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Does “Get Big Fast” Apply to the Nitrogenous Fertilizer Sector?
Modelling Series: Does “Get Big Fast” Apply to the Nitrogenous Fertilizer Sector?

Bryane Michael, CASE-Poland and Oxford

Abstract

The nitrogenous fertilizer segment hardly represents the most glamorous sector – even if it represents one of the most important. Simply put, we could not eat (or live) without nitrogen feeding our crops. Yet, economists have paid little attention to modelling this boring yet vital market segment. In this paper, we provide a model of the nitrogenous fertilizer sector (SIC codes 2870-3). We develop a simple dynamic model of segment profitability as a function of revenue in a differential equations framework. We show that any strategy to “get big fast” (in revenue terms) depends on structural factors affecting firm profitability. Our models and results should serve other analysts studying the fertilizer industry and potentially a range of other sectors – in the US and in places like Poland.

Keywords: nitrogen fertilizer, differential equations, get big fast.
JEL Codes: C61, M21, N50, Q13
Modelling Series

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Introduction

Amazon (a large US-based internet retailer) made the phrase “get big fast” a mantra for company development in the technology sector. According to this approach, a company which can increase its sales (revenues) dramatically in its industry faster than rivals can achieve higher levels and rates of profitability. The mantra differs from previous views of profitability – which view such profitability as a function of asset sizes or market shares – in that revenue represents the maximand. Such a mantra appears to affect the US nitrogen fertilizer industry – which has only a few large-scale producers. And despite the concentrated nature of the industry, recent low fertilizer prices have prompted calls for further consolidation. To what extent does the US nitrogenous fertilizer industry follow the “get big quick” mantra? Does size (as proxied by revenues) actually help these companies increase their profits?

In this paper, we find that US nitrogenous fertilizer companies’ profitability benefits from expanding sales only under certain conditions. We find that profit depends on a “mystery variable” which does not depend on revenue. We describe – in a differential equations setting -- that variable and discuss the circumstances (parameter values) under which increasing revenues in the industry lead to higher profits. In the first section of this paper, we provide a brief overview of the sector and describe the relevant literature. The second section looks at whether getting big fast actually affects growth. In other words, we look at whether revenue and profit levels influence their rates of change. The third section assess whether changes in revenues correlate with (influence) changes the US nitrogenous fertilizer industry’s profitability. The fourth section explores the profit maximising level of revenues across time – and derives the conditions under which changes in those revenues lead to positive changes in profits. The fifth section looks at the way revenues and profits are distributed over time. To the extent what the past reflects the future, the industry looks neither posed to get big, nor fast. The final section concludes.

Stylized Facts and Literature Review

The nitrogen fertilizer sector represents an important and under-modelled segment of the economy. Figure 1 shows the supply and demand trends of the fertilizer industry in general. As shown in the figure, revenue has remained – and should remain – relatively constant at around $24 billion. Domestic demand for fertilizer (in general) will continue to exceed supply (expressed in revenue and thus market price terms) by about $6 billion per year. Figure 2 shows these trends in more detail. Value-added in the industry (the

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1 The Modelling Series presents models motivating my policy work in the areas of business, law, politics and (of course) economics. The purpose of the series aims at fostering deeper academic debate on the economic models guiding practical recommendations presented in other fora. Special thanks to the CASE for providing the space and dissemination of this series.
market value of the final products minus the costs of inputs) comes to about $3.7 billion per year – or about 15% of the value of sector’s revenue. The trade balance (defined as exports minus imports) comes to a deficit of about $6 billion per year -- providing the fertilizers which top up domestic production in order to meet total demand. Estimates do not foresee any boast in these revenues.

What about the nitrogenous fertilizer segment in particular (SIC codes 2870 to 2873)? While the major US companies in this segment only generated about $44 million in 2013 (and about $7 million in profits), nitrogen fertilizers feed much of the corn and other agricultural commodities US consumers eat every day. Interestingly, no large company has emerged as the undisputed champion in this space. Agrium generated roughly $15.7 million in revenues in 2013, while Mosaic and the Potash Corporation of Saskatchewan generated about $7.5 million in sales. CF Industries generated about $5.5 million – but smaller players like American Vanguard earned only $381,000. No one seems particularly interested or able to get really big really fast.

Little of the extent literature looks at the effect of revenue size on profitability – particularly in the nitrogen fertilizer sector. Many of the studies from the economics literature look at maximising the yield of nitrogen fertilizers (Huang et al., 1992; Tumusiime et al., 2011). Another branch of this literature looks at the environmental impacts of use fertilizer use (Yadav et al., 1997; Fishman et al., 2009). If economic studies look at the determinants of the profitability of fertilizer production and use, such
studies focus on lower income, impoverished economies (Beaman et al., 2013; Duflo et al., 2008). Very few studies analyse specific fertilizer companies in-depth – and model their profitability – with studies like Lie (2008) providing a welcome exception. Except for the odd industry forecasting model – like Vromeen (1991) – few industry-wide models seek to explain revenues and profitability in the nitrogen fertilizer industry.²

Few studies look at the US nitrogen fertilizer industry in particular. Kim et al. (2001) represents an exception to this rule. They specifically look at oligopoly pricing power among US nitrogen fertilizer firms. Using a decomposed negative binomial regression model, they find that certain firms display oligopolistic price leadership. Declining profit margins – in their view – historically have caused producers to consolidate, thereby amassing more revenue on a per-firm basis. Their model and its results obviously imply that increasing revenues should bring increasing profits – otherwise why should these fertilizer producers consolidate in the first place? Laudably, they look prices and the effect these prices have on firms’ sizes and profitability. Thus, indirectly, they paint a story about revenue – to the extent that price determines revenue. Yet, they do not specifically address the top line of the balance sheet in their model.

The Kim and co-authors’ model suffers from several drawbacks. First, they do not actually model the way that revenues and profits evolve from any kind of micro-foundations. They slap a Poisson regression model – an empirical method – on to the data and see what the regression turns up. Indeed, the choice of model already builds in certain assumptions about the way the industry should behave. Second, they – like many in this strand of the literature – look at plant size rather than firm revenue sizes. Such a focus on plant size (and the physical fundamentals driving production) ignore the strongly economics-focus of managerial decision making. Senior managers and investors keep an eye on revenues more than plant sizes and concentration. Thus, any “story” about this segment should include revenues as a key actor.

Stepping back from Kim et al., the existing literature (overall) suffers from several drawbacks which this paper hopes to address. First, we hope to provide a truly dynamic view of profitability. Most researchers “jam” dynamic effects into their models with simple two-stage models, because they lack the intuitions behind differential equations methods they studied as graduate students. Having provided the obligatory model, they proceed to use the standard time series methods we all learn in school. Their modelling provides few true insights that guide their empirical methods. Second, the management literature – and its derivatives in international development – provides data and analysis, without much modelling. Lack of modelling keeps these studies from having a deep understanding of the dynamics driving the industry. Some of the studies from international development do provide models, but in a context very different from the mass produced, large-scale nitrogen fertilizer production employed in the US and other

² The industrial organisation literature serves as an exception. This branch of the literature represents a tapestry of very specific models which explain various aspects of companies’ revenue and profitability. We omit a discussion of this literature because of its generality and its specificity (namely it does not compare with the macro-level analysis we seek to do in this working paper). Slade (2004) provides one example of how – depending on your model of market structure – revenue and profitability relate to each other.
advanced economies. Researchers looking at developed OECD economies will find their studies of limited value.

Our paper adds to the existing literature in two ways. First, we show – using differential equations methods not widely understood even by academic economists – how to decide when revenue growth leads to profitability. To the extent other academics find our methods (or at least the final model we derive) useful, we have contributed to the academic study of the nitrogen fertilizer industry and potentially other industries. Second, we disprove the widely held notion that nitrogen fertilizer companies need to expand sales in order to achieve profit growth. We find that such folk wisdom only holds true under limited parameterizations of our model.

The Data and Methods

To gather the segment data we used for this study, we have downloaded revenue and profit data from the Wharton Research Data Service (WRDS). We added together revenues and profits for each year across the 10 US companies classified by SIC codes 2870-2873. We transformed these totals by differencing and finding the natural log of these totals – in order to run the regressions needed to construct our differential equations. Figure 3 shows the data we used as well as the characteristics of that data.

<table>
<thead>
<tr>
<th></th>
<th>Valid N</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>10</td>
<td>33,740.51</td>
<td>16,519.3</td>
<td>48,892.83</td>
<td>12,124.42</td>
</tr>
<tr>
<td>Profit</td>
<td>10</td>
<td>4,807.26</td>
<td>862.1</td>
<td>9,504.66</td>
<td>3,339.20</td>
</tr>
<tr>
<td>Diff Revenue</td>
<td>10</td>
<td>3,101.95</td>
<td>-11,833.5</td>
<td>11,603.53</td>
<td>7,324.79</td>
</tr>
<tr>
<td>Diff Profit</td>
<td>10</td>
<td>722.15</td>
<td>-3,665.0</td>
<td>5,542.66</td>
<td>2,870.96</td>
</tr>
<tr>
<td>Ln Rev</td>
<td>10</td>
<td>10.36</td>
<td>9.7</td>
<td>10.80</td>
<td>0.40</td>
</tr>
<tr>
<td>LN profit</td>
<td>10</td>
<td>8.16</td>
<td>6.8</td>
<td>9.16</td>
<td>0.93</td>
</tr>
<tr>
<td>Ln diff Rev</td>
<td>7</td>
<td>8.63</td>
<td>7.5</td>
<td>9.36</td>
<td>0.72</td>
</tr>
<tr>
<td>Ln Diff Profit</td>
<td>10</td>
<td>722.15</td>
<td>-3,665.0</td>
<td>5,542.66</td>
<td>2,870.96</td>
</tr>
</tbody>
</table>

Our approach consisted of fitting the data to the various differential equations we describe in the paper. In that way, we could find the best class of differential equation to describe the evolution of the nitrogen fertilizer segment. We use the data from 2003 to 2013 as a “window in time” – providing a class of models which may serve researchers analysing this segments, other segments, or the fertilizer industry in other countries. We report the best fitting model and provide no information about the goodness of fit, either of our parameter estimates or the regression (in general) we used to estimate the differential equations we present in this paper. We did not want to distract attention from our approach by loading the paper with regression analysis. We provide enough information for other researchers to repeat our analysis in case they (you) want to critique our methods.
Does the nitrogen fertilizer industry (in aggregate) reflect the way that a get-big-fast philosophy might affect companies operating in that industry? Figure 4 shows the correlation in revenues between companies making up the aggregate we model in this paper. Revenue tends to rise and fall relatively similarly across US fertilizer companies. As such, “playing” with aggregate revenue figures tell us something about the individual companies in the industry segment. We do not show the data, but profit behaves similarly. We encourage other researchers disgruntled with our lack of reporting on the statistical side of our reporting to repeat our work.

Figure 4: Most Companies’ Revenue Correlates with Other Companies’ Revenue

<table>
<thead>
<tr>
<th>Means</th>
<th>Std.Dev.</th>
<th>AVD</th>
<th>FMC</th>
<th>Potash</th>
<th>SMG</th>
<th>RTK</th>
<th>TNH</th>
<th>AGU</th>
<th>SOYL</th>
<th>MOS</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>236.6</td>
<td>82.0</td>
<td>1</td>
<td>0.96</td>
<td>0.77</td>
<td>0.53</td>
<td>0.90</td>
<td>0.76</td>
<td>0.936</td>
<td>-0.72</td>
<td>0.65</td>
<td>0.94</td>
</tr>
<tr>
<td>2832.8</td>
<td>675.6</td>
<td>0.96</td>
<td>1</td>
<td>0.85</td>
<td>0.69</td>
<td>0.95</td>
<td>0.82</td>
<td>0.98</td>
<td>-0.78</td>
<td>0.75</td>
<td>0.95</td>
</tr>
<tr>
<td>5647.6</td>
<td>2449.1</td>
<td>0.77</td>
<td>0.85</td>
<td>1</td>
<td>0.64</td>
<td>0.75</td>
<td>0.97</td>
<td>0.84</td>
<td>-0.50</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>2709.1</td>
<td>397.7</td>
<td>0.90</td>
<td>0.95</td>
<td>0.75</td>
<td>0.64</td>
<td>1</td>
<td>0.58</td>
<td>0.62</td>
<td>-0.43</td>
<td>0.80</td>
<td>0.55</td>
</tr>
<tr>
<td>140.2</td>
<td>118.2</td>
<td>0.90</td>
<td>0.95</td>
<td>0.75</td>
<td>0.64</td>
<td>1</td>
<td>0.78</td>
<td>0.89</td>
<td>-0.68</td>
<td>0.62</td>
<td>0.83</td>
</tr>
<tr>
<td>602.9</td>
<td>177.5</td>
<td>0.76</td>
<td>0.82</td>
<td>0.97</td>
<td>0.58</td>
<td>0.78</td>
<td>1</td>
<td>0.80</td>
<td>-0.40</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>8966.0</td>
<td>5462.3</td>
<td>0.94</td>
<td>0.98</td>
<td>0.84</td>
<td>0.62</td>
<td>0.89</td>
<td>0.80</td>
<td>1</td>
<td>-0.85</td>
<td>0.72</td>
<td>0.98</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>-0.72</td>
<td>-0.77</td>
<td>-0.50</td>
<td>-0.43</td>
<td>-0.68</td>
<td>-0.40</td>
<td>-0.85</td>
<td>-0.38</td>
<td>-0.77</td>
<td>1</td>
</tr>
<tr>
<td>7563.8</td>
<td>2872.2</td>
<td>0.65</td>
<td>0.75</td>
<td>0.88</td>
<td>0.80</td>
<td>0.62</td>
<td>0.83</td>
<td>0.72</td>
<td>-0.38</td>
<td>1</td>
<td>0.78</td>
</tr>
<tr>
<td>3436.9</td>
<td>1789.4</td>
<td>0.94</td>
<td>0.95</td>
<td>0.88</td>
<td>0.55</td>
<td>0.83</td>
<td>0.84</td>
<td>0.98</td>
<td>-0.77</td>
<td>0.78</td>
<td>1</td>
</tr>
</tbody>
</table>

The figure shows the correlation coefficients for revenues earned by the companies shown in the figure from 2003-2013. We highlight correlation coefficients NOT significant at the 5% level. The tickers shown in the figure represent American Soil Technologies (SOYL), Mosaic (MOS), CF Industry Holdings Inc (CF), Agrum (AGU), American Vanguard Corporation (AVD), Scotts Miracle Grow (SMG), Rentech (RTK), Terra Nitrogen (TNH), Potash Corp. of Saskatchewan (POT) and FMC Corp (FMC).

**Does revenue and profit exhibit a size effect?**

In a “get big fast” industry, revenue would increase over time as a function of existing revenue. Equations (1) and (1a) describe the relationship (as found by a best-fit regression) between the yearly change in revenue $y_R$ and profits $y_\pi$ from 2003 to 2013 in the US nitrogenous fertilizer industry. In both cases, a quadratic equation (a Riccati equation) best fits the data we described previously. Equations (2) and (2a) show the solution to each of these equations of motion.

\[
y_R = 0.008y_R^4 + 0.0001y_R^3 + 0.007y_R^2 + 16.881y_R + 138746 \tag{1}
\]

\[
y_\pi = 0.0004y_\pi^4 + 0.0006y_\pi^3 + 0.005y_\pi^2 + 12.4y_\pi - 7930 \tag{1a}
\]

\[
\ln y_R = \left[ \frac{460 + 443i}{0.01 - 0.01i} \right] - \left[ \frac{460 - 443i}{0.01 - 0.01i} \right] + \left[ \frac{450 + 470i}{0.01 - 0.01i} \right] + \left[ \frac{450 - 470i}{0.01 - 0.01i} \right] - 0.002t + c \tag{2}
\]

\[
\ln y_\pi = \left[ \frac{6.3 + 1.30i}{0.01 - 0.01i} \right] - \left[ \frac{6.3 - 1.30i}{0.01 - 0.01i} \right] + \frac{122}{0.08 - 0.01i} + \frac{135}{0.07 - 0.01i} - 0.002t + c \tag{2a}
\]
The solution to these equations exhibit several interesting features. First, any solution for the data coming from the US nitrogen fertilizer industry requires an imaginary number – and thus an extra degree of freedom – to solve.\textsuperscript{3} Using revenues and profits simply isn’t good enough – even before confronting the model seriously with more elaborate data. Another variable affects the way revenues and profits change over time. Second, the appearance of conjugates suggests that, whatever the omitted variable from our model, that variable likely consists of two competing tendencies. For example, if plant sizes affect US fertilizer industry revenues and profits (just to take an example), then the effect of this competition likely exhibits contradictory tendencies – such as economies of scale and X-inefficiencies coming from size.\textsuperscript{4} We can not know, without looking at the panoply of possible independent variables affecting revenues and profits, what such a factor might be.

The model driving the US nitrogen fertilizer industry probably simplifies to a simple expression. We can express revenues and profits as a function of time and the unknown variable (which we relabel as $z$). We can also remove the higher-order coefficients from our model, in order to simplify it. Equations (3) and (3a) result from these simplification to the base model portrayed in equations (1) and (1a). Removing the specific parameters we used from the fertilizer industry from 2003 to 2013 and using parameters instead, we can portray the evolution of revenues and profits over time as shown in equations (4) and (4a).

\begin{align*}
y_R &= 16800 + 7270 * \left[ z \left( e^{(10175r-5000)z} - e^{(10175r+5000)z} \right) \right] \\
y_\pi &= 31 + 40 * \left[ 6z \left( e^{(1.3r-2500)z} - e^{(-1.3r+2500)z} \right) \right] \\
y_i &= a e \left( \frac{\alpha}{\beta} \right)^z i \quad \text{for} \quad i \in [R, \pi] \\
y_i &= a + \beta e^{\pi + \phi} \quad \text{for} \quad i \in [R, \pi]
\end{align*}

The differences between equation (4) and (4a) show how excluding higher-order terms from the model can affect a model of the nitrogen fertilizer industry’s revenues and profits. In equation (4), revenues and profits grow as the result of a growth factor $\alpha$, yet fall as the result of a constraining factor $\beta$. In the simplified model (which excludes higher order independent variables), the factor $\phi$ and $\gamma$ may move in the same – or opposite directions. If $\phi$ is negative, then $\phi$ works the same way as $\beta$ in the first expanded expression.

\textsuperscript{3} No consensus exists among mathematicians or economists about how to interpret imaginary numbers. Svetunkov (2012) provides a useful guide to using complex numbers in economic analysis.

\textsuperscript{4} If the Kim et al study truly represents a collectively exhaustive model of the segment, then industrial concentration and price movements may represent such omitted variables in our own depiction of this fertilizer industry segment.
Second, unlike the previous studies, we find that over time revenues and profits decrease (as denoted by $1/t$) in the absence of scale and other factors. In the simplified model, revenues and profits may increase or decrease over time – depending on the model’s parameterization. Third, the variable $z$ plays a much larger role in explaining profits and revenues in equation (4a) than in equation (4). As an illustration of Occam’s razor, any empirical model which fits the data more closely actually results in a simpler empirical model. Given our analysis from a previous study, equation could very well explain the data better – even if the simpler more may prove more useful to other academics in other sectors.  

The model predicts the way revenues and profits change when the mystery variable and time change. Taking equation (4a), we can differentiate by the mystery variable $z$ and by time to come up with the equations (5). Equation (5) suggests that a small change in the omitted variable should cause revenues and profits to change by the multiplier $\phi$. Thus, researchers using the standard regression formula will find a beta coefficient whose value equals equation (5) so that we arrive at equation (6).

$$\frac{\partial y_i}{\partial z} = \beta e^{\phi} (\varphi + 1)$$  
$$y_i = \alpha + \beta e^{\phi} (\varphi + 1)z$$

The reader will notice from equation (6) that the beta coefficient represents a complex version of the dependent variable itself. The econometrician can not resolve the lack of independence between regressor and explanandum by conventional means. Moreover, the dependent variable does not represent a simple linear function of $z$. Any errors in the regression likely would also reflect $z$ (as a function of $y$) ensuring the break-down of the standard regression model. Taking logs or differencing – while potentially producing a stable ARIMA model – clearly would not solve the problem illustrated in equation (6).

**Does revenue get-big-fast affect profits?**

Does profit depend on revenues? The short answer is yes, and no. Equations (7) and (7a) show the relationship – as derived from the best model in regression analysis -- in the data between total profits and revenues between 2003 and 2013. Equations (8) and (8a) show the relationship between revenues and profits when allowing for both variables to determine revenue and profit respectively. As shown by equations (7) and (7a), profits clearly grow as a function of revenue over time. The data show what we already knew – that profit levels come to about one-fourth of revenue levels and that the growth in profits comes to about one-third the growth in revenues. Equation (8) shows the way that the growth over time of profits depends on the level of revenues and profits in the US.

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5 This working paper accompanies a Seeking Alpha article in which we reviewed the dynamics of this segment. See Bryane Michael, Terra Nitrogen and the Nitrogenous Fertilizer Industry, available online at: [http://seekingalpha.com/article/2698345-terra-nitrogen-and-the-nitrogenous-fertilizer-industry](http://seekingalpha.com/article/2698345-terra-nitrogen-and-the-nitrogenous-fertilizer-industry)
fertilizer segment from 2003 to 2013. Profits decrease rapidly with changes in either level, while revenues grow with diminishing marginal returns to either variable.

\[
y_{\pi} = -3810 + \frac{1}{4} y_R \quad \text{(7)}
\]

\[
y_{\pi} = -330 + \frac{1}{3} y_{R(t)} \quad \text{(7a)}
\]

\[
y_{\pi} = -70 y_R^{2.52} y_{\pi}^{6.25} \quad \text{(8)}
\]

\[
y_{R(t)} = 7.7 y_R^{-0.64} y_{\pi}^{0.95} \quad \text{(8a)}
\]

The steady state solution profit level – once we take revenues into account – does not require an external variable in order to figure out if fertilizer firms should get bigger fast (more quickly). Equation (9) shows the steady-state solution to equation (8). We can find the way state-state profits change with revenues – as shown in equation (9a). In our particular case, \( a = -1.7 \times 10^{-15} \), \( b = 8.6 \times 10^{-16} \), \( c = 1.5 \times 10^{50} \) and \( d = 2.5 \times 10^{49} \). The coefficients \( a \) and \( b \) clearly eclipse \( c \) and \( d \), making the entire expression a relatively large constant in \( y \). However, the \( y \) term in the numerator and the \( y \) term in the denominator make \( x \) relatively small to \( y \). As the steady state value of \( y \) increases, the steady state value of \( x \) probably increases microscopically – meaning it does not pay to get big fast in the fertilizer industry.

\[
y_{\pi} = \frac{1.5}{(875 y_R^{3.5} - 44e)^{1/5}} \quad \text{(9)}
\]

\[
y_{\pi} = \frac{((-a - bi)(1.5cy^5 + 2.5d)y^2)^{2/7}}{y^2} \quad \text{(9a)}
\]

Equation (10) shows the way that profits and revenues determine each other in this segment, while equations (11) and (11a) show the transition path and steady state values of nitrogen fertilizer profits and revenues. As for the transition path, if one graphically plots the solution (which we do not do here), we see qualitatively that both profits and revenue tend to drift down over time – with a sudden boost occurring at a point which depends on time. From a modelling point of view, the general equation in (4) appears to reflect the data better than (4a). Again, we see that the steady state relationship depends on a variable not included in our analysis.

\[
\begin{bmatrix}
\ln y_{\pi} \\
\ln y_R
\end{bmatrix} =
\begin{bmatrix}
252 & 625 \\
-0.64 & 0.95
\end{bmatrix}
\begin{bmatrix}
\ln x \\
\ln y + \ln(7.7)
\end{bmatrix} \quad \text{(10)}
\]

\[
\bar{y}_{\pi} = e^{250t} - 0.003e^{2.55t} + (0.14 + 0.001z) \quad \text{(11)}
\]

\[
\bar{y}_R = -0.93e^{250t} + 0.37e^{2.55t} + (2.26 + 0.007z) \quad \text{(11a)}
\]
Just like last time, we can generalize to arrive at a general solution to equations (11) and (11a). Equation (12) shows the solution, with parameters inserted to replace the specific parameter estimates we derived from the nitrogen fertilizer industry. As shown – and unremarkably – profits depend on some function of time and what we call the mystery variable $z$. This equation shows us that, when we look at the way revenue and profits move together, the mystery variable we left out of our analysis determines far more importantly whether fertilizer producers should “get big fast” in revenue terms.

$$y_{\pi} = ae^{at} - b3e^{bt} + (c + dz) \quad (11)$$

**What is the profit-maximising level of revenue over time?**

The optimal level of profit for the US nitrogen fertilizer consists of figuring out the profit maximising level of revenues for each time “slice” into the future. The first step – as shown in equation (12) -- consists of working out a relationship between profits, revenues, and the change in revenues during the period we analysed. Equation (12b) shows the set up for finding the optimal profit over time, using the Euler method. Equation (12c) shows the resulting solution. As shown, just like in the previous cases, the optimal level of profits depends on a mystery variable $z$, which looking at profits and revenues alone can not uncover.

$$y_{\pi}^* = -15x^{2.3} - 11x^{1.3} \cdot x^{\pi} - 3.3x^{1.3} \quad (12)$$

$$y_{\pi}^* = -34.5x^{1.3} - 15x^{0.3} \cdot x^{\pi} = \frac{d}{dt} \left[-6.6x^{1.3} \cdot x^{\pi} + 4.3x^{0.3} \right] \quad (12a)$$

$$y_{\pi}^* = e^{(-25+0.97z)t} + e^{(-25-0.97z)t} + 2 \quad (12c)$$

What do we know about the solutions to this equation? We know that the optimal value of the mystery value (the value which corresponds to the optimal level of nitrogenous fertilizer industry profits) equals equation (13). Assuming that $q(y)$ such that $q(y) = 0.5y - 1 - 5(y^2 - 4y + 4 - 4e^{-5t})$, then equation (13a) shows that the optimal value of $x$ – for our parameterization -- probably decreases with time $t$. Equation (13b) shows more specifically the way that $x$ changes over time.

$$x_{\pi}^* = -\frac{0.25(t + 4 \ln(0.5y - 1 - 5\sqrt{y^2 - 4y + 4 - 4e^{-5t}}))}{t} \quad (13)$$

$$x_{\pi}^* = -\frac{0.25(t + 4 \ln q(y))}{t} \quad (13a)$$

$$\frac{dx_{\pi}^*}{dt} = -\frac{1}{t} \left(0.25 \left(1 - \frac{2e^{-5t}}{\sqrt{y^2 - 4y - 4e^{-5t}q(y)}} + \frac{25(t + 4 \ln q(y))}{t^2} \right) \right) \quad (13b)$$
Empirical Aspects Driving the Empirical Analysis of the Nitrogen Fertilizer Segment’s Profit and Revenue

If we could discover the mystery variable $z$ which solves equation 4(a), we could effectively model the fertilizer industry’s revenues and profits. What would such a variable look like? In other words, after screening hundreds of different potential variables, how to do we know we have arrived at the “true” explanatory variable? Doing such math on equation (4) produces a solution for $z$ as a relatively simple function of constants and parameters, as shown in equation (12). While the term Lambert W looks scary, the solution ready be inserted into easy-to-find internet based applications (apps) in order to find the numerical solution. The solution to equation (12) lies well outside the realm of most economists’ math. Yet, software packages like Mathematica and Maple can readily find the answer. Simply comparing that answer with the variables used during the data mining process can ready discover the variable driving the profitability of the US nitrogenous fertilizer industry.

$$z = \frac{\text{LambertW} \left( \frac{-y + a + e^{-y}}{b\phi} \right)}{\phi} \quad (12)$$

What about the way that revenues and profits change over time? How do we know the extent to which revenue “gets big fast” in this industry? What about profits? Equations (13) and (13a) show the closest fitting distributions onto revenues and profits, using their Schwarz Information Criterion scores. As shown, the curve of best fit consists of a beta distribution, which itself, is a composite of exponential functions. The differences between the first distribution (7.6) and the second (0.82) comes to about 6.78 – a relatively large difference. Getting big fast does not seem to come from the same distribution as profiting from it.

$$y_R = B(0.4,0.17) = \frac{\Gamma(.17)\Gamma(.4)}{\Gamma(.57)} = \frac{e^{1.695}e^{0.79668}}{e^{0.4461}} = \int_0^\infty \frac{t^{0.43}}{e^t} \frac{t^{0.6}}{0.83} \quad (13)$$

$$y_\pi = B(0.93,0.64) = \frac{\Gamma(.64)\Gamma(.93)}{\Gamma(1.57)} = \frac{e^{0.3394}e^{0.0445}}{e^{0.160}} = \int_0^\infty \frac{t^{0.57}}{e^t} \frac{t^{0.07}}{0.36} \quad (13a)$$

Conclusion

In this paper, we have shown the conditions under which the US nitrogenous fertilizer industry profits from getting big fast. We derive a model of revenue and profit growth – and show that “size” (as measured by revenues) can decrease or increase profits, depending on the structural parameters defining this market segment. We show that US fertilizer industry profits most likely result from a third variable which our analysis did not include. However, we know roughly how that “mystery variable” should behave – pointing the way for further empirical research. We also know that – for most model
parameterizations – that likelihood of getting bigger fast (and profiting from it – remains slim.

References


