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Does it all add up? What's the story with mathematical literacy and numeracy? Where to next?

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ACER

Improving Learning

Australian Council *for* Educational Research

The PISA Part of the Story

Lessons about Mathematical Literacy from PISA 2012

Ross Turner

PISA 2012 major headline findings

- Australian student performance in mathematics has declined in absolute terms
- Australian students have slipped relative to other countries
- More Australian students performing at the lowest levels, fewer performing at the highest levels
- Wider gap for girls, and for Indigenous students

[comparisons are with PISA 2003]

A sample (released) PISA item

Sauce

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil:	60 mL
Vinegar:	30 mL
Soy sauce:	10 mL

How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

Answer: ……………….. mL

Some features of this item:

- ‘real world’ context
- Requires some thinking to formulate as a mathematical problem
- [formulate, employ, interpret]
- Little guidance given as to what kind of mathematical knowledge is required
- Level of mathematics not high – the kind of knowledge useful at work and in daily life

56% of Oz kids could do this item – substantially below the OECD average percent correct.

Best-performed by Oz kids:

- Of the 10 items where Oz kids had highest percent correct
  - 5 were among the easiest internationally
  - 5 others were done better by Oz kids than in many other countries
- Comparison with a basket of countries
  - Top performers (Shanghai, Korea)
  - Middle performers, above OECD average (Canada, Australia)
  - Middle performers, near or below OECD average (Denmark, New Zealand, UK, USA)

Oz kids did relatively well with …

- Interpreting data in a table, involving rates (82%, 3rd highest of 69 countries
- Understanding probability outcomes of tossing coins (84%, 7th of 69)
- Interpreting data and extracting specified information from a table (94%, 11th of 69)
- Reasoning about patterns in a straight-forward spatial context (82%, 12th of 69)
- Spatial reasoning – translating a 3-D representation into a birds-eye view (80%, 14th of 69)
- Interpreting a graph – finding and extracting specified data (80%, 23rd of 52)

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Worst-performed by Oz kids:

- Of the 10 items where Oz kids had the lowest percent correct
  - 6 were among the most difficult internationally
  - 4 others were done worse by Oz kids than in many other countries
- Items on which Oz kids performed near to or below the international average
- Items on which Oz kids performed less well than the comparison group

Oz kids did relatively poorly with …

- Formulating an equation that captures a pattern in a spatial context (11%, 13th of 69 countries)
- Formulating a model connecting the diameter of a wheel and the number of rotations needed to travel a given distance (16%, 13th of 69)
- Identifying a pattern in spatial context, formulate it as a relationship to extend the pattern (20%, 22nd of 69)
- Interpreting relationships between variables in an equation (21%, 30th of 52)
- Formulating a geometric relationship by applying Pythagoras's rule in a construction context (4%, 29th of 52)
- Adding sets of 3 three-digit numbers and identifying the highest total (85%, 49th of 69)

One workplace example …

- Drip Rate
  - Medical (nursing) context – setting up an infusion
    - Given a formula connecting drip rate (*D* drops per minute) to drop factor (*d* drops per mL), volume of infusion (*v* mL), and infusion time (*n* hours)

$$D = \frac{dv}{60n}$$

"A nurse wants to double the time an infusion runs for. Describe precisely how *D* changes if *n* is doubled but *d* and *v* do not change."

- What is needed?
  - Reasoning, interpreting and understanding relationships between variables in a formula; writing a conclusion

The key lessons?

How do we encourage our students to see their world through mathematical lenses?

What will it take to increase our students' experience grappling with real-world situations and problems?

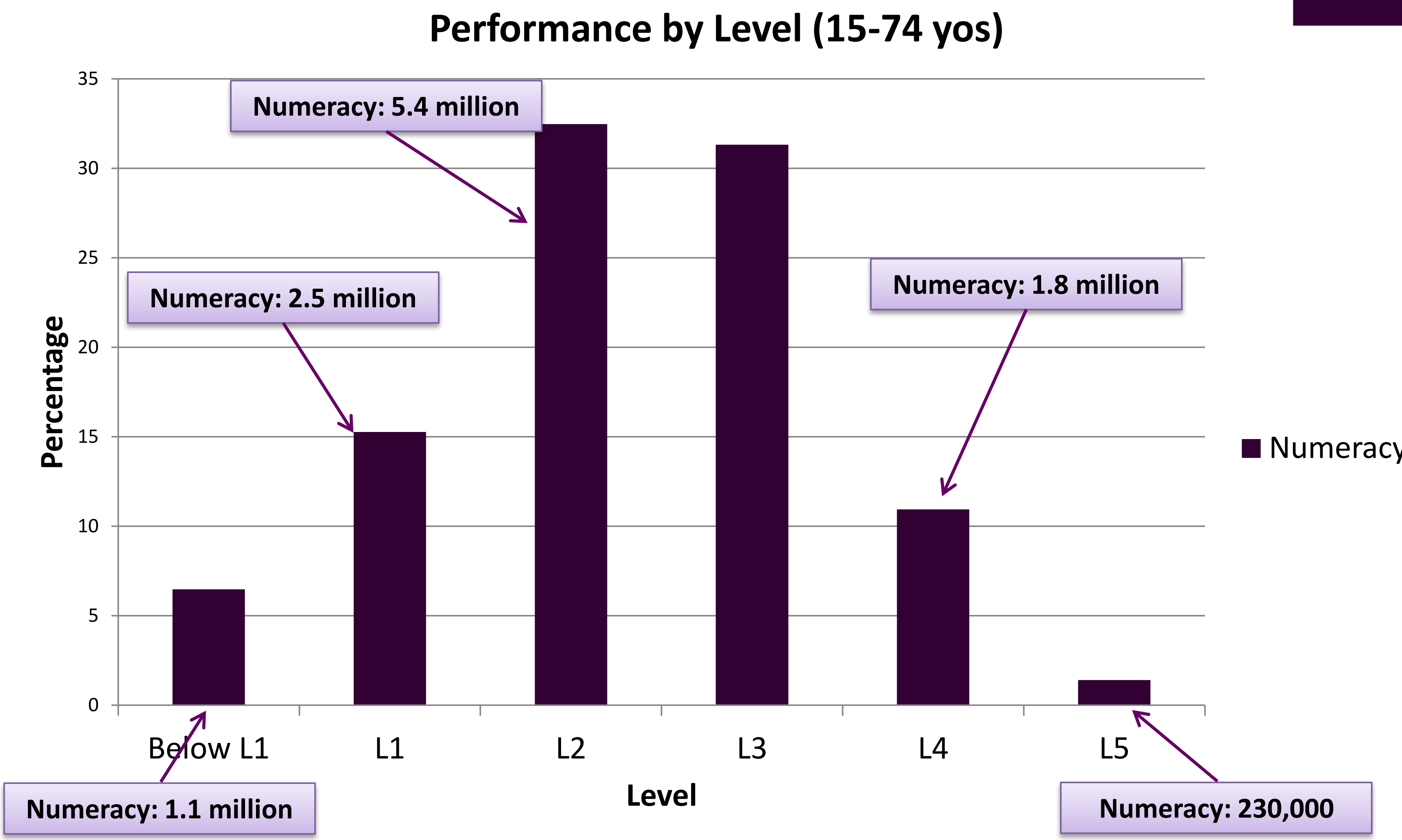
In particular, how can we ensure our students grapple with unusual problems, that are ill-formulated and that require the problem solver to transform the problem into a form amenable to mathematical treatment?

Does it all add up? What's the story with Mathematical Literacy and Numeracy? Where to next?

The Adult Story!

Dave Tout, ACER

The Adult Story!



Proportions of persons in Literacy and Numeracy in PIAAC. Total Australian population aged 15-74 years.

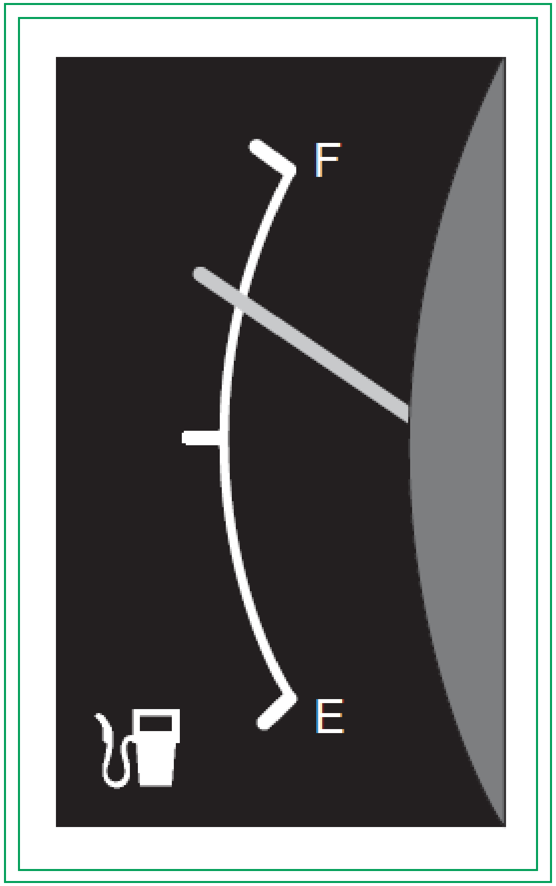
The Adult Story!

Adults were asked to look at a photograph containing two cartons of coca cola bottles (changed to water bottles for PIAAC) and give the total number of bottles in the two full cases.

This was a **Pre-Level 1** item:
Tasks at this level are set in concrete, familiar contexts where the mathematical content is explicit with little or no text or distractors and that require only simple processes such as counting, sorting, performing basic arithmetic operations with whole numbers or money, or recognizing common spatial representations.

1.1 million Australians aged 15-74 years of age are operating at this level.

The Adult Story!



Adults were asked to look at the petrol gauge image. The task states that the petrol tank holds 48 litres and asks the respondent to determine how many litres remain in the tank. A range of answers are allowable as correct.

This was a **Level 2** item:
Tasks in this level require the respondent to identify and act upon mathematical information and ideas embedded in a range of common contexts where the mathematical content is fairly explicit or visual with relatively few distractors. Tasks tend to require the application of two or more steps or processes involving, for example, calculation with whole numbers and common decimals, percents and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables and graphs.

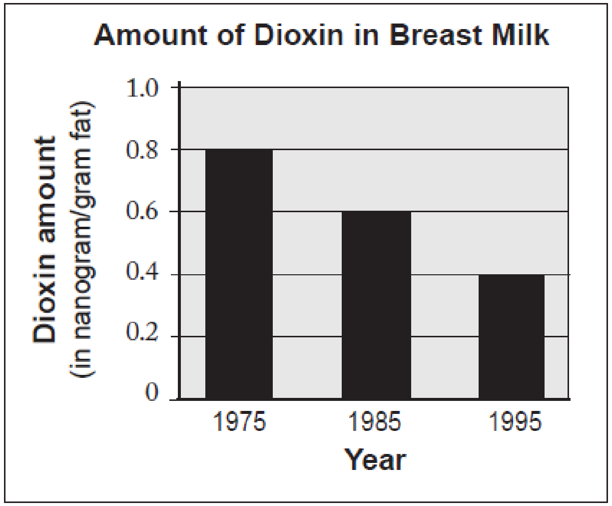
About 3.6 million Australians aged 15-74 years of age could NOT answer this question.

The Adult Story!

Is breast milk safe?

Since the 1970s, scientists have been worried about the amount of Dioxin, a toxin in fish caught in the Baltic sea. Dioxin tends to accumulate in breast milk and can harm newborn babies.

The diagram shows the amount of Dioxin in the breast milk of North European women, as found in studies done from 1975 to 1995.



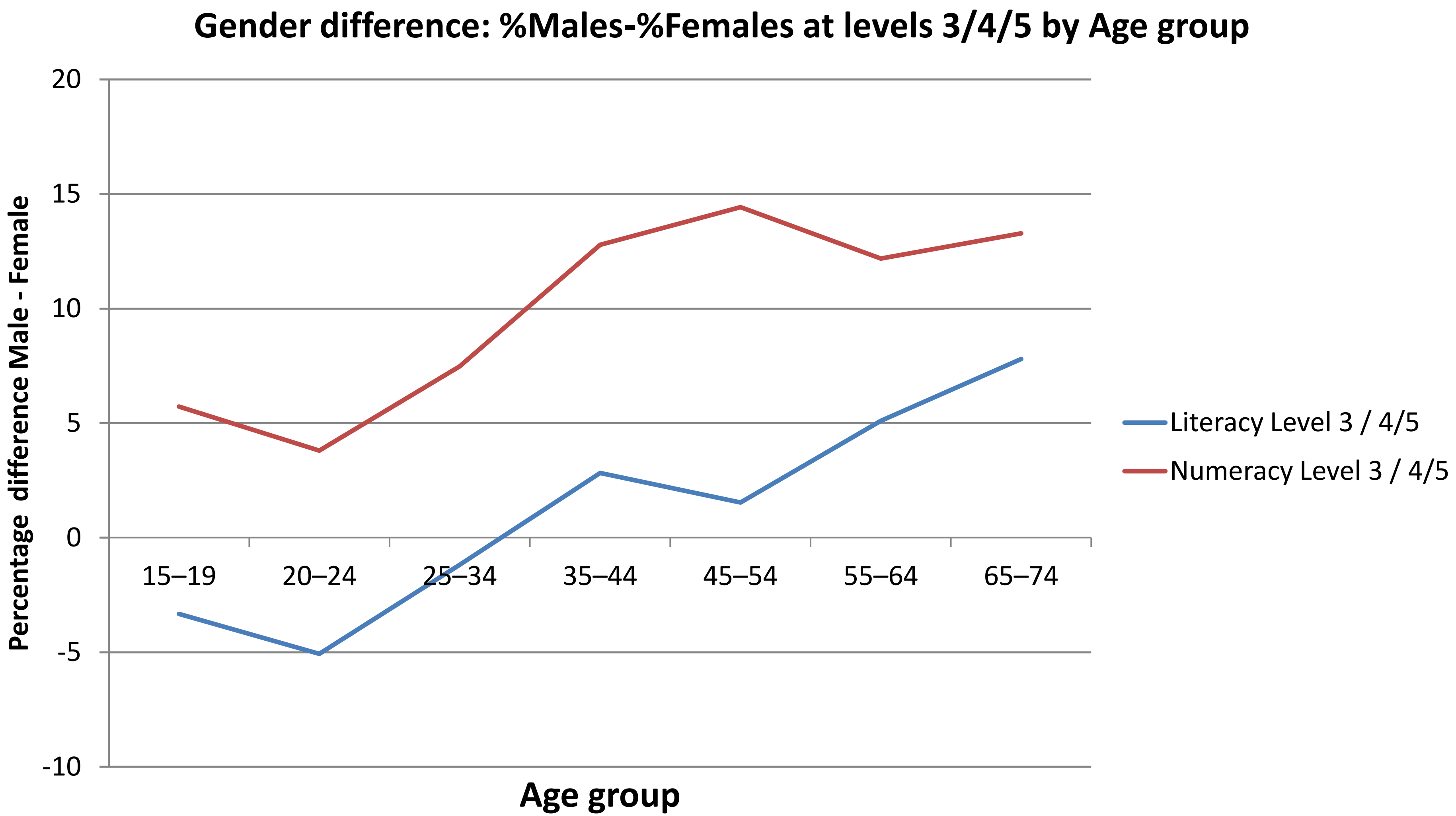
Adults were asked to look at this article and compare the per cent of change in Dioxin level from 1975 to 1985 to the per cent of change in Dioxin level from 1985 to 1995 and argue which per cent of change is larger, and why.

This was a borderline **Level 4/5** item. Level 4 description is:

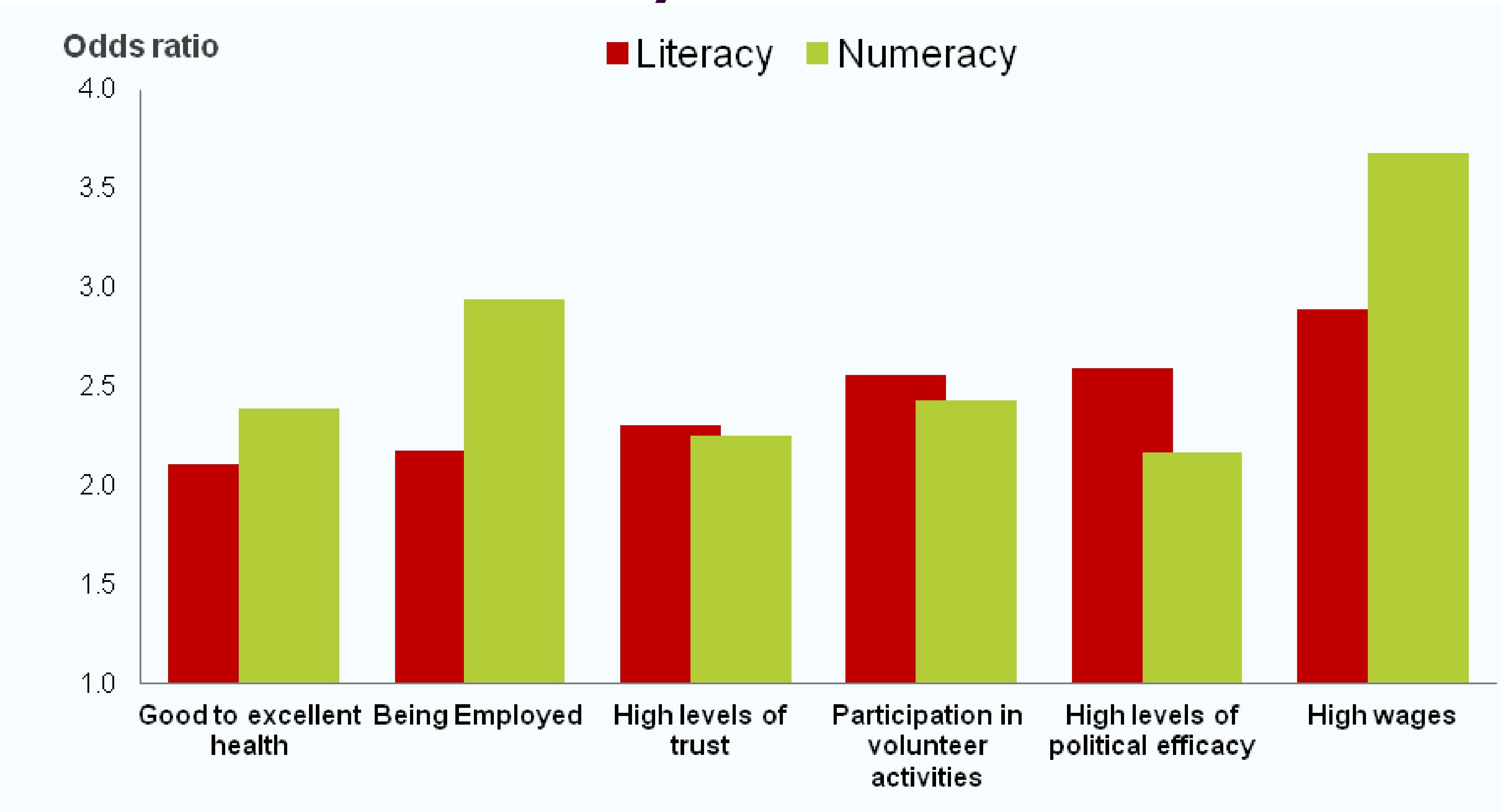
Tasks in this level require the respondent to understand a broad range of mathematical information that may be complex, abstract or embedded in unfamiliar contexts. These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies and processes. Tasks tend to require analysis and more complex reasoning about, for example, quantities and data; statistics and chance; spatial relationships; change; proportions; and formulas. Tasks in this level may also require comprehending arguments or communicating well reasoned explanations for answers or choices.

Only about 230,000 Australians aged 15-74 years of age COULD answer this question.

The Adult Story!



The Adult Story!



As an example of the analytic potential of PIAAC this graph shows that adults with high proficiencies in literacy and in numeracy are much more likely, compared to those with lower skills, to report good health, to be employed, to have higher earnings, and to have positive social dispositions and take part in community life. And that **numeracy** appears to be a more potent predictor of social and economic outcomes such as **health**, **employment**, and **high salary**, compared with literacy.

The Adult Story!

- So some research is indicating numeracy can play a more important role than literacy in both human and social capital terms.
- Another example is the Byrnner and Parsons research from the UK that indicates that for women, while the impact of low literacy and low numeracy is substantial, low numeracy has the greatest negative effect, even when it is combined with competent literacy. … Poor numeracy skills make it difficult to function effectively in all areas of modern life, particularly for women. (Bynner & Parsons, 2005, p. 7)

The Adult Story!

- Other research argues that owing to globalisation and the introduction of technology, workplace numeracy demands are growing rapidly and more workers are now engaged in maths-related tasks of increasing sophistication (e.g. Hoyles et al 2002)
- Need to research more about the important role of maths in work and in life – how important and powerful is it? More of the above research. Are demands and expectations increasing?
- Don’t lower our standards or expectations – counter the community/cultural attitude that it’s OK to not be good at maths. It counts socially and economically.

The key lessons?

- Investing in improving the mathematical literacy/numeracy skills of young people and adults has significant benefits – for the individual, for society and for the economy.
- And it seems that numeracy counts more than literacy – but maybe there are bigger challenges to face?

The key lessons?

How do we encourage our students/adults to see their world through mathematical lenses?

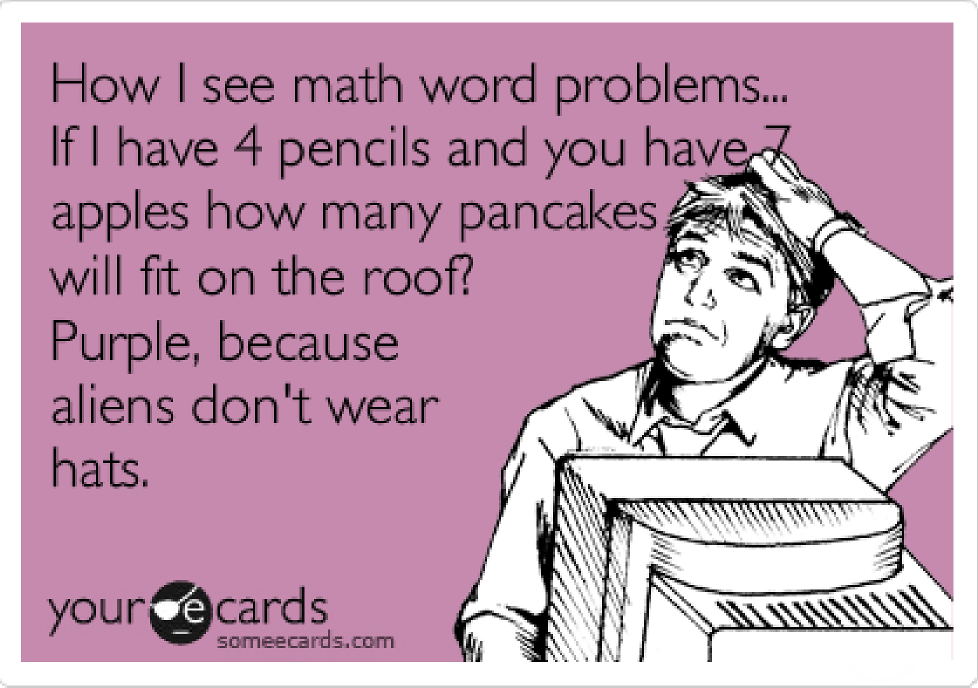
What will it take to increase our students'/adults' experience grappling with real-world situations and problems?

In particular, how can we ensure our students/adults grapple with unusual realistic problems, that are ill-formulated or complex and that require the problem solver to transform the problem into a form amenable to mathematical treatment?

And why do this …

A drum of petrol containing 480 litres was shared between 5 drivers. The first driver took ⅔ of the contents of the drum, the second took ¼ of what was left, and the remainder was shared equally between the last three drivers. How many litres did each of the remaining drivers receive?

Four horses cost as much as 3 cows, 4 sheep as much as 2 horses, and 3 lambs as much as 1 sheep. How many cows could I exchange for 40 lambs?



when we could do this …
… use PISA/PIAAC type items like this

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 pm.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

The key lessons?

- Lynn A. Steen, probably the most articulate spokesperson for “Quantitative Literacy” in the US, states that:

  "...numeracy is not the same as mathematics, nor is it an alternative to mathematics. Today's students need both mathematics and numeracy. Whereas mathematics asks students to rise above context, quantitative literacy is anchored in real data that reflect engagement with life's diverse contexts and situations.

And for the engaged and the disengaged – the context can provide both the challenge and the motivation/purpose.