

Appendix A – Maya Mathematics

The motivation for increasing the knowledge of the cycles of existence led the Maya to develop a systematic way of mapping and knowing the cycles of existence and the interrelations between the cycles. As we have seen in chapter 2, the Maya developed a system of mathematics that allowed them to measure some of these cycles with great precision. The key mathematical tools were the vigesimal number systemⁱ and the arithmetic.ⁱⁱ This appendix serves as a venue for becoming acquainted with the arithmetical computations the Maya certainly used to arrive at their precise accounting of the planetary and other cycles.

The use of such tools allowed them to investigate the mathematical and physical connections between them. For instance, the Maya's desire to know and calculate the cycles of existence—agricultural, meteorological, divine, and physiological cycles, among others—went along with the desire to control these cycles. Maya astronomy, then, developed making use of this competent mathematical system with the purpose to know and control the cycles. Hence, the desire to know the essence of interconnected cycles was the force behind the Maya number system and, in turn, the mathematics they developed allowed them to further their inquiries into the precise nature of those cycles.

The aim of this appendix is to familiarize the reader with how the vigesimal system worked and to practice some simple arithmetical computations. Doing so will prepare the reader for a more robust understanding of the key Maya philosophical conceptions.

In our base ten system we use ten symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to create all the numbers. After we use each of the symbols, we begin a new cycle or place value using those ten basic symbols. After 9, we create a new place value for multiples of ten in front of the single units and we write 10. The units place value begin a new cycle with zero, while the tens place value becomes 1. After 99, we create another place value for multiples of one hundred. So 100 will follow 99 and has three place values: hundreds, tens, and units. The number 100 has 1 in the hundreds place value, zero for the tens and zero for the units. For instance, we can write number 324 is $3 \times (\text{hundreds}) + 2 \times (\text{tens}) + 4 \times (\text{units})$. So our basic cycle in arithmetic is marked by a 10 digit cycle using ten different symbols.

The Maya used three symbols in their base 20 number system instead:ⁱⁱⁱ [some of this material should be familiar for it appeared in chapter 2, but I repeat it here for the sake of continuity and in case someone would chose to read it separately]

For zero they used several symbols, but the shell glyph is one of the most common:



For units a dot or as some prefer to think it, a cacao bean:



For groups of five units as a bar or a branch or five compressed cacao beans:



So they would write number 7:



and number 16 like this:



If you think of numbers in cycles, the unit becomes the standard of measure for each cycle. Five means five cycles of unity. Twenty is twenty cycles of unity or five cycles of four, or four cycles of five or ten cycles of two. Some of these cycles were of great importance to the Maya and so were the numbers associated with these cycles. The number 20 is not only the basis for the numerical system, but the number of digits in our extremities, the number of days in the solar calendar and in the ritual calendar. For the Maya the number five is also a symbol and a cycle of importance since like the number of fingers and toes in each hand and foot, the cycle of five repeats four times in the number twenty which would be represented as four bars. The Mayas figure out that if they kept adding bar after bar and dot after dot would limit the use of mathematics to small numbers.

In their vigesimal system, the Maya also created place values usually on top or in front after the number 19. In this text, I have chosen to put the place values on top, but the choice is arbitrary. So the number 20, was written like this:

	$1 \times (\text{twenty}) = 20$
	$0 \times (\text{units}) = 0 \text{ adds to } 20$

Twenty-one is:

●	1x(twenty)	= 20	
●	1x(units)	= 1	adds to 21

Three hundred forty-two is:

● ●			
≡≡	17x(twenty)	= 340	
● ●	2x(units)	= 2	adds to 342

So we would expect then that larger numbers would demand another place value multiple of twenty in the same way that after 99 we need another place value to create 100. What happens, however, is that since the mathematical system is supposed to serve as a map of the cycles in time and since the most important cycle of existence is the sun, the Maya felt that the solar cycle had to be reflected in their arithmetic. The second place setting (multiples of 20) does not go beyond 360, for 360 (plus five unlucky days) is the number of days it takes for the sun to return to the same location in the sky (approximately, since they knew that it actually takes 365.2422 days). The number 360 would be written as:

●	1x(20x18=360) = 360
⊖	0x(20) = 0
⊖	0x(units) = 0

With larger numbers, this modification of the limit of the second place value to numbers no larger than 17, implies that larger place settings will be modified as well. So we will start at the bottom with multiples of units until we reach 19. At the second place setting, the multiples of 20 will go up until they reach 359 ($17 \times 20 + 19 \times 1$). The third place setting, above the second place setting (as in the previous example) will go up until it reaches 7199 ($19 \times 360 + 17 \times 20 + 19 \times 1$).

The number 16,804 will be:

● ●	2x7200=14400
●	6x360=2160
—————	
● ●	12x20=240
—————	
—————	
● ● ● ●	4x1=4

which adds to 16,804.

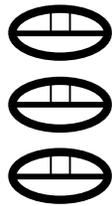
This would be a good time for you to practice writing a few Maya numbers.

Answer in them in your notebook and be sure to bring it to the next class.

Ex 7 (4): Write down the number 476 in Maya numerals.

Ex 8 (4): Write down the number 342,879 in Maya numerals.

Ex 9 (4): What is the following number in Arabic numerals?



Ex 10 (4): What is the following number in Arabic numerals?

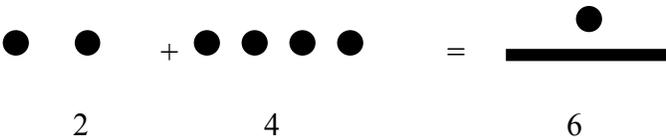


Adding and subtracting becomes an operation of adding and subtracting dots and bars in the appropriate place values. Even though we do not know for certain how the Maya computed with their number system, there is enough indirect evidence to support that they did compute with large numbers and that they could have used multiplication and division.^{iv} We can imagine, then, how addition may work as a matter of adding dots and bars following some simple rules:

- There are three symbols:  for zero or completion,  or unity,  for five 
- Five will always be written as: 
- In each place value, the numbers will range from zero to nineteen. Any multiple of 20 will be carried over to the next place value with one exception:
 - The second place value goes up to seventeen only.

Keeping these rules in mind, let's try our hand at Maya arithmetic.

Let's start with a simple operation:

$$2 + 4 = 6$$


Try it on your own before looking at the answer on this next one :

$$7 + 12 = 19$$

Answer:

$$\begin{array}{c} \bullet \quad \bullet \\ \hline 7 \end{array} + \begin{array}{c} \bullet \quad \bullet \\ \hline \hline 12 \end{array} = \begin{array}{c} \bullet \bullet \bullet \bullet \\ \hline \hline \hline 19 \end{array}$$

Let's try:

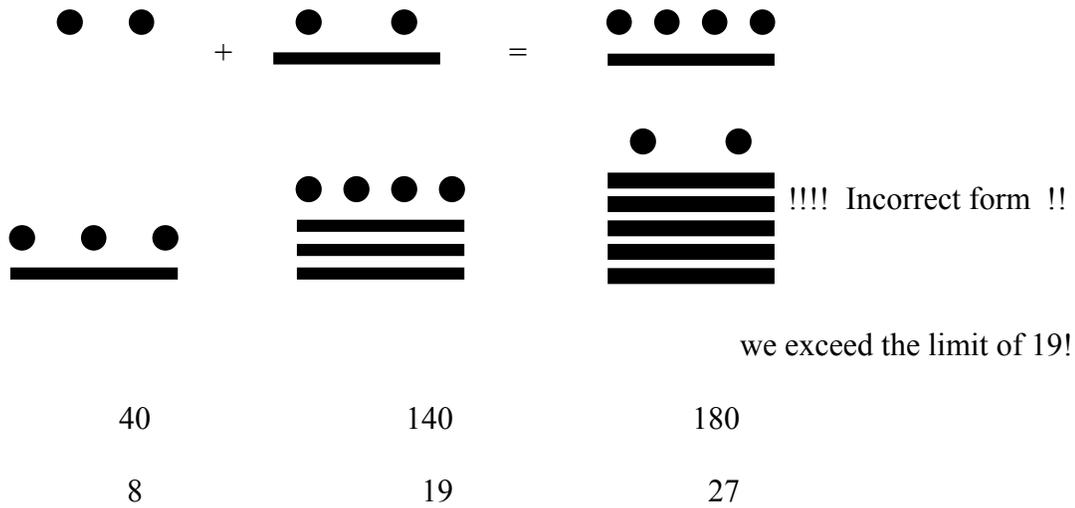
$$13 + 23 = 36$$

Answer:

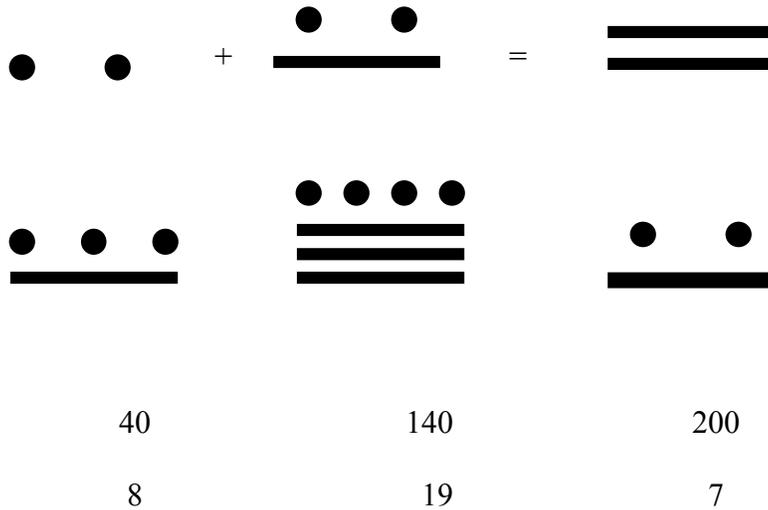
$$\begin{array}{c} \text{⊖} \\ \bullet \bullet \bullet \\ \hline 0 \end{array} + \begin{array}{c} \bullet \\ \bullet \bullet \bullet \\ 20 \end{array} = \begin{array}{c} \bullet \\ \bullet \\ \hline \hline 20 \end{array}$$
$$\begin{array}{r} + \underline{13} \\ 13 \end{array} \quad \begin{array}{r} + \underline{3} \\ 23 \end{array} = \begin{array}{r} + \underline{16} \\ 36 \end{array}$$

Also,

$$48 + 159 = 207$$



Remember that we can only write up to nineteen on each place holder, (except the second one, where we use up to 17). So we bring up 20 units up to the upper place value, as a dot, and we get:



Ex 11 (4): Add the following numbers, using Maya symbols. Then, write the sum in the decimal system using Arabic notation:

												
		+			=							

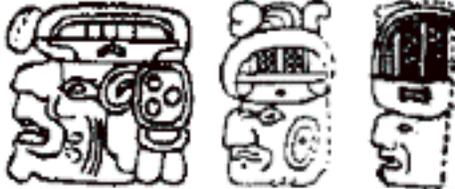
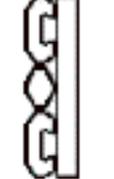
Often the Maya used several different symbols and glyphs to represent the same concept. This is no different with numbers. In some contexts, the Maya preferred to use different variants for numbers. Below are the glyphs, head variants and Yucatec names for the numbers: (Notice how the numbers ten and above are formed)

If you don't see the pictures below, please go to the following sites online:

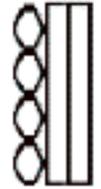
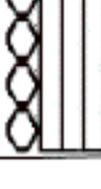
<http://www.halfmoon.org/.cal/num1.gif>

<http://www.halfmoon.org/.cal/num2.gif>

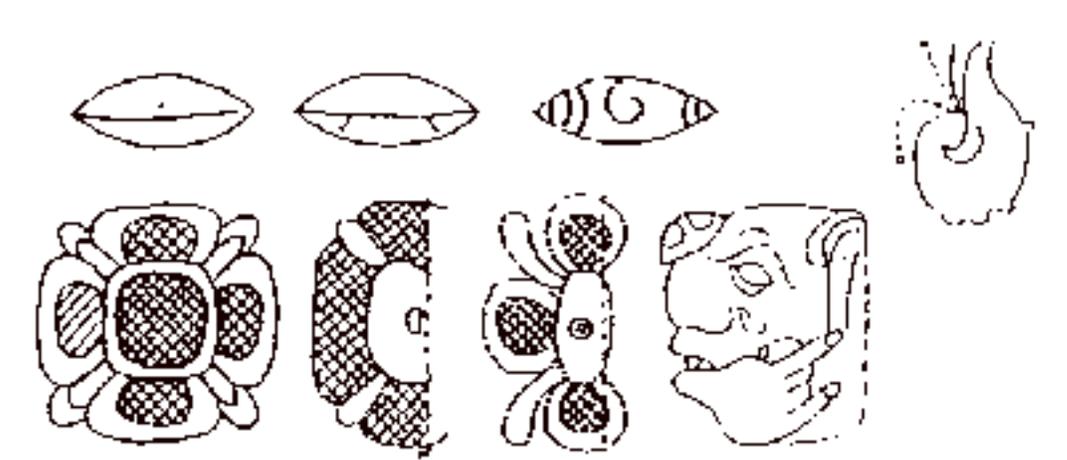
<http://www.halfmoon.org/.cal/num3.gif>

NUMBER ENGLISH & YUCATEC	BAR & DOT MONUMENT	DOT CODEX	HEAD VARIANT'S
ZERO MI			 [SEATING GLYPH]
ONE HUN		•	
TWO KA		• •	
THREE OX		• • •	
FOUR KAN		• • • •	
FIVE HO		—	
SIX WAB		— •	

NUMBER ENGLISH & YUCATEC	BAR & DOT		HEAD VARIANTS
	MONUMENT	CODEX	
SEVEN UX			
EIGHT WAKAH			
NINE BOLON			
TEN LAHUN			
ELEVEN MILIK			
TWELVE LABAH			
THIRTEEN OKLAHUN			

NUMBER ENGLISH & YUCATEC	BAR & DOT MONUMENT ¹ CODEX		HEAD VARIANT'S		
FOURTEEN KANLAHUN					
FIFTEEN HOLAHUN					
SIXTEEN WABLAHUN					
SEVENTEEN UKLAHUN					
EIGHTEEN WAKABLAHUN					
NINETEEN BOLONLAHUN					
COMPLETION					

The Maya also used different symbols for zero or completion. One of them is a conch or shell, another a half of a four-petaled flower:



The Maya were one of the few cultures in the world that created the concept of zero in their mathematics. They were using the concept of zero several hundred years before Europeans learned it from Islam and India. The symbol of zero was helpful for the Maya to indicate that a certain place-value had reached completion and that a new place-value was needed. Such a simple device allows for the computation of large numbers that otherwise would become undoable. As we will see, the concept of zero will carry further significance as completion of cycles. So zero, instead of simply meaning “empty” or “nothing” it indicated to the Maya that a new regenerative process was about to begin. Zero was the beginning, the setting of what was to come.

Below you can find some useful websites to hone your mathematical skills and for further investigation. Some of these sites, however, use the 400 mathematical model, rather than the 360 we are using. So please beware. In this chapter we used the 360 model because (1) it is the model used in calendrical computations and (2) we do not have direct evidence that the Maya actually used the 400 model, even though, they probably did:

<http://www.mayacalendar.com/mayacalendar/mayamath.html>

http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Mayan_mathematics.html

http://math.ucsd.edu/programs/undergraduate/history_of_math_resource/history_papers/math_history_07.pdf

Notes

ⁱ Lambert, Joseph B.; Ownbey-McLaughlin, Barbara and McLaughlin, Charles D. Maya arithmetic. *Amer. Sci.* **68** (1980), no. 3, 249--255.

ⁱⁱ Ifrah, G. *A Universal History of Numbers: From Prehistory to the Invention of the Computer*. London, 1991. Ifrah points out that even though there is only evidence for the 360 model of counting instead of the 400, it would be reasonable to assume that merchants may have used this simpler model.

ⁱⁱⁱ This maya notation gifs are borrowed from <http://www.michielb.nl/maya/math.html>.

^{iv} Lambert, Joseph B.; Ownbey-McLaughlin, Barbara and McLaughlin, Charles D. Maya arithmetic. *Amer. Sci.* **68** (1980), no. 3, 249--255.