2014

Negative Leakage

Kathy Baylis, *University of Illinois at Urbana-Champaign*
Don Fullerton, *University of Illinois at Urbana-Champaign*
Daniel H. Karney, *Ohio University*

Negative Leakage

Kathy Baylis, Don Fullerton, Daniel H. Karney

Abstract: Our analytical general equilibrium model solves for effects of a small increase in carbon tax on leakage—the increase in emissions elsewhere. Identical consumers buy two goods using income from endowments that are mobile between sectors. Usually an increase in one sector's tax raises output price, so consumption shifts to the other good, causing positive leakage. Here, we find a new negative effect not recognized in existing literature: the taxed sector substitutes away from carbon into clean inputs, so it may absorb resources, shrink the other sector, and reduce their emissions. This "abatement resource effect" could offset some or all of the positive effect. We show that this effect can substantially affect estimates of leakage and is robust to model extensions.

JEL Codes: H23, Q37, Q54, Q56

Keywords: Cap and trade, Carbon tax, Climate change, Climate policy, Emissions abatement, Factor mobility, Global warming, International trade

A COMMON CONCERN with a pollution restriction in one region or one sector is that abatement will be offset by "leakage," the increase in pollution elsewhere. In a trade context, the regulating country puts itself at a competitive disadvantage. Within a country as well, a policy such as cap and trade may apply only to one sector such as electricity, which raises its price and shifts demand to other goods. Purely domestic leakage may offset some of the regulated sector's abatement. In the context of climate policy, computational general equilibrium (CGE) models are often used to calculate

Kathy Baylis is at Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign (baylis@illinois.edu). Don Fullerton is at Department of Finance and IGPA, University of Illinois at Urbana-Champaign (dfullert@illinois.edu). Daniel H. Karney is at Department of Economics, University of Illinois at Urbana-Champaign (dkarney2@illinois.edu). We are grateful for comments and suggestions from Mustafa Babiker, Jared Carbone, Fernando Pérez-Cervantes, Brian Copeland, Carolyn Fischer, Larry Goulder, Corbett Grainger, Garth Heutel, Mark Jacobsen, Larry Karp, Carol McAusland, Gilbert Metcalf, Sergey Paltsev, Sebastian Rausch, David Weisbach, Martin Weitzman, Niven Winchester, and various seminar and conference participants.

Received January 10, 2014; Accepted February 24, 2014; Published online May 20, 2014.

JAERE, volume 1, number 1.
2333-5955/2014/0101-00XX $10.00 http://dx.doi.org/10.1086/676449

leakage from a partial limit on carbon emissions such as under the Kyoto Protocol. Paltsev (2001) finds a leakage rate of 10%, whereas Babiker (2005) finds rates as high as 130%. In that case, a carbon tax in one region raises worldwide emissions. More typical of other recent estimates, Elliott et al. (2010) find a 20% carbon leakage rate.

In this study, we demonstrate a new and substantial negative effect on leakage that has not been identified in prior literature.[1] To do so, we use a simple analytical general equilibrium model in the style of Harberger (1962) with only two outputs, each produced using two inputs: carbon emissions and one clean input (which could be labor, capital, or a composite of both). Our model can be taken to represent two countries that each produce one good or a closed economy that produces two goods. We then suppose an increase in one sector's carbon tax (or permit price). It raises the price of that output, so consumers substitute to the other output, with a positive effect on leakage we call the "terms-of-trade effect" (TTE). However, that carbon policy also induces firms to abate carbon per unit of output by using more of the clean input for abatement. The taxed sector draws resources away from the other sector or country, which reduces their output and emissions. We call this an "abatement resource effect" (ARE). Its size depends on parameters. If consumers can shift their purchases easily, then positive leakage may be high. Even then, however, leakage may be overstated in models that do not allow for substitution in production. If consumer flexibility is low compared to producers' ability to abate pollution by use of other resources, however, then we show that overall leakage may be negative.

The negative effect on leakage we identify here is based on three reasonable and general assumptions. First, the two goods are not perfect substitutes. Consumers still cause leakage when they substitute toward the other good, but not perfectly. Second, the firm has some ability to substitute out of carbon and into the clean input. The elasticity of substitution in production is not zero, so firms can reduce carbon per unit of output by using "abatement resources." Third, the clean input is mobile between the two sectors or countries. These assumptions do not represent a special case. Rather, they are generalizations of prior models that assume capital is not mobile, or that firms cannot change carbon per unit of output, or that the two goods must be identical.

Given the simplicity of this result, we wonder why previous literature has not identified it. First, some papers assume a fixed ratio of emissions to capital (e.g., Ogawa and Wildasin 2009). Second, many models in the Heckscher-Ohlin-Samuelson tradition assume that goods are traded between countries but factors are not, and this tradition is adopted in CGE models that assume capital and labor are mobile between sectors within a country but not between countries (e.g., Elliott et al. 2010; Böhringer, Carbone, and Rutherford 2011; Winchester, Paltsev, and Reilly 2011). Those three

1. Below we clarify how this new effect differs from negative effects on leakage identified by others.

studies simulate a tax on all carbon in a country, so the firms cannot draw resources from an untaxed sector.[2] Third, some CGE models may incorporate all three of our key assumptions but then report a single net leakage result, obscuring the fact that our ARE offsets some of the positive leakage. In any case, we do not find any prior study that derives analytical expressions for leakage in a model where firms can substitute into a clean input that is mobile.[3]

Given findings that leakage is positive, academics have searched for particular cases with counterintuitive results. We cannot review all such literature, but we list a few examples. First, Felder and Rutherford (1993) build a CGE model with five regions and 10-year intervals, finding that leakage can be negative after several decades if the carbon tax leaves enough unused oil to delay the other region's switch to carbon-intensive synthetic fuel. Second, Copeland and Taylor (2005) show how negative leakage can arise through endogenous policy: if one region cuts emissions, then the other region gains income and could choose more environmental quality by cutting their own emissions. Third, negative leakage can arise through endogenous technology: the carbon tax may induce R&D on new abatement technology that can be used by the unregulated sector, especially if patents are poorly protected.[4] Fourth, Chua (2003) shows that a carbon tax can induce abatement that might itself be either labor or capital intensive, which can change the wage-rental ratio. If the dirty sector intensively uses the factor whose relative price has fallen, then Karp (2013) shows how this change in wage-rental ratio can reduce the cost of producing the dirty good, reduce imports of it, and thus reduce emissions elsewhere.[5] In contrast, our simple

2. Even without international factor mobility, CGE models can include an ARE if they are used to simulate a carbon tax that does not apply to all sectors (as in Carbone 2013) or to all of the United States (as in Caron, Rausch, and Winchester 2012; or Winchester and Rausch 2013). Fischer and Fox (2010) simulate a US carbon tax in "covered" sectors. They report positive leakage in other countries, and positive leakage overall, but domestic uncovered sectors reduce emissions. This result can be attributed to an "input-output effect": the carbon tax raises the price of electricity and gasoline, both used as inputs in the domestic uncovered sectors. Those uncovered sectors then reduce their energy use and, therefore, their emissions.

3. Burniaux and Oliveira Martins (2012) also find that negative leakage is possible in their simulation model, based on variations in parameters via sensitivity analysis, not analytical expressions.

4. See Di Maria and Smulders (2004), Golombek and Hoel (2004), Gerlagh and Kuik (2007), and Di Maria and van der Werf (2008).

5. Karp (2013) uses analytical general equilibrium models of a small open economy that produces both a clean good and a dirty good facing a carbon tax, and he investigates several possibilities for negative leakage. Factors are mobile between sectors but not between countries. He describes an income effect that also operates in our model: the tax reduces real income, so consumers may buy less of both goods (which reduces imports of the dirty good). To the extent that results are similar, note that his effort and ours were simultaneous and independent

model employs no endogenous policy, no induced technology, no change in wage-rental ratio, and no fall in the cost of producing the carbon-intensive good. It just involves a clean input (like labor or capital) that can be used for abatement in the regulated sector instead of in production of the other good.

Our intent is to demonstrate this abatement resource effect using the simplest possible model, not to measure actual leakage. We therefore abstract from many important issues such as materials production and intermediate inputs (e.g., Felder and Rutherford 1993), endogenous number of firms (Gurtzgen and Rauscher 2000), oligopolistic competition (Babiker 2005), and strategic interaction (Fowlie 2009). Such features could affect leakage, but none would remove the ARE in our expressions. In any model with the three assumptions above, results would still include this negative leakage term. We also abstract from distributional effects or allocation of permits, as many identical consumers receive lump-sum rebate of all carbon tax or permit revenue.

Thus the model is too simple to identify all effects or to measure actual leakage, but it is useful nonetheless to show the simple intuition for an effect that does operate in more complicated models and in actual policy settings. For example, a carbon policy could apply to electricity generation and not to other industries, as in President Obama's recent greenhouse gas (GHG) initiative, or it could apply to one region within a country.[6] If labor or capital can be used for abatement, then it might not be used in the unregulated sector.

We intentionally eschew derivations of welfare effects, because readers would naturally focus on those results and the normative implications. Here we wish to focus on the positive implications of climate policy for emissions quantities. We therefore put welfare extensions into a different paper that follows this one.[7] Also, others have pointed out multiple effects on leakage not captured here. In a later paper, we extend

---

(the first version of both working papers appeared in the same month of 2010). The papers differ in other respects. For example, Karp relaxes the assumption of separability in production between emissions and the two factor inputs to capture a "production effect" from a change in relative factor prices. He also shows how partial equilibrium models may overstate leakage found in a comparable general equilibrium model. In contrast, we solve for closed-form expressions that identify the "abatement resource effect."

6. For example, the Regional Greenhouse Gas Initiative (RGGI) is a subset of US states that agree to limit carbon emissions. Sue Wing and Kolodziej (2008) find that leakage rates exceed 50%, due to electricity imports from non-RGGI states. Also, the case of California is analyzed by Caron et al. (2012).

7. See Baylis, Fullerton, and Karney (2013a). The carbon tax always reduces the quantity of the taxed good, and net leakage can only be negative with reductions in the other good as well. Since those are the only two consumption goods, net negative leakage necessarily means a loss in utility from consumption. With net positive leakage, however, that paper shows when the effect on welfare is positive or negative.

this model to include six different leakage effects (two positive and four negative). Those extensions are used to "unpack" results from CGE models.[8] But our goal here is to explore the ARE, not to build a model with all the bells and whistles.

To explore potential magnitudes of the ARE, we use empirical estimates of parameters and existing CGE models with modifications to allow for the ARE. Those modified models find positive net leakage over many parameter values, but allowing for the ARE in some cases can substantially reduce the leakage rate (defined as the change in emissions elsewhere as a percentage of abatement in the regulated sector). Allowing for the ARE reduces that rate from 10% to 3% in one model, and from +15% to −8.5% in another.

Then, to demonstrate robustness of our result, we extend the basic model in three ways. First, we allow for an upward-sloping supply of carbon, which implies another positive source of leakage but which does not affect the ARE.[9] Second, we extend the basic model to allow for a variable factor supply. Third, we consider imperfect mobility of the clean input (labor and capital). All three extensions can affect the amount of leakage, but the ARE remains.

The next section presents our basic model, and section 2 differentiates equations to linearize the model and solve for effects of an increase in one sector's pollution price. We identify the ARE in a closed-form expression for the change in carbon leakage. Section 3 provides a numerical example using the simple model, and it cites new modifications to CGE models that also demonstrate the existence of the ARE. Section 4 provides further discussion, and section 5 provides extensions.

1. THE BASIC MODEL

Two competitive sectors ($i = X, Y$) each use clean input K_i and carbon emissions C_i with decreasing marginal products in a constant returns to scale production function, $X = X(K_X, C_X)$ and $Y = Y(K_Y, C_Y)$. We call the clean input "capital," but it

8. Baylis, Fullerton, and Karney (2013b) start with the model of this paper but add a fuel price effect (FPE): a fall in the taxed sector's demand for oil may reduce its price and induce other regions to increase their use of oil. They also add a pure income effect (PIE), policy normalization effect (PNE), and input-output effect (IOE). They employ data from a CGE model to determine the corresponding parameters for the expanded analytical model and to determine the implied size of each of the six leakage terms. Thus the one number for net leakage in the CGE model can be unpacked into six different terms using the analytical model.

9. If the supply of fossil fuel were perfectly inelastic in this static model, then its price would adjust so that any fuel not used by the taxed sector must be used by the other sector: exactly 100% leakage. Our basic model represents the other extreme where supply is perfectly elastic, but the truth is in between. Our extension in section 5 provides an intermediate fuel supply elasticity. It yields a positive effect on leakage but does not remove the ARE term's negative effect on leakage.

can be labor, capital, or a composite of both. It is perfectly mobile and earns the same return, p_K, in either sector. It is initially in fixed supply $(\bar{K} = K_X + K_Y)$, but an extension below shows that variable factor supply does not affect the ARE. The only existing taxes are on carbon in each sector. Each sector faces a carbon tax τ_i and can choose any amount of C_i. Below we consider an extension to change this flat supply curve to an upward-sloping supply curve; we find that it affects leakage but not the ARE. Competition and constant returns to scale imply zero profits, so $p_X X = p_K K_X + \tau_X C_X$ and $p_Y Y = p_K K_Y + \tau_Y C_Y$.

In response to an increase in τ_Y, firms can reduce carbon per unit of output by substitution from C_Y into K_Y for abatement. In the electricity sector, for example, firms can reduce emissions per kilowatt hour by investing in natural gas plants, wind turbines, or solar power. With no uncertainty, however, the resulting choice for carbon quantity at tax rate τ_Y can equally represent a policy with that number of permits at price τ_Y. All revenue is returned via lump-sum rebate $R \equiv \tau_X C_X + \tau_Y C_Y$.

Emissions from each sector add to total carbon, $C \equiv C_X + C_Y$, which negatively affects utility in a separable manner. Many identical households earn income from the clean factor K and the rebate of revenue, taking as given the total carbon and all market prices (p_X, p_Y, and p_K). They maximize homothetic utility by choice of X and Y:

$$\max_{\{X,Y\}} U(X, Y; C) \text{ subject to } p_K \bar{K} + R \geq p_X X + p_Y Y.$$

We have no need to specify which sector initially has the higher carbon tax rate, and so we simply investigate effects of a small increase in τ_Y with no change in τ_X. We compare the new long-run equilibrium to the initial one, ignoring adjustments during the transition. The increase in τ_Y reduces equilibrium emissions in sector Y, and so leakage is defined as the effect on emissions in sector X.

This simple model can be interpreted at least two ways. First, it can represent an international context where Y is produced in one country or set of countries that raises its carbon tax, while X is produced in the "rest of world." In this case, we suppose that all consumers have the same utility function. Capital is owned by these identical worldwide consumers, and it can be used to produce either region's output.[10] A more complete trade model might have both regions produce both outputs, with the same type of negative leakage that we identify (so long as firms can substitute into mobile capital).

Alternatively, the model can represent a closed economy in which sector Y faces a raised price of carbon. The transportation sector is not a good example, because a

10. Our model with a single type of worldwide consumer is not adequate to analyze effects on welfare in each country, but our goal here is only to look at effects on carbon emissions in each country.

limited supply of oil is traded worldwide. A carbon tax would reduce demand for oil and its price, increasing consumption and emissions elsewhere. In this basic model, we omit this "fuel price effect." A better example is a tax on carbon emissions of coal-fired power plants, where coal is not scarce (its world price depends primarily on extraction cost). Any US carbon policy might start just with electricity generation, as did the US sulfur dioxide permit market. Also, the European Union's Emission Trading System (EU-ETS) applies to electricity and major industries, including only about 45% of total GHG emissions.[11] Any agreement between the EU and United States or other nations may apply GHG pricing to a similar subset of outputs (with the usual problems of aggregation). This example is particularly appropriate because electricity has relatively inelastic demand, which means a low elasticity of substitution in utility (decreasing the positive TTE term and making net negative leakage more likely).

2. SOLVING FOR EQUILIBRIUM EFFECTS

Given this setup, we now linearize the model to solve for n linear equations in n unknowns. Totally differentiate the resource constraint $\bar{K} = K_X + K_Y$, and use the "hat" notation to denote a proportional change in any variable (e.g., $\hat{K}_X \equiv dK_X/K_X$):

$$0 = \alpha_X \hat{K}_X + \alpha_Y \hat{K}_Y, \tag{1}$$

where $\alpha_i \equiv K_i/\bar{K}$ is the share of capital in production of i ($i = X, Y$), and $\alpha_X + \alpha_Y = 1$. Then totally differentiate each production function to show how changes to inputs affect final output:

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XC}\hat{C}_X, \tag{2}$$

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YC}\hat{C}_Y, \tag{3}$$

where θ_{ij} is the factor share of income for input j in the production of good i (e.g., $\theta_{XK} \equiv (p_K K_X)/(p_X X)$). Then $\theta_{XK} + \theta_{XC} = 1$ and $\theta_{YK} + \theta_{YC} = 1$. Next, totally differentiate the zero profit conditions, and use the firm's profit maximizing first-order conditions:

$$\hat{p}_X + \hat{X} = \theta_{XK}\left(\hat{p}_K + \hat{K}_X\right) + \theta_{XC}\left(\hat{\tau}_X + \hat{C}_X\right), \tag{4}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}\left(\hat{p}_K + \hat{K}_Y\right) + \theta_{YC}\left(\hat{\tau}_Y + \hat{C}_Y\right). \tag{5}$$

11. See http://ec.europa.eu/clima/policies/ets/index_en.htm. Even in a well-designed climate policy, Metcalf and Weisbach (2009) find that only 80% or 90% of GHG emissions can be feasibly covered.

Each production function has only two inputs, so factor intensity responds to a change in relative input prices according to each elasticity of substitution, σ_X and σ_Y. We define these elasticities to be positive. Differentiating their definitions yields

$$\hat{C}_X - \hat{K}_X = \sigma_X\left(\hat{p}_K - \hat{\tau}_X\right), \tag{6}$$

$$\hat{C}_Y - \hat{K}_Y = \sigma_Y\left(\hat{p}_K - \hat{\tau}_Y\right). \tag{7}$$

Finally, under the assumption that pollution is separable in utility, we can use a single parameter σ_U to define the elasticity of substitution in utility between X and Y. Differentiating the definition of σ_U yields

$$\hat{X} - \hat{Y} = \sigma_U\left(\hat{p}_Y - \hat{p}_X\right). \tag{8}$$

Suppose β is the share of income spent on Y, and η_{YY} is the usual own-price elasticity of demand (with no change in any other prices). Then one can show $\eta_{YY} = -[\beta + \sigma_U(1-\beta)]$. In other words, for a good like electricity with inelastic demand, the trade-off between that good and all other goods can be represented by a small σ_U.

Equations (1)–(8) are the linear system for general equilibrium effects of a small change in policy. Specifically, we consider a small increase in sector Y's carbon tax, while holding the carbon tax in sector X constant; that is, $\hat{\tau}_Y > 0$ and $\hat{\tau}_X = 0$. We define capital as numeraire ($\hat{p}_K = 0$), which leaves the eight numbered equations above with eight unknowns (changes in X, Y, their two prices, and the four input quantities). Sector X experiences no change in relative input prices $\left(\hat{\tau}_X = \hat{p}_K = 0\right)$, so equation (6) simplifies to $\hat{C}_X = \hat{K}_X$. Note that we do not assume Leontief production in X. Those firms have a positive σ_X, but they choose not to alter input ratios because they face no relative input price changes. In addition, unchanged input prices means no change in the break-even output price, so $\hat{p}_X = 0$ (from eqs. [2] and [4]).

Next, observe from (3) and (5) that $\hat{p}_Y = \theta_{YC}\hat{\tau}_Y > 0$. The additional carbon tax always raises the price of Y relative to the price of X. Further algebra reveals

$$\hat{Y} = -\left[\alpha_X\sigma_U + \alpha_Y\sigma_Y\right]\theta_{YC}\hat{\tau}_Y < 0. \tag{9}$$

Since all parameters in this equation are positive, the negative sign out front means that the increase in τ_Y unambiguously reduces the Y output—to an extent that depends on substitution elasticities and the carbon share of production. Algebra also yields an expression for the change in the taxed sector's carbon:

$$\hat{C}_Y = \left[\underbrace{-\left(\alpha_X\sigma_U + \alpha_Y\sigma_Y\right)\theta_{YC}}_{\substack{\text{Output}\\ \text{Effect}}} - \underbrace{\theta_{YK}\sigma_Y}_{\substack{\text{Substitution}\\ \text{Effect}}}\right]\hat{\tau}_Y < 0. \tag{10}$$

The second term inside the large brackets is the "substitution effect," since the tax changes relative input prices and induces substitution through the elasticity σ_Y. Firms reduce carbon per unit of output. Then the first term is just $\hat{Y}$, from (9). It represents an "output effect," since the tax raises output price and reduces demand. The tax on carbon reduces carbon emissions through both of these channels, and so (10) shows that $\hat{C}_Y$ is unambiguously negative.

In the other sector, however, two leakage effects operate in different directions:

$$\hat{C}_X = \alpha_Y(\sigma_U - \sigma_Y)\theta_{YC}\hat{\tau}_Y = \left[\underbrace{\sigma_U\alpha_Y\theta_{YC}}_{TTE} - \underbrace{\sigma_Y\alpha_Y\theta_{YC}}_{ARE}\right]\hat{\tau}_Y \gtrless 0. \tag{11}$$

The first effect in (11) is a terms-of-trade effect (TTE), because the higher price of Y induces consumer substitution into X (to an extent that depends on σ_U). Alone, it would raise production of X and therefore raise C_X (positive leakage). The other term in (11) is what we call the abatement resource effect (ARE). It depends on σ_Y, because the firms in Y substitute from carbon into capital for abatement and thus bid capital away from X. Since $\hat{\tau}_X = \hat{p}_K = 0$, those firms choose not to substitute and instead reduce both K_X and C_X. This term is a negative leakage term.

Clearly, from (11), the relative size of these offsetting effects depends on the relative size of σ_U and σ_Y. If consumers can substitute easily between goods, then the terms-of-trade effect dominates, and net leakage is positive. This effect would be large for the case with international trade in close substitutes. Using the Armington (1969) assumption, for example, σ_U would be large, and net leakage is positive. Even then, however, researchers might overstate leakage if they do not allow for any negative effect through home substitution into abatement capital (the ARE effect).

In other cases such as the pricing of carbon permits in the electricity sector, demand is inelastic, and so σ_U is small. If technology allows for easy abatement per unit of output, then σ_Y may exceed σ_U, and overall leakage is negative. In this case, models that ignore the ARE would find the wrong sign for overall leakage. If leakage is negative, the net effect of unilateral pollution regulation could be overall pollution reduction beyond what is achieved within the regulating sector, region, or country.

Equation (11) for $\hat{C}_X$ is a simple linear function of the elasticities, and it clearly indicates whether leakage is positive or negative. The change in C_Y in (10) is also a linear function of elasticities. However, the rate of leakage is normally defined as the change in the other sector's emissions as a fraction of abatement in the taxed sector:

$$-\frac{dC_X}{dC_Y} = -\frac{\hat{C}_X}{\hat{C}_Y}\left(\frac{C_X}{C_Y}\right). \tag{12}$$

This expression is not a linear function of elasticities. Also, it does not depend on the size of $\hat{\tau}_Y$, which is in both the numerator from (11) and the denominator from (10).

3. NUMERICAL MAGNITUDES

For an illustration of how leakage depends on elasticities, we use data from Elliott et al. (2010) to set parameters for equations above. Suppose an international agreement to tax GHG emissions from electricity generation, Y, where X is the rest of production in a closed economy (the whole world).[12] The electricity sector might be the initial staging ground for carbon policy, just as it was for sulfur dioxide in the United States. The illustration here uses the entire world, for comparability with the closed economy of our analytical model, but of course worldwide agreement is hardly "likely." In these data, electricity is quite carbon intensive ($\theta_{YC} = 0.147$) compared to the rest of the economy ($\theta_{XC} = 0.01$). But electricity is a small sector (the share of labor and capital used in electricity is only $\alpha_Y = 0.018$). Thus, the other sector still uses more carbon than does electricity ($C_X/C_Y = 1.66$). These choices are simple, but they allow for a visual representation of how leakage depends on σ_U and σ_Y.

Using these parameter values in equations (10)–(12), figure 1 shows the percentage rate of leakage on the vertical axis ($-dC_X/dC_Y \times 100$), plotted against the elasticity of substitution in production on the horizontal axis (σ_Y). The top dotted line uses $\sigma_U = 1.5$ to show how leakage declines from about +2.0% to −0.1% as σ_Y varies from 0.1 to 2.0. Thus, net leakage can be negative, even with a high σ_U, if σ_Y is high enough. The middle dashed line uses $\sigma_U = 1.0$ and shows again that leakage declines with σ_Y. The bottom solid line is for $\sigma_U = 0.5$, where leakage declines from +1.1% to −0.4% as σ_Y varies from 0.1 to the high value of 2.0.

These results demonstrate the potential importance of including the ARE. Setting $\sigma_Y = 0$ effectively ignores it and yields only positive leakage, while the figure shows how the ARE reduces leakage. With σ_Y high enough, the ARE can more than offset the TTE.

So far, we have shown computed outcomes from our analytical two-sector general equilibrium model, which essentially represents a simple CGE model. But what is the likely size of the ARE based on estimated magnitudes or other more complicated models?

Available data do not provide enough carbon price variation for empirical estimation of the elasticity of substitution between carbon and other inputs, σ_Y, especially for each different application or for each sector. Yet we have many estimates of the price elasticity of demand for electricity, η_{YY}, and this literature suggests −0.40 as a reasonable central estimate.[13] We showed above that $\eta_{YY} = -\beta + \sigma_U(1 - \beta)$, and households

12. Elliott et al. (2010) use the Global Trade Analysis Project (GTAP) data from 2004 for values of labor, capital, and output. We find CO_2 emissions for that year and assume a price of $15 per ton to get hypothetical payments for carbon.

13. Reiss and White (2005) find an average elasticity of −0.39 for California residential consumers. Alberini and Filippini (2011) note that estimates of the long-run elasticity for US

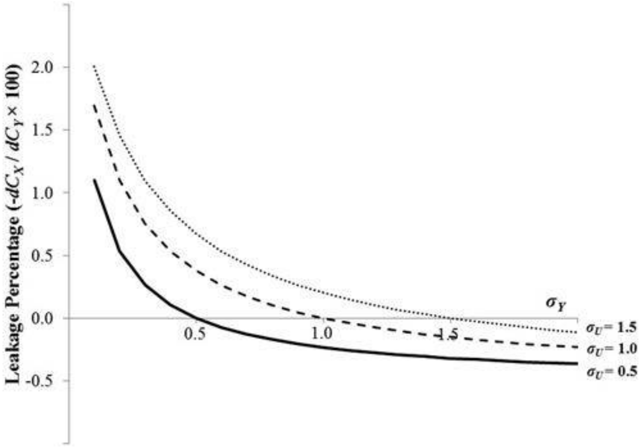

Figure 1. Leakage rises with σ_U and falls with σ_Y ($\alpha_Y = 0.018$, $\theta_{YC} = 0.147$)

spend approximately $\beta = 0.03$ of income on electricity, so we calculate $\sigma_U = 0.38$. For that choice of σ_U, equation (11) implies that net leakage would be negative for any σ_Y greater than 0.38.

What do complicated CGE models imply about the ARE and net leakage? Typically, CGE models are built by several researchers over several years, and each can be used to calculate leakage for a variety of parameters. Our introduction above lists several models that assume factors are mobile between sectors within a region but not between regions; then the models are used to simulate a tax on all carbon in a region. Since firms cannot draw resources from an untaxed sector, the ARE cannot arise. Fortunately, however, authors of three such models saw our initial working paper (Fullerton, Karney, and Baylis 2011) and have modified their models to study the question.

First, Winchester and Rausch (2013) consider a carbon tax in just the western portion of the United States.[14] The model includes nested substitution in production, so firms in the West can abate carbon by altering use of three fossil fuels and other

residential electricity vary from −0.3 to −0.8 (and their own estimate ranges from −0.45 to −0.75). In an extensive survey of hundreds of estimated long-run price elasticities, Dahl (2011) finds a mean of −0.48 and median of −0.37.

14. California's Global Warming Solutions Act (AB32) establishes a tradable emissions cap that declines over time and that applies to major sources of GHG emissions such as refineries, power plants, industrial facilities, and transportation fuels (http://www.arb.ca.gov/cc/ab32/ab32.htm). Also, see Goulder (2013).

inputs.[15] Their model has many other features, and they always report net leakage that is positive, but the case that corresponds most closely to our analytical model is their simulation where labor and capital are mobile across regions and fuel supply is elastic. In that case, their figure 2*A* shows how net leakage falls from about 10% when σ_Y is zero to about 3% as σ_Y rises to 2.0 or higher. Because the ARE depends directly on σ_Y in our equation (11), we see that it can offset two-thirds of leakage in this model.[16]

Second, the CIM-EARTH model of Elliott et al. (2010) also assumes that capital and labor are not mobile between regions when they simulate a tax on all carbon in one region. Their model has 16 world regions that each produce 16 outputs using labor, capital, various kinds of natural resources, and intermediate inputs from other sectors. To look for an ARE, and to convert this CGE model to a closed economy, Elliott and Fullerton (2014) suppose that the entire world agrees to a carbon tax only on the electricity sector. Then they vary the elasticity in that model that corresponds most closely to our σ_Y, and they show results in a figure that looks very much like figure 1 above. When that elasticity is high enough, net leakage turns negative.[17]

Third, Carbone (2013) uses the model of Böhringer et al. (2011) to simulate a tax on carbon used in electricity, also for a single world region. His case that corresponds most closely to ours appears in the last column of his table 1B with high fuel supply elasticity. Leakage is +15% with high substitution between electricity and other goods (analogous to our σ_U), but it falls to −8.5% when that elasticity is low and σ_Y is high.

The two sectors in our analytical model can represent two different regions (such as East and West in Winchester and Rausch [2013]) or two different outputs (such as electricity and all other goods as in Carbone [2013] and Elliott and Fullerton [2014]). In either type of simulation, using these three CGE models, an abatement resource effect can offset some or all of the resulting leakage.

15. They use the US Regional Economic Policy model described by Rausch et al. (2010), a multiregion, multisector CGE model calibrated to 2006 data. Each output is produced using capital, labor, coal, crude oil, natural gas, and intermediate inputs, all in a nested constant elasticity of substitution (CES) function. The utility function for each region is also a nested CES function of commodities that enter final demand.

16. When fuel supply is not so elastic, leakage is larger because of the fuel price effect. Even in those cases, however, increases in σ_Y indicate that the ARE still offsets up to 7 percentage points of leakage.

17. Both Winchester and Rausch (2013) and Elliott and Fullerton (2014) include figures where net leakage starts positive, falls with σ_Y, and then levels off. An implication is that σ_Y does not need to be very high for the ARE to affect leakage; any increase above 2.0 has little effect. See figure 1 above.

4. DISCUSSION

Even our basic model raises at least four questions, which we now address.

4.1. How Can Factor Prices in Sector X Remain Unchanged?

Some have wondered why the increased demand for abatement resources does not raise the price of K. We choose K as numeraire ($\hat{p}_K = 0$), but our solution yields $\hat{p}_X = 0$. Thus, the choice of X as numeraire would yield exactly the same analytical results. Yet we also assume that $\hat{\tau}_X = 0$, which means that policy holds constant the carbon tax in sector X relative either to their output price or to their input price. Indeed, if p_K were to rise relative to τ_X, then leakage would increase because of a cut in the other sector's real carbon tax.[18] Leakage is best defined by an increase τ_Y with no change in the real τ_X. What matters is not the choice of numeraire but the choice of what defines a constant τ_X in real terms. Suppose that policy were to hold τ_X constant relative to p_Y (or to some overall weighted average price). Since p_Y rises, that would raise τ_X relative to our numeraire (p_K or p_X), which would reduce leakage. In this respect, our simple assumption yields a conservative expression for net negative leakage.

4.2. Does the Demand for X Slope Up?

When we find that net leakage $\hat{C}_X$ is negative, the output of X declines even though the relative price of X falls (recall that $\hat{p}_Y > 0$, while $\hat{p}_X = 0$). Is X a Giffen good? No. The usual own-price elasticity of demand for X with no change in other prices is derived as $\eta_{XX} = -[(1-\beta) + \sigma_U \beta]$, which is clearly negative. A fall in p_X alone would raise X, partly because it increases real income. In contrast, the increase in p_Y reduces real income. In fact, the cross-price elasticity of demand for X with respect to a change in p_Y is $\eta_{XY} = \beta(\sigma_U - 1)$, which can have either sign.[19]

18. The setting of τ_X is an assumption, as explored in the "policy normalization effect" of Baylis et al. (2013b). Of course, actual policy might allow the real τ_X to fall. A carbon cap or tax could increase global demand for relatively clean inputs like natural gas, shifting other sectors toward carbon-intensive inputs like coal. The increased price of clean inputs could reduce the real τ_X, which might increase both output and carbon intensity in the unregulated sector. We do not mean to predict what actually would occur. Rather, we conduct a conceptual experiment to measure leakage, defined as the effect of increasing τ_Y with no change in τ_X, where the best interpretation of "no change in τ_X" is no change in the real τ_X. Then any reduction in the real τ_X is conceptually a different policy change with its own effects.

19. Net negative leakage in this model means that both commodities fall, which reduces economic welfare from consumption of X and Y (even if it provides separable benefits from a better environment). As shown in Baylis et al. (2013a), net positive leakage can be associated with a welfare gain or loss.

4.3. Why Doesn't Compensation Ensure That X Rises When Its Price Falls?

Consumers receive back all of the tax revenue, so how can this compensated increase in p_Y/p_X reduce X? Recall that consumers earn $I = p_K\bar{K} + R$, where $R \equiv \tau_Y C_Y + \tau_X C_X$. The answer is that the rebate of revenue is not enough to reach the same indifference curve, especially since the increase in the input tax worsens production inefficiency. Because of this "cost of abatement," an increase in τ_Y may reduce utility from consumption. In fact, increasing deadweight loss from an input tax is the reason for a Laffer Curve, where revenue is a hump-shaped function of the tax rate. Initial increases in τ_Y may raise positive additional revenue, but successive increases yield zero and then negative marginal revenue.

4.4. Is the Sign of Leakage Related to the Sign of the Change in Revenue?

We wonder if our negative leakage result is related to this insight about the Laffer curve. As it turns out, the set of parameters for which leakage is negative is not a subset of the parameters for which the effect on revenue is negative, nor vice versa.

Since the capital stock and its price are fixed, the only change to income is the change in the rebate of revenue. We totally differentiate that expression for R and find

$$\hat{R} = \{\alpha_Y[\sigma_U - \sigma_Y]\theta_{YC} - \delta_Y[\theta_{YC}(\sigma_U - 1) + \theta_{YK}(\sigma_Y - 1)]\}\hat{\tau}_Y, \qquad (13)$$

where $\delta_Y \equiv \tau_Y C_Y / R$ is the share of total tax revenue from sector Y.

To see how the change in revenue and leakage each depend on substitution parameters, figure 2 plots σ_Y on the horizontal axis and σ_U on the vertical axis. First note that $\hat{C}_X$ in equation (11) has a term $(\sigma_U - \sigma_Y)$ times $\hat{\tau}_Y$, so leakage is zero

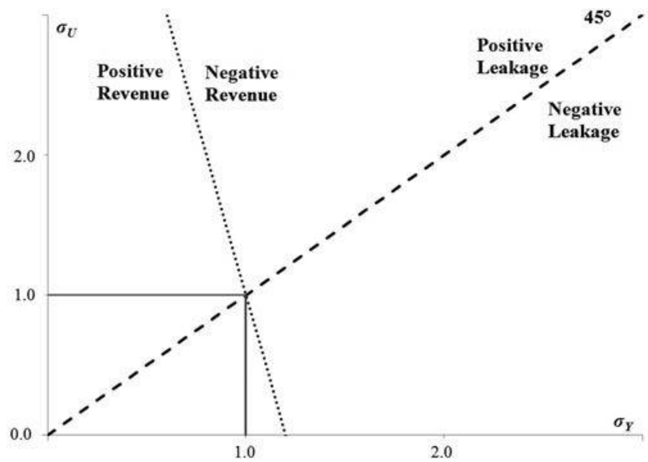

Figure 2. Effects on leakage and revenue (when the share of tax revenue from sector Y exceeds its share of capital: $\delta_Y = 0.75$, $\alpha_X = \alpha_Y = 0.5$, and $\theta_{YC} = 0.5$)

whenever $\sigma_U = \sigma_Y$ (on the 45 degree dashed line in fig. 2). Net leakage is positive to the upper left of that line (with higher σ_U) and negative to the lower right (with higher σ_Y).

To find areas for positive or negative changes in revenue in figure 2, we set $\hat{R} = 0$ in equation (13) and solve

$$\sigma_U = \left(1 - \frac{\delta_Y}{\theta_{YC}(\delta_Y - \alpha_Y)}\right)\sigma_Y + \left(\frac{\delta_Y}{\theta_{YC}(\delta_Y - \alpha_Y)}\right). \tag{14}$$

This line has a slope that depends on the sign and magnitude of $(\delta_Y - \alpha_Y)$. If the share of tax revenue from sector Y exceeds its share of capital $(\delta_Y > \alpha_Y)$, then this slope can be negative. Figure 2 depicts the case where $\delta_Y = 0.75$ and $\alpha_Y = 0.5$, using a dotted iso-revenue line. To the lower left, where both σ elasticities are small, the increase in τ_Y raises revenue; to the upper right of this line, the larger responsiveness means that an increase in τ_Y reduces the tax base by enough that revenue falls. If $\delta_Y = \alpha_Y$, then the dotted line is vertical, and if $\alpha_Y > \delta_Y$ the slope is positive. The iso-leakage and iso-revenue lines always intersect where σ_U and σ_Y both equal one. In any case, the figure clearly shows four different areas: the signs of $(\hat{C}_X, \hat{R})$ can be $(+, +)$, $(+, -)$, $(-, +)$, or $(-, -)$.

So far, leakage seems unrelated to revenue. But as the initial τ_Y approaches zero, so does the initial tax revenue from that sector, δ_Y. Then equation (14) shows that the iso-revenue line has an intercept of zero and a slope of one. In this case, it is coincident with the iso-leakage line. In other words, an increase in τ_Y from near zero necessarily has both negative net leakage and negative net revenue whenever $\sigma_Y > \sigma_U$. That increase in τ_Y induces sector Y to substitute into abatement, which draws capital away from sector X. The output of X shrinks, along with both of its inputs. Less C_X means negative leakage, and it also means less revenue from $\tau_X C_X$.

5. EXTENSIONS

We now extend the basic model three ways, to consider an upward-sloping supply of carbon, variable factor supply, and imperfect mobility of the clean factor. In all three cases, we show that the extension may affect leakage, but it does not affect the existence of the ARE. That term remains as a negative effect on leakage.

5.1. An Upward-Sloping Supply of Carbon

So far, because we assume unlimited carbon emissions, the supply curve for C is horizontal, and the TTE is our only positive form of leakage. If the supply of a fossil fuel were instead vertical in this static model, and if carbon always has a positive marginal product, then all such fuel would be used. Any fuel not used in one sector will be used by the other sector, and leakage is necessarily exactly 100%.

Must the carbon supply curve be either horizontal or vertical? The extension in this subsection considers an intermediate case where the two sectors use carbon-based fuel produced with a rising marginal cost curve. If the carbon tax reduces demand for fuel in one sector, that fuel can now be used in the other sector. The point of this extension is to see if an upward-sloping fuel supply affects the negative term given by the ARE identified above.

In the basic model, C represents total carbon emissions. In this extension, we want C also to represent the total amount of carbon-based fuel, so we define one unit of fuel as the amount that emits one unit of carbon. Then C_X and C_Y are the carbon-based fuels used in sectors X and Y, respectively, as well as emissions in each sector. The production function for fuel is given by $C = K_C$, where K_C is capital and labor used in production of C. The price of this fuel is p_C, but sector i faces a cost of $p_C + \tau_i$ (for $i = X, Y$), where the carbon tax is a per unit tax. This model does not have storage, so fuel supply equals fuel demand, $C = C_X + C_Y$.

First, we modify the resource constraint such that

$$0 = \alpha_X \hat{K}_X + \alpha_Y \hat{K}_Y + \alpha_C \hat{K}_C, \tag{15a}$$

where $\alpha_C \equiv K_C/\bar{K}$ is the share of capital in production of C, and $\alpha_X + \alpha_Y + \alpha_C = 1$. Next, we differentiate the production function for fuel, the market clearing condition, and a zero profits condition for the fuel sector:

$$\hat{C} = \hat{K}_C, \tag{15b}$$

$$\hat{C} = \rho_X \hat{C}_X + \rho_Y \hat{C}_Y, \tag{15c}$$

$$\hat{p}_C + \hat{C} = \hat{p}_K + \hat{K}_C, \tag{15d}$$

where $\rho_i \equiv (C_i/C)$ is the fraction of fuel initially used in sector i, and $\rho_X + \rho_Y = 1$.

Since the carbon tax is a per unit tax, modifications are also required to the zero-profit conditions and elasticities of substitution definitions in both sectors ($i = X, Y$). After applying the price $p_C + \tau_i$ to carbon fuel C_i for sector i, the zero-profit conditions become $p_i i = p_K K_i + (p_C + \tau_i) C_i$. Total differentiation yields

$$\hat{p}_X + \hat{X} = \theta_{XK}\left(\hat{p}_K + \hat{K}_X\right) + \theta_{XC}\hat{C}_X + \theta^p_{XC}\hat{p}_C + \theta^\tau_{XC}\hat{\tau}_X, \tag{15e}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}\left(\hat{p}_K + \hat{K}_Y\right) + \theta_{YC}\hat{C}_Y + \theta^p_{YC} p_C + \theta^\tau_{YC}\hat{\tau}_{Y.} \tag{15f}$$

Here $\theta^p_{XC} \equiv p_C C_X / p_X X$ is the carbon-price share and $\theta^\tau_{XC} \equiv \tau_X C_X / p_X X$ is the carbon tax share in sector X, so that $\theta_{XC} = \theta^p_{XC} + \theta^\tau_{XC}$ (and similarly for sector Y). In addition, the substitution definitions are modified as

$$\hat{C}_X - \hat{K}_X = \sigma_X\left(\hat{p}_K - \frac{\theta^p_{XC}}{\theta_{XC}}\hat{p}_C - \frac{\theta^\tau_{XC}}{\theta_{XC}}\hat{\tau}_X\right), \tag{15g}$$

$$\hat{C}_Y - \hat{K}_Y = \sigma_Y \left(\hat{p}_K - \frac{\theta^p_{YC}}{\theta_{YC}} \hat{p}_C - \frac{\theta^\tau_{YC}}{\theta_{YC}} \hat{\tau}_Y \right), \tag{15h}$$

using the new factor shares just defined. This model is solved using the eight equations (15a)–(15h), along with equations (2), (3), and (8) from the basic model.

Since $C = K_C$, and capital is numeraire, we find $\hat{p}_C = \hat{p}_K = 0$. The relative price of carbon rises, however, because the use of capital has a rising opportunity cost as it becomes scarcer. In other words, carbon fuel has an upward-sloping marginal cost curve in general equilibrium. We solve this extended model for leakage and find

$$\hat{C}_X = \left[\underbrace{\sigma_U \alpha_Y \theta^\tau_{YC}}_{\text{TTE}} - \underbrace{\sigma_Y \alpha_Y \theta^\tau_{YC}}_{\text{ARE}} + \underbrace{\alpha_C \rho_Y \left[\sigma_U \theta_{YC} + \sigma_Y \theta_{YK} \right] \frac{\theta^\tau_{YC}}{\theta_{YC}}}_{\text{FRE}} \right] \hat{\tau}_Y, \tag{16}$$

where $\theta^\tau_{YC} \equiv \tau_Y C_Y / p_Y Y$ is the carbon tax share in the Y sector. Comparing equation (16) with (11) identifies a positive leakage term we call the fuel resource effect (FRE), reflecting the fact that the increase in carbon tax reduces the use of fuel by sector Y and allows more use of fuel by sector X. Note that the TTE and ARE are the same as in the basic model, except that here they include θ^τ_{YC} instead of θ_{YC}.[20]

5.2. Variable Factor Supply

Our abatement resource effect operates through mobility of a fixed total factor supply, so we wonder how the ARE is affected by variable factor supply. Indeed, carbon policy can change factor returns, labor supply, and future capital stocks. In the context of our static model with one factor, these behaviors can be represented by a simple extension that allows factor supply to depend on prices and income.

In particular, suppose household utility depends on leisure or home production (K_H). We define η_K as the elasticity of factor supply with respect to the real net wage or factor price, and η_I as the elasticity of factor supply with respect to real income. We solve for the change in market factor supply, $K_M = K_X + K_Y$, and we solve the new expression for leakage. These complications raise the number of equations and unknowns from 8 to 12. All derivations appear in the appendix, but the intuition is clear enough simply to state the results here in words: variable factor supply can affect total leakage through two additional terms of opposite sign. The carbon tax raises the price of the Y output relative to the numeraire, and so it reduces the real net factor price and therefore might reduce factor supply (if $\eta_K > 0$).

---

20. In the limit, the FRE goes to zero as $\alpha_C \equiv K_C/\bar{K}$ goes to zero. Also, if the only fuel costs are taxes, then $\theta^\tau_{YC} = \theta_{YC}$, and the TTE and ARE would revert to the basic model as well.

Less market factor supply means less available for K_Y or for K_X, a negative effect on output of X and on leakage. The carbon tax also reduces household real net income, however, and so it might increase factor supply (if $\eta_I < 0$), which would have a positive effect on leakage. Neither of these additional terms affects the negative ARE term.

5.3. Imperfect Mobility of Capital

The ARE requires that substitution in production is not zero, that substitution in utility is not infinite, and that labor or capital is mobile. Figure 1 shows how it depends on elasticities, so here we show how it depends on mobility. Many existing models assume no factor mobility between countries, so they miss the ARE, but we find here that leakage is not necessarily overstated in that case. With imperfect mobility, the price of capital in one sector may differ from that in the other sector ($p_{KX} \neq p_{KY}$). We thus have one more unknown, relative to the basic model of sections 1–4, but we can use one more equation: $K = K(K_X, K_Y)$, where σ_K is the elasticity of K_X/K_Y with respect to p_{KY}/p_{KX}. Total capital here is fixed $\left(\bar{K} = K_X + K_Y\right)$, but it is partially mobile.[21] Its movement depends on σ_K. With these changes and setting p_{KX} as numeraire, the equation for leakage becomes

$$\hat{C}_X = \frac{\alpha_Y \left[\sigma_U - \sigma_Y\right] \theta_{YC} \hat{\tau}_Y}{\left[1 + \dfrac{\left(\sigma_U \theta_{YK} + \sigma_Y \theta_{YC}\right)}{\sigma_K}\right]} \gtrless 0. \tag{17}$$

The numerator of (17) matches exactly our previous expression for leakage in (11), so the only change here is the denominator. As σ_K approaches infinity (perfect mobility of capital), this denominator approaches one, and our new expression reduces exactly to our previous expression. Yet lower values of σ_K mean a larger denominator, and thus smaller values for both the positive TTE and the negative ARE.[22] While the existence of the newly identified ARE does depend on the mobility of capital, so does the TTE.

---

21. McKibbin, Morris, and Wilcoxen (2012) consider a carbon tax in US electricity using a CGE model where labor is immobile between regions, financial capital is perfectly mobile, but physical capital is specific to sectors and regions. In the long run, physical capital is mobile because of depreciation with reinvestment elsewhere. Burniaux and Oliveira Martins (2012) also look at effects of mobility on leakage.

22. Interestingly, σ_K approaching zero (immobility) means the denominator goes to infinity, and all leakage disappears. We said above that existence of the ARE depends on mobility of capital, but the simplicity of this model implies that existence of the TTE also depends on that mobility. Because p_{KX} is numeraire, no change in τ_X means no change in relative input prices in X. Firms there want unchanged relative factor use. With no change

Prior CGE models often assume that capital is mobile between sectors but not between countries. When those authors simulate a tax on all carbon in one region, they miss the negative effect on leakage of the ARE. Yet capital immobility does not imply that leakage is overstated. When Winchester and Rausch (2013) find net positive leakage and then increase capital mobility, they find larger positive leakage. That result is explained by equation (17), where a larger σ_K implies a smaller denominator and, thus, a larger value for any net leakage in the numerator.

6. CONCLUSION

Leakage from climate policy has been evaluated using many different CGE models. Some disallow the possibility of negative leakage and some report a single positive amount of leakage that might be the difference between a larger gross positive leakage term and a negative leakage term. This paper solves analytical expressions for leakage that show not only a positive term but also a new negative term we call the "abatement resource effect" (ARE). We explain, and we discuss interpretations. We show that various extensions do not affect the existence and sign of the ARE.

The point of our study is not that leakage must be negative. Various other extensions might reduce the size of our negative leakage effect or introduce other positive effects. Rather, we show that in some cases leakage might be negative. More importantly, policy makers and economists who ignore the abatement resource effect might misstate the size of carbon leakage.

APPENDIX
DETAILS FOR THE SECTION 5.2 EXTENSION TO VARIABLE FACTOR SUPPLY

Section 5.2 extends our basic model to consider variable factor supply, where household utility depends on leisure or home production (K_H). Our utility function becomes $V(U(X, Y), K_H; C)$, where subutility $U(X, Y)$ is homothetic and separable from leisure, and where damages from carbon emissions are also separable from both consumption and leisure. Define p_U as the price of $U(X, Y)$, an index of the prices of goods X and Y (so that $p_U U = p_X X + p_Y Y = I$). To derive the additional linear equations for this model, first totally differentiate the price index and rearrange to get

$$\hat{p}_U = \beta\hat{p}_Y + (1-\beta)\hat{p}_X. \tag{A1}$$

As above, $\beta = p_Y Y/I$ is the share of income spent on good Y.

---

in K_X, they choose no change in C_X. The only effect of higher τ_Y is to raise p_Y, so consumers get less Y. But complicated CGE models can certainly get leakage without capital mobility.

Second, define K_M as the market supply of capital ($K_M = K_X + K_Y$). Following Fullerton and Metcalf (2001), we can then define the factor supply function as

$$K_M = K_M\left(\frac{p_K}{p_U}, \frac{R}{p_U}\right).$$

We define η_K as the uncompensated elasticity of factor supply with respect to the first argument (any change in the real wage or other factor return). If the labor supply curve is not backward bending, for example, then $\eta_K > 0$. And we define η_I as the elasticity of labor or other factor supply with respect to the second argument (any change in real exogenous income such as the lump-sum tax rebate, R). If leisure is a normal good, then $\eta_I < 0$. Totally differentiate this function to yield

$$\hat{K}_M = \eta_K\left(\hat{p}_K - \hat{p}_U\right) + \eta_I\varphi\left(\hat{R} - \hat{p}_U\right), \tag{A2}$$

where $\varphi = R/I$ is the share of income from the tax rebate. Third, totally differentiate the definition of the lump-sum tax rebate to get

$$\hat{R} = \delta_X\left(\hat{C}_X + \hat{\tau}_X\right) + \delta_Y\left(\hat{C}_Y + \hat{\tau}_Y\right). \tag{A3}$$

As above, $\delta_Y \equiv \tau_Y C_Y/R$ is the share of total tax revenue from sector Y, and δ_X is similarly defined. Finally, we replace equation (1) in the basic model with two new ones. Totally differentiate the market supply equation and the definition of K_H:

$$\hat{K}_M = \alpha_X\hat{K}_X + \alpha_Y\hat{K}_Y, \tag{A4}$$

$$\hat{K}_H = \left(-K_M/K_H\right)\hat{K}_M, \tag{A5}$$

where $\alpha_i \equiv K_i/K_M$, and $\alpha_X + \alpha_Y = 1$. Therefore, the variable factor supply model consists of equations (A1)–(A5) and equations (2)–(8) from the basic model. Solving for the leakage term, we find

$$\left(1 - \eta_I\varphi\right)\hat{C}_X = \left[\underbrace{\alpha_Y\sigma_U\theta_{YC}}_{TTE} - \underbrace{\alpha_Y\sigma_Y\theta_{YC}}_{ARE} + \underbrace{\varphi\delta_Y\left(\underbrace{-\eta_I\left(\varphi + \left(\sigma_U\theta_{YC} + \sigma_Y\theta_{YK} - 1\right)\right)}_{RIE} - \underbrace{\eta_K}_{RPE}\right)}_{VFE}\right]\hat{\tau}_Y.$$

For convenience, the denominator $(1 - \eta_I\varphi)$ is moved to the left-hand side. Note that if leisure is a normal good ($\eta_I < 0$), then this denominator is positive.

This closed-form solution contains the same TTE and ARE terms as before, but it now includes a new term we call the "variable factor effect" (VFE), which itself can be divided into a real price effect (RPE) and a real income effect (RIE). These two effects have different signs under most parameterizations (where $\eta_K > 0$ and $\eta_I < 0$). The RPE reduces leakage, because the carbon tax raises the price of the Y output, so it reduces the real wage or factor return and therefore reduces factor supply. The RIE raises leakage, however, under the usual assumptions where leisure is a normal good and where the carbon tax is on the normal side of the Laffer curve. In that case, the increase in carbon tax reduces household real income and thus increases factor supply, which allows more output of X.

If both of the newly introduced elasticities are zero ($\eta_K = \eta_I = 0$), then this new leakage expression reduces exactly to equation (11) of our basic model above. This new model also reduces to the basic model in the limit as the share of tax revenue in total income (φ) goes to zero. That is, no initial carbon tax in either sector means that a small increase in the carbon tax in sector Y has no VFE because it has no first-order effect on the real net factor return or on real income. It has no effect on the real net factor return because $\hat{p}_Y = \theta_{YC}\hat{\tau}_Y$, and no initial carbon tax means no initial carbon share ($\theta_{YC} = 0$). It has no effect on real income because the initial equilibrium has no tax distortions; the increase in tax rate from zero has no deadweight loss, and so the marginal cost of abatement is zero.

REFERENCES

Alberini, Anna, and Massimo Filippini. 2011. Response of residential electricity demand to price: The effect of measurement error. *Energy Economics* 33, no. 5:889–95.

Armington, Paul S. 1969. A theory of demand for products distinguished by place of production. *IMF Staff Papers* 16:159–76.

Babiker, Mustafa H. 2005. Climate change policy, market structure, and carbon leakage. *Journal of International Economics* 65, no. 2:421–45.

Baylis, Kathy, Don Fullerton, and Daniel H. Karney. 2013a. Leakage, welfare, and cost-effectiveness of carbon policy. *American Economic Review, Papers and Proceedings* 103, no. 3:332–37.

———. 2013b. A model of the model: Unpacking CGE results on leakage from climate policy. Unpublished manuscript, University of Illinois.

Böhringer, Christoph, Jared C. Carbone, and Thomas F. Rutherford. 2011. Embodied carbon tariffs. NBER Working Paper 17376, National Bureau of Economic Research, Cambridge, MA.

Burniaux, Jean-Marc, and Joaquim Oliveira Martins. 2012. Carbon leakages: A general equilibrium view. *Economic Theory* 49, no. 2:473–95.

Carbone, Jared C. 2013. Linking numerical and analytical models of carbon leakage. *American Economic Review, Papers and Proceedings* 103, no. 3:326–31.

Caron, Justin, Sebastian Rausch, and Niven Winchester. 2012. Leakage from sub-national climate initiatives: The case of California. MIT Joint Program Report no. 220, Cambridge, MA.

Chua, Swee. 2003. Does tighter environmental policy lead to a comparative advantage in less polluting goods? *Oxford Economic Papers* 55, no. 1:25–35.

Copeland, Brian R., and M. Scott Taylor. 2005. Free trade and global warming: A trade theory view of the Kyoto Protocol. *Journal of Environmental Economics and Management* 49:205–34.

Dahl, Carol A. 2011. A global survey of electricity demand elasticities. Paper presented at the 34th IAEE International Conference: Institutions, Efficiency, and Evolving Energy Technologies, Stockholm School of Economics.

Di Maria, Corrado, and Sjak A. Smulders. 2004. Trade pessimists vs. technology optimists: Induced technical change and pollution havens. *Advances in Economic Analysis and Policy* 4, no. 2, article 7.

Di Maria, Corrado, and Edwin van der Werf. 2008. Carbon leakage revisited: Unilateral climate policy with directed technical change. *Environmental and Resource Economics* 39, no. 2:55–74. doi:10.2202/1538-0637.1344.

Elliott, Joshua, Ian Foster, Samuel Kortum, Todd Munson, Fernando Perez Cervantes, and David Weisbach. 2010. Trade and carbon taxes. *American Economic Review, Papers and Proceedings* 100, no. 2: 465–69.

Elliott, Joshua, and Don Fullerton. 2014. Can a unilateral carbon tax reduce emissions elsewhere? *Resource and Energy Economics* 36, no. 1:6–21.

Felder, Stefan, and Thomas F. Rutherford. 1993. Unilateral CO_2 reductions and carbon leakage: The consequences of international trade in oil and basic materials. *Journal of Environmental Economics and Management* 25, no. 2:162–76.

Fischer, Carolyn, and Alan K. Fox. 2010. On the scope for output-based rebating in climate policy: When revenue recycling isn't enough (or isn't possible). Resources for the Future Discussion Paper 10-69, Washington, DC.

Fowlie, Meredith L. 2009. Incomplete environmental regulation, imperfect competition, and emissions leakage. *American Economic Journal: Economic Policy* 1, no. 2:72–112.

Fullerton, Don, Daniel H. Karney, and Kathy Baylis. 2011. Negative leakage. NBER Working Paper 17001, National Bureau of Economic Research, Cambridge, MA.

Fullerton, Don, and Gilbert E. Metcalf. 2001. Environmental controls, scarcity rents, and pre-existing distortions. *Journal of Public Economics* 80, no. 2:249–67.

Gerlagh, Reyer, and Onno Kuik. 2007. Carbon leakage with international technology spillovers. FEEM Working Paper 33.2007, Fondazione Eni Enrico Mattei, Milan.

Golombek, Rolf, and Michael Hoel. 2004. Unilateral emission reductions and cross-country technology spillovers. *Advances in Economic Analysis and Policy* 4, no. 2, article 3. doi:10.2202/1538-0637.1318.

Goulder, Lawrence H. 2013. Markets for pollution allowances: What are the (new) lessons? *Journal of Economic Perspectives* 27, no. 1:87–102.

Gurtzgen, Nicole, and Michael Rauscher. 2000. Environmental policy, intra-industry trade and transfrontier pollution. *Environmental and Resource Economics* 17, no. 1:59–71.

Harberger, Arnold C. 1962. The incidence of the corporation income tax. *Journal of Political Economy* 70, no. 3:215–40.

Karp, Larry. 2013. The income and production effects of leakage. Working Paper, University of California, Berkeley.

Metcalf, Gilbert E., and David Weisbach. 2009. The design of a carbon tax. *Harvard Environmental Law Review* 33, no. 2:499–506.

McKibbin, Warwick J., Adele C. Morris, and Peter J. Wilcoxen. 2012. Pricing carbon in the U.S.: A model-based analysis of power sector only approaches. Climate and Energy Economics Discussion Paper, Brookings Institution, Washington, DC.

Ogawa, Hikaru, and David E. Wildasin. 2009. Think locally, act locally: Spillovers, spillbacks, and efficient decentralized policymaking. *American Economic Review* 99, no. 4:1206–17.

Paltsev, Sergey V. 2001. The Kyoto Protocol: Regional and sectoral contributions to the carbon leakage. *Energy Journal* 22, no. 4:53–79.

Rausch, Sebastian, Gilbert E. Metcalf, John M. Reilly, and Sergey Paltsev. 2010. Distributional implications of alternative U.S. greenhouse gas control measures. *B. E. Journal of Economic Analysis and Policy* 10, no. 2, article 1. doi:10.2202/1935-1682.2537.

Reiss, Peter, and Matthew W. White. 2005. Household electricity demand, revisited. *Review of Economic Studies* 72, no. 3:853–83.

Sue Wing, Ian, and Marek Kolodziej. 2008. The regional greenhouse gas initiative: Emissions leakage and the effectiveness of interstate border adjustments. Kennedy School Regulatory Policy Program Paper RPP-2008-03, Harvard University.

Winchester, Niven, Sergey Paltsev, and John M. Reilly. 2011. Will border carbon adjustments work? *B. E. Journal of Economic Analysis and Policy* 11, no. 1, article 7. doi:10.2202/1935-1682.2696.

Winchester, Niven, and Sebastian Rausch. 2013. A numerical investigation of the potential for negative emissions leakage. *American Economic Review, Papers and Proceedings* 103, no. 3:320–25.