Producers

The functional form of the producer production function is CES (Solow)[41]. While not the most general function available, this function has several useful properties. First it is flexible, allowing substitution to occur among inputs. Second, it has a derivable, closed form dual unit cost function. Other more flexible functions, such as the Translog, have dual forms but not analytically derivable ones.[42] The CES (Solow) function is "self-dual," meaning the cost form has the same algebraic form and structure as the production function form. This also assists in introducing technology parameters into both forms [see equation (E.13)]. The functional form for production is:

$$Y = \left\{ a\left[\left(\tau_{K_X} K_X\right)^{\alpha}\left(\tau_{L_X} L_X\right)^{1-\alpha}\right]^{\rho} + (1-a)\left[\tau_{XR}\left(\tau_{K_R} K_R\right)^{\beta}\left(\tau_{L_R} L_R\right)^{1-\beta}\right]^{\rho}\right\}^{1/\rho} \tag{F.3}$$

Here, we distinguish between effective resources used in production, R, and the raw resources drawn from natural capital, R_{raw}:

$$\begin{aligned} R &= \tau_{XR}\left(\tau_{K_R} K_R\right)^{\beta}\left(\tau_{L_R} L_R\right)^{1-\beta} \\ R_{raw} &= \left(\tau_{K_R} K_R\right)^{\beta}\left(\tau_{L_R} L_R\right)^{1-\beta} \end{aligned} \tag{F.4}$$

the parameter τ_{XR} being the parameter that reflects the efficiency with which the productive economy uses raw resources (reflecting both process efficiency and, for energy resources, the conversion of energy content into exergy, or useful work). From (F.3) and (F.4) can be seen the link between output production and resource use (extraction).

[41] Author note: I use the moniker "CES(Solow)" to emphasize the fact the origin of the "constant elasticity of substitution" (CES) production function is properly credited to Robert Solow (Solow, 1956, page 77), not, as is commonly mis-attributed, to Arrow et al, 1961, whose article appeared 5 years later (and included Solow as co-author, ironically). I have no idea why this misconception is so widespread, but it is such a beautiful and useful function that it should be beyond dispute that the actual inventor deserves to be widely credited for it.

[42] The Translog cost function widely used for econometric work has a production function dual form (often called the "primal" form), but indirect techniques have to be used to derive production-side quantities (Saunders, 2008, 2015, 2017).

The corresponding cost function is introduced later.

Natural Capital

The function (F.3) must be further modified to account for scarcity (Ricardian) rent extracted by natural capital. As seen in Appendix A, the natural capital production function is

$$N_t = N_{t-1} + \left(\text{¡}_t - R_t \right) + \left(A_t - W_t \right) \qquad \text{(F.5)}$$

As shown in Appendix A, the dual unit cost function is a fixed scalar for any combination of values for the elements of (F.5), and can arbitrarily be set to unity $c_N = 1$.

However, as natural capital is limited, it is to be expected that it will extract rent from the productive economy in the form of increased effort needed to supply raw resources for use in production. Accordingly, what is needed is a price of natural capital defined as follows

$$p_N = c_N + rent_N \qquad \text{(F.6)}$$

Further, if it is the case that natural capital has a lower limit below which it experiences catastrophic collapse, $N_{critical}$, the rent term in (F.6) must accommodate this so that

$$p_N = p_N \left(c_{N,} N_t, N_{critical}, \text{¡}, R, A, W \right) \qquad \text{(F.7)}$$

Further, we would like it to be the case that as N_t approaches $N_{critical}$, natural capital becomes stingier in the resources it provides to producers. The following functional form exhibits these properties

$$p_N = c_N + \frac{1}{\theta} \ln \left(\frac{N_{capacity} - N_{critical}}{N_t - N_{critical}} \right) \qquad \text{(F.8)}$$

where θ is a shaping factor determining how quickly p_N rises as it approaches the critical value, and $N_{capacity}$ is a reference point determining overall p_N magnitude. When $N_t = N_{capacity}$, the above expression becomes $p_N = c_N$: the supply of natural capital is at the maximum possible for the planet (which would reflect a condition where human economic activity is absent – no raw resources extracted, and no waste inflicted on natural capital).

This function can be better understood by way of graphical illustration:

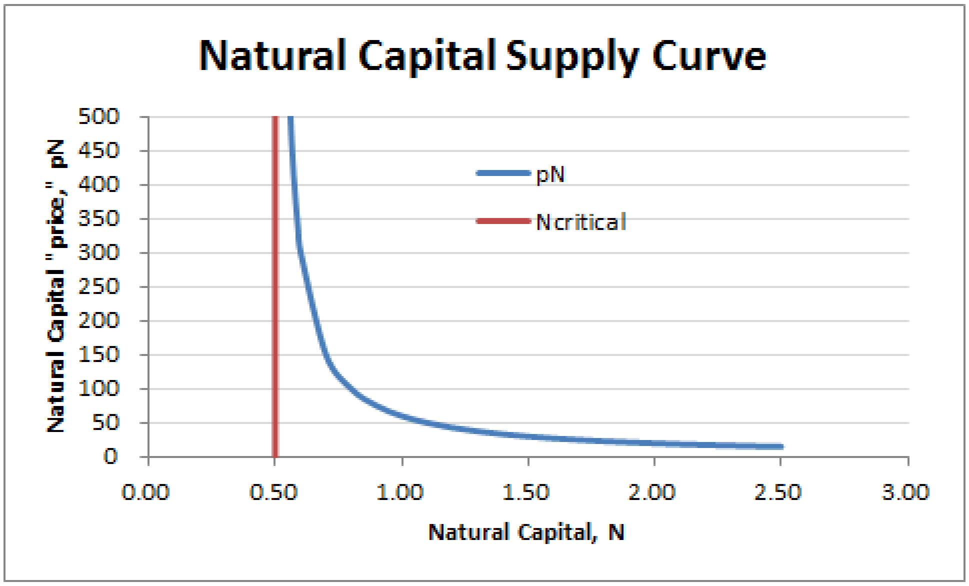

Figure E.1. The Natural Capital Supply Curve

As the level of natural capital in place increases to the right in Figure A2, the Ricardian rent decreases. But as instead natural capital decreases toward the critical value, rent extracted becomes asymptotically large. As the value of $N_{critical}$ increases, this curve shifts rightward, indicating that the rent extracted by natural capital will be higher for any given value of N_t and will rise to its asymptotic value at a larger value of N_t than before the shift in $N_{critical}$, meaning nature is extracting more rent at lower values of N_t.

To accommodate its meaning for producers, Ricardian rent must reflect the added effort required to extract raw resources. As shown in the discussion surrounding equation (B.13), the prices of all elements of (F.5) each carry a price equal this unit cost and are fixed to each other and to unit cost. Accordingly we say $p_R = p_N$.

This dynamic must be introduced to the resource sector of the production function (F.3). To accomplish this, we must introduce a scalar quantity reflecting the difficulty producers face in extracting raw resources from natural capital (comparable to the scalar technology terms for other factor inputs). Accordingly, we define such multiplier (which we can call τ_R) based on (F.8) as follows:

$$\tau_R = \kappa_{base}\left[c_N + \frac{1}{\theta}\ln\left(\frac{N_{capacity} - N_{critical}}{N_t - N_{critical}}\right)\right] \tag{F.9}$$

Further, we would like to have it be the case that for some base case reference value of $N_t = N_{base} = 1$, this multiplier is unity, given base case choices for $N_{capacity}$, $N_{critical}$, and θ. Specifically, we can define this normalizing value κ_{base} as follows:

$$
\begin{aligned}
\tau_R &= 1 = \kappa_{base}\left[1+\frac{1}{\theta}\ln\left(\frac{N_{capacity}-N_{critical}}{1-N_{critical}}\right)\right] \\
\Rightarrow \kappa_{base} &= \frac{1}{\left[1+\frac{1}{\theta}\ln\left(\frac{N_{capacity}-N_{critical}}{1-N_{critical}}\right)\right]}
\end{aligned} \tag{F.10}
$$

When this τ_R is introduced to the production function, (F.4) becomes

$$
R_{raw} = \frac{1}{\tau_R}\left(\tau_{K_R}K_R\right)^{\beta}\left(\tau_{L_R}L_R\right)^{1-\beta} \tag{F.11}
$$

This says producers must apply extra capital and labor to produce the same raw resources as natural capital's Ricardian rent becomes larger (natural capital becomes scarcer). Note that while p_R is stated as a price, it can easily be understood as a price containing a resource exhaustion premium extracted by natural capital from the human economy.

Accordingly, the production function corresponding to (F.3) becomes

$$
Y = \left\{a\left[\left(\tau_{K_X}K_X\right)^{\alpha}\left(\tau_{L_X}L_X\right)^{1-\alpha}\right]^{\rho} + (1-a)\left[\frac{\tau_{XR}}{\tau_R}\left(\tau_{K_R}K_R\right)^{\beta}\left(\tau_{L_R}L_R\right)^{1-\beta}\right]^{\rho}\right\}^{1/\rho} \tag{F.12}
$$

This says producers can become more efficient using raw resources by implementing technology gains τ_{XR} but resource producers become less effective as Ricardian rent burdens (τ_R) become greater.

The unit cost function that is dual to (F.12) is[43]

$$
c = \left\{\hat{a}\left[\left(\frac{r}{\tau_{K_X}}\right)^{\alpha}\left(\frac{w}{\tau_{L_X}}\right)^{1-\alpha}\right]^{\frac{\rho}{\rho-1}} + \hat{b}\left[\frac{\tau_R}{\tau_{XR}}\left(\frac{r}{\tau_{K_R}}\right)^{\beta}\left(\frac{w}{\tau_{L_R}}\right)^{1-\beta}\right]^{\frac{\rho}{\rho-1}}\right\}^{\frac{\rho-1}{\rho}} \tag{F.13}
$$

where

[43] That technology gains as specified on the production function side can be introduced this simple way into the cost function (for any CRS function) is shown in Saunders, 2005, Supporting Proofs, Theorem 4. The formal derivation (long, tedious) of c from Y is available from the author, but follows the method outlined in Saunders, 2014, Appendix B (supplementary materials). Note the simple way in which factor-augmenting technology gain parameters in the production function instead simply divide the factor prices. A powerful benefit of duality theory.

$$\hat{a} = a^{\sigma}\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\right]^{1-\sigma}$$
$$\hat{b} = \left(1-a\right)^{\sigma}\left[\beta^{\beta}\left(1-\beta\right)^{1-\beta}\right]^{1-\sigma} \quad \text{(F.14)}$$

To be complete, we must specify functional forms for the elements of (F.5). To keep things simple, we choose the following functional forms:

$$\begin{aligned} \text{¡}_t &= \zeta N_t \\ R_t &= R_{raw} \\ A_t &= \xi N_t \\ W_t &= \varepsilon e^{-\lambda_W} R_t \end{aligned} \quad \text{(F.15)}$$

where ζ, ξ, ε are constants (that can be changed for emulation purposes to show sensitivities). These relationships imply the following: the replenishment capacity of natural capital depends directly on the supply of natural capital in place; resources drawn from natural capital are determined by the productive economy's demand for them; the ability of natural capital to absorb waste thrown off by producers again depends directly on the supply of natural capital in place; and the waste generated by producers is directly related to on quantity of resource used to produce total economic output, Y_t. This latter function incorporates a technology improvement parameter λ_W that allows for waste production to become more efficient over time (i.e., less waste produced per unit of resource extracted for production – and/or, waste recycled). From (F.15) it seen that absorption capacity is limited via natural capital supply. There is also a restriction that absorption itself cannot exceed waste ($A_t \leq W_t$) for the identity (F.5) to be physically meaningful.

Obviously, these are overly simplistic functional forms. For example, the waste generation function assumes all waste is of a single aggregate type, uniformly absorbable by the natural economy. Present economy-climate models treat this in a much more sophisticated way. But in the cause of generating initial insight, we ignore that here, leaving others to more properly specify this – and the other components driving the formation dynamics of natural capital.

But with this, the specification of the simulation model is complete and the eco-economic system is fully defined.

Utility-maximizing Conditions (extended from previous published paper)

In addition, specific utility-maximizing considerations allow the development of equations that also appear in the model. In particular, equations for consumption, savings and leisure time are readily developed:

Households are assumed to exhibit a household utility function of the following form shown in (F.1):

$$u(C,S,l) = u_0 C^{\gamma} S^{\nu} l^{\varpi} N^{\pi} \tag{F.16}$$

On the household side, the following equations describe household utility-maximizing behavior, assuming knowledge of certain quantities and prices delivered from the production-side optimization:

$$\begin{aligned} &\max_{C,S,l} u = f(C,S,l) \\ &\textit{subject to}: \\ &cC + cS = w(\bar{L} - l) + rK \\ &\bar{L} - l = L \geq 0 \\ &S \geq 0 \end{aligned} \tag{F.17}$$

Utility is maximized subject to two primary constraints. The first of these is the household budget constraint: the amount consumed and saved (in nominal terms) must equal wage earnings plus investment returns on the total savings they hold claim to. (K is the sum of all past savings from earned income, and the sum of all past savings from both earned and unearned income, reduced by depreciation). The second constraint is essentially a statement that total labor supplied to the productive economy cannot exceed the amount of labor hours available in a day from the household, accounting for the fact that households by necessity take leisure time – even if only for sleep, to eat, and for time dedicated to basic household management and child care.

Solving this utility-maximizing problem involves forming the Lagrangian and calculating the first-order partials:

$$\mathcal{L} = -u(C,S,l) + \lambda\left[cC + cS + wl - w\bar{L} - rK\right] + \mu(l - \bar{L}) \tag{F.18}$$

So that for $\frac{\partial \mathcal{L}}{\partial C} = 0$, $\frac{\partial \mathcal{L}}{\partial S} = 0$ and $\frac{\partial \mathcal{L}}{\partial l} = 0$,

$$\begin{aligned} \frac{\partial u(C,S,l)}{\partial C} &= \lambda c \\ \frac{\partial u(C,S,l)}{\partial S} &= \lambda c \\ \frac{\partial u(C,S,l)}{\partial l} &= (\lambda w + \mu) \end{aligned} \tag{F.19}$$

And the budget constraint must be satisfied

$$cC + cS + wl = w\bar{L} + rK \tag{F.20}$$

Also, from the Kuhn-Tucker theorem, the inequality constraint must satisfy

$$\mu\left(l-\bar{L}\right)=0 \tag{F.21}$$

For the Cobb-Douglas utility function used for the simulations ($u\left(C,S,l\right)=u_0 C^{\gamma}S^{\nu}l^{\omega}$, where $\omega=1-\gamma-\nu$), the first-order conditions (F.19) become:

$$\begin{aligned}
\frac{\partial u\left(C,S,l\right)}{\partial C} &= a\gamma C^{\gamma-1}S^{\nu}l^{1-\gamma-\nu} = \lambda c \\
\frac{\partial u\left(C,S,l\right)}{\partial S} &= a\nu C^{\gamma}S^{\nu-1}l^{1-\gamma-\nu} = \lambda c \\
\frac{\partial u\left(C,S,l\right)}{\partial l} &= a\omega C^{\gamma}S^{\nu}l^{-\gamma-\nu} = \lambda w \\
cC+cS+wl &= w\bar{L}+rK
\end{aligned} \tag{F.22}$$

One solves this system by testing combinations of the Lagrange multipliers to see under what conditions the system constraints are satisfied. If it is assumed that the leisure constraint is non-binding (household time available $\bar{L}$ is always greater than leisure time taken), so that $\mu=0$, it is discovered that the resulting value of λ delivers a solution. In particular, after some algebraic manipulation the solution is

$$\begin{aligned}
C &= \frac{\gamma}{c}\left(w\bar{L}+rK\right) \\
S &= \frac{\nu}{c}\left(w\bar{L}+rK\right) \\
l &= \frac{\omega}{w}\left(w\bar{L}+rK\right)
\end{aligned} \tag{F.23}$$

where

$$\begin{aligned}
&\mu=0 \\
&\lambda=a\left(\frac{\gamma}{c}\right)^{\gamma}\left(\frac{\nu}{c}\right)^{\nu}\left(\frac{\omega}{w}\right)^{\omega}>0
\end{aligned} \tag{F.24}$$

Equations (F.23) are used directly in the emulation model.

Appendix F: Relation to Georgescu-Roegen Functional Forms, Structure and Functioning

The neoclassical eco-economy model draws heavily on the Georgescu-Roegen Flow-Fund model, with some extensions and a few modifications.[44]

[44] As noted in the main text, I am grateful to Herman Daly for suggesting inclusion of Georgescu-Roegen insights, principles, and methods.

Like Georgescu-Roegen, the neoclassical model treats capital and labor as funds (or stocks) from the perspective of producers. Further, resources are likewise treated as a flow to producers and waste as an outflow from producers. The only possibly meaningful departures from the Georgescu-Roegen picture are:

1. *Labor* is treated by Georgescu-Roegen (GR) as a fund producers call on; the neoclassical eco-economy model (call it NEEM) treats it likewise. However, in addition to a fund producers call on, NEEM also treats labor as a flow from households to producers each period.
2. *Resources* are treated by GR as a flow; NEEM treats it likewise. However NEEM also treats resources as a fund originating in natural capital and drawn on as a flow.
3. *Intermediate Materials* is treated by GR as a flow; NEEM does not capture this flow explicitly, except inasmuch as the production function (B.5) incorporates in aggregate form the interactions among intermediates producers in the X (intermediates) component of the production of final goods and services Y, and also treats extracted resources R as an intermediate input drawn from primary sources.
    a. (The designator "X" is used to characterize the intermediates sector to convey that it is properly modeled at a lower aggregation level by way of Input-Output modeling techniques that identify "crisscross" intermediates flows that would feed a GR-style production model at each node.)
4. *Maintenance* is treated by GR as an input flow to producers; NEEM does not capture this flow explicitly, except inasmuch as a certain amount of capital and labor is set aside each period to refresh and replenish deteriorating (depreciating) capital.
    a. (Again, a lower aggregation level model would explicitly treat new capital and labor flows as specifically tied to the Input-Output nodes in a way that would feed a GR-style production model.)

NEEM does incorporate other funds and flows beyond those in GR, as can be seen in the comparison Table A1 below:

	Neoclassical Eco-economy Model		**Georgescu-Roegen Flow-Fund Model**	
Descriptor	*Fund*	*Flow*	*Fund*	*Flow*
Consumption		**C**: flow from producers to households	N/A	N/A
Savings		**S**: flow from households to producers	N/A	N/A
Output		**Y**: flow from producers to households and intermediates producers		**Q**: flow from production

Capital	**K**: fund for producers		**K**: fund for producers	
Labor	**L**: fund for producers	**L**: flow from households to producers	**H**: fund for producers	
Resources	**R**: fund for natural capital	**R**: flow from natural capital to producers		**R**: input flow to producers
Waste		**W**: flow from producers to natural capital		**W**: output flow from producers
Replenishment		$\mathcal{R}$: regenerative flow from natural capital	N/A	N/A
Absorption		**A**: flow from natural capital offsetting waste	N/A	N/A
Intermediate Materials		Not explicit, except for extracted resources		**I**: input flow to producers
Maintenance	N/A	Not explicit, except that **I** and **L** go to replenish capital and offset depreciation.		**M**: input flow to producers
Land	Part of natural capital, **N**		**L**: fund for producers	
Natural Capital	**N**: stock/fund		Not explicit	

Table A1. Comparison of the Eco-Economy Model to Georgescu-Roegen Structure

My claim is that this framework honors the essence and spirit of the Georgescu-Roegen formulation.

Appendix G: A Note on Resource Rebound Dynamics

In the predecessor paper to this (Saunders, 2014), attention was given to the phenomenon of "resource rebound," wherein increases in resource use efficiency may deliver resource use reductions less than naïvely expected from pure engineering calculations. But the inclusion of the natural economy changes these dynamics in interesting ways.

First, the emulation model shows rebound dynamics persist even when the natural economy is included, but in a curious way. When the Base Case (zero growth, technology gains absent) is given a resource use efficiency gain, this reduces initial resource use (rebound less than 100%), meaning the draw on natural capital is initially reduced, causing natural capital to grow. This boost in natural capital has the ancillary effects of increasing its replenishment and absorption capacities (compared to the Base Case), which makes more natural capital available to the human economy.

But over time, two other dynamics come into play. One, the resource efficiency gain reduces the "effective" price of resource use, spurring its use in standard neoclassical rebound fashion.[45] But a further dynamic comes into play. That is, Ricardian rent dynamics makes itself evident. By increasing the supply of natural capital (in the short run), Ricardian rent is reduced as constraints on natural capital are reduced, thus spurring further resource use. In the emulations, it happens, this increased resource use is allowed by the increased natural capital supply made available.

Eventually, the emulation model suggests, resource rebound returns to a 100% value, but alongside a larger natural capital resource base to supply it.

A different thing happens when technology gains are deployed to the production of intermediate goods and services. In this case, 100% resource rebound still obtains but natural capital draws are diminished in a secular way, even with rebound in operation.

Finally, an interesting thing happens when the resource substitution elasticity σ is altered. Theory to date says that the larger is σ, the larger will be rebound.[46] However, when σ is reduced in the emulation model, long-term rebound is *increased*. What happens here is that resource efficiency gains decrease resource use more in the short term with lower σ, just as theory would predict (lower rebound); however, the reduced resource draw then allows expansion of natural capital, which because this reduces Ricardian rent allows greater resource draw over time (and expansion of the economy therewith). Explicit introduction of the natural economy changes the dynamics of resource rebound.

Caution: It is **crucial** to note that the above emulations do not apply only to zero-growth conditions: household consumption, savings, and utility can continue growing even while being sustainably fed by growing natural capital if biophysical boundaries are honored. However, a far different picture emerges when you force consumption (and savings) to remain at fixed levels (procedure described in the User Guide of the model). For instance, when either resource use technology gain efficiencies or efficiencies of production are combined with "satiation" levels of consumption/savings, natural capital can grow even while household leisure time increases – resource rebound dynamics (while still in play) notwithstanding. Something like a utopian-like result for many environmental humanists?

Inclusion of the natural economy changes many common conceptions, it appears.

---

[45] Thus making resource draw from natural capital, and its use, function equivalently to the "exergy/energy" distinction drawn by Ayres (1997) and Ayres and Warr (2005,2009). Perhaps we need a more general moniker for resources in use ("exer-resources"? Awkward.)

[46] See e.g., Saunders (1992,2008,2014) and Sorrell (2007).

Second Caution: Results reported from the emulation model rely on a very simple (simplistic) depiction of the components driving natural capital (even though the natural capital identity (5) itself is mathematically unassailable). More sophisticated depictions of natural capital replenishment and absorption dynamics could change this picture entirely. (Note, however, that the emulation model allows for accumulation of waste, even in this simple depiction, should one wish to explore the waste dynamic.)

All of which points to a large and complex degree of subtlety in regard to this (contentious) issue of "rebound effects" (Nordhaus, 2016).