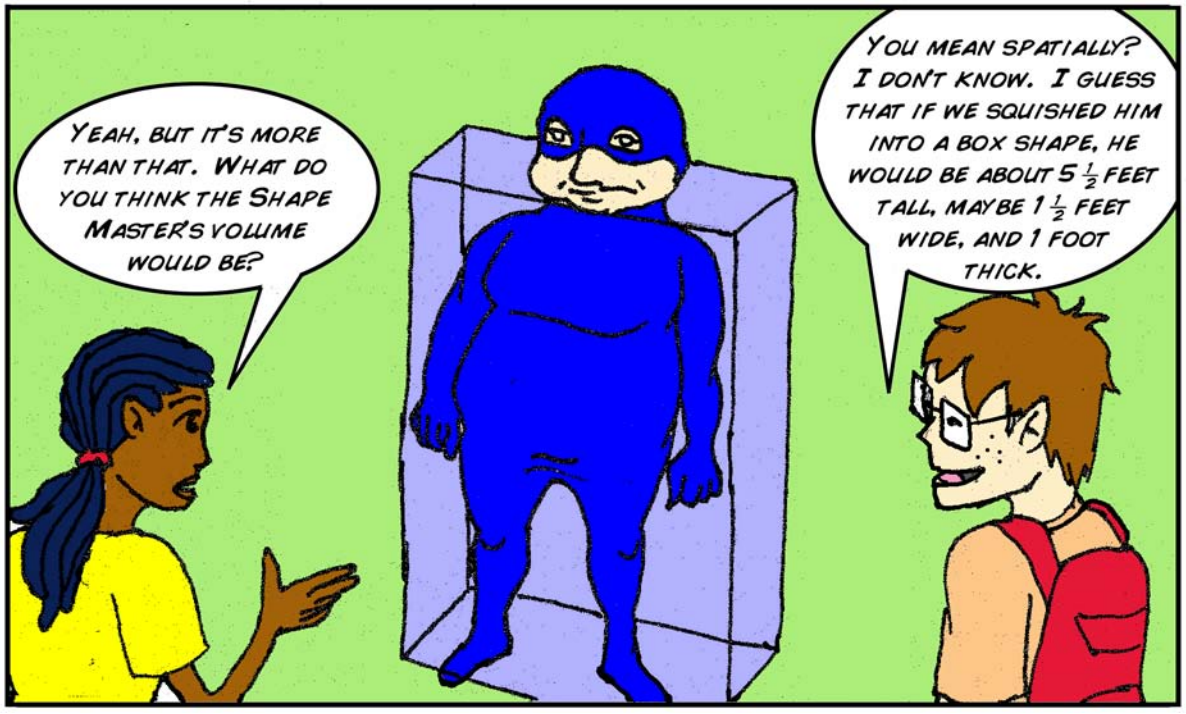
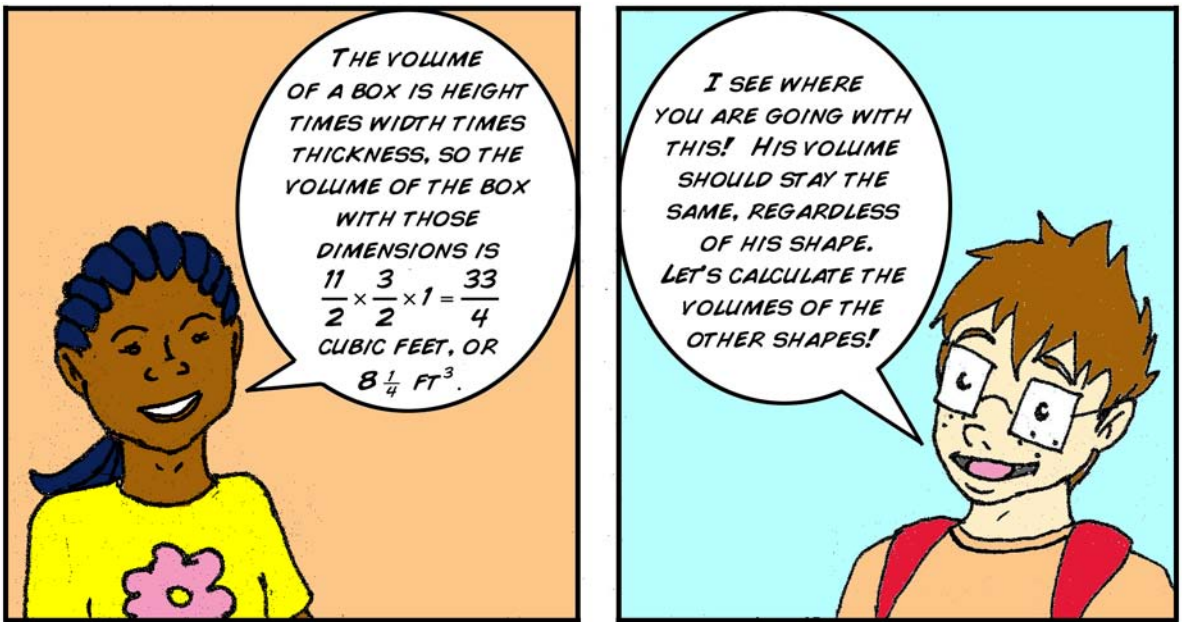
Teacher Supplement
to
Operation Comics, Issue #2

The purpose of this supplement is to provide content support for the mathematics embedded into the second issue of Operation Comics, and to show how the mathematics addresses the content standards provided by the National Council of Teachers of Mathematics (NCTM) for grades 3–5 and 6–8. The mathematics can be segmented into four blocks: the three volume problems worked by Claire and Dillon, and the dimension discussion between Claire and Dillon. The four problems and the content that they illustrate are given below, along with excerpts from the comic. All four problems address both the NCTM standard for problem solving and the NCTM standard for reasoning and proof, for both grade ranges, in addition to the standards addressed below.

Volume of a Rectangular Prism



Claire's inquiry into the volume of the Shape Master shows an understanding of physics as well as mathematics. Although the ability to change shapes is not currently possible to a human being, it stands to reason that such an ability would have to obey the laws governing the conservation of mass. The concept of mass seemed a bit too much for this age range, but having the same mass is equivalent to having the same volume, provided that his density is always constant (same amount of mass per cubic unit of the solid material), so that assumption is in place here. Dillon estimates the Shape Master's dimensions if squished into a box shape as shown above, and Claire quickly calculates his volume using the formula for the volume of a rectangular prism, $V = lhw$, where l, h, and w represent length, width, and heights, respectively, as shown below.



"Divisibility with Claire and Dillon"

The student supplement that accompanies this section of content has the students using the volume formula for rectangular prisms to calculate volumes. Solutions are given below.

1) 4 ft^3 2) 32 ft^3 3) 256 ft^3
Super-Tricky: If the lengths of all of the sides are increased by a factor of 2, then its volume is increased by a factor of 8. If the lengths of all of the sides are increased by a factor of x, then its volume is increased by a factor of x^3.

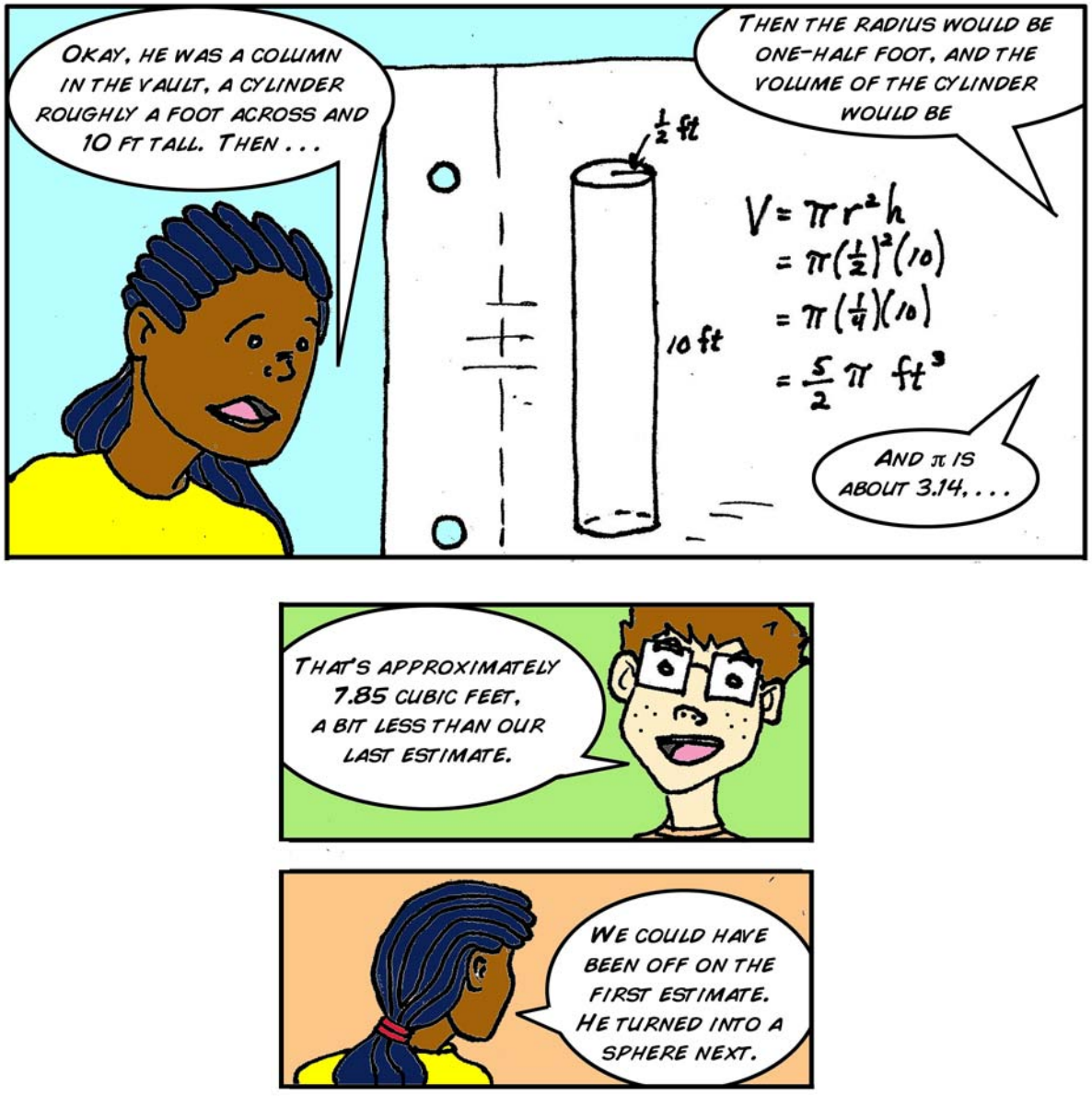
NCTM Standards

This problem addresses the NCTM standard for geometry for grades 3–5, which says that "in grades 3–5, all students should use geometric models to solve problems in other areas of mathematics, such as number and measurement" and "should recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life." The conversion between mixed numerals and improper fractions addresses the NCTM standard for number and operations for grades 3–5, which says that "in grades 3–5, all students should recognize and generate equivalent forms of commonly used fractions, decimals, and percents." The NCTM standard for measurement for grades 3–5, which says that "in grades 3–5, all students should understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute", "understand that measurements are approximations and how differences in units affect precision", "select and use benchmarks to estimate measurements", and "develop strategies to determine the surface area and volumes of rectangular solids" is also addressed.

The problem also addresses the NCTM standard for geometry for grades 6–8, which says that "in grades 6–8, all students should understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects" and "recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life", and the NCTM standard for number and operation for grades 6–8, which says that "in grades 6–8, all students should work flexibly with fractions, decimals, and percents to solve problems",

"select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods", and "develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results." It also addresses the NCTM standard for measurement for grades 6–8, which says that "in grades 6–8, all students should "understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume", "use common benchmarks to select appropriate methods for estimating measurements", and "develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders."

Volume of a Cylinder



The second problem is very straightforward, with Claire and Dillon using estimations of the height and radius of the cylinder to perform their calculations, and then converting their symbolic answer to a decimal for comparison to the first answer.

"Finding Volumes with Claire and Dillon - Cylinders"

The student supplement that accompanies this section of content has the students using the formula for the volume of a cylinder. Solutions are given below.

1) $2\pi \text{ ft}^3 \approx 6.28 \text{ ft}^3$ 2) $16\pi \text{ ft}^3 \approx 50.27 \text{ ft}^3$ 3) $128\pi \text{ ft}^3 \approx 402.12 \text{ ft}^3$

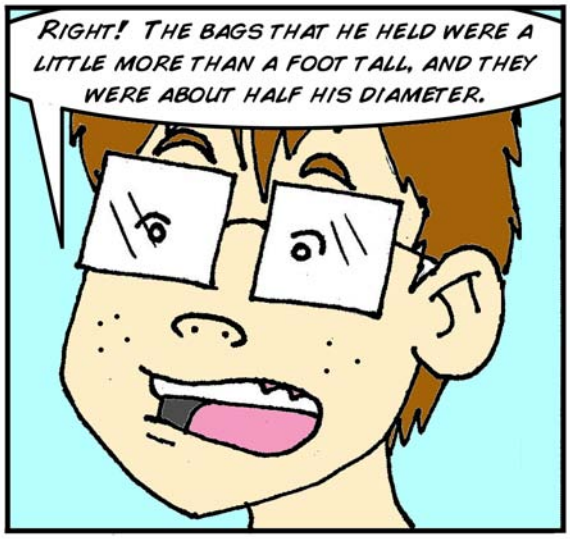
Super-Tricky: If the lengths of all of the sides are increased by a factor of 2, then its volume is increased by a factor of 8. If the lengths of all of the sides are increased by a factor of x, then its volume is increased by a factor of x^3.

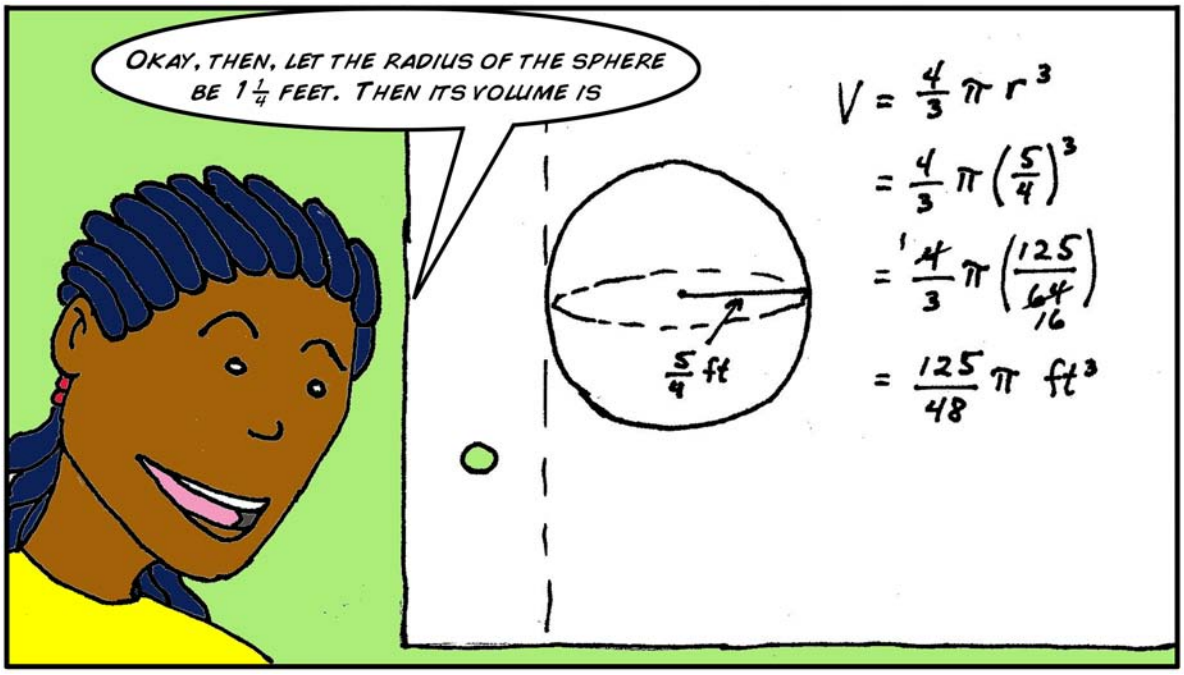
NCTM Standards

This problem addresses the NCTM standard for geometry for grades 3–5, which says that "in grades 3–5, all students should use geometric models to solve problems in other areas of mathematics, such as number and measurement" and "should recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life." The NCTM standard for measurement for grades 3–5, which says that "in grades 3–5, all students should understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute", "understand that measurements are approximations and how differences in units affect precision", and "select and use benchmarks to estimate measurements" is also addressed.

The problem also addresses the NCTM standard for geometry for grades 6–8, which says that "in grades 6–8, all students should understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects" and "recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life", and the NCTM standard for number and operation for grades 6–8, which says that "in grades 6–8, all students should work flexibly with fractions, decimals, and percents to solve problems", "select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods", and "develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results." It also addresses the NCTM standard for measurement for grades 6–8, which says that "in grades 6–8, all students should "understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume", "use common benchmarks to select appropriate methods for estimating measurements", and "develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders."

Volume of a Sphere



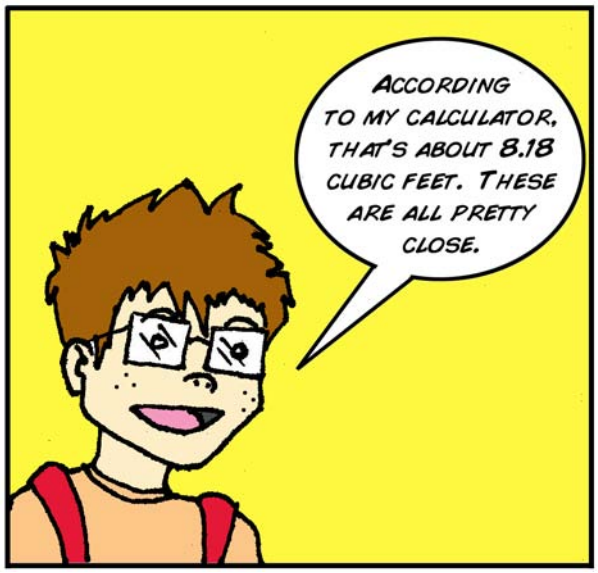


The third problem is also very straightforward, with Claire and Dillon using estimations of the radius of the sphere to perform their calculations, and then converting their symbolic answer to a decimal for comparison to the first two answers.

<u>"Finding Volumes with Claire and Dillon - Sphere"</u>

The student supplement that accompanies this section of content has the students using the formula for the volume of a sphere. Solutions are given below.

1) $\frac{4}{3}\pi \text{ ft}^3 \approx 4.19 \text{ ft}^3$ 2) $\frac{32}{3}\pi \text{ ft}^3 \approx 33.51 \text{ ft}^3$ 3) $\frac{256}{3}\pi \text{ ft}^3 \approx 268.08 \text{ ft}^3$



Super-Tricky: If the lengths of all of the sides are increased by a factor of 2, then its volume is increased by a factor of 8. If the lengths of all of the sides are increased by a factor of x, then its volume is increased by a factor of x^3.

NCTM Standards

This problem addresses the NCTM standard for geometry for grades 3–5, which says that "in grades 3–5, all students should use geometric models to solve problems in other areas of mathematics, such as number and measurement" and "should recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life." The NCTM standard for measurement for grades 3–5, which says that "in grades 3–5, all students should understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute", "understand that measurements are approximations and how differences in units affect precision", and "select and use benchmarks to estimate measurements" is also addressed.

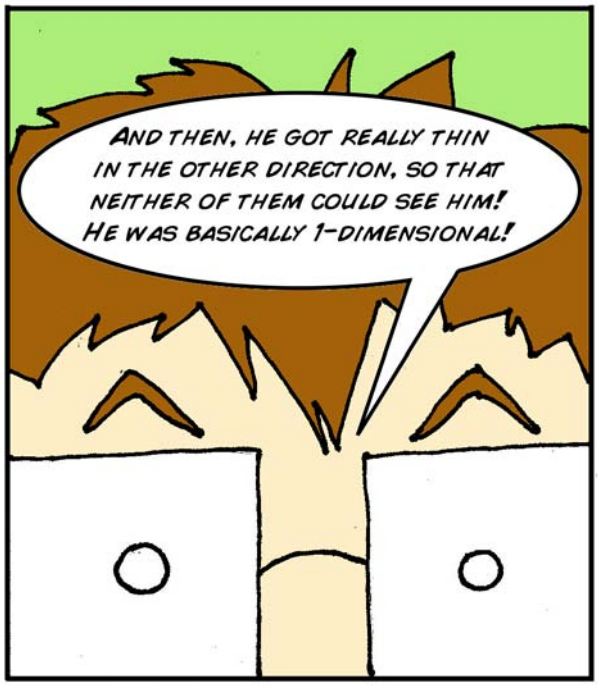
The problem also addresses the NCTM standard for geometry for grades 6–8, which says that "in grades 6–8, all students should understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects" and "recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life", and the NCTM standard for number and operation for grades 6–8, which says that "in grades 6–8, all students should work flexibly with fractions, decimals, and percents to solve problems", "select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods", and "develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results." It also addresses the NCTM standard for measurement for grades 6–8, which says that "in grades 6–8, all students should "understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume", and "use common benchmarks to select appropriate methods for estimating measurements."

2-Dimensional and 1-Dimensional Objects

This last problem was not a computational problem, but one of understanding the physical properties of 1- and 2-dimensional objects, and how they would be interpreted in a 3-dimensional world. I chose not to include a student supplement along with this problem.

NCTM Standards

This problem addresses the NCTM standard for geometry for grades 3–5, which says that "in grades 3–5, all students should use geometric models to solve problems in other areas of mathematics, such as number and measurement" and "should recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life." The conversion between mixed numerals and improper fractions addresses the NCTM standard for number and operations for grades 3–5, which says that "in grades 3–5, all students should recognize and generate equivalent forms of commonly used fractions, decimals, and percents." It also addresses the NCTM standard for geometry for grades 6–8, which says



that "in grades 6–8, all students should understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects" and "recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life."

One last note . . .

While my purpose here is to produce a comic book with embedded mathematical content, and mathematics is my background, I did endeavor to write a good story, and hopefully, the comics can be used for their literary elements as well. The following are a few examples of subtle things at work in the story.

- There are some catch-phrases that are becoming commonplace for Wonderguy to use – "Have a Wonder-ful day!" and "Great googly-moogly!". One could ask students about the advantages and disadvantages of a superhero-type having regularly-used catch-phrases. One disadvantage that comes to mind is that it could jeopardize his secret identity if he continued to use the phrases when not in costume.
- Wonderguy has a tendency to use his great strength as the solution to every problem. One could ask students for examples in the story where this was not the case. The bank manager stopping him from ripping off the vault door, and his "popping" of Wonderguy at the end would be good examples.
- There are plot points and context clues in the story that pertain to how strong Wonderguy really is. One could ask students to give examples that demonstrate how strong Wonderguy is. Good examples would be catching the minivan, his confidence that he could rip the vault door off its hinges, and his comment that the Shape Master "only" weighed "two or three hundred pounds".
- There is some foreshadowing in the story, when the government scientist makes the comment "You ride around on a motorcycle? We should work on that." I have an idea for a future episode where the scientist develops an alternate, more efficient, form of

transportation for Wonderguy. One could ask the students to come up with their own ideas for this new mode of transportation.

This document, as with the student supplements, is a work in progress. Please contact me with corrections or suggestions, and I will make the needed changes. Thanks for inviting Wonderguy into your classrooms, and please encourage your students to contact me with their comments and suggestions for future episodes.