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Newtonian Time from Timeless Dynamics

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Abstract

The role of a time parameter is vital to physics, yet time is often taken for granted. Newtonian physics assumes an idealized time variable that is unmeasurable. In this paper, we answer calls for clearer accounts of the emergence of Newtonian time from timeless mechanical systems (Smolin 2013). We consider a simple three-particle system in the timeless formalism of Jacobi. Time is abstracted from paths in configuration space and can be viewed as analogous to Mach's principle of universal inertial reference frames. Our physical demonstration of how Newtonian time emerges completes Mach's arguments about clocks and time in classical, non-relativistic systems.

1 Introduction

Newtonian time and its emergence from more fundamental physical systems continues to elicit calls for explanation and reconciliation with those more fundamental descriptions (Smolin 2013 and Callender 2017). Mach famously critiqued Newton's notion of time but did not provide a physical demonstration of his critique (Mach 1919). This paper explains the emergence of Newtonian time from timeless systems by using a simple system that demonstrates Mach's claims regarding time and re-articulates the relevance of those claims to current discussions of time. Mach described how to start with Newtonian mechanics and eliminate time altogether; we return to the logical consequence of not having needed time in the first place. We consider a simple mechanical system in the timeless framework of Jacobi (Lanczos 1970). The system's Lagrangian is defined independently of any time. We construct a clock internal to the system by selecting a particle in the system and taking multiple measures of its position. Time thus reduces to the measurement of particle positions in the system. Rather than beginning with absolute time and eliminating it, we demonstrate the logical point of Mach's critique by showing how the system need not have been formulated with ideal time to begin with and yet will birth Newtonian-like time as a feature of its dynamical behavior.

Section two presents Newton's views on absolute time, Mach's response to Newton, and situates this paper as clearer articulation and demonstration of Mach's point. Section three presents the timeless dynamical system. Section four provides the canonical equations of motion and the Dirac-Bergmann observables. Section five shows the emergent Newtonian clock. We conclude in section six with some implications and next steps.

2 Mach, Newton, and Current Discussions

Newton thought it necessary to posit absolute time. Mach argued Newton was wrong. Since their famous debate, calls and attempts to explain the emergence of Newtonian time and reconcile it with Relativity Theory and Quantum Mechanics continue (Smolin 2013, Barbour 2001, Callender 2017).[1]

Newton says of time that, "absolute, true, and mathematical time, of itself, and by its own nature, flows uniformly on, without regard to anything external. It is also called *duration*."[2] He elaborates

[1] Our daily experiences of the apparent passing of time is sometimes associated with absolute, or Newtonian, time, called 'manifest time' (see Sellars 2017 and Callender 2017, among others).

[2] Newton *The Principia*–all quotes of Newton in this section are from *The Principia*, all the translations are taken from Mach's *The Science of Mechanics*, pp. 222-223.

further on this absolute time saying, "the flow of absolute time cannot be changed. Duration, or the persistent existence of things, is always the same, whether motions be swift or slow or null" (Newton). Newton places these descriptions of absolute time in contrast to what he calls "relative, apparent, and common time," which he takes to be "some sensible and external measure of absolute time (duration), estimated by the motions of bodies, whether accurate or inequable" and which is used in place of true time, such as in the case of hours, days, months and so on (Newton). And these things which are taken to measure absolute time, while often treated as flowing at an ideal and equal rate of motion, are in fact unequal. Thus, all attempts to measure absolute time are at best approximations.

Mach critiques Newton's "medieval" notions of absolute time — where things change *in time* and where time is an invisible thing we attempt to measure indirectly. He argues that Newton ought instead to have viewed time as an abstraction from the changes of things, changes which we observe and of which we are a part (Mach 1919, p. 224). Mach notes that

> time is an abstraction, at which we arrive by means of the change of things; made because we are not restricted by any one *definite* measure, all being interconnected. A motion is termed uniform in which equal increments of space described correspond to equal increments of space described by some motion with which we form a comparison, as the rotation of the earth. A motion may, with respect to another motion, be uniform (Mach 1919, p.24).

In practice, clocks are produced by comparing positions between chosen features of our system. From that comparison we construct an abstracted time which we then use to make additional observations, comparisons, and measurements about the system. So we ought not, Mach argues, imagine anything like Newton's absolute time. Rather,

> the question whether a motion is *in itself* uniform, is senseless. With just as little justice, also, may we speak of an "absolute time" — *of a time independent of change.* This absolute time can be measured with no motion; it has therefore neither a practical nor a scientific value; and no one is justified in saying he knows aught about it. It is an idle metaphysical conception.[3]

In short, we have no good way of measuring an invisible, absolute time. What we do, in practice (aware or unaware), is construct an abstract time based on a clock, which is itself just the comparison of positions of stuff in the physical system.[4] If we take Mach's claims seriously, we must conclude that a clock is a constructed part of the system and does not measure time — it measures position.[5] It is an abstraction, at least on the face of it, because what we measure and observe is not time but, rather, positions of basic stuff within a physical system. With the simple model in this paper, we

[3]Mach 1919, p.24. First, Mach is making a point related to Aristotle's claim that time is inseparable from change and that time is unknowable without change (*Physics III*). Second, Mach proceeds to make a similar argument about space (Mach, 1919, pp. 226-238). It would be helpful to develop a parallel case study to this paper where the focus is on demonstrating the development of abstract space as a metric for articulating relations and features of a system. Third, in this paper we have taken the concept of time to be less fundamental than space and have offered the detailed example of how it is developed in a physical system but our starting assumption should not be taken as a denial of Mach's further and similar point about space.

[4]There remain some metaphysical questions about the conceptual need for time (for more on the metaphysical debates see Maudlin 2007, Deasy 2015, Crisp 2003, Prior 2008, Tallant 2010 and 2017, among others). Whether metaphysicians choose to dither and insist on the logical need for real time, be that in the form of numerical and physical sequence or some more robust notion of time, abstract time is beyond the requirements of a physical system and therefore beyond the scope of this paper as well. The need to sequence events and physical configurations is one motivation for positing an abstract time. However, it is worth noticing that sequencing of events is not necessarily a need in physical spaces so much as in the perceptual and psychological spaces.

[5]Some philosophers of time already press for this sort of understanding of time (see Barbour 2001, Rovelli 2018, Corish 2009, and Gentry 2021, among others).

provide the completion of Mach's argument against Newton's idealized, absolute time by showing how starting in a timeless system we arrive at Newtonian time.

From the outset it is important to distinguish two things we might mean by "Newtonian time," which Newton appears to have treated as two facets of the same concept. First is the idealized, absolute time which is so severely criticized by Mach as "medieval." For Newton, this absolute time is the ontological basis for temporal measurements and experience. Second is the "Newtonian time" implicit in his first and second laws and which is about "good clocks." For Newton, "good clocks" measure time such that isolated bodies move at constant speed. In practice, this second notion of time involves isolating some part of a physical system to function as a good clock. Newton views good clocks as measuring idealized absolute time (if only approximately). While Newton seems to have conflated these two notions – idealized time and "good clocks" – they are distinct. Our paper will not suggest that idealized, absolute time emerges from physical systems but we will demonstrate how good clocks, ones that yield Newton's first and second laws, are constructed from timeless systems. Because these systems are timeless, any choice of clock will do; good clocks are just one of infinitely many valid clocks one could choose.

3 A timeless dynamical system

We consider a simple mechanical system comprised of three identical particles, each of mass m and moving in one dimension.[6] The particle's locations are denoted $x^j \in \mathbf{R}^3$, $j = 1, 2, 3$, which defines the configuration space of the system. The paths taken by the particles in the configuration space are defined as critical points of the action integral:

$$S = \int_I^F ds = \int_I^F \sqrt{2mK(x)\left[(dx^1)^2 + (dx^2)^2 + (dx^3)\right]}, \tag{1}$$

where $K(x) = K(x^1, x^2, x^3)$ is any chosen function; it determines the interactions of the particles with each other and their environment. It should be made clear at the outset that the term "path" will denote a smooth sequence of positions. No parametrization of the paths is assumed. Speaking more physically, there is no time which labels the sequence of positions. The critical points are computed by varying the action over all paths which begin and end at given initial and final points x^j_I and x^j_F, respectively. Conceptually, the action integral defines a notion of dynamical "length" of a path in configuration space and the physically allowed paths are those that minimize that length. Mathematically, this variational principle determines the geodesics in the geometry defined by a line element conformal to the Euclidean line element:

$$ds^2 = 2mK(x)\left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]. \tag{2}$$

What we have been describing is the Jacobi variational principle for mechanics [24]. There is no notion of time present.[7] As mentioned above, the paths in configuration space determined by the Jacobi variational principle are, mathematically speaking, "unparametrized". Physically, the paths are simply continuous sequences of position and there is no information about how they are to be traced out in time. So, for example, the Jacobi variational principle will tell us that the paths taken by the planets are ellipses, but it will not tell us at what speed the planets are moving along the ellipses.

[6] Generalizations such as: adding dimensions to the configuration space, using a non-Euclidean configuration space, or allowing differing masses for the particles are straightforward and do not bring any conceptually new features into play.

[7] While we have done away with time, we certainly have not done away with space. The configuration space is simply assumed to be Euclidean, although any configuration space geometry could be used. This assumption of a fixed spatial geometry is not essential, but allows us to focus on issues associated with time alone. A class of models which generalize the Jacobi-type models and, in particular, have no "absolute space", have been provided by Barbour and Bertotti (Barbour and Bertotti 1982).

For subsequent mathematical analysis, it is nonetheless mathematically convenient to introduce a parametrization, which is, however, arbitrary and has no physical significance.[8] For example, one can parametrize the paths by their arc length s, defined implicitly by (2). In general, if the parameter is denoted τ, the curves are denoted by $x^i = x^i(\tau)$, and the action takes the form

$$S = \int_{\tau_I}^{\tau_F} \sqrt{2mK(x(\tau))\left[\left(\frac{dx^1}{d\tau}\right)^2 + \left(\frac{dx^2}{d\tau}\right)^2 + \left(\frac{dx^3}{d\tau}\right)^2\right]}\, d\tau. \tag{3}$$

The Euler-Lagrange equations for the parametrized geodesics, $x^i = x^i(\tau)$, are then

$$\sqrt{\frac{m\left[\left(\frac{dx^1}{d\tau}\right)^2 + \left(\frac{dx^2}{d\tau}\right)^2 + \left(\frac{dx^3}{d\tau}\right)^2\right]}{2K(x)}}\frac{\partial K}{\partial x^i} - \frac{d}{d\tau}\left(\sqrt{\frac{2mK(x(\tau))}{\left(\frac{dx^1}{d\tau}\right)^2 + \left(\frac{dx^2}{d\tau}\right)^2 + \left(\frac{dx^3}{d\tau}\right)^2}}\frac{dx^i}{d\tau}\right) = 0, \quad i = 1, 2, 3. \tag{4}$$

These equations are rather complicated, but they must be so that they do not depend upon the choice of the parameter τ. A manifestly parameter-independent form of the equations determining the geodesics is:

$$\sqrt{\frac{m\left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]}{2K(x)}}\frac{\partial K}{\partial x^i} - d\left(\sqrt{\frac{2mK(x)}{(dx^1)^2 + (dx^2)^2 + (dx^3)^2}}\, dx^i\right) = 0, \quad i = 1, 2, 3. \tag{5}$$

These equations only depend upon positions and displacements in position.

As mentioned earlier, the function $K(x)$ determines the interactions among the particles and between the particles and their environment. In the simplest case where $K(x)$ is a constant, the Euler-Lagrange equations define straight lines in the Euclidean geometry defined by (2). This can be seen by using the arc length parametrization, defined by

$$\left(\frac{dx^1}{d\tau}\right)^2 + \left(\frac{dx^2}{d\tau}\right)^2 + \left(\frac{dx^3}{d\tau}\right)^2 = 2mK(x), \tag{6}$$

whence the Euler-Lagrange equations (with $K(x) = \text{const.}$) are

$$\frac{d^2x^i}{d\tau^2} = 0. \tag{7}$$

A manifestly parameter-invariant form of this result is to observe that the equations (5) imply

$$dx^i = c^i, \tag{8}$$

where c^i, $i = 1, 2, 3$ are constants. Therefore the ratios of displacements of the x^i must remain the same throughout the curve, thus defining straight lines in the x^i variables.

From the point of view of the Jacobi variational principle, dynamical behavior is represented by unparameterized curves in configuration space. Time evolution is a fiction which we introduce for convenience. In particular, Newton's description of motion is just one of infinitely many possibilities. Time evolution according to Newton's second law is obtained by choosing the parameter τ such that

$$\frac{m}{2}\left[\left(\frac{dx^1}{d\tau}\right)^2 + \left(\frac{dx^2}{d\tau}\right)^2 + \left(\frac{dx^3}{d\tau}\right)^2\right] = K(x), \tag{9}$$

[8]The parametrization allows us to discuss "motion" using differential equations.

whence the equations of motion are

$$m\frac{d^2x}{d\tau^2} = \frac{\partial K}{\partial x^i}. \tag{10}$$

This takes the familiar form of Newton's second law when we make the identification $K(x) = E - V(x)$, where $V(x)$ is the potential energy function and the constant E is interpreted as the value of the total energy. For a given curve in configuration space, the Newtonian time t and the arc length s are related by

$$ds = 2Kdt. \tag{11}$$

As our aim is to consider the fate of this timeless model in the quantum domain, it will be useful to have the Lagrangian,

$$\mathcal{L} = \sqrt{2mK(x)\left[(\dot{x}^1)^2 + (\dot{x}^2)^2 + (\dot{x}^3)^2\right]}, \tag{12}$$

and its Euler-Lagrange equations expressed in phase space form.[9] The canonical momenta are

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}^i} = \sqrt{\frac{2mK}{(\dot{x}^1)^2 + (\dot{x}^2)^2 + (\dot{x}^3)^2}}\,\dot{x}^i = \sqrt{\frac{2mK}{(dx^1)^2 + (dx^2)^2 + (dx^3)^2}}\,dx^i. \tag{13}$$

The last equality in (13) makes manifest the fact that the momenta are defined independently of any time parameter.

The momenta in (13) can be interpreted as normalized infinitesimal displacements in position. Here "normalized" means

$$p_1^2 + p_2^2 + p_3^2 = 2mK(x), \tag{14}$$

a condition satisfied by virtue of the definition of the momenta. This condition represents a constraint to be imposed on the phase space variational principle. We write the constraint as

$$\mathcal{H} = 0, \tag{15}$$

where

$$\mathcal{H} = \frac{1}{2m}\left(p_1^2 + p_2^2 + p_3^2\right) - K(x). \tag{16}$$

Because the Lagrangian $\mathcal{L}$ is homogeneous of degree one in the velocities, the Hamiltonian vanishes,[10] and the phase space Lagrangian L is given by

$$L = p_1\dot{x}^1 + p_2\dot{x}^2 + p_3\dot{x}^3 - \lambda\mathcal{H}, \tag{17}$$

where λ is a Lagrange multiplier.

The meaning of the Lagrange multiplier can be established from the Euler-Lagrange equation for p_i:

$$\dot{x}^i = \frac{\lambda}{m}p_i. \tag{18}$$

From the constraint $\mathcal{H} = 0$ it follows that λ is the rate of change of Newtonian time t with respect to the arbitrarily chosen parameter time τ:

$$\lambda = \frac{1}{2K}\frac{ds}{d\tau} = \frac{dt}{d\tau} \tag{19}$$

The condition $\lambda \neq 0$ may be assumed to ensure only proper parametrizations are being used.

[9] Here a dot denotes a derivative with respect to the arbitrary time τ.

[10] This is a well-known feature of timeless dynamical systems (Sundermeyer, 1982).

4 Canonical equations of motion and Dirac-Bergmann observables

Just as the Lagrangian (12) determines paths in the configuration space variables x^i, so the Lagrangian (17) determines paths in the phase space variables (x^i, p_i). It is from such paths that clocks will emerge, including clocks which tell Newtonian time. Moreover, the set of such paths has a fundamental role to play in the quantum theory.

The parametrized paths are determined by the Euler-Lagrange equations of (17), which are

$$\dot{x}^i = \frac{\lambda}{m} p_i, \quad \dot{p_i} = \lambda \frac{\partial K}{\partial x^i}. \tag{20}$$

The form of the equations – and hence the form of their solution – depends upon a choice for the function $\lambda = \lambda(\tau)$, which amounts to a specification of the parameter τ.[11] The need to specify the function $\lambda(\tau)$ corresponds to the mathematical need to select a parametrization of the solution curves if we are to use differential equations to describe them. The choice of λ is however, completely arbitrary (except as dictated by convenience), and does not, *a priori*, correspond to any particular clock-measurement scheme. However, if a particular observable is used as a clock, then one can infer the corresponding choice of λ. For example, $\lambda = 1$ corresponds to using a clock which reads Newtonian time, while $\lambda(\tau) = 1/(2K(x(\tau))$ corresponds to using arc length along the geodesics as a "time".

To the equations (20) we must adjoin the constraint $\mathcal{H} = 0$. The constraint function $\mathcal{H}$ is easily checked to be a constant of the motion (for any choice of λ) and so is independent of any notions of time.

According to the theory of constrained Hamiltonian systems, the set of curves satisfying (20) and $\mathcal{H} = 0$ will be uniquely characterized by 4 constants of the motion [38]. These constants of motion are often called *Dirac-Bergmann observables.* These observables express relational information about the behavior of the system and, being constants of motion, do not involve any *a priori* notions of time. In the quantum description of constrained Hamiltonian systems one only promotes Poisson algebras of Dirac-Bergmann observables (and functions thereof) to operator algebras on the Hilbert space of states (Sundermeyer 1982, Rovelli 1991 and 1991b, Gambini *et al.* 2009). These observables always exist, at least locally, but their explicit analytic form is usually not available, so let us consider a simple example.

We suppose that the function $K(x) \equiv k =$ constant. This means that the particles do not interact. The Dirac-Bergmann observables can be now be constructed from: (1) the usual angular coordinates in momentum space on the sphere S^2 of radius $\sqrt{k}$, according to the solution of the constraint (15), and (2) from the corresponding infinitesimal generating functions for rotations of this S^2. Explicitly, we define

$$p_1 = \sqrt{2mk}\sin(\beta)\cos(\alpha), \quad p_2 = \sqrt{2mk}\sin(\beta)\sin(\alpha), \quad p_3 = \sqrt{2mk}\cos(\beta), \tag{21}$$

or

$$\alpha = \tan^{-1}\left(\frac{p_2}{p_1}\right), \quad \beta = \cos^{-1}\left(\frac{p_3}{p_1^2 + p_2^2 + p_3^2}\right), \tag{22}$$

along with

$$J_1 = x^2 p_3 - x^3 p_2, \quad J_2 = x^3 p_2 - x^2 p_3, \quad J_3 = x^1 p_2 - x^2 p_1. \tag{23}$$

It is straightforward to verify that α, β, J_1, J_2, and J_3 are indeed constants of motion. The J_i obey an identity: $x^1 J_1 + x^2 J_2 + x^3 J_3 = 0$, so only 2 of the three J's are independent. It is possible to construct 2 independent functions of the J_i, but is usually more convenient to work with this redundant – but globally defined – set of three observables. The Dirac-Bergmann observables $(\alpha, \beta, J_1, J_2, J_3)$ label the unparametrized phase space paths defined by the equations of motion.

[11] We emphasize that the unparametrized paths are unambiguously determined, a parametrization of the paths is completely arbitrary.

5 An emergent Newtonian clock

We now show how time can emerge from the timeless dynamics associated to the Jacobi variational principle. Suppose a degree of freedom, say x^3, is weakly coupled to the remainder of the system. One can idealize this by neglecting the presence of this variable in $K(x)$. In this case the equations of motion have x^3 behaving as a free particle,

$$\dot{x}^3 = \lambda \frac{p_3}{m}, \quad \dot{p}_3 = 0. \tag{24}$$

The dynamical variable

$$T = \frac{m x^3}{p_3} \tag{25}$$

will serve as a convenient clock[12] for the following reasons. The equations of motion for T imply

$$\dot{T} = \lambda. \tag{26}$$

Consequently, if T is used to parametrize the curves in configuration space that represent the motion of the system then $\lambda = 1$. As mentioned earlier, the clock being used is "Newtonian" when $\lambda = 1$, that is, the equations of motion relative to T are given by Newton's second law with $K(x) = E - U(x)$, where E is the value of the total mechanical energy and $U(x)$ is the potential energy function. The clock T allows Newtonian time to "emerge" from relational observations of positions and momenta. These relations are dictated by the *unparametrized* paths taken by the particles in phase space.

To see how this works in a little more detail, let us return to our simple example of non-interacting particles where $K(x) = k =$ constant. In terms of the clock T, the equations of motion and the constraint imply

$$x^1 = \frac{P_1}{m} T + X^1, \quad x^2 = \frac{P_2}{m} T + X^2, \tag{27}$$

$$p_1 = \sqrt{2mk}\sin(\beta)\cos(\alpha) \equiv P_1, \quad p_2 = \sqrt{2mk}\sin(\beta)\sin(\alpha) \equiv P_2, \quad p_3 = \sqrt{2mk}\cos(\beta) \equiv P_3, \tag{28}$$

where the X^1 and X^2 denote the values of the position of particles 1 and 2 when $T = 0$. We can now construct Dirac-Bergmann observables which, when measured, answer the following question: What are the values of x^1 and x^2 when the clock T reads the value c? These observables (still assuming $K(x) = k$) are given by

$$Q^1 = x^1 - \frac{P_1}{m}(T - c), \quad Q^2 = x^2 - \frac{P_2}{m}(T - c). \tag{29}$$

It is easily verified that: (1) Q^1 and Q^2 are constants of the motion, and (2) $Q^i = x^i$, $i = 1, 2$, when $T = c$. These observables are examples of "evolving constants of motion" [29], which exhibit time evolution as a purely relational construction independent of any non-dynamical, externally prescribed time parameter. Such observables have been shown to feature in a conditional probability approach to time evolution in quantum theory [17]; a proposition we plan to explore in a future article.

6 Conclusion

Following Mach, we have used a relatively simple model to show absolute time is not essential to Newtonian mechanics, and that Newtonian clocks can be constructed from the observables of timeless dynamical systems. Time is constructed from the configurations of particles. Effectively, then, the boundaries of any dynamical system should extend to include the clock that we use to

[12] The motion of particle 3 provides a clock only in regions of phase space where $p_3 \neq 0$. For motion which includes $p_3 = 0$ one should select some other dynamical variable to serve as a clock.

measure values of 't'. We have shown in the preceding discussion that this extension is easily done in classical mechanics. The analogous extension in quantum mechanics is problematic, where the clock is treated classically while the rest of the system is non-classical! We do not know how to extend the boundaries of a quantum system to include clocks because it is not clear how best to interpret the 't' that appears. The conditional probability approach of (Gambini *et al* 2009) seems most promising in this regard and we will explore this approach in the context of the Jacobi variational principle in a future work.

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