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Specification testing in panel data with instrumental variables

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Abstract

I show that specification tests for correlated fixed effects developed by Hausman and Taylor extend in an analogous way to panel data sets with endogenous regressors. Given panel data, different sets of instrumental variables can be used to construct the test. For a simple class of models, the test in many cases is asymptotically more efficient if an incomplete set of instruments is used. However, in small samples one may do better using the complete set of instruments. Monte Carlo results demonstrate the likely gains for different assumptions about the degree of between and within variance in the data.

Key words: Econometrics; Panel data; Instrumental variables; Specification testing
JEL classification: C12; C15; C23

1. Introduction

Following early work by Hausman (1978) and Hausman and Taylor (1981), specification testing for correlated fixed effects in panel data models is now a standard tool for researchers. To date, the test as derived in the latter paper has not allowed for the possibility of endogenous right-hand side variables. In this paper, I extend the results of Hausman and Taylor (1981) to the case where right-hand side variables are assumed to be endogenous (specifically, correlated with the time-varying component of the error structure). It turns out that the IV analogous specification tests for correlated fixed effects given in Hausman and Taylor (1981) are applicable in this context. However, it is important to specify the instrument set appropriately for the test. Perhaps surprisingly, for a simple

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class of models that I consider, the more powerful test statistic in many cases uses an inefficient estimator. Asymptotically, while the variance used to construct the test statistic will be greater than the variance associated with using a more efficient estimator, its asymptotic bias will also be greater as the null hypothesis of no correlation is violated. The increase in bias more than offsets the increase in variance thereby leading to a more powerful test statistic.

The next two sections derive this asymptotic result for a simple model. I then consider the small-sample properties of the different test statistics under different assumptions about the quality of the instrument and the degree of correlation between the fixed effects and the instrument by means of a Monte Carlo experiment. Finally there is a brief conclusion.

2. The model and test

The model under consideration is

$$Y = X\beta + \alpha \otimes e_T + \varepsilon, \tag{1}$$

where Y is an $NT \times 1$ vector, X an $NT \times k$ matrix, α an $N \times 1$ vector of individual effects (α_i i.i.d. with mean 0 and variance σ_α^2), and ε an i.i.d. random vector with mean 0 and covariance matrix $\sigma_\varepsilon^2 I_{NT}$. The vector e_T is a $T \times 1$ vector of ones. The data are stacked by individuals over time. That is, $Y' = [Y_1' \; Y_2' \; \ldots \; Y_N']$, where Y_i is a $T \times 1$ vector of observations on the ith individual. This equation is part of a simultaneous system and by assumption some columns of X are correlated with ε. It is assumed that was some (possibly all) columns of X are also correlated with the individual effects. There is a set of instruments Z, a matrix $NT \times L$, $L \geqslant k$, valid in the sense that Z is correlated with X but uncorrelated with ε. It is assumed that columns of X which are uncorrelated with ε are contained in Z. The present purpose is to test whether Z is correlated with the individual effects. Specifically, I consider the hypotheses:[1]

$$\mathrm{H_0}\colon \quad \operatorname*{plim}_{N\to\infty} \left\{ \sum_{i=1}^{N} Z_{it}' \alpha_i / N \right\} = 0, \quad \forall t,$$

$$\mathrm{H_A}\colon \quad \operatorname*{plim}_{N\to\infty} \left\{ \sum_{i=1}^{N} Z_{it}' \alpha_i / N \right\} \neq 0, \quad \forall t.$$

[1] An alternative null hypothesis which appears less restrictive is $\operatorname{plim}_{N\to\infty} \sum_{i=1}^{N} \{\bar{Z}_1' \cdot \alpha_i / N\} = 0$ where $\bar{Z}_i$ is the average over time of the observations of Z_{it}. However, Amemiya and McCurdy (1986) note that the assumptions under the two null hypotheses are equivalent if one also assumes that the estimator for β continues to be consistent when estimated using any $T-1$ of the T time periods. While there may be circumstances in which this second set of $T-1$ assumptions fails to hold while the assumption that $\operatorname{plim} \sum_{i=1}^{N} \bar{Z}_i' \cdot \alpha_i / N = 0$ holds, it seems reasonable to believe this is an unusual case. Hence I argue that for our purposes the null hypothesis in the text is not overly restrictive. Whether the additional $T-1$ assumptions implied by this null hypothesis hold or not is in fact an empirical manner. A specification test can easily be constructed to test their validity.

Given the loss of information resulting from the use of the within or first difference estimators to eliminate correlated fixed effects, there is a large gain possible if one can assume the null hypothesis. In this case, the GLS-IV estimator will be more efficient.

Letting $u = \alpha \otimes e_T + \varepsilon$, then

$$E(uu') \equiv \Omega = T\sigma_\alpha^2 P_v + \sigma_\varepsilon^2 I_{NT}, \tag{2}$$

or

$$E(uu') = \sigma_1^2 P_v + \sigma_\varepsilon^2 Q_v, \tag{2'}$$

where $\sigma_1^2 = T\sigma_\alpha^2 + \sigma_\varepsilon^2$, $P_v = (I_N \otimes e_T e_T')/T$, and $Q_v = I - P_v$. For future reference, I use the fact that $\Omega^{-1/2} = \sigma_1^{-1} P_v + \sigma_\varepsilon^{-1} Q_v$ and denote $\Omega^{-1/2}$ by H. $P_v X$ replaces the observations for each column of X by the average of the observations for each individual over time. $Q_v X$ replaces the observations by the deviations from the time averages.

First note that the Hausman-type specification test, comparing the GLS-IV estimator with the fixed effects (within) estimator, can be constructed using the within and the between estimators. Define the operator P_A as the projection operator: $P_A = A(A'A)^{-1}A'$. If Z is a set of variables uncorrelated with ε, there are different possible instrument sets that I can use. Following the general approach of Cornwell, Schmidt, and Wyhowski (1993) (CSW), I consider instrument sets of the form $\tilde{Z} = [Q_v Z, P_v B]$ where B is defined as a matrix of potential instruments. The GLS-IV estimator is given by

$$\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} = (X'H'P_{\tilde{Z}}HX)^{-1} X'H'P_{\tilde{Z}}HY. \tag{3}$$

Some simple algebra (see Appendix A) shows that the GLS-IV estimator is a matrix weighted average of the within IV estimator ($\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}}$) and the between IV estimator ($\hat{\beta}_{\mathrm{B}}^{\mathrm{IV}}$) . That is,

$$\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} = \Lambda \hat{\beta}_{\mathrm{W}}^{\mathrm{IV}} + (I - \Lambda)\hat{\beta}_{\mathrm{B}}^{\mathrm{IV}}, \tag{4}$$

where

$$\Lambda = [\sigma_\varepsilon^{-2} X' P_{Q_v Z} X + T\sigma_1^{-2} \bar{X}' P_{\bar{B}} \bar{X}]^{-1} \sigma_\varepsilon^{-2} X' P_{Q_v Z} X, \tag{5}$$

$$\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}} = [X' P_{Q_v Z} X]^{-1} [X' P_{Q_v Z} Y], \tag{6}$$

$$\hat{\beta}_{\mathrm{B}}^{\mathrm{IV}} = [\bar{X}' P_{\bar{B}} \bar{X}]^{-1} [\bar{X}' P_{\bar{B}} \bar{Y}]. \tag{7}$$

In Eqs. (5) and (7), $\bar{X} = [\bar{X}_{1.}' \bar{X}_{2.}' \ldots X_N']'$, where $\bar{X}_{i.}$ is the mean of the T observations on X for the ith individual (and similarly for B, Y, and Z). That Eq. (4) holds should not be surprising as it is simply the IV analog to the result for OLS estimators derived in Maddala (1971).

Under the null hypothesis, $\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}}$ and $\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}}$ are consistent estimators of β with $\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}}$ the more efficient estimator, while under the alternative, $\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}}$ is consistent and $\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}}$ is inconsistent. A Hausman test statistic of the form

$$m = (\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{W}}^{\mathrm{IV}})' V(\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{W}}^{\mathrm{IV}})^{-1} (\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{W}}^{\mathrm{IV}}) \tag{8}$$

can be constructed. Under the null, m is distributed as a chi-square statistic with k degrees of freedom. Simple algebra using Eq. (4) shows that m can be written as

$$m = (\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{B}}^{\mathrm{IV}})' (V_{\mathrm{W}} + V_{\mathrm{B}})^{-1} (\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{B}}^{\mathrm{IV}}). \tag{9}$$

One advantage of the latter formulation of the test statistic is that the covariance matrix of the difference between the between and within estimators is easier to compute. While the $\mathrm{cov}(\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{W}}^{\mathrm{IV}})$ is equal to $V_{\mathrm{W}} - V_{\mathrm{GLS}}$ if $\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}}$ is asymptotically efficient, the estimated difference of the covariance matrices may not be positive definite in small samples. This equivalent formulation of the chi-square test statistic generalizes a result of Hausman and Taylor (1981) to allow for IV estimation.

To this point, I have considered instrument sets of the general form $[Q_v Z, P_v B]$. Now I turn my attention to the choice of B. An obvious choice for B is Z itself. Then $\tilde{Z} = [Q_v Z, P_v Z]$. In other words, the instruments Z are used twice: first as deviations from their time means and then as the time means themselves. This is essentially the Hausman–Taylor (HT) estimator discussed in Breusch, Mizon, and Schmidt (1989) and Cornwall, Schmidt, and Wyhowski (1993). However, since the values of Z_{it} are uncorrelated with the individual effects for each t under H_0, then each of the T $N \times L$ matrices Z_t, where $Z_t = [Z'_{1t}, \ldots, Z'_{Nt}]'$, can be used as instruments for X_t. As a result, more instruments are available which cannot decrease the efficiency of the GLS-IV estimator. Under the null hypothesis that individual values of Z are uncorrelated with the individual effects for all values of t, then the instrument set $B = Z^*$ provides more efficient estimates of β, where Z^* is formed as in Breusch, Mizon, and Schmidt (1989).[2] That is Z^* is an $NT \times TL$ matrix:

$$Z^* = \begin{bmatrix} Z_{11} & \cdots & Z_{1T} \\ Z_{11} & \cdots & Z_{1T} \\ \vdots & & \\ Z_{N1} & \cdots & Z_{NT} \\ Z_{N1} & \cdots & Z_{NT} \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} Z \\ Z \end{matrix}}\right\} T \text{ times} \\ \\ \\ \\ \end{matrix} .$$

Note that $Q_v Z^* = 0$ and $P_v Z^* = Z^*$. Hence, $\tilde{Z} = [Q_v Z, Z^*]$ and $\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}}$ constructed using Z^* will be more efficient than if constructed using $P_v Z$. CSW refers to this estimator as the Amemiya–McCurdy (AM) estimator.

[2] Z^* is defined as S^* in Eq. (7) of Breusch, Mizon, and Schmidt (1989). The instrument set $[Q_V Z, Z^*]$ defined below is analogous to set B^0 in Theorem 2 of their paper. While a more efficient instrument set may exist, analogous to the set D in Eq. (8) of B-M-S, for the purposes of this paper all that is needed is that the estimator using Z^* be *more* efficient than the estimator using $P_v Z$.

One might imagine that the appropriate specification test should employ the more efficient GLS estimator to obtain greater power. However, this need not be the case. A reduction in the variance of the difference of the within and between estimators will indeed increase the power of the test. Given the greater efficiency of the between estimator using Z^* rather than Z, the variance will decrease. However, the asymptotic bias may also increase when a less efficient estimator is used. If, for example, the bias increases at the same rate as the variance as the null is violated when using a less efficient estimator, then the power will increase since the test statistic is quadratic in bias but linear in variance. The next section considers the asymptotic efficiency of the test statistic formed with different instrument sets for a particularly simple class of data processes to illustrate this point.

3. Asymptotic efficiency

To illustrate the point that the statistic for testing correlated fixed effects may be more powerful when an inefficient estimator is used, I consider a simple scalar model with one explanatory variable and one instrument:

$$x_{it} = \gamma_x x_{it-1} + v_{it}, \tag{10a}$$

$$z_{it} = \gamma_z z_{it-1} + \eta_{it}, \tag{10b}$$

$$\operatorname*{plim}_{N\to\infty}\left(\frac{1}{NT}\, v'\eta\right) = \sigma_{v\eta} \neq 0, \tag{10c}$$

$$\operatorname*{plim}_{N\to\infty}\left(\frac{1}{NT}\, v'v\right) = \sigma_v^2 > 0, \tag{10d}$$

$$\operatorname*{plim}_{N\to\infty}\left(\frac{1}{NT}\, \eta'\eta\right) = \sigma_\eta^2 > 0, \tag{10e}$$

$$|\gamma_i| < 1, \qquad i = x, z. \tag{10f}$$

I will occasionally refer to σ_x^2 and σ_z^2 which equal $\sigma_v^2/(1-\gamma_x^2)$ and $\sigma_\eta^2/(1-\gamma_z^2)$, respectively. I assume that η is uncorrelated with ε. However, it may be correlated with α:

$$\operatorname*{plim}_{N\to\infty}\left(\frac{1}{N}\, z_t'\alpha\right) = \sigma_{z\alpha}, \tag{11}$$

when $\sigma_{z\alpha}$ equals zero under the null hypothesis and is nonzero otherwise.[3]

[3] This model easily extends to the matrix case of k explanatory variables and L instruments where the autoregressive parameters γ_x and γ_z are scalar. For details see Metcalf (1992).

This is a particularly simple structure for the data generation process, but it has the appealing property that as γ_i increases from 0 towards 1 an increasing fraction of the variance of the random variable is due to the variation across individuals.[4] Since panel data are often slow-moving over time, the performance of the specification test at high levels of γ_i is of considerable interest. I exclude the possibility that $\gamma_i = 1$. A more general model would allow some variables to be nonstationary. However, the greater model complexity would obscure the essential results without adding much in the way of insights.

Define the between estimator using the mean of the instrument as $\hat{\beta}_{\mathrm{HT}}$ and the between estimator using Z^* as $\hat{\beta}_{\mathrm{AM}}$. Under the null hypothesis, the asymptotic variance of $\sqrt{N}\hat{\beta}_{\mathrm{HT}}$ is given by

$$V_{\mathrm{HT}} = \sigma_u^2 \frac{T^2(2\sum_t b_t - T)}{(\sum(a_t + b_t) - T)^2} \frac{\sigma_z^2}{(\sigma_{xz})^2}, \tag{12}$$

where

$$a_t = \sum_{s=1}^{t} \gamma_x^{s-1} \quad \text{and} \quad b_t = \sum_{s=1}^{t} \gamma_z^{s-1}.$$

The asymptotic variance for $\sqrt{N}\,\hat{\beta}_{\mathrm{AM}}$ is given by

$$V_{\mathrm{AM}} = \sigma_u^2 \frac{T^2}{FB^{-1}F'} \frac{\sigma_z^2}{(\sigma_{xz})^2}, \tag{13}$$

where $F = [a_T + b_1 - 1 \;\; a_{T-1} + b_2 - 1 \; \ldots \; a_1 + b_T - 1]$ and B is the $T \times T$ matrix

$$B = \begin{bmatrix} 1 & \gamma_z & \gamma_z^2 & \gamma_z^3 & \cdots & \gamma_z^{T-1} \\ \gamma_z & 1 & \gamma_z & \gamma_z^2 & \cdots & \gamma_z^{T-2} \\ & & \vdots & & & \\ \gamma_z^{T-1} & \gamma_z^{T-2} & \gamma_z^{T-3} & \gamma_z^{T-4} & \cdots & 1 \end{bmatrix}.$$

Consider local alternative of the form $\sigma_{z\alpha} \neq 0$ and $\sqrt{N}\sigma_{z\alpha} \to \psi < \infty$ as N approaches ∞. Under the null hypothesis, the probability limit (as $N \to \infty$) of $\hat{\beta}_{\mathrm{w}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{b}}^{\mathrm{IV}}$ is zero and m is chi-square with one degree of freedom. Under the alternative hypothesis, m is distributed as a noncentral chi-square random variable with one degree of freedom and noncentrality parameter δ where

[4] For example, the between variance for X as a fraction of the total variance equals $(2\sum_t a_t - T)/T^2$ where $a_t = \sum_{s=1}^{t} \gamma_x^{s-1}$. This fraction varies between $1/T$ and 1 as γ_x increases from 0 to 1.

$\delta^2 = \bar{q}' M^{-1} \bar{q}$, $\bar{q}$ is the probability limit of $\sqrt{N}(\hat{\beta}_{\mathrm{w}}^{\mathrm{IV}} - \hat{\beta}_{\mathrm{b}}^{\mathrm{IV}})$, and M is its asymptotic variance (see Scheff, 1959). Let $\bar{q}_i$ equal $\bar{q}$ with $\hat{\beta}_i$ substituted for $\hat{\beta}_{\mathrm{b}}^{\mathrm{IV}}$ ($i = \mathrm{HT}$, AM). The asymptotic biases for the two estimators using the different set of instruments are

$$\bar{q}_{\mathrm{HT}} = \frac{T^2}{\sum(a_t + b_t) - T} \frac{\sigma_{z\alpha}}{\sigma_{xz}} \tag{14}$$

and

$$\bar{q}_{\mathrm{AM}} = \frac{T \cdot FB^{-1} e_T}{FB^{-1} F'} \frac{\sigma_{z\alpha}}{\sigma_{xz}}. \tag{15}$$

If $\gamma_x = \gamma_z$, it is straightforward to show that $V_{\mathrm{HT}} = V_{\mathrm{AM}}$ and that $\bar{q}_{\mathrm{HT}} = \bar{q}_{\mathrm{AM}}$ leading to the following proposition:

Proposition 1. Given the model in Eqs. (10)–(11) and the assumption that $\gamma_x = \gamma_z$, the two estimators $\hat{\beta}_{\mathrm{HT}}$ and $\hat{\beta}_{\mathrm{AM}}$ are equally efficient asymptotically and the power of the specification test of the hypothesis that the instrumental variables are uncorrelated with the individual effects is unaffected by the choice of instrument set.

Proof. See Appendix B.

As $\gamma_x - \gamma_z$ diverges from zero, the variances and power of the specification test begins to differ. Since $V_{\mathrm{HT}} - V_{\mathrm{AM}}$ is positive definite, it would appear that the power of the specification test should increase using Z^* as the set of instruments. However, it will turn out that $\bar{q}_{\mathrm{HT}}$ may also be greater than $\bar{q}_{\mathrm{AM}}$ which will increase the power of the test using the means of Z as instruments. For $\gamma_x > 0$, this result is formalized in the following proposition.

Proposition 2. If $\gamma_x > 0$ and $\gamma_z = 0$, then the asymptotic power of the test statistic using $\bar{Z}$ as instruments is greater than the power using Z^* as instruments.

Proof. See Appendix B.

Proposition 2 illustrates very clearly the trade-off between decreased efficiency and greater bias in moving from an efficient to an inefficient set of instruments. Increasing the asymptotic bias ($\bar{q}_i$) will increase the power of the test. With $\gamma_x > \gamma_z = 0$, the bias of each statistic is proportional to the variance ($\bar{q}_i = V_i \lambda$, $i = \mathrm{HT}$, AM, where λ is a $k \times 1$ vector). Since the power of the test statistic is quadratic in bias and linear in the inverse of the variance, the power must increase as the bias and inverse of the variance increase at the same rate.

I now turn my attention to the case where $\gamma_x = 0$ and $\gamma_z > 0$. Some tedious algebra shows that the asymptotic bias for the test statistic under the alternative

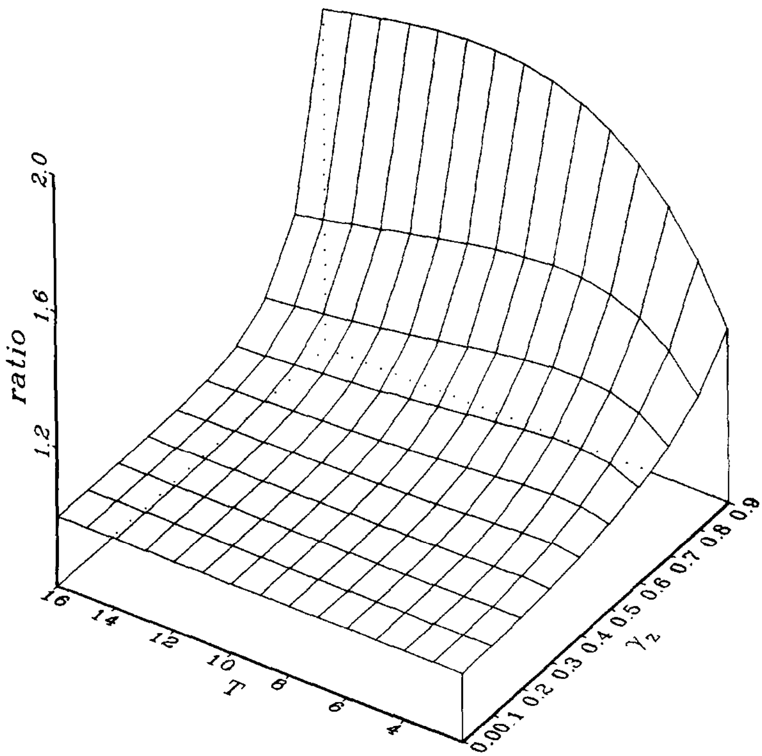

Fig. 1. Power ratio of m_{HT} to m_{AM}.

hypothesis is greater when $\bar{Z}$ is used as the set of instruments. Again, the variance of the between estimator is greater when $\bar{Z}$ is used. In this case it is difficult, however, to show that the power of the test is greater when $\bar{Z}$ is used as the instrument set rather than Z^*. In the simple case where $k = L = 1$ a grid search shows that the test statistic using $\bar{Z}$ is more powerful than when Z^* is used.

The increase in power is quite dramatic as illustrated in Fig. 1. Let the covariance of Z and α be equal to half the variance of Z. At $\gamma_z = 0.8$ and $T = 7$, the increase in the number of rejections is 41% (power equals 0.14 versus 0.10) and declines to 26% at $T = 16$ (power is 0.33 versus 0.26). At $\gamma_z = 0.9$, the test using the mean of the instruments rejects nearly twice as often as when the instruments for each time period are used separately. Note that at $\gamma_z = 0.9$ and $T = 7$, 80% of the variation in the data occurs across individuals rather than for individuals across time. It is quite typical for many panel data applications to lose 80% of the variance in the data when using the fixed effects estimator.

Fig. 2 graphs the efficiency gains from using the means of the instruments when $T = 5$ and γ_x and γ_z vary between 0.1 and 0.9. As pointed out above, the

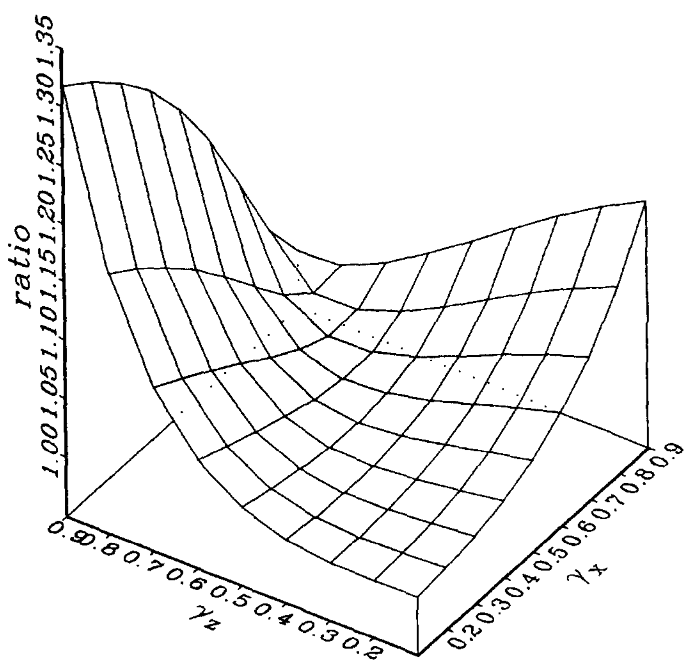

Fig. 2. Power ratio of m_{HT} to m_{AM}.

tests perform equivalently when $\gamma_x = \gamma_z$ and the test using the means of the instruments performs better as the two autocorrelations move apart. However, the improvement is not dramatic with a maximum improvement of less than 32%. This raises the issue of the performance of the tests in small samples. I turn my attention to this issue in the next section.

4. Small-sample characteristics of the test

Specification test statistics in general have been criticized for having low power (e.g., Holly, 1982; Newey, 1985). One might expect that the power of the test would deteriorate further as a result of the additional noise from the instrumenting of variables in X. To consider how well the test works in practice, I present results from a Monte Carlo experiment. I consider a simple model with $k = L = 1$, set β equal to 1 in Eq. (1) and take draws from a normal distribution for X_{it}, Z_{it}, ε_{it}, and α_i, each with mean 0. The first three variables have variance 1 and α_i has variance $1/T$. The covariance of Z and ε is zero while the other covariances vary from experiment to experiment.[5] After generating the data, I compute the within and between estimates of β, their variances, and the chi-square test (which has one degree of freedom).[6] I repeat the process 1000 times for each model.

For the first set of results, I set $\gamma_x = \gamma_z = 0$. With these assumptions, the noncentrality parameter, δ^2, is given by the formula

$$\delta^2 = \psi^2 T^2 \left(\frac{T-1}{2T-1}\right). \tag{16}$$

The asymptotic power of the test increases with more time periods, and with a higher correlation between the time means of the instruments and the individual effects. Note that the tests should perform equally well based on the results from the last section. Table 1 presents Monte Carlo results with $N = 200$ and $T = 5$. $\mathrm{Cov}(X_{it}, \varepsilon_{it}) = 0.4$ and $\sigma_{z\alpha}$ and σ_{xz} vary from 0 to 0.06 and 0.1 to 0.7, respectively. The numbers in each cell show the fraction of times the null hypothesis is rejected due to m in Eq. (9) exceeding the 5% critical value for a chi-square random variable with one degree of freedom. The top number in each cell presents results using the mean of Z as the instrument set, while the bottom number uses the set Z^*. For future reference, call the first test statistic m_{HT} and the second statistic m_{AM}. The first column in the table shows the computed size of the test. Note that neither of these tests has a computed size near 5% at very low levels of correlation between Z and X. This is suggestive of the results of Nelson and Startz (1990a, b) who have shown that the distribution of IV estimators diverges dramatically from the asymptotic distribution in the presence of poor instruments.

The remaining columns in Table 1 show the power of the test in the face of increasing correlation of Z with α. In nearly every case, the power of m_{AM} is higher than that of m_{HT}. The increase in power can be significant, particularly with poor instruments. These results are striking given the number of individuals in the data set ($N = 200$) as well as the fact that m_{AM} has the same distribution asymptotically as m_{HT}. Clearly, in the case where $\gamma_x = \gamma_z = 0$, the main advantage of m_{AM} over m_{HT} lies in its performance in the presence of poor instruments.

These results show that in the case where asymptotically the two formulations of the chi-square test should give equivalent results, the test statistic using the more efficient estimator is a more powerful test statistic. However, Proposition

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[5] I have also experimented with varying the variance of α_i. The results are not qualitatively different.

[6] Equivalently, I could take the square root of the statistic and use the standard normal distribution. Constructing the experiment with one degree of freedom allows me to avoid issues of direction in defining the local alternative which affect the power of the test.

Table 1
Computed power and size, $\gamma_x = \gamma_z = 0$

	$\sigma_{z\alpha}$			
σ_{xz}	0.0	0.02	0.04	0.06
0.1	0.004	0.029	0.082	0.146
	0.018	0.058	0.120	0.196
0.3	0.032	0.170	0.457	0.754
	0.044	0.184	0.475	0.744
0.5	0.045	0.174	0.497	0.786
	0.050	0.196	0.512	0.775
0.7	0.051	0.181	0.459	0.803
	0.063	0.205	0.489	0.807

This table presents the fraction of rejections of the null hypothesis that $\sigma_{z\alpha} = 0$ out or 1000 replications. The top entry in each cell uses $\bar{\bar{Z}}$ as an instrument for $\bar{X}$ while the bottom entry uses Z^*. The covariance of X and ε equals 0.40, N equals 200, and T equals 5. The nominal size of the test is 0.05.

Table 2
Computed power and size, $\gamma_x = 0.9$, $\gamma_z = 0$

	$\sigma_{z\alpha}$			
σ_{xz}	0.0	0.06	0.12	0.18
0.1	0.002	0.007	0.021	0.023
	0.003	0.005	0.012	0.011
0.3	0.026	0.124	0.310	0.362
	0.040	0.101	0.242	0.261
0.5	0.042	0.256	0.514	0.715
	0.058	0.191	0.421	0.639
0.7	0.043	0.328	0.813	0.942
	0.046	0.259	0.608	0.791

This table presents the fraction of rejections of the null hypothesis that $\sigma_{z\alpha} = 0$ out or 1000 replications. The top entry in each cell uses $\bar{\bar{Z}}$ as an instrument for $\bar{X}$ while the bottom entry uses Z^*. The covariance of X and ε equals 0.40, N equals 200, and T equals 5. The nominal size of the test is 0.05.

2 states that in cases where $\gamma_x > 0$ and $\gamma_z = 0$, then the test statistic using the less efficient estimator is more powerful. Table 2 presents Monte Carlo results in the case that $\gamma_x = 0.9$ and $\gamma_z = 0$. Recall that this implies that 80% of the variance in X is lost when the fixed effects estimator is used. In all other respects, the model is the same as in the first experiment. Table 2 shows the power of the two test

Table 3
Computed power varying γ_x and γ_z

	γ_x				
γ_z	0.00	0.25	0.50	0.75	0.90
0.00	0.979	0.967	0.942	0.837	0.606
	0.980	0.962	0.929	0.748	0.430
0.25	0.991	0.976	0.969	0.899	0.730
	0.990	0.972	0.956	0.847	0.583
0.50	0.987	0.981	0.984	0.913	0.806
	0.983	0.978	0.973	0.877	0.695
0.75	0.975	0.957	0.956	0.879	0.770
	0.977	0.963	0.953	0.861	0.682
0.90	0.848	0.854	0.814	0.745	0.550
	0.961	0.938	0.926	0.821	0.670

This table presents the fraction of rejections of the null hypothesis that $\sigma_{z\alpha} = 0$ out or 1000 replications. The top entry in each cell uses $\bar{Z}$ as an instrument for $\bar{X}$ while the bottom entry uses Z^*. The covariance of X and ε equals 0.40, N equals 200, and T equals 5. The nominal size of the test is 0.05. $\sigma_{z\alpha} = 0.09$ and $\sigma_{zx} = 0.70$.

statistics. The clear advantage of m_{HT} over m_{AM} is evident here. While both test statistics have the correct size at moderate levels of correlation between Z and X, the power of m_{HT} is greater than the power of m_{AM} in every case conditional on the alternative. The increase in power can be quite substantial even in the presence of good instrumental variables. The power of the test using the inefficient between estimator in the case where $\sigma_{xz} = 0.7$ and $\sigma_{zx} = 0.18$ is 0.942 compared to a power of 0.791 when the efficient between estimator is used to construct the chi-square test. This suggests that in cases where there is significant time variation for the instrumental variable, while there is little time variation for the explanatory variable, one should consider using the inefficient between estimator construct the chi-square statistic to test for correlation between the instrumental variables and the individual effects.

The final sets of Monte Carlo results provide guidelines for generalizing the results of Propositions 1 and 2 along with the Monte Carlo results of Tables 1 and 2. In Table 3, I fix the convariance of Z and α at 0.09, the covariance of Z and X at 0.70 and the covariance of X and ε at 0.40 and vary γ_x and γ_z from 0 to 0.90. In all cases where $\gamma_x > \gamma_z$, m_{HT} has higher power than m_{AM}. Again, the increase in power can be quite dramatic (e.g., $\gamma_x = 0.9, \gamma_z = 0.25$). This suggests that where is more time variation in the instrumental variables than in the explanatory variables, the test statistic should be computed using the inefficient between estimator. Where γ_x equals γ_z, there is no clear result with

Table 4
Computed power when model is overidentified

	γ_1				
γ_2	0.00	0.25	0.50	0.75	0.90
0.00	0.945	0.936	0.951	0.931	0.902
	0.922	0.912	0.941	0.920	0.900
0.25	0.952	0.945	0.958	0.954	0.925
	0.943	0.932	0.937	0.937	0.933
0.50	0.962	0.968	0.958	0.963	0.932
	0.943	0.950	0.944	0.949	0.930
0.75	0.938	0.934	0.941	0.921	0.909
	0.920	0.927	0.936	0.917	0.917
0.90	0.855	0.888	0.852	0.856	0.782
	0.891	0.917	0.910	0.911	0.897

This table presents the fraction of rejections of the null hypothesis that $\sigma_{z\alpha} = 0$ out of 1000 replications. The top entry in each cell uses $\bar{Z}$ as instruments for $\bar{X}$ while the bottom entry uses Z^*. The covariance of X and ε equals 0.40, N equals 200, and T equals 5. The nominal size of the test is 0.05. $\sigma_{z\alpha} = 0.05$ and $\sigma_{zx} = 0.50$. There is one explanatory variable (x) with a serial correlation coefficient of 0.5. There are three instrumental variables, two with autoregression coefficient γ_2 and one with coefficient γ_1.

both test statistics performing about the same, as Proposition 1 suggests they should. As γ_z becomes larger than γ_x, it becomes more likely that the m_{AM} outperforms m_{HT} though the improvement is not large until γ_z is much greater than γ_x.

Given the large number of possible configurations to consider, it is difficult to generalize broadly to cases where k and L exceed 1. As a first pass at broadening the findings of the previous tables, Table 4 presents results from a model in which $k = 1$ and $L = 3$. I fix γ_x at 0.5 and range γ for the instrumental variables. The first instrument has an autocorrelation coefficient equal to γ_1 and the next two instruments have a coefficient of γ_2. In cases where both γ_1 and γ_2 are less than γ_x, m_{HT} has higher power than m_{AM}. Similarly, when both γ_1 and γ_2 are greater than γ_x, m_{AM} has higher power than m_{HT} in three of four cases. These results are consistent with the results in Tables 2 and 3. In cases when $\gamma_2 > \gamma_x > \gamma_1$, m_{AM} outperforms m_{HT} when γ_2 is very large (0.9). When $\gamma_1 > \gamma_x > \gamma_2$, m_{HT} tends to outperform m_{AM}, though not by a large margin. These results suggest a tendency for m_{HT} to outperform m_{AM} when there are a number of instrumental variables with a greater fraction of within group variation than between groups variation – again consistent with the results in Tables 2 and 3.

While the results presented here are from a particularly simple model, they illustrate an important point. In certain circumstances one can improve on the power of a specification test for correlated fixed effects by using an inefficient estimator.

5. Conclusion

Testing for correlated individual effects has become increasingly important with the greater use of panel data sets. This paper shows that the type of specification test often employed in models where all the explanatory variables are considered exogenous carries over in a straightforward manner to models with endogenous explanatory variables. However, greater attention must be paid to the quality of the instruments used for the explanatory variables if the actual size and power of the test statistic is to correspond to the theoretical size and power.

The between estimator used in the specification test can be constructed with different sets of instruments. In many cases, a larger set of instruments leads to a more powerful test statistic. However, it is often the case that the more powerful test statistic uses a reduced set of instruments for the between estimator. While the variance of the test statistic is driven up in this case, so is the asymptotic bias which can more than offset the increase in variance. Such a case happens when the explanatory variables are slow moving over time while the instruments are not. In this case, there is a distinct advantage to constructing the specification test using the less efficient estimator to take advantage of its greater asymptotic bias.

These results have been shown in a simple model with one explanatory variable and one instrument. The model is easily extended to the matrix case where the autoregressive parameter is scalar. While the model does not directly generalize to allow for independent autoregressive paramters for individual explanatory variables or instruments or for nonstationary variables, I conjecture (and show in one example) that the basic insight is unchanged: in many cases more powerful statistics for testing correlated fixed effects can be constructed by using less efficient estimators.

Appendix A

Relationship between GLS-IV, within-IV, and between-IV estimators

Define instrument set $\tilde{Z} = [Q_v Z, P_v B]$, where B is an $NT \times M$ matrix with the property that the columns of $P_v B$ are legitimate instruments for the regression in Eq. (1) (i.e., uncorrelated with ε but correlated with X). As noted in the text, the GLS-IV estimator is given by

$$\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} = (X'H'P_{\hat{Z}}HX)^{-1}X'H'P_{\hat{Z}}HY. \tag{A.1}$$

It can easily be shown that $H'P_{\hat{Z}}H = \sigma_\varepsilon^{-2}P_{Q_v Z} + \sigma_1^{-2}P_{P_v B}$ and so

$$\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} = [\sigma_\varepsilon^{-2}X'P_{Q_v Z}X + \sigma_1^{-2}X'P_{P_v B}X]^{-1} \times [\sigma_\varepsilon^{-2}X'P_{Q_v Z}Y + \sigma_1^{-2}X'P_{P_v B}Y]. \tag{A.2}$$

If we define the $N \times k$ matrix $\bar{X} = [\bar{X}_1' \, \bar{X}_2' \, \dots \, X_N']'$, where $\bar{X}_i$ is the mean of the T observations on X for the i individual (and similarly for B, Y, and Z), we can rewrite (A.2) using the fact that $P_v B = \bar{B} \otimes e_T$. Making this substitution and some simple algebra leads to

$$\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} = [\sigma_\varepsilon^{-2}X'P_{Q_v Z}X + T\sigma_1^{-2}\bar{X}'P_{\bar{B}}\bar{X}]^{-1} \times [\sigma_\varepsilon^{-2}X'P_{Q_v Z}Y + T\sigma_1^{-2}\bar{X}'P_{\bar{B}}\bar{Y}]. \tag{A.2$'$}$$

Now the within estimator ($\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}}$) follows first from eliminating α from Eq. (1) in text:

$$Q_v Y = Q_v X\beta + Q_v \varepsilon. \tag{A.3}$$

We premultiply this by $P_{\hat{Z}}$ and, noting that $P_{\hat{Z}} = P_{Q_v Z} + P_{P_v B}$, obtain

$$P_{Q_v Z}Y = P_{Q_v Z}X\beta + P_{Q_v Z}\varepsilon, \tag{A.4}$$

and therefore

$$\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}} = [X'P_{Q_v Z}X]^{-1}[X'P_{Q_v Z}Y]. \tag{A.5}$$

This is simply the 2SLS regression of $Q_v Y$ on $Q_v X$ using $Q_v Z$ as instruments. The between estimator ($\hat{\beta}_{\mathrm{B}}^{\mathrm{IV}}$) is similarly derived and is given by

$$\hat{\beta}_{\mathrm{B}}^{\mathrm{IV}} = [\bar{X}'P_{\bar{B}}\bar{X}]^{-1}[\bar{X}'P_{\bar{B}}\bar{Y}]. \tag{A.6}$$

Again, this is simply the 2SLS regression of $\bar{Y}$ and $\bar{X}$ using $\bar{B}$ as instruments. Defining

$$\Lambda = [\sigma_\varepsilon^{-2}X'P_{Q_v Z}X + T\sigma_1^{-2}\bar{X}'P_{\bar{B}}\bar{X}]^{-1}\sigma_\varepsilon^{-2}X'P_{Q_v Z}X, \tag{A.7}$$

then it follows immediately that

$$\hat{\beta}_{\mathrm{GLS}}^{\mathrm{IV}} = \Lambda\hat{\beta}_{\mathrm{W}}^{\mathrm{IV}} + (I - \Lambda)\hat{\beta}_{\mathrm{B}}^{\mathrm{IV}}. \tag{A.8}$$

Eq. (A.8) shows that the GLS-IV estimator can be written as a matrix weighted average of the within-IV and the between-IV estimators and is the IV analog to the result for OLS estimators presented in Maddala (1971).

Appendix B

Proof of Proposition 1. Let $\gamma_x = \gamma_z = \gamma < 1$. Therefore $a_t = b_t$ and B^{-1} is given by

$$B^{-1} = (1-\gamma^2)^{-1}\begin{bmatrix} 1 & -\gamma & 0 & & \cdots & 0 \\ -\gamma & 1+\gamma^2 & -\gamma & 0 & \cdots & 0 \\ & & \vdots & & & \\ 0 & \cdots & & -\gamma & & 1 \end{bmatrix}. \tag{B.1}$$

Then

$$FB^{-1} = (1-\gamma^2)^{-1}\,[F_1 - \gamma F_2 (1+\gamma^2)\, F_2 - \gamma(F_1 - F_3) \cdots F_T - \gamma F_{T-1}], \tag{B.2}$$

where F_i is the ith element of F.

Since $a_t = \sum_{s=1}^{t}\gamma^{s-1}$, it is easy to show that $a_t - \gamma a_{t-1} = 1$. Therefore,

$$F_1 - \gamma F_2 = a_T - \gamma(a_{T-1} + \gamma) = 1 - \gamma^2.$$

Similarly,

$$F_T - \gamma F_{T-1} = 1 - \gamma^2.$$

The expression $(1+\gamma^2)F_T - \gamma(F_{t-1} + F_{t-1})$, $t = 2, \ldots, T-1$, can be written as $-\gamma(a_{T-t+2} - \gamma a_{T-t+1}) + (a_t - \gamma a_{t-1}) + 2\gamma - 1 - \gamma^2 + (a_{T-t+1} - \gamma a_{T-t}) - \gamma(a_{t+1} - \gamma a_t) = 1 - \gamma^2$. Therefore,

$$FB^{-1} = e_T', \tag{B.3}$$

$$FB^{-1}F' = 2\sum_{t=1}^{T} a_t - T, \tag{B.4}$$

$$FB^{-1}e_T = T. \tag{B.5}$$

Substituting Eq. (B.4) into Eq. (13) shows V_{HT} equals V_{AM} and substituting (B.4) and (B.5) into (15) shows $\bar{q}_{\mathrm{HT}}$ equals $\bar{q}_{\mathrm{AM}}$. ■

Proof of Proposition 2. Define δ^2_{HT} as the noncentrality parameter for the test using $\bar{\bar{Z}}$ as the instrument set. Similarly, define δ^2_{AM} for the test using instrument set Z^*. Let V_W be the variance of the within estimator. First, I note that $V_{\mathrm{HT}} - \mathrm{V}_{\mathrm{AM}}$ is a positive definite matrix, assuming $\gamma_x > 0$:

$$V_{\mathrm{HT}} - V_{\mathrm{AM}} = T^3\sigma_{\bar{u}}^2\,\frac{\sigma_z^2}{(\sigma_{xz})^2}\left[\frac{1}{(\sum a)_t^2} - \frac{1}{T\sum a_t^2}\right] > 0, \tag{B.6}$$

by Chebyshev's Inequality.

It is easily shown that $V_{\mathrm{HT}}^{-1}\,\bar{q}_{\mathrm{HT}} = V_{\mathrm{AM}}^{-1}\,\bar{q}_{\mathrm{AM}} = (\sum a_t/T)\,(\sigma_{xz}\sigma_{z\alpha}/\sigma_z^2) \equiv \lambda$. Therefore,

$$\delta_{\mathrm{HT}}^2 - \delta_{\mathrm{AM}}^2 = \lambda'(R_{\mathrm{HT}} - R_{\mathrm{AM}})\lambda, \tag{B.7}$$

and is greater than zero if $R_{\mathrm{HT}} - R_{\mathrm{AM}}$ is positive definite, where R_i equals $[V_i^{-1}V_w V_i^{-1} + V_i^{-1}]^{-1}$, $i = \mathrm{HT,AM}$. $R_{\mathrm{HT}} - R_{\mathrm{AM}}$ will be positive definite if $R_{\mathrm{AM}}^{-1} - R_{\mathrm{HT}}^{-1}$ is positive definite. But

$$R_{\mathrm{AM}}^{-1} - R_{\mathrm{HT}}^{-1} = (V_{\mathrm{AM}}^{-1} - V_{\mathrm{HT}}^{-1})V_W(V_{\mathrm{AM}}^{-1} - V_{\mathrm{HT}}^{-1}) + (V_{\mathrm{AM}}^{-1} - V_{\mathrm{HT}}^{-1}). \tag{B.8}$$

Each of the bracketed terms in (B.8) is positive definite, so $R_{\mathrm{AM}}^{-1} - R_{\mathrm{HT}}^{-1}$ is positive definite and $\delta_{\mathrm{HT}}^2 > \delta_{\mathrm{AM}}^2$. ■

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