January 2015

Modelling Course II
Cointegration Test

When variables depend on one another

1. Test y & x if AR
2. Run normal regression and see if E is AR
3. If E~AR(1), use
\[ \Delta y = \beta + \Delta x + \varepsilon \]
   
   "Siméon regression" when regress y~AR & x~AR

Dickey-Fuller is test
theory, reality, assumption, proposition, premise, conclusion, certainty, definitely, assuming, precon
modelling inequality empirically in Excel

\[ y = 18 - 4x^{1.4} \]
\[ y = 3x^{1.6} \]

Determine all interior points for dataset

@ Fit non-linear line to max & min

3 write down resulting inequality
How to use differentiation and integration:

Don't just find derivatives, evaluate at one or more interesting points:

\[ \frac{dy}{dx} = 2x^2 - 3x - 6 \]

Integrate and evaluate at a certain point:

\[ \int y = 48k^2 L^2 + C \]
\[ = 650,000 \text{ output gains} \]

Integration involving only when evaluated over a range.
Comparative Statics in Practice (Part I)

1. Regress some 2 indicators on 2 trades:
   \[ T = 3I^2NX^4 \]
   \[ C = 2I^2NX^4 \]

2. Put into matrix form and solve:
   \[
   \begin{bmatrix}
   0 & 1 & 1 & 0 & 2I^2NX^4 & 12I^2NX^2 \\
   0 & 0 & 0 & 1 & 0 & 0.02I^2NX^2
   \end{bmatrix}
   \begin{bmatrix}
   I \\
   N \end{bmatrix}
   \]

3. Regress indicator on "smoothing static variable"
   \[ I = 0.75 \]
   \[ N = -0.62 \]

4. \[ I = 40 + 26^2 \]
   \[ N = 75 - 36^3 \]
Creating a corresponding strategy:

\[ y = a \cdot k \]

- \( a \in \mathbb{R} \)
- \( k \in \mathbb{R}^+ \)

2. To set by coordinates set by...

...cases
writing Excel equations implicitly

1. find entry equation
   \[ y = 2x^2 \quad y = 3 \pm 2\sqrt{x} \]

2. multiply to get real whole numbers
   \[ 10y = 4 + 60x^2 \quad y^2 = 9 + 16x \]

3. rewrite
   \[ 60x^2 + 10y = 4 \quad 16y^2 = 9 \]

Use implicit form when messy coefficients/intercepts
1. put new regression into location
\[
\begin{bmatrix}
6(90+26^2)^2(75-56^2) & \cdots \\
8(70+26^2)^2(75-56^2) & \cdots 
\end{bmatrix}
\]

5. put new regression on LHS and take derivative
\[
C = \begin{bmatrix}
\sum_i (x_i - \bar{x})^2 \\
\sum_i (x_i - \bar{x})(y_i - \bar{y}) \\
\end{bmatrix}
\]

6. cross each different
\[
\frac{dF}{dG} = \frac{4G^2}{76^2} + \frac{1}{36^2} + \frac{1}{8}
\]
\[
\frac{dN}{dG} = \frac{8G^2}{76^2} + \frac{36^2}{8} + \frac{1}{8}
\]
Philosophy of Standard Economics Paper

Produce one yes/no answer

Model to resolve identity

\[ y = \beta_0 + \beta X + \varepsilon \]

\( \varepsilon \sim N(0, \sigma^2) \)

\( \sigma^2 \) must be non-negative

Standardised to get out quickly doesn't do ever!
Writing a theoretical economics paper:

1. Gather data
2. Print equations
3. Analyze equations
4. Formulate theory
5. Test through theory

Next steps:
Charting probability

% default vs mortgage rate

68% probability of default at current mortgage rate.
Second possibility

72% Microsoft
61% Nextel
98% Qsource

\[ \frac{AUS}{\text{ND}} \] money fall of caravel
Probability Grid

68.2%

Use model risk to plot pdf

(can also make df by hand)

- show my choice regression

82% if proxy passed x
How to "shift" a line in Excel

\[ Y = 6 + 8W \]
\[ L = 2 + 4W \]

\[ y_5 = aX^b \]
\[ y_5 = b_0 + b_1 x + b_2 x^2 + \ldots + \]
\[ \rightarrow \text{SVMs} \]
\[ b = 1.8 \]

\[ y_5 = aX^b \]
\[ = b_0 + b_1 x + b_2 x^2 + \ldots + \text{known} \]
\[ \rightarrow \text{SVMs} \]
\[ \Delta = -2.5 \]

1. Regress each variable w/ "shift".
2. Draw out new variable.
3. Calculate equilibrium of new level (if markets).
Use discrete math and set notation anytime do ANOVA

\[ X = \begin{cases} \emptyset & 0 < y < 4 \\ \{5 \leq y < 6\} & 4 \leq y \leq 8 \end{cases} \]

\[ \theta = f(q > 2 | w > 5) \]
Statistics is plop the \( E[\cdot] \)

\[
E[aX] = a E[X]
\]

\[
\text{var} [x] = E[X^2] - (E[X])^2
\]

\[
\text{cov} [x, y] = E[xy] - E[x]E[y]
\]

\[
\text{var} [x+y] = \text{var} [x] + \text{var} [y] + 2 \text{cov} [x,y]
\]

\[
\text{cov} [aX, Ay] = ac \text{ cov} [x,y]
\]
\( \text{Indeed, we have} \)
\[
\text{joint } = \text{pdf } A \text{ and } f(x,y) = f_x(x) \cdot f_y(y)
\]
\[
\text{Risk } \approx \text{mod } = \sigma^2
\]
\[
\text{Use Expectation operator we have} \quad \text{mod } = \mathbb{E}[\sigma^2]
\]
\[
\sigma^2 = \text{cov}(x, y) + \text{var}(y)
\]
\[
x = a \cdot x + b \cdot X
\]
\[
y = c \cdot y + \epsilon
\]
L'Hopital's Rule

1. McKee's only works
   \[ x = 0 \]
   \[ \frac{e^x}{\sin x} \]
   \[ \cos x \]

2. Other methods:
   \[ x = \frac{\sqrt{5}}{\theta} x^\theta \]
   \[ \theta = 5^\theta \]
   \[ \theta = 10 (x-5)^{\theta} \]
   \[ \theta = \frac{\sqrt{5}}{\theta} (x-5)^\theta = (x-5)^\theta \]
   \[ \theta = \sqrt{-\frac{1}{10} \left( \frac{\sqrt{5}}{\theta} - 5^\theta \right)} = x - 5 \]
   \[ x = 5 + \sqrt{-\frac{1}{10} \left( \frac{\sqrt{5}}{\theta} - 5^\theta \right)} \]
How to kill Exponent.

1. Set exponent to 1

2. Do another derivative and take 17

3. Subtract if same exponent

4. Numerically (put in values)

5. Number at x = 0
model proof

$y = \frac{6\times x}{ht}$

Any "proof" just says of logic steps (like this)

- Usually thought explained and economic relations
- Mix "real analysis" and economic reasoning

Corollary: Menu or production bundle into bits
Writing an econometric model

1. Use log regression to find formula for 2-3 variables

2. Use math to calculate FOCs and optimal values

3. Repeat with second equation

4. Use result to make predictions
**Cointegration**

- "Cointegration" just describes if two AR processes regressed against each other

- Cointegration is a weaker description (like non-linearity, robust steadiness, etc.)

- "Unit root" is descriptive

- AECMA/GARCH is descriptive for the unit root is greater.
Series has "feel" of Fourier

\[ x = 9x^2 + 4x^3 \]

M.C. Cheng  \( x = 0 \)

Power  \( x = c \)

Linearisation: Find slope at \( x = c \)

\[ y = \frac{5 + 6x}{5} \]
\[ U = \frac{1}{1-q} C \]

Search

Model

Problems

Engines

Entity

Model to model

Entity

Engines

Model to model

Entity
Math of Time Series

1. regress $y_t = \beta_0 + \beta_1 x_t + \epsilon_t$

2. put in some form

\[ \sum_{t=1}^{T} \frac{\epsilon_t}{x_t} \]

3. Analysis: when converge/diverge

"red analysis" - comparing one or testing about $x, \gamma$, etc.
From model to analysis

**Model**

- **Proposition 1:**
  \[ y \leq \frac{\beta}{\sigma} \text{ when } \sigma > 1 \]

**Regression**

- **Hypothesis 1:**
  \[ \beta_y > 0 \]

- **Proposition 2:**
  \[ \frac{\mu_y}{\mu_x} = \min(p, q) \quad q > \beta \]

- **Hypothesis 2:**
  \[ \frac{d_x}{d_y} < 0.5 \]

Theoretical propositions translate into empirical predictions (hypotheses)
Math - use set math to describe data processing

\[ f = 1 \times 2.053 \]

Literature - out examples in each topic sector

Model - one

Regression - It can't be a paper w/o any regression
Two regressions will be able to solve for \( x, y \)

\[
\begin{align*}
y &= 32 + 4x \\
y &= 7 - 5x
\end{align*}
\]

interests are outliers

\[
\begin{align*}
-4x + y &= 32 \\
3x + y &= 7
\end{align*}
\]

Solving \( y \):

\[
0 = 6x^2
\]

So \( x = 0 \)?

\[-1.6 = 31x \]

\[-\frac{1.6}{31} = 11x \]

[?]
Structured Form

\[ y = \beta_1 C + \beta_2 I \]
\[ y_{\text{r.w.d.}} = \beta_3 C + \beta_4 I \]

Reduced Form

\[ C = \frac{(\beta_4 + \delta_4) I}{\beta_3} \]

Reduced Form coding two opposite forces solves for a variable
How to find joint - need to know conditional. 

Marginal: integrate over the separate variable.

Can leave as abstract

\[
\frac{\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} \, dx \cdot e^{-y^2/2}}{\int_{-\infty}^{\infty} ye^{-y^2/2} \, dy} = g(x, y)
\]
bivariate dist. = joint dist.

For joint, conditional

use ANOVA

cat

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<thead>
<tr>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>152</td>
<td>152</td>
</tr>
</tbody>
</table>

\[ P(D=x_2 | \cdot=2) = \checkmark \]

\[ P(D=x_2 | \cdot=2) = \checkmark \]
Transfer word into end model II

\[ \int e^{x^2} \, dx = \frac{1}{2} \int e^{y^2} \, dy \]

\[ \int e^{-x^2} \, dx = \frac{1}{2} \int e^{-y^2} \, dy \]

\[ \text{var} \, z = E(x^2) - [E(x)]^2 \]

\[ E(x) = \int x \, dx = \int e^{at} \, dx - (2e^{at}) \]

\[ \text{risk} \, z \text{ at } \delta \text{ for } \gamma \text{ in terms of finding } \gamma \text{ mode} \]
\frac{\text{polar rect}}{y = x + 1}

\begin{align*}
\text{0} & \Rightarrow 3\pi \\
y &= \frac{1}{\sqrt{x}} \\
\end{align*}
integrate equality

opposite of differential equality
equality of action, not make

\[ \int_{y}^{\infty} \text{d}(2, 2) \times \text{e}^{y} \text{e}^{-2y} \]

\[ y = \int \text{w}(x) \times dx \]

wages for x
households

- leave in form leave to analytical solution
- wait to leave (motion)
form (don't know wage reaction)
Conditional Prob

$$p_{X|Y}(x|y) = \frac{p(x, y)}{p(y)}$$

Leave this in abstract during modeling.

$$E(Y|X) = \mu_y + \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x)$$

$$\sigma^2_{\mu_x} = \sigma_y^2 (1 - \rho^2)$$

Just plug these into these formulas.
\frac{dx}{dx} vs. \frac{dy}{dx}

like puzzle pieces

\frac{dy}{dx} is each piece of the puzzle
How to model non-linear relation

1. Plot scatter plot
2. Eyeball which best fits
3. Find linear form
   \[ y = x (a-x) \]
   \[ y = \ln x + h \]
4. Regress linear form
5. See if \( \beta = 1 \)
   (or use non-linear in statistics)
Concave or Convex?

Concave

Convex when observed

depends when observed (data) is/are
Identification problem

Drawing study case using data, giving false flat.

Needed to control for other variables.

Can actually put in b-value line to show true and add trend to data.

Use EW to show "true" vs. apparent scatter.
Can bring sun in

But not
denominator

\[ x^2 - 9 = 0 \]

Not \( x^2 + 14x = 0 \) !

My domain = 0

\[ \frac{9x}{x^2 + 2} = 0 \]

But...

\[ \frac{9x}{x^2 + e} = 0 \]
Like CS 50
page has 11 equations

- write page in pseudo-code (often policy
  brisk)
- translate English into maths
- clean up sets, conditional algorithms, etc.
- more parts not part of file conductair
why economics are so boring/unreadable?

- must want reader to believe up is actually down

- review model (identified boxes & arrows)
- review empathy
- robustness

- gives minute so check double check work

not supposed to be narrative
Can save probability
only in relation to distribution

and profit > 810m

and that the
possess has 3
and

62% probability banking crisis in UA

82% end of career drug money
Dividing with 0

\[ 0 < \frac{dy}{dx} < \frac{3x^2 + 18x + 9k}{2\sqrt{x}} \]

Don't divide 0 by denominator!

Beat us to sad + over inequality
using standard economics pages in my usual work

Figure 30 (text box)

Question: How does cat eat deer?

Model: words only

Method: The answer:

Yes $70$

Put "standard" page as Appendix or separate page

Think of economy page as audit "uniting page"
Always define range when presenting variable.

\[ x \text{ is best holding } \]

\[ x \geq 0 \]

\[ x \in [0,1] \]

\[ x \in \{ \frac{1}{3}, \frac{1}{2}, \frac{1}{4} \} \]

Several different range strategies available.
Graph Theory

Can also be concepts/ideas

USA democracy
UK tyranny
Free society
Doing Great
Theory

Solving Nica 201
USA

Solving 13
Ching USA 415

Ching USA 7

need

From To

Number #
$y = \cos 9x$

outer petals

inner petals
\[
\text{polar plot}
\]
\[
y = x + 1
\]
\[
y = \frac{1}{\sqrt{x}}
\]

\[
\theta \geq \pi
\]

\[
\theta \geq \pi
\]

\[
0 \leq \theta \leq 2\pi
\]

\[
0 \leq \theta \leq 2\pi
\]
Cointegration Test

When variables depend on one another
1. Test y & x if AR
2. Run normal regression and see if E is AR
3. If E ~ AR(1), we
   Δy = y + Δx + δ

"Spurious regression" when regress y ~ AR & E ~ AR

Dickey-Fuller is test
Use matrix Econometrics (Granger) to explain and correct regression modeling (non-linearity, heteroskedasticity, etc.)

\[ x \begin{pmatrix} \beta \end{pmatrix} = \nu \]

\[ s = 0 \]
Vector Autoregressions (VARs)

just regression of N variables on each other

\[ \Delta y = a + \beta \Delta x \times \Delta z \]

\[ \Delta x = a - \beta y \times \Delta z \]

\[ \Delta z = a + \beta x \times \beta y \]

USE structural equation USER
To make sure regression results say what you think
Simulate w/ Excel and regress results

model: \( y = \alpha x \)

regression: \( y = \beta_1 x_1 + \beta_2 x_2 + \ldots \)

\( \Delta C = x_1 + 1 \, x_2 \)
Exponents Across the Equals Sign

\[
\frac{3^4}{(x^4)^\frac{1}{4}} = \frac{3^4}{x}
\]

\[
(3^2)^4 = (15^2)
\]

\[
\frac{5^2}{(5^2)^\frac{1}{2}} = \frac{5^2}{\sqrt{25}} = \frac{25}{5} = 5
\]

For \(-x\), minor is \(-\frac{1}{x}\) keeps the sign, just recomputates.
Linear Approximation

\[
F(x) \approx \frac{yA}{\theta} C^0 + \frac{yB}{\theta} (x-1) + yB \theta (x-1)^2
\]

\[
= \frac{yB}{\theta} - yB \theta + yB x
\]

Always linearize at \( x = 1 \)
Joint Probability Distribution (multi-variable)

Can do math on random variables just like random variables...

\[ z = x^2 \]
\[ E[z] = \sigma_h x + \sigma_y \]
\[ \text{Var}[z] = \sigma_x \text{Var}[h] + \beta \text{Var}[h, y] \]

\[ x \sim N \]
\[ y \sim N \]
\[ t = xty \]
Conservative Stokes

Two variables

\[ \frac{\partial^2 f}{\partial y \partial y} = \frac{\partial f}{\partial x} \]

\[ f = \sigma m - (G + A) \sigma - k \sigma \cdot Y - a = 0 \]

\[ \frac{\partial^2 f}{\partial x \partial x} = (G + A) \frac{\partial f}{\partial x} - \frac{\partial f}{\partial x} = 0 \]

\[ \frac{\partial f}{\partial x} = -\frac{k}{2m} \]

When indirectly demonstrated, you must flip "minus flip!"

Use Planck's

Do not take density twice

Instead take implicit derivative!
Also use functional form why don't wastewater equation mess

\[
y = \beta \frac{x^3 y^2}{55} + \left(\frac{x y}{55}\right)^2
\]

\[
y = \beta f(x, y, x, t) + \left[ g(x, y, x, t) \right]^2
\]

\[
y - \beta f(x) + s(x)^2 = 0
\]

\[
\frac{dy}{dx} = -\frac{y_x - \beta f_x + 2 s g x}{1 - \beta g_x^2 s g}
\]
Only take $x = 1$ otherwise doesn't work.

\[ \frac{\phi}{\phi} \times \frac{\phi}{\phi} \]

Amidst the variable and exponent
Hits a demand penalty for lower income

\[ P \]

\[ Q \]

Marshall is "one"
Intervals

1. Doesn't include
   \((5,10)\) \(5 < x < 10\)

2. Includes
   \([5,10]\) \(5 \leq x \leq 10\)

3. Includes members
   \(\{\text{cats, dogs}\}\)

\(e \in \mathbb{U} \cup \mathbb{A}\) or \(e \in \mathbb{A} \cup \mathbb{B}\) and
Describing joint probabilities

\[
\frac{\text{unit (rad)}}{\text{ANS}}
\]

A/\text{\small{S}}

\text{\small{corr}}
\\text{\small{e}}
A/\text{\small{not S}}

\text{\small{D}}
\text{\small{irec}}
\text{\small{AN}}
\\text{\small{St}}

\text{\small{insecting}}
\\text{\small{ANS}}

\text{\small{h}}
\\text{\small{prod}}
\\text{\small{ANS}}

\text{\small{h A \times h B}}
\\text{\small{ANS}}

\text{\small{h - h B}}
Matrix eigenvalues & only 2

\[ y = \mathbf{A} y \]
\[ \mathbf{A} = \begin{bmatrix} 5x^2 & 6xy \\ -9x^2 & 9y \end{bmatrix} \]

\[ \frac{dy}{dx} = -3x^2 y + \frac{dy}{dx} \]
\[ -9x^2 y + \frac{dy}{dx} \]
How to find probability?

25  30

Yes p(A) (x ≤ 25)

1. Use statistics calculator
2. Use binomials etc.
   because easier.

\[ \frac{\text{binom}}{\text{total}} \]

"inverse lookup" - give probability
production = consumption

y = \frac{1 - \theta}{\theta(1 - \gamma)}

\begin{align*}
y_1, y_2 &= c_1, c_2 \\
y_1 &= \frac{1 - \alpha}{\alpha} \cdot \frac{1 - \beta}{\beta} \\
y_2 &= c_1 \cdot c_2 \\
\end{align*}

\begin{align*}
y_2 &= \left[ \frac{c_1}{c_2} \right] \\
y_2 &= \frac{1 - \beta}{\beta(1 - \gamma)} \\
\end{align*}

marginal\text{\ }condition\text{\ }U.I.T.\text{\ }constraint\text{\ }of\text{\ }good\text{\ }2
Solving Jacobi

\[ \text{Use Mode to find solutions. Usually plot fit easy. Find and use the instead.} \]
5 kinds of models

1. X-Y models (and flavors like Dickey-Q)

2. Probability models (Bijes, normal, gamma)

3. Split the pie models (probability, sin, cosine)

4. Variance models

5. Long-term (good, discrete)
model geometry and work ball

- draw graph
- work backward to variable
- tell about geometric similarity in abstract

"Show" results using simulation

- draw and define geometry
decide in math symbols
play symbols

No algebraic manipulation required
modeling w/ state-dependent variables

1. Think about states and interactions

\[ S = \{ \text{good, bad} \} \]
\[ U = \{ (S_1, S_2) \} \]

<table>
<thead>
<tr>
<th>State</th>
<th>Utility</th>
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<tbody>
<tr>
<td>H</td>
<td>30</td>
</tr>
<tr>
<td>L</td>
<td>12</td>
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</table>

2. Express as math
\[ y = 3 + 5(x) \]
\[ U = \begin{cases} 
30 & \text{if } S_{\text{good}} \text{ and } U = 30 \\
15 & \text{if } S_{\text{bad}} \text{ and } U = 15 \\
12 & \text{if } S_{\text{good}}, \text{bad} \text{ and } U = 12 \\
10 & \text{if } S_{\text{bad}}, \text{bad} \text{ and } U = 10 
\end{cases} \]
The Process

1. Regress 2-5 x variables on 2-5 y

2. Equilibrium: solve for the unknown

\[ x = \frac{a - b + c}{d} \]

3. Optimizing: use equilibrium to find best value

y: Comparative Statics:
Go back and cut in fourth variable
Funding Question

- Forget notices

- $G = G(x, y, z, t)$

- \[ G = \frac{1}{2} \]

- \[ G_x = \alpha G_y \]

- \[ y = G \]
Use functional form notation $f(x)$
when there unobservable, unmeasurable variable
(such as capital, productivity, etc.)

\[ \frac{df}{dy} = -\frac{f_z}{Fe} \]

Find derivative by minus drop drop drop
Knock the socks of

\[ \Pr \left( x > y \right) = \int \frac{(x-a)^{\gamma-1}(b-x)^{\delta-1}}{\Gamma(\gamma) \Gamma(\delta)} \, dx \]

\[ y = y + \beta x + \Delta x \]

\[ x \sim N(\mu, \sigma^2) \]

\[ F = \frac{\frac{z}{\sqrt{\sigma^2}}}{\sqrt{\frac{\mu^2}{2\sigma^2} + \frac{\sigma^2}{\mu^2} - 1}} \]

\[ \text{Fit y to} \]

\[ y + \text{de Czako etc} \]
Algebra right or wrong... as long as you have economic intuitions and arguments, you have a paper
Using probability

Present, the set of $\mathcal{D}$:

\[ \sim U[0, R] \]
\[ \sim N[\mu, \sigma] \]

\[ D = N(\mu, \sigma) \]

\[ D, \text{IR}, < D, \text{IR} \]

\[ E[D, \text{IR}] < E[D, \text{IR}] \]

\[ \mu_2 < \mu_1 \]

Can also integrate over age

\[ u = \int_{-\infty}^{\infty} \]
sets and inequalities

\[ y = L^x \]

If \( L = 10 \) for \( x < 0.5 \)
\[ L = 5 \text{ for } x > 0.5 \]
\[ x = \frac{\ln 5}{\ln L} < 3 \]

No intersection

\[ \frac{x-1}{x} \text{ for } x < x < 2x \]
Cointegration

\[ y_{t+1} = \beta y_t + \delta_{t+1} \]

\[ \text{Prob}(y_{11} = 32 | y_t = 30, x_{11} = 0) = 62\% \]

\[ y \sim AR(1,2) \]

Cointegration = \text{linear} + \text{linear}
Heavy-tail Distributions

Model when low prob but many rare events also likely.

F: dist, folded dist

Log-normal and gamma also fall.

Cauchy - large tails

Tukey - 2 - no tails
Don't make a mess - solve (direct + indirect)
implicitly

In a multivariate context,

\[ \frac{\partial f}{\partial x} \] means total derivative (indirect + direct)
How to do Lasering

\[ u = c \]

\[ x, y \]

\[ I = I_0 e^{-a x} \]

\[ \Delta x = x - x_0 \]

\[ \Delta y = y - y_0 \]

Degree the direct independent variable for same degree

\[ D \text{ act in } \Pi = x y - [x y] \]

\[ \text{ do corrective states} \]
From equilibrium to optimization

Use regression to "weed out" each curve, then solve for equilibrium using Gnome's rule.

Use only data for two new regression optimize.
Modeling with matrices

1. Form system of equations

\[ \begin{align*}
&x = 3 + \frac{1}{2}t \\
&y = \frac{x^2}{\sqrt{t}}
\end{align*} \]

\[ \text{the angles vary} \]

\[ 2 \begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix} \]

2. Rate dot, cross, cos/sin etc.

3. Use for learning, revision, etc.
When to use $\frac{\partial a}{\partial x}$ and $\frac{da}{dx}$

$\frac{\partial a}{\partial x}$ is direct effect

Use when don't run through other variables.

$\frac{da}{dx}$ is direct + indirect effect

(\text{influence} \cdot \text{interaction})
Economics Paper

Theory (Thesis)

Economic reasons in food

... anything not to make economic sense... "show" order in models...
Modelling

1. Pick 3 variables
2. Regress until $3 \times 12$

3. Choose math branch
4. Write out
5. Write up
Geometry

1. Find equations
   \[ x(t) = \frac{e^{it}}{t^2(t-1)} \]

2. Split into geometric elements
   circles, hyperbolas, etc.

3. Take "cuts" of metric space

4. Use geometric analysis to "pose" superposition

Geometry just another way to pose superposition
visualse a
Lagrange

think of objective
finding as a field

constraint as specific
line/plane etc.

it's job to scale line
up/down to match
movement in field
Finding an instrument in a covariance matrix

\[ Z = \beta_1 X + \beta_2 Y \]

(associated with another variable but not order)

\[ Z = 0.13 + 0.99 Y \]

(associates with another variable but not order)
Get pretty regression by two steps

\[ y = \beta_0 + \beta_1 x \]

\[ \begin{aligned}
E[y_{\text{pred}}] &= \hat{\beta}_0 + \hat{\beta}_1 x \\
&= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_1 x_i \\
&= \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) + \sum_{i=1}^{n} \hat{\beta}_0 + \sum_{i=1}^{n} \hat{\beta}_1 x_i
\end{aligned} \]

Loge denom in \( X \)

\[ \frac{\partial}{\partial \alpha} = -y + \sum_{i=1}^{n} \frac{\partial (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)}{\partial \alpha} + 2 \sum_{i=1}^{n} \frac{\partial \hat{\beta}_0}{\partial \alpha} + 2 \sum_{i=1}^{n} \frac{\partial \hat{\beta}_1}{\partial \alpha} \]

\[ \frac{\partial}{\partial \alpha} = \frac{-y + \sum_{i=1}^{n} \frac{\partial (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)}{\partial \alpha} + 2 \sum_{i=1}^{n} \frac{\partial \hat{\beta}_0}{\partial \alpha} + 2 \sum_{i=1}^{n} \frac{\partial \hat{\beta}_1}{\partial \alpha}}{\partial \alpha} \]
Argument strategy

1. Formulate and solve in functional form
   \[ g'(\pi, k, b, c, s) \frac{g'}{g'} \]

2. Choose specific functional form

3. Fit in estimable parameters

4. Run simulation

5. Compare with actual data

6. Change 9 parts
   - \( \pi = 0 \) works
   - \( \pi = 5 \) works too

7. \( \alpha \) not in control
Relationships of marginal and elasticities

\[ MR = P \left( \frac{1}{E} + 1 \right) \]

\[ \frac{dMR}{dQ} = P \left( \text{inverse elasticity} + 1 \right) \]

When have two multiplied variables, derivative with \( Q \) equals the product of \( MR \) and \( \frac{dP}{dQ} \times 1 \)

\[ P \left( \frac{dP}{dQ} + 1 \right) \]

\[ MR \]
Frankenstein Model

Variables: B, C, D, \( \delta, \beta, \gamma \)

Use gradient ascent

Major components...

\[ \frac{\text{new} - \text{old}}{\text{step}} \]

Can have alphabet soup b/c we don't solve analytically

Just run model and compare with...
Models for checks

- draw a picture
- describe in words
- use common sense to describe changes
- write in words - describe as function, proposition, etc.
- close parameters slow

Extra credit: show generally out of analysis
\( \frac{d}{dt} \) means total derivative.

This affects several variables and you report all channels or influence, you're talking about total.

Otherwise why channels you specifically want to keep on sides, they use \( \frac{d}{dt} \) constant.
Theory of value

Sections: make new seeds for each new math twist

Oppositions

Each interesting bit of math play

\[ y_1 \rightarrow k^{n} \]

Support w/ economics when they need. Can use cites.

\( \Rightarrow \) Propositions are\( \square \) comments

ideas w/ math of theory
Income/Substitution Effect

\[ U = xy \]

\[ 3x + 4y = 12 \]

- Find \( x \) \& \( y \)
- Use old utility in new price
- Find first order conditions

\[ X_{old} = 5 \text{ units} \]
\[ X_{new} = 2 \text{ units} \]

3 difference

In Excel
1. Find like normal
2. Use new price for old utility

I split this as income/substitute
Proving now with cases...

If no data, try model
\[ y = ax + b + q \]

Cases
Case 1: Chicago 1968
Case 2: Espadrille 1984
How to compare to abstract regimes

1. Define the regime(s) (like into availability)

2. Assume variable disease

   \[ L_1 > L_2 \] when

   more info because

   we trust one

3. Do analysis on \( L_1, L_2 \)

No need to model specific proxy for variable

\[ \text{modelling by } \]

\[ \text{implication} \]
model variance

\[ Y \sim \mathcal{N}(\mu, \sigma^2) \]

\[
\begin{cases} 
\eta_k < 1 \text{ is normally} \\
\eta_k = 1 \text{ are one} \\
\eta_k > 1 \text{ is overfitting}
\end{cases}
\]

\[ Y = \left( \frac{\gamma h}{\eta k} \right)^{1-\mu} \]

So...

\[ T_{\mu, q_1 < q_2} \]

\[
\begin{align*}
\text{blabla blabla blabla blabla}
\end{align*}
\]
Making a Calico Model

\[
Y = \begin{cases} 
\frac{x^2}{2} & \text{if } 0 < \frac{x^2}{2} < \frac{3}{16} \\
\frac{x^2}{2} & \text{if } \frac{3}{16} \leq \frac{x^2}{2} < \frac{27}{32} \\
\frac{x^2}{2} & \text{if } \frac{27}{32} \leq \frac{x^2}{2} < \frac{9}{8} \\
x^2 & \text{if } \frac{9}{8} \leq \frac{x^2}{2}
\end{cases}
\]

1. Define each variable using piecewise categories.

2. Inequalities set "trigger" for end assignment.
Why pull through several graphs?

\[
W = 80 + \frac{1}{2}q + \epsilon \\
L = 3W \\
L_0 = y^2 + \frac{3y}{x} + y
\]

\[
W = 80 + \frac{1}{2}q + \epsilon \\
L = 3W
\]

\[
y = -y \left( \frac{320 + \frac{320}{x}}{2x + \frac{320}{x}} \right) + \frac{\epsilon}{x^2} + \frac{130}{x^2}
\]

Pulling through several regressions allows for comparative statics.
Apparent vs. red relationship

\[ y = \frac{8.0 \times 9.9^2}{\sqrt{2} \times 1.8} \]

\[ \frac{17}{0.8} \approx 20.5 \]

Only get "red" equation by rolling through several regressions.
when agents have a decision to make

1. Use max/min

\[ r^* = \max \begin{cases} \{ \text{tasks}, \text{chance} \\ r \land b, 0 \} \end{cases} \]

2. Use choice

\[ r = \begin{cases} 1 & \text{if } \gamma / \beta > 1 \\ 0 & \text{if } \gamma / \beta < 1 \end{cases} \]

Non-linear relations help produce semantics

[Diagram with two rules for \( y \):]

- \( y = \frac{a}{b} \) when \( r < 1 \)
- \( y = \frac{v_0}{\delta^2} \) when \( r > 1 \)
Talking about elasticity

\[ \frac{dY}{dC} = \frac{\sigma Y}{\lambda} \]

\[ \frac{dY}{dC} \frac{C}{Y} = \frac{\sigma Y}{\lambda} \frac{C}{C} \div \frac{\sigma Y}{\lambda} \]

= \frac{1}{Y}

Elasticity as way of ameliorating variables

After calculating derivative, next critical aspect as elasticity.

Theoretical: derivative
Practical: elasticity
\[ \frac{dy}{dx} \text{ vs } \frac{dx}{dx} \]

\[ \frac{dy}{dx} = \frac{dy}{dx} + \frac{dx}{dx} = \frac{dy}{dx} + 1 \]

\[ \frac{dx}{dx} \text{ is the total differential} \]

\[ \frac{dx}{dx} = \frac{dx}{dx} + \frac{dx}{dx} + \cdots \]

\[ dx = dx \text{ only when } one \]
Difference between
$y = x$ \& $y = \int x \, dx$

Why $\frac{70}{x}$ $y = \frac{1}{2}x^2$?

$y = \frac{1}{4}x^4 \times x^{\frac{1}{2}}$ $\int_{a}^{b} dx$ $y = x^2$ $dx$

mode at $x$: varying somewhere
like $x^3 - x$, just mode
flat
Random Variable Functions

\[ y = x^2 + 1 \quad x \sim \mathcal{N}(3, 4) \]

\[ P(Y \leq y) \]

1. Find CDF

\[
P(x \leq \sqrt{y} - 1) = \int_{-\sqrt{y} - 1}^{\infty} 4e^{3x} \, dx
\]

\[ = -e^{3\sqrt{y} - 1} \]

2. Find new PDF

\[ f(y) = \frac{1}{4y} e^{3\sqrt{y} - 1} \]

Probability below point is function of new variable
Turning world into probability model

\[ x^2 = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-\frac{y^2}{2}) \, dy \]

\[ \text{Joint prob dist} \]

1. Regress relation between variables over a range

2. Set equal to 1 to get joint prob dist.

3. Any joint can integrate for each, find diff for conditional etc.

\[ P(x < 2 | y < 5) = \frac{\int_{-\infty}^{2} \exp(-\frac{y^2}{2}) \, dy}{\int_{-\infty}^{5} \exp(-\frac{y^2}{2}) \, dy} \]