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The Good Samaritan and Traffic on the Road to Jericho

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> "A certain man went down from Jerusalem to Jericho, and fell among thieves, which stripped him of his raiment, and wounded him, and departed, leaving him half dead. And by chance there came down a certain priest that way: and when he saw him, he passed by on the other side. And likewise a Levite, when he was at the place, came and looked on him, and passed by on the other side. But a certain Samaritan, as he journeyed, came where he was: and when he saw him, he had compassion on him, And went to him, and bound up his wounds, pouring in oil and wine, and set him on his own beast, and brought him to an inn, and took care of him." *Parable of the Good Samaritan, New Testament, Luke 10: 30-34*

Driving along a lonely road, you come upon a stalled car and a motorist who appears to have run out of gas. You consider stopping to offer help, although this may cost you several minutes and some extra driving. Would your decision be different if the road were heavily travelled? If you were to run of gas, would you prefer that it be on a busy street or on a lonely road?

1 Equilibrium with Identical Travelers

Let us try to develop our understanding with a simple game-theoretic model. Cars approach a stranded motorist's location according to a random Poisson process with arrival rate λ. Passing travelers are sympathetic to this

*I am grateful to Greg Leo and Rod Garratt of UCSB and to Stefano Barbieri and John Edwards of Tulane University for stimulating discussions and helpful advice.

motorist's plight, but stopping is costly and they believe that other potential helpers will arrive in the future. Every passing traveler attaches a cost $c > 0$ to stopping to help, and a psychic cost of vt to the prospect that the stranded motorist must wait for an expected length of time t before being rescued.

In deciding whether to stop, passers-by compare the cost of stopping to the psychic cost of not stopping. They choose to stop if $c < vw$ where w is the expected amount of time that the stranded motorist will have to wait if the passer-by does not stop. If a motorist expects all future passers-by to stop, then the expected waiting time for the stranded motorist will be

$$w = \int_{\ell}^{\infty} te^{-\lambda t} = \frac{1}{\lambda}. \tag{1}$$

Passing travelers would always stop if $vw = v/\lambda > c$. Therefore if the road is so little-traveled that $\lambda < \frac{v}{c}$, then, in Nash equilibrium, every passing motorist would stop, and the expected waiting time for a stranded motorist would be $1/\lambda$.

Where traffic is frequent enough that $\lambda > v/c$, there cannot be a Nash equilibrium in which everyone stops, nor can there be an equilibrium in which no one stops. The only symmetric Nash equilibrium is a mixed strategy equilibrium in which each passing traveler stops with some probability p, where $0 < p < 1$. The appearance of a driver who will stop and help is then a Poisson process with arrival probability λp. Thus if the currently passing traveler does not stop, the expected waiting time until the stranded motorist is helped is

$$w = \frac{1}{\lambda p}. \tag{2}$$

In a symmetric mixed-strategy Nash equilibrium, all passing travelers must be indifferent between stopping and not stopping. This occurs if $c = vw$ or equivalently if

$$w = \frac{c}{v}. \tag{3}$$

From Equations 2 and 3, it follows that in equilibrium,

$$p = \frac{1}{\lambda}\left(\frac{v}{c}\right). \tag{4}$$

Thus changes in λ lead to inversely proportional changes in p, so that the equilibrium expected waiting time for the arrival of a passing motorist *who is willing to help* does not change with traffic density.

As traffic becomes more dense, the expected number of travelers who pass the stranded motorist before one of them stops to help will increase. Where λ is traffic density and p is the probability that any passer-by will stop, the expected number of motorists to drive past a stranded motorist is $(1-p)/p$.[1] From Equation 4, it follows that the relation between traffic density λ and the expected number of motorists that drive by before help arrives is given by

$$\frac{1-p}{p} = \lambda\frac{v}{c} - 1 \tag{5}$$

We now have answers to our opening questions.

Proposition 1. *If all travelers have the same cost ratio, c/v, and the arrival rate of traffic is λ, then in symmetric Nash equilibrium:*

- *Over the range of traffic densities such that $\lambda < c/v$, all passing travelers will stop, and the expected waiting time for rescue decreases with increased traffic density.*
- *Over the range of traffic densities such that $\lambda > c/v$:*
  - *the only symmetric equilibrium is one in which the stopping probability is $p = (1/\lambda)(v/c)$, which is between 0 and 1.*
  - *the probability of stopping declines with traffic density so that the expected waiting time for a stranded motorist is invariant to changes in traffic density.*
  - *the expected number of cars that pass by before help arrives is $\lambda(v/c) - 1$.*

2 Equilibrium When Costs and Sympathies Differ

Let us add realism by allowing those who travel on the road to differ in their costs of stopping, and in their sympathy for the plight of strangers.

[1] The probability that help first arrives at time t is $\lambda p e^{-\lambda p t}$. If help arrives at time t, the expected number of motorists to pass by before help arrives is $(1-p)\lambda t$. Therefore the expected number of motorists to drive past before help arrives is

$$\begin{aligned} \int_{\ell}^{\infty}(1-p)\lambda t\lambda p e^{-p\lambda t}dt &= (1-p)\lambda\int_{\ell}^{\infty}(\lambda p)te^{-\lambda p t}dt \\ &= \frac{1-p}{p}. \end{aligned}$$

Attention to such differences leads to qualitatively different conclusions and to interesting comparative statics that are not found when passing travelers are identical. In equilibrium, for the model presented here, all consumers choose pure strategies, and the expected waiting time for a stranded motorist decreases as traffic density increases.

2.1 Passing strangers and incomplete information

We model the situation as a symmetric game of incomplete information. All passing travelers are aware of the density λ of traffic, and they know their own ratios c/v of the cost of stopping to the value they place on a stranded motorist's time. They do not know the cost ratios of other travelers, but they share a common belief that the cost ratio c/v of each subsequent passer-by is an independent random draw from a continuous distribution, $F(\cdot)$, with density function, $f(\cdot)$.

A strategy for any passing motorist is a mapping from his or her own cost ratio c/v to one of the two actions *Stop* and *Don't stop.* Under these assumptions, there will be a symmetric Nash equilibrium in which players observe their own cost ratio and compare it with some threshold ratio $(c/v)^*$. The strategy of every player is to stop if and only if $c/v < (c/v)^*$.

Where w is the expected waiting time for the stranded motorist, a passer-by with cost ratio c/v will stop only if if $c < wv$, or equivalently $c/v < w$. Therefore if expected waiting time is w, the probability that a randomly selected motorist will stop is

$$p = F(w). \tag{6}$$

If the probability that any subsequent traveler will stop is p, then the Poisson arrival rate of passers-by who will stop is λp and the expected waiting time for the stranded motorist is

$$w = \frac{1}{\lambda p}. \tag{7}$$

In Nash equilibrium, the stopping probability p and the expected waiting time w must satisfy Equations 6 and 7.

In this model, we find that if some passers-by are kinder or less busy than others, you are better off running out of gas on a busy street than on a lonely road. Stated more formally:

Proposition 2. *In the model described in this section, with a continuous distribution, F, of cost ratios, for any arrival rate $\lambda > 0$, there is a unique equilibrium expected waiting time $w_F(\lambda)$ for a stranded motorist, and $w_F(\lambda)$ is a decreasing function of λ.*

Proof. Equations 6 and 7 imply that in equilibrium it must be that in equilibrium,

$$wF(w) = \frac{1}{\lambda}. \tag{8}$$

Since $wF(w)$ is a continuous increasing function that ranges from 0 to ∞ as w ranges from 0 to ∞, for any λ, there must be exactly one solution to Equation 8. Since the left side of Equation 8 is increasing in w and the right side is decreasing in λ, it must be that this solution is decreasing in λ. □

A Graphical Exposition

Figure 1: Traffic Density and Expected Waiting Time with Varying Costs

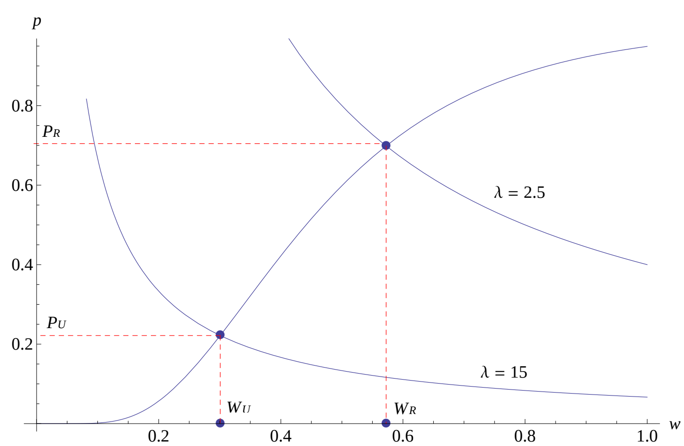


Figure 1 shows the effect of traffic density on the equilibrium stopping threshold and expected waiting time for a stranded traveler. The horizontal axis plots the stranded motorist's expected waiting time, while the vertical axis shows the probability that a random passer-by will stop. The upward-sloping curve is a "stopping-response curve", showing the probability p that a random passer-by will stop if the expected waiting time is w. A passer-by with cost ratio c/v will stop if $c/v < w$. Therefore the stopping-response curve is just the graph of the cumulative density, $F(w)$. The curve drawn in this figure is the cdf of a log normal distribution with mean $1/2$.[2]

[2]The diagram was drawn with *Mathematica* for a distribution in which the logarithm of c/v has mean $\mu = \log(1/2) - 1/8$ and standard deviation $\sigma = 1/2$. The resulting log

The figure shows two downward-sloping "expected waiting-time curves." These curves relate the expected waiting-time $w = 1/\lambda p$ of a stranded motorist on a road with density λ to the probability p that a random passer-by will stop. The higher of these two curves is drawn for a "rural" highway with relatively low traffic density, $\lambda = 2.5$. The lower downward-sloping curve is drawn for an "urban" highway with higher traffic density, $\lambda = 15$. For each of the two highways, the equilibrium outcome is found at the intersection of the stopping-response curve with the corresponding expected waiting-time curve. For this example, the graph shows that a random passer-by on an urban highway is less likely to stop than on a rural highway, but because travelers pass by more often, the expected waiting time for the stranded motorist is shorter on the urban than on the rural highway.

Figure 2: Equilibrium with Uniform and Varying Costs

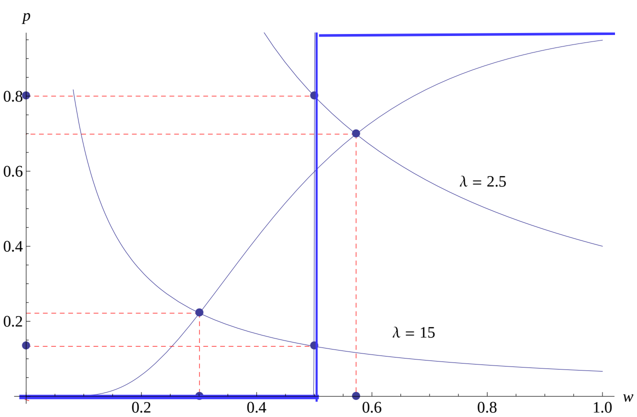


Figure 2 compares the equilibria described in Figure 1 to equilibria in the case where all travelers have identical cost ratios with the same mean, $c/v = 1/2$. The downward-sloping curves show the expected waiting-time function, $w = 1/\lambda p$, for two alternative traffic densities, $\lambda = 2.5$ and for $\lambda = 15$. The smooth upward-sloping curve is the same as that shown in Figure 1, while the thick piecewise linear "curve" shows the stopping-response correspondence for the case of uniform cost ratios. As this curve shows, if expected waiting time is $w < 1/2$, no motorists would stop. If $w > 1/2$, all motorists would stop and if $w = 1/2$, all motorists are indifferent between stopping and not

normal distribution has mean $1/2 = e^{\left(\mu+\sigma^2/2\right)}$ and standard deviation $(1/2)\sqrt{e^{1/4}-1} = e^{\left(\mu+\sigma^2/2\right)}\sqrt{e^{\sigma^2}-1}$.

stopping. and the smooth upward-sloping curve shows the stopping-response curve for the case where cost ratios are log-normally distributed with mean 1/2.

As Figure 2 shows, when all travelers have identical cost ratios, the downward-sloping curves must intersect the stopping-response curve in its vertical section, with an expected waiting time of $w = 1/2$. Thus the equilibrium adjustment to a change in λ must take the form of an offsetting change in the mixed-strategy probability p so that $w = 1/\lambda p$ is the same on the quiet rural road as on the busy urban highway.

2.2 Comparative statics results

The elasticity of waiting time with respect to density

Proposition 2 tells us that when cost ratios are continuously distributed, the equilibrium expected waiting time for a stranded passenger is shorter when traffic is more dense. Here we quantify the relation between the dispersion of cost ratios and the effect of traffic density on expected waiting time. Where F is the distribution of cost ratios, we define the *elasticity of equilibrium waiting time with respect to traffic density* as

$$\eta_w(\lambda, F) = \frac{d \ln w_F(\lambda)}{d \ln \lambda} = \frac{\lambda}{w_F(\lambda)} \frac{\partial w_F(\lambda)}{\partial \lambda}. \tag{9}$$

and the *elasticity of the cost distribution function* $F(\cdot)$ at the point w as

$$\sigma_F(w) = \frac{d \ln F(w)}{d \ln w} = \frac{w f(w)}{F(w)}. \tag{10}$$

We have the following result, which is proved in the Appendix.

Proposition 3. *The elasticity of expected waiting time for the stranded traveler with respect to traffic density lies between* -1 *and* 0*. If the distribution function of cost cost ratios is* F*, it must be that*

$$\eta_w(\lambda, F) = \frac{-1}{1 + \sigma_F\left(w_F(\lambda)\right)}. \tag{11}$$

Simple mean-preserving spreads and expected waiting time.

Michael Rothschild and Joseph Stiglitz [14] defined the notion of a *mean-preserving spread* to capture the idea of "taking weight from the center of a probability distribution and shifting it to the tails, while keeping the mean of the distribution constant."[3] Stiglitz and Peter Diamond [9] describe a special class of such spreads as follows: a distribution function G is said to be a *simple mean-preserving spread* of a distribution function F if the two distribution functions have the same mean, and if they are related by a single crossing property such that for some $\hat{x}$, $G(\hat{x}) = F(\hat{x})$, while if $x < \hat{x}$ then $G(x) > F(x)$ and if $x > \hat{x}$, then $G(x) < F(x)$.[4]

The log-normal distribution shown in Figure 2 is a simple mean-preserving spread of the distribution in which all travelers have the same cost ratio. In this example, travelers on the rural road have a longer expected waiting time with the spread-out distribution than with the concentrated distribution, while travelers on the more heavily-traveled urban road would have a shorter expected waiting time with the more spread-out distribution. This observation generalizes to show that, broadly speaking, greater dispersion of the distribution of cost ratios tends to reduce the expected waiting time of stranded motorists on heavily traveled roads and increase their expected waiting time on less traveled roads.

Proposition 4. *Let the distribution function G be a simple mean-preserving spread of the distribution function F, where a single crossing point at $\hat{x}$. Let $\hat{\lambda} = 1/\left(\hat{x}F(\hat{x})\right)$. Then on roads where $\lambda < \hat{\lambda}$, equilibrium waiting time for the stranded motorist is* longer *if the distribution of cost ratios is G than if it is F. If $\lambda > \hat{\lambda}$, then equilibrium waiting time for the stranded motorist is* shorter *if the distribution is G than if it is F.*

Proof. Let $w_F(\lambda)$ and $w_G(\lambda)$ be equilibrium waiting times with the distributions F and G respectively. Suppose that $\lambda > \hat{\lambda}$. Then

$$w_F(\lambda)F\left(w_F(\lambda)\right) = \frac{1}{\lambda} < \frac{1}{\hat{\lambda}} = \hat{x}F(\hat{x}).$$

Since $zF(z)$ is strictly increasing in z, it follows that $w_F(\lambda) < \hat{x}$ and hence $G\left(w_F(\lambda)\right) > F\left(w_F(\lambda)\right)$. Therefore

$$w_F(\lambda)G\left(w_F(\lambda)\right) > w_F(\lambda)F\left(w_F(\lambda)\right) = 1/\lambda = w_G(\lambda)G\left(w_G(\lambda)\right).$$

[3] Formally, a random variable Y is a mean-preserving spread of the random variable X if and only if Y is equal in distribution to $X + Z$ for some random variable Z such that $E(Z|X) = 0$ for all values of X.

[4] Diamond and Stiglitz show that every simple mean-preserving spread is a mean-preserving spread, but not every mean-preserving spread is a simple mean-preserving spread. In general, the crossing point of the two distributions need not necessarily be the same as their common mean.

Since $G(w_G(\lambda)) > 0$, it follows that $w_F(\lambda) > w_G(\lambda)$.

A similar argument shows that if $\lambda < \hat{\lambda}$, then $G(w_F(\lambda)) < F(w_G(\lambda))$ and that $w_F(\lambda) < w_G(\lambda)$.

$\square$

Equilibrium preserving spreads and elasticities

We have seen that a simple mean-preserving spread of the distribution of cost-ratios implies increased expected waiting time on little-travelled roads and decreased expected waiting time on busy roads. A mean-preserving spread can be thought of a stretch of the density function in both directions away from the mean. Here we consider a spread in the cost-ratio distribution that leaves the equilibrium expected waiting time unchanged but stretches the density function out in both directions away from the equilibrium corresponding to a fixed density λ.

Let us define the distribution function G to be a *simple equilibrium-preserving spread* of the distribution function F around $w_F(\lambda)$ if $w_G(\lambda) = w_F(\lambda) = \hat{w}$ and if $G(w) < F(w)$ for $w > \hat{w}$ and $G(w) > F(w)$ for $w < \hat{w}$.[5]

Proposition 5. *If the cost ratio has a continuous distribution function F and if the distribution function G is a simple equilibrium-preserving spread of F, then the elasticity of waiting time with respect to λ is greater in absolute value for a population with distribution function G than for one with distribution function F.*

Proof. If G is a simple equilibrium-preserving spread of F, then it is immediate that $G'(\hat{w}) = g(\hat{w}) < F'(\hat{w}) = f(\hat{w})$. Then, since $F(\hat{w}) = G(\hat{w})$, it follows that

$$\sigma_F(\hat{w}) = \frac{\hat{w} f(\hat{w})}{F(\hat{w})} < \frac{\hat{w} g(\hat{w})}{G(\hat{w})} = \sigma_G(\hat{w}) \tag{12}$$

From Equations 12 and 3, it then follows that it follows that

$$\eta_w(\lambda, F) = \frac{-1}{1 + \sigma_F(w_F(\lambda))} > \eta_w(\lambda, G) = \frac{-1}{1 + \sigma_G(w_G(\lambda))}. \tag{13}$$

and since both elasticities are negative, $\eta_w(\lambda, G)$ is higher in absolute value.

$\square$

[5]For example, it is is not hard to show that where $\hat{w}$ is an equilibrium waiting time and

$$G(w) = F\left(\bar{w} + \frac{w - \bar{w}}{\epsilon}\right),$$

it must be that G is a simple equilibrium-preserving spread of F if $\epsilon > 1$.

3 Ethical Guidelines for When to Stop and Help

Perhaps the priest and the Levite who hurried past the injured traveler had good excuses. Maybe they had important things to do and realized that if they did not stop, someone less busy would soon be likely to appear and perform the rescue.

When stopping costs differ among travelers, it is not necessarily efficient for travelers to stop every time that they encounter someone in distress. Efficiency would be better served by a convention that passing travelers should stop if and only if their costs fall below some threshold level. Since stopping costs of passing travelers are not likely to be transparent to others, such a rule could not be enforced by external sanctions. But experimental evidence [3] suggests that many people act by self-imposed ethical rules that dictate behaving sympathetically toward others. It is therefore interesting to explore the social effects of alternative ethical rules. In this discussion we examine the nature of an "ethical ideal", that is, a rule that, if followed by everyone, would lead to an efficient outcome in environments like the Road to Jericho game.

To avoid difficulties in making interpersonal utility comparisons, we assume sufficient symmetry for the population so that there will be unanimous agreement about the "best" symmetric ethical rule. All persons are assumed to have the same the cost vt of being stranded for a length of time t. We assume that all have the same travel frequencies, all are equally likely to be stranded, and all are equally likely to encounter a stranded motorist on any road. The costs of stopping to perform a rescue will differ from occasion to occasion, but for all individuals, these costs are assumed to be independent draws from the same distribution function $F(c)$.

In this environment, an efficient ethical rule would require passing motorists to stop and help if and only if their stopping costs are below some threshold, c^*. If all individuals abide by this rule, then for any stranded motorist, the Poisson arrival rate of a motorist who will help is $\lambda F(c^*)$ and expected waiting time is $1/\lambda F(c^*)$. The expected total cost of each incident of a stranded motorist includes the expected waiting cost for the stranded motorist and the expected cost c for the first passer-by who has a cost below the threshold c^*. This total is:

$$\frac{\int_\ell^{c^*} cf(c)dc}{F(c^*)} + \frac{v}{\lambda F(c^*)} \tag{14}$$

Given our symmetry assumptions, all individuals bear the same expected total cost. Therefore the expected costs of all community members would be minimized by a rule that set a stopping threshold of c^*, where c^* minimizes Expression 14. We will call a rule with this stopping threshold an *ideal ethical stopping rule.*

Differentiating Expression 14 with respect to c^* yields the first order necessary condition:

$$c^* - \frac{\int_\ell^{c^*} cf(c)dc}{F(c^*)} = \frac{v}{\lambda F(c^*)}. \tag{15}$$

Equation 15 has a straightforward interpretation as a marginal efficiency condition. This equation requires that for a traveler with costs equal to the threshold c^*, the difference between c^* and the expected cost of the next traveler who would be required to stop is equal to the expected additional cost of waiting for the stranded traveler if the current passer-by does not stop.

Proposition 6, which is proved in the Appendix, shows if all community members follow an ideal ethical stopping rule, then as traffic density increases, individual passers-by will be less likely to stop, but the average waiting time for help to arrive will also be smaller.

Proposition 6. *In the symmetric community described in this section, if the entire population abides by an ideal ethical stopping rule, then*

- *on busier roads, the probability that a randomly selected passer-by will stop is lower.*
- *if the distribution F of costs is log-concave, then on busier roads, the expected waiting time for help to arrive for a stranded traveler is lower.*

The assumption that cumulative distribution function F is log-concave is not a strong requirement. Essentially all commonly-known distribution functions have log-concave cumulative distribution functions. Log concavity of the density function is sufficient but not necessary for log-concavity of the distribution function [4].[6]

Having found a socially efficient cost threshold for potential helpers, we can ask how this ethical rule might be expressed in common language. An interesting candidate rule is: "Treat the misfortune of others as if it were your own." A passing motorist who practiced this rule would stop whenever his cost of stopping was less than the expected cost of further waiting to the

[6] For example, the log-normal distribution does not have a log-concave density function, but has a log concave distribution function [4].

stranded motorist. If travelers all abide this rule and have a stopping cost threshold $\bar{c}$, the probability that a passing motorist will stop is $F(\bar{c})$ and the expected waiting cost to a stranded motorist would be $v/\lambda F(\bar{c})$. Therefore the cost-threshold $\bar{c}$ would be an equilibrium for motorists abiding by the rule "treat the misfortune of others as your own" only if

$$\bar{c} = \frac{v}{\lambda F(\bar{c})}. \tag{16}$$

Comparing Equation 16 with Equation 15 leads us to conclude that:

Proposition 7. *In the symmetric society posited in this section, if travelers applied the rule "act as if the misfortune of a stranded traveler is your own," they would stop* less *frequently than if they applied the the ideal ethical stopping rule.*

Proof. Define $G(c) = c - \frac{v}{\lambda F(c)}$. From Equations 15 and 16 it follows that

$$G(c^*) - G(\bar{c}) = \frac{\int_\ell^{c^*} cf(c)dc}{F(c^*)} > 0.$$

Since G is an increasing function of c, it follows that $c^* > \bar{c}$. A higher stopping threshold implies that each passing traveler is more likely to stop. □

There is a simple explanation of the result of Proposition 7. The benefits that one confers on others by stopping to help include not only the benefit to the stranded individual, but also the benefit to another traveler who otherwise would have felt obliged to stop and perform a rescue. If travelers take only the former effect into account, they underestimate the total benefits that their action confers on others.

4 Publicly supported rescue patrols

Community members may decide to support publicly-funded highway patrols that cruise the streets and help any stranded travelers that they encounter. The presence of such patrols will typically shorten the expected waiting time for those in need of help. This effect is weakened by the fact that public patrols typically "crowd out" some private rescues. However, this crowding-out also has the beneficial effect of reducing the total costs borne by private travelers stopping to help a stranded motorist.

4.1 Comparative Statics and Crowding Out

On a highway with traffic rate λ, suppose that sufficient rescue patrols are provided to arrive at a Poisson rate μ. If the fraction of ordinary passing travelers who will stop to aid a stranded motorist is p, then for a stranded motorist, the Poisson arrival rate of a vehicle that will stop to help is $\mu + \lambda p$ and the expected waiting time for help to arrive is

$$w = \frac{1}{\mu + \lambda p}. \tag{17}$$

Let us normalize the sympathy variable v so that $v = 1$ and hence $vw = w$ is the psychic cost of leaving a motorist stranded for an expected length of time w. We assume that the cost of stopping for private travelers is a random variable with distribution function $F(\cdot)$ and density function f defined on the interval $[\ell, \infty]$. Then it follows from Equation 17 that at an interior equilibrium where some, but not all, private travelers will stop, it must be that

$$w = \frac{1}{\mu + \lambda F\left(w\right)}. \tag{18}$$

or equivalently,

$$w\mu + \lambda w F\left(w\right) = 1. \tag{19}$$

For all $\lambda > 0$, the expression on the left side of Equation 19 is strictly increasing in w and in μ. For all $\lambda > 0$, and all $\mu \geq 0$, the value of this expression ranges from 0 to ∞ as w ranges from 0 to ∞. Therefore for all $\lambda > 0$ and all $\mu \geq 0$, Equation 19 has a unique solution for w. This solution implicitly defines the function $w(\mu, \lambda)$ representing equilibrium waiting time on a road with traffic rate λ and arrival rate μ of public rescue vehicles. Differentiating Equation 19 and rearranging terms, we find that the marginal effect of the rate of rescue patrolling on expected waiting time is

$$\frac{\partial w(\mu, \lambda)}{\partial \mu} = \frac{-w(\mu)}{\mu + \lambda\left(F\left(w(\mu)\right) + wf\left(w(\mu)\right)\right)} < 0. \tag{20}$$

Where $p(\mu, \lambda)$ is the equilibrium probability that a private traveler will stop, the expected arrival rate of private travelers who will stop is $\lambda p(\mu, \lambda) = \lambda F\left(w(\mu, \lambda)\right)$. Therefore the the rate at which public patrols crowd out private help is

$$\lambda \frac{\partial p(\mu, \lambda)}{\partial \mu} = \lambda f\left(w(\mu, \lambda)\right) \frac{\partial w(\mu, \lambda)}{\partial \mu} = \frac{-\lambda w f\left(w(\mu)\right)}{\mu + \lambda\left(F\left(w(\mu)\right) + wf\left(w(\mu)\right)\right)}. \tag{21}$$

From Equations 20 and 21, we conclude that:

Proposition 8. *If the distribution F of stopping costs for private travelers is continuous, then, at an interior equilibrium, an increase in the arrival rate of a public rescue service will reduce expected waiting time for stranded travelers, but will also crowd out some private help.*

A special case with closed-form solution

Equation 19 defines the function $w(\mu)$ only implicitly. For most familiar forms of the distribution function, there is no simple closed-form expression for $w(\mu)$. There is, however a closed-from solution for $w(\mu)$ in the case of a Pareto distribution taking the special form

$$F(c) = 1 - \frac{A}{c}$$

over the range $c \in [A, \infty]$. In this case Equation 19 becomes

$$w\mu + \lambda(w - A) = 1 \tag{22}$$

and hence

$$w(\mu, \lambda) = \frac{1 + \lambda A}{\mu + \lambda}. \tag{23}$$

Then we have

$$\frac{\partial w(\mu, \lambda)}{\partial \mu} = -\frac{1 + \lambda A}{(\mu + \lambda)^2} = \frac{-w(\mu, \lambda)}{\mu + \lambda} \tag{24}$$

and the rate at which private help is crowded out by public patrols is

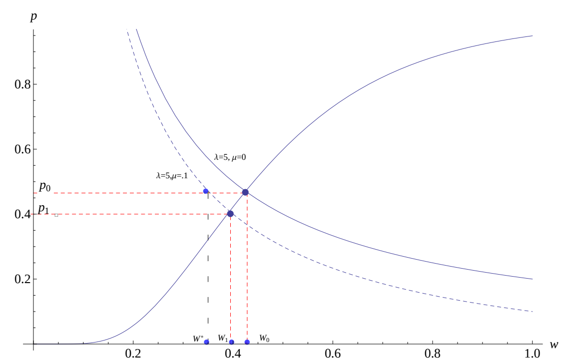
$$\lambda \frac{\partial p(\mu, \lambda)}{\partial \mu} = \lambda f\left(w(\mu, \lambda)\right) \frac{\partial w(\mu, \lambda)}{\partial \mu} = \frac{-\lambda A}{1 + \lambda A}. \tag{25}$$

Equation 25 tells us that for this special distribution, the degree of crowding out of private rescues by a public rescue service is independent of μ and increases continuously with the density of traffic. There is almost no crowding-out with very thin traffic and almost full crowding out with very dense traffic.

Graphical Exposition

Figure 3 illustrates the comparative statics effects of changes in the frequency of public rescue patrols. On the horizontal axis, this figure shows expected waiting time for a stranded traveler the vertical axis shows the probability that a private traveler will stop to help. The upward-sloping curve is the graph of the distribution function $F(\cdot)$. The solid downward-sloping curve shows how the probability p that a private traveler will stop influences the expected waiting time if there are no public rescue patrols. This function is given by $p = 1/\lambda w$ where, in the figure, we have set $\lambda = 5$. The dashed downward-sloping curves shows expected waiting as a function of p when the frequency of rescue patrols is $\mu = 1/2$. In this case, expected waiting time is given by $w = 1/(\mu + \lambda p)$. The intersections of these two downward-sloping curves with the graph of the distribution function F determine show, with and without the presence of the rescue services, the equilibrium probabilities that private travelers will help and equilibrium expected waiting times for a stranded traveler. In this graph, we see that the presence of the rescue patrol reduces expected waiting time of a stranded traveler from W_0 to W_1 and reduces the probability that a private traveler will stop form p_0 to p_1. If there had been no crowding-out effect, so that private travelers continued to help as often as in the absence of public rescue patrols, then the addition of the rescue patrols would have reduced waiting times to the point W^* on the graph.

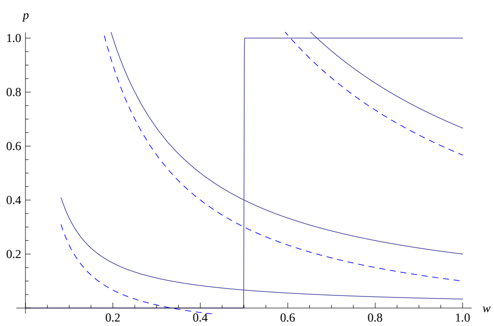
Figure 3: Effect of Public Rescue Patrol



In the special case where travelers have identical cost distributions, the marginal crowding-out effects are quite simply described. Where traffic rates are very low and rescue vehicles are infrequent, there is no crowding out because all private travelers will stop even with modest increases in the amount of public patrols. On heavily travelled roads with frequent public rescue patrols, there will be no marginal crowding out because no private travelers will stop with or without incremental changes in the frequency of public patrols.

The three solid downward-sloping curves in Figure 4 relate expected waiting times for a stranded traveler to private stopping probabilities with there is no public rescue service for three different levels λ of traffic density. For each of these three traffic densities, the corresponding dashed downward-sloping curve relates expected waiting time to private stopping probabilities if public rescue patrols arrive at the rate 0.1λ. The reaction correspondence, showing the fraction of the population willing to stop, given the expected waiting time of a stranded traveler is shown by the piecewise linear curve running rom the origin to $(1/2, 0)$, then vertically from $(1/2, 0)$ to $(1/2, 1)$ and then horizontally from $(1/2, 1)$ to $(1, 1)$.

Figure 4: Public Rescue Patrol with Identical Travelers



The highest of the downward-sloping curve shows $w = 1/\lambda p$ for a relatively isolated road with traffic arrival rate 1.5. As the graph shows, if there are no public rescue patrols, all private travelers would stop to help. In the example shown here, we also see that if a public rescue patrol arrives at the rate 0.1λ, all private travelers would continue to stop, so there would be no crowding out. The middle downward-sloping curve is drawn for an intermediate density, $\lambda = 5$. We see from the intersection of the dotted line just below this curve that for this traffic density, there is complete crowding out. The equilibrium probability of stopping by private travelers falls by exactly the probability $\lambda\mu$ that a rescue vehicle will appear, and the expected waiting time of the stranded traveler is unchanged by the presence of the rescue patrol. The lowest of the solid curves corresponds to a relatively busy road with traffic arrival rate of 30. The dashed curve just below this curve relates expected waiting time to the probability that a private motorist will stop, given that the road features public rescue patrols that arrive at the rate 0.1λ. We see from these curves that the addition of the public patrol eliminates private stopping altogether and it reduces the expected waiting time to a period shorter than that which would induce private travelers to stop.

4.2 Benefit-cost for Public Rescue Patrols

Calculating marginal benefits

An increase in the frequency of public rescue patrols will benefit travelers by reducing the expected amount of time that a stranded traveler must wait for help. The greater frequency of rescue patrols is partially counteracted by a reduction of private rescue efforts, a "crowding-out effect." But crowding-out also yields benefits. Those private travelers who were crowded out are spared the cost of performing a rescue.

Let us assume that the probability that every motorist faces a Poisson probability of ρ of being stranded while traveling. Then on a road with traffic density λ, trended motorists will appear according to a Poisson process with arrival rate $\rho\lambda$. The expected number of motorists to be stranded on a road with traffic density λ, during a period of length T, is $\rho\lambda T$. Let $\mu\lambda$ be the Poisson arrival rate of traffic patrols, let $w(\mu)$ be the corresponding equilibrium waiting time for motorist who is stranded, and let v^* be the cost per unit of expected waiting time for those who are stranded. Then, on a road with traffic density λ, over a time interval of length T, the expected total waiting cost for stranded motorists is $v^* w(\mu)\rho\lambda T$.

Total costs of travelers include not only the cost of waiting if stranded, but also expected costs of stopping to help those in distress. Where $p(\mu) = F\left(w(\mu)\right)$ is the fraction of passing travelers with stopping costs low enough that they will stop and where ℓ is the lower bound of the support of $F(\cdot)$, the average stopping cost incurred by those who stop to help will be

$$A(\mu) = \frac{\int_\ell^{w(\mu)} cf(c)dc}{F(w\left(\mu\right))}. \tag{26}$$

The fraction of all stranded motorists who are helped by private travelers is

$$S(\mu) = \frac{F\left(w(\mu)\right)}{\mu + F\left(w(\mu)\right)}. \tag{27}$$

Then, for each stranded motorist, the expected cost of rescues by private travelers is $A(\mu)S(\mu)$. Therefore over a time interval of length T, on a road with traffic density λ, the expected total rescue costs borne by private motorists must be

$$A(\mu)S(\mu)\rho\lambda T = \left(\frac{\int_\ell^{w(\mu)} cf(c)dc}{\mu + F\left(w(\mu)\right)}\right)\rho\lambda T. \tag{28}$$

It follows that over a time interval of length T, on a road with traffic density λ, the expected total cost borne by stranded motorists and those who stop to help them is

$$C(\mu, T) = \left(v^* w(\mu) + \frac{\int_\ell^{w(\mu)} cf(c)dc}{\mu + F\left(w(\mu)\right)} \right) \rho\lambda T \tag{29}$$

From Equations 18 and 29 it follows that

$$C(\mu, T) = \left(v^* w(\mu) + \lambda w(\mu) \int_\ell^{w(\mu)} cf(c)dc \right) \rho\lambda T \tag{30}$$

The marginal benefits $MB(\mu)$ from an increase in μ for a period of time T are equal to the resulting marginal reduction in costs. Differentiating Equation 30, we find that

$$MB(\mu) = -w'(\mu) \left(v^* + \lambda \int_\ell^{w(\mu)} cf(c)dc + \lambda w^2(\mu) f\left(w(\mu)\right) \right) \rho\lambda T. \tag{31}$$

The three terms in parentheses on the left side of Equation 31 show three distinct benefits of the reduction in expected waiting time induced by additional public patrols. The first term, v^*, is the benefit to stranded travelers of reduced waiting time. The second term measures the expected reduced cost to private travelers that results when a public rescue vehicle arrives before any willing private traveler. The third term represents the reduction in private rescue costs that results from reducing the average stopping cost of those travelers whose cost falls below the stopping threshold.

4.2.1 Equating marginal costs with marginal benefits

Let us assume that the cost of maintaining rescue patrols that traverse a highway with frequency $\mu\lambda$ for a period of time T is $c\mu\lambda T$. Then the marginal cost of increasing the rate μ is simply

$$MC(\mu) = c\lambda T. \tag{32}$$

Efficiency calls for equality between marginal benefits and marginal costs of changing μ. From Equations 31 and 32 we see that this implies that

$$\frac{c}{\rho} = -w'(\mu) \left(v^* + \lambda \int_\ell^{w(\mu)} cf(c)dc + \lambda w^2(\mu) f\left(w(\mu)\right) \right). \tag{33}$$

The efficiency condition in Equation 33 is stated only in implicit form. Even for simple distribution functions, the resulting expression does not have a closed-form solution, for the efficient relative frequency μ of public patrols, but would need to be solved numerically. However in this expression is greatly simplified in the special case where traffic is so frequent that efficient levels of public rescues fully crowd out private rescues and in the case where traffic is so infrequent that, even with efficient availability of public rescue patrols, almost all private travelers will stop.

5 Are country folk more helpful than city folk?

A large body of field studies in social psychology explore what Nancy Steblay [15] calls the "rather simple hypothesis that 'country people are more helpful than city people.'" Stebley examines 65 studies, of which 46 support greater rural helpfulness, 9 support greater urban helpfulness and 10 report no significant differences.

Paul Amato [2] conducted a series of field experiments in 55 randomly selected Australian communities stratified by size and isolation.[7] In each community, Amato and his co-workers staged a number of situations that tested the willingness of random passers-by to help a stranger. One of Amato's staged events bears a close similarity to the Road to Jericho game. Amato described the set-up as follows:

> The episode began with the investigator walking along the sidewalk with a noticeable limp. A suitable pedestrian approaching from the opposite direction was selected to be the subject ... the investigator would suddenly drop to the sidewalk with a cry of pain. Then while half kneeling, the investigator would reveal a heavily bandaged leg, with ... bandage generously smeared with a fresh application of theatrical blood.

A confederate observed whether the subject offered to help and scored the response of the subject on a scale of "prosocial responsiveness." Amato found that the percentage of individuals who offered to help the injured person declined steadily with population size, from a helping rate of about 50 per cent in communities with populations below 5,000 to about 15 percent in larger cities. In another study [1], Amato conducted the hurt-leg experiment in several small Northern California cities and also in San Francisco. His

[7] About 1/4 of these communities came from each of the size ranges: < 1000, $1000 - 5,000$, $5,000 - 20,000$, $> 20,0000$.

findings were similar to those for Australia. In the small Californian cities, 43% of the subjects offered to help and in San Francisco, 20% offered help.

Robert Levine and his co-workers [11] [12] conducted two series of field experiments in a large number of small, medium, and large U.S. cities, once in the early 1990's and again 13-15 years later. These experiments included an episode similar to Amato's hurt-leg experiment.

> "Walking with a heavy limp and wearing a large and clearly visible leg brace, experimenters 'accidentally' dropped and then unsuccessfully struggled to reach down for a pile of magazines..."

Both studies found that in places with larger populations, people were less likely to help. Evidence from the first of these studies suggested that population density had a stronger effect than population size. In the second study, there was insufficient independent variation of size and density to allow them to statistically distinguish the effects of size from those of density.

Our theoretical model suggests that when the hurt-leg experiment is performed on more busily traveled sidewalks, the fraction of passers-by who offer to help would be smaller, but that the average amount of time between offers to help would be shorter. As far as I know, none of these studies calculated the effects of a direct measure of the traffic rate on either the probability that an individual would stop, or on the average amount of time that the "victim" would have to wait for help.[8] While it is likely that the cities with larger population had more frequent pedestrian traffic on the sidewalks where the experiments were performed, this correlation is not likely to be perfect. Levine makes a partial correction for this effect by using population density as well as population size as a variable.

Suppose that we want to test the hypothesis that "big city life leads to public apathy and a lack of concern for the well-being of others." Our discussion suggests that it would not be sufficient simply to find whether "country people are more helpful than city people" by finding the relation between population size and probability of helping, nor would it be sufficient to find out whether people are less likely to stop on busier sidewalks. Our model has it that even if people everywhere have the same distribution of sympathies for others, those who travel on busy city sidewalks are less likely to stop than those on less busy small-town sidewalks. But this model also predicts that the average amount of time between offers of help would be

[8] Amato [2] (page 578) reports that he and his associates recorded pedestrian traffic rates at the sites of their experiments, but he does not appear to have related this observation to outcomes, nor does he estimate the expected waiting time before an injured pedestrian would receive help.

smaller, the busier the sidewalk. If experimenters were to discover that the expected amount of time between offers of help is greater on the busy sidewalks of large cities, this evidence would suggest that those in big cities may tend to have less sympathy or higher costs of stopping than those in small towns.

6 Related Theoretical Work

6.1 The Volunteer's Dilemma game

In 1964, a young woman named Kitty Genovese was attacked and stabbed to death on the streets of New York City. According to news accounts, the murder was witnessed by a large number of people whose apartments overlooked the site of the crime. Yet, none of them came to help her or even bothered to call the police. Commentators offered this event as evidence that big city life leads to public apathy and a lack of concern for the well-being of others. Andreas Diekmann, a sociologist, suggested that these sad events might better explained as the result of a coordination problem that arises whenever well-meaning observers are aware that several other potential helpers are available and that the help of only one is needed.

Diekmann modeled this situation as a game which he called the *Volunteer's Dilemma Game*. In this game, n players choose simultaneously whether or not to take a costly action. If one or more persons act, everyone who acted must pay a cost of c, while all n persons, including those who did not act, will receive a benefit $b > c$. If nobody acts, all receive a payment of zero. Where there are n identical players, Diekmann suggested that the most plausible outcome of the game is the symmetric mixed Nash equilibrium, which will be unique in this case. He showed that, in equilibrium, as the number of player increases, each individual is less likely to take action. More surprisingly, he also found that as the number of players increases, the probability that *nobody* takes action increases. Thus, it would not be surprising to find that even if urban people are just as concerned about the welfare of their neighbors as rural people, crime victims are less likely to be helped in more densely populated cities.

Joseph Harrington [10] and Martin Osborne [13], (pp 131-134) offer further interesting discussions of the Volunteer's Dilemma game. Jeroen Weesie [16] presented a thorough study of this game, along with a provocative discussion of its applications. Weesie showed that, even with symmetric payoffs and complete information, the Volunteer's Dilemma Game will have a large number of asymmetric equilibria. He characterized the full set of complete information Nash equilibria for symmetric and asymmetric versions of the Volunteer's Dilemma.

Weesie [17] presented a version of the Volunteer's dilemma that is a symmetric game of incomplete information in which players' costs differ. Players know their own costs and all believe that the costs of other players are independent draws from a commonly-known uniform distribution. For this distribution, he shows that in symmetric equilibrium, the probability that nobody acts is increasing in n for small n and decreasing in n for large n. Xiaopeng Xu [18] considers a similar model and finds similar results.

Stefano Barbieri and David Malueg [5] study a broader class of games in which the amount of a public good provided to a group is determined by the maximum effort, "Best Shot", made by a group member. This generalizes the Volunteer's Dilemma in which effort levels can take only one of two values, 0 or 1. Barbieri and Malueg find the mixed strategy equilibria for symmetric, complete information Best Shot games. Their results for the complete information game are qualitatively similar to those for the discrete Volunteer's Dilemma. As the number of active players increases, individuals stochastically reduce their efforts and the realized maximum effort is also stochastically reduced. Barbieri and Malueg go on to analyze symmetric games in which the costs of effort differ and where players' own costs being private information. For these games, the symmetric equilibrium is in pure strategies. In equilibrium, as the number of players increase, individual contribution functions are point wise reduced, but players' payoffs are increased. Barbieri and Malueg also show that greater heterogeneity of the group increases players' payoffs.

6.2 Waiting to take action

Weesie [16] studied a related game, which he called the Volunteer's Timing Dilemma. In Diekmann's Volunteer's Dilemma, players move simultaneously and must decide whether to act without being able to observe what others have done. In contrast, players in the Volunteer's Timing Dilemma choose when to act and can wait to see whether others have acted before taking their own actions. In this game, Weesie assumes that everyone would prefer that action be taken earlier rather than later. A strategy in this game is a choice of when to act, conditional on nobody else having acted previously. Assuming that all players have complete information, Weesie finds equilibrium solutions for this game, both for the case of identical payoff functions and for differing payoffs.

In a paper titled "Dragon Slaying and Ballroom Dancing: The Private

Supply of a Public Good", Christopher Bliss and Barry Nalebuff [7] study a game with a payoff structure to similar to that of Weesie's Volunteer's Timing Dilemma. They motivate their discussion by scenarios in which the first person to take a costly action provides benefits to several others who might have taken this action.[9] Bliss and Nalebuff model this as a "game of attrition" in which players have incomplete information about others' costs. Players know their own costs of taking action, but each player views the costs of the other players as independent random draws from a distribution that is common knowledge. Players need not act immediately, but delay is costly to all. Players are able to observe whether any one else has acted, before taking action themselves. A strategy for any player is a mapping from the player's type to the time at which this player will take action if no one else has yet acted. Bliss and Nalebuff show that the game has a Nash equilibrium in which each player's strategy is the same function from his own type to the time at which he will take action if no one else has yet acted. In equilibrium, the first to act will be the player with the greatest net benefit from action being taken. Bliss and Nalebuff find that as the number of players increases, the expected net payoff to each possible volunteer increases. However, for small groups, the length of time before someone takes action may either increase or decrease with the number of players. In a study that summarizes results of Volunteer's Dilemma games and Volunteer's Timing Games [17] with complete and incomplete information, Weesie finds results similar to those of Bliss and Nalebuff.[10]

Marc Bilodeau and Al Slivinski [6] show that if the Dragon Slaying and Ballroom Dancing story of Bliss and Nalebuff is modeled as a stationary game with complete information and an infinite horizon, then it will have an infinite number of subgame perfect equilibria. However, with complete information, if individuals have a finite time horizon, there is a unique subgame perfect equilibrium equilibrium and in that equilibrium, the individual with the highest benefit-cost ratio acts immediately.

6.3 Comparison of results

The sequential arrival structure of passing travelers in the "Road to Jericho" story produces a game that differs qualitatively from both the Volunteer's

[9] As examples, they suggest the awkward period before some couple is first to start dancing in a ballroom full of shy dancers, or where someone first to make the effort to open a window in an overheated lecture hall.

[10] Weesie's model differs slightly from that of Bliss and Nalebuff in the way in which the cost of delay is modeled.

Dilemma, the Best Shot game, and the Volunteer's Timing Dilemma. In the Volunteer's Dilemma game and the Best Shot game, actions are taken simultaneously and there is a significant probability that costly efforts will be duplicated, even though the effort of only one player is needed. In the Volunteer's Timing Dilemma game and in the Ballroom Dancing game, there is no duplicated effort in equilibrium. In these games, all players observe the need for action at the same time, but all have the option of waiting to act at a later time and can choose to act only if no one else has yet acted. The Road to Jericho game also avoids the possibility of wasteful duplicated effort, since travelers see the stranded motorist only if he has not been helped. In this game, however, travelers do not have the option of waiting, but must either offer immediate help or drive on.

In the Volunteer's Dilemma with identical players, the equilibrium expected payoff to potential volunteers is unchanged as the number of volunteers increases, but the recipient is worse off, since help is less likely to arrive. In the Volunteer's Timing Dilemma, as more individuals are added, the expected payoff to potential volunteers increases, but the recipient may or may not have to wait longer to receive assistance. In the Road to Jericho game, as traffic levels increase, travelers who are not stranded are better off, since they are less likely to have to pay the cost of stopping. If stopping costs differ among travelers, stranded motorists can expect shorter waiting times on more heavily travelled roads, while if stopping costs are the same for all travelers, expected waiting times do not change with the density of traffic.

7 Conclusion: Fables and Games

Ariel Rubenstein asserts that:

> "Game theory is about a collection of fables. Are fables useful or not? In some sense, you can say that they are useful, because good fables can give you some new insight into the world and allow you to think about a situation differently. But fables are not useful in the sense of giving you advice about what to do tomorrow..."[8]

This paper tells a simple tale. I will consider it successful if it meets Rubenstein's criterion for a good fable: giving new insight to a common situation and allowing one to think about it differently. The fable told here offers a fresh look at the familiar *Parable of the Good Samaritan.* In this version, I apply game theory to the riddle of who would help someone in need if everyone believes that others are willing to do so. This paper probes the relation between social efficiency and ethical norms when more than one person could help a stranger. The fable told here proposes a theoretical explanation for observations that people tend to be act more helpfully in less populous places, but it also predicts that help is likely to arrive more quickly in more densely populated places. The discussion also suggests an improvement in the design of field experiments that may lead to a sharper test of the hypothesis that urban dwellers are less neighborly than their country cousins.

8 Appendix

Proof of Proposition 3

Proof of Proposition. Equilibrium requires that

$$w_F(\lambda)F(w_F(\lambda)) = \frac{1}{\lambda}. \tag{34}$$

Taking logarithms of both sides of Equation 34 and differentiating with respect to $\log \lambda$, we find that:

$$\frac{d \ln w_F(\lambda)}{d \ln \lambda} + \frac{d \ln F(w_F(\lambda))}{d \ln w} \frac{d \ln w_F(\lambda)}{d \ln \lambda} = -1 \tag{35}$$

Rearranging terms of Equation 35 results in the expression

$$\eta_w(\lambda, F) = \frac{-1}{1 + \sigma_F(w_F(\lambda))}. \tag{36}$$

Since η_F is necessarily non-negative, it follows from Equation 36 that the elasticity of equilibrium expected waiting time with respect to the arrival rate λ is negative and lies in the interval between -1 and 0.

□

Proof of Proposition 6

Our proof of the second assertion of Proposition 6 uses the following Lemma.

Lemma 1. *Suppose that the distribution function F is log-concave. Then the function*

$$\delta(x) = x - \frac{\int_\ell^x t f(t) dt}{F(x)}$$

is monotone increasing in x.

A proof of this result is found in Bagnoli and Bergstrom [4].

Proof of Proposition. Let $c^*(\lambda)$ be the ethical ideal stopping threshold when the Poisson arrival rate of traffic is λ. From Equation 15 it follows that

$$c^*(\lambda)F(c^*(\lambda)) - \int_\ell^{c^*(\lambda)} cf(c)dc = \frac{v}{\lambda} \tag{37}$$

Differentiating both sides of Equation 37 with respect to λ and rearranging terms, we find that

$$c^{*\prime}(\lambda) = -\frac{v}{\lambda^2 F(c^*(\lambda))} < 0. \tag{38}$$

The effect of traffic density on the probability that any single passer-by will stop is therefore

$$\frac{d}{d\lambda}F\left(c^*(\lambda)\right) = f\left(c^*(\lambda)\right)c'(\lambda) = -\left(\frac{v}{\lambda^2}\right)\frac{f\left(c^*(\lambda)\right)}{F\left(c^*(\lambda)\right)} < 0. \tag{39}$$

This proves the first assertion of the theorem.

Let $w(\lambda) = 1/\lambda F\left(c^*(\lambda)\right)$ be the expected waiting time for a stranded traveler if passing travelers stop when and only when their their stopping costs are below $c^*(\lambda)$. Let us define

$$\delta(c^*) = c^* - \frac{\int_\ell^{c^*} cf(c)dc}{F(c^*)}. \tag{40}$$

From Equations 15 and 40 it follows that

$$\delta\left(c^*(\lambda)\right) = vw(\lambda). \tag{41}$$

From Equation 41 it follows that

$$w'(\lambda) = \frac{1}{v}\delta'\left(c^*(\lambda)\right)c^{*\prime}(\lambda) \tag{42}$$

According to Lemma 1, the assumption that F is log-concave implies that $\delta'\left(c^*(\lambda)\right) > 0$. According to Expression 38, it must be that $c^{*\prime}(\lambda) < 0$. Therefore Equation 42 implies that $w'(\lambda) < 0$. It follows that the more dense is traffic on the road, the shorter is the equilibrium expected waiting time for a stranded traveler.

□

Proof of Proposition 8

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