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Senior Center Network Redesign Under Demand Uncertainty

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Senior centers offer a variety of services to facilitate independent living of older adults. In the U.S., increasing suburbanization and aging of suburban residents necessitate reconfiguring senior services. We propose a two-echelon network of senior centers across large study areas and formulate a stochastic facility location/allocation model with mixed-integer recourse. We apply our model to Allegheny County, Pennsylvania, which has one of the oldest population in the U.S. Our model shows that a two-echelon network design is appropriate for increasing the occupancy of senior centers as community focal points while maintaining customized and accessible programming in small neighborhood areas.

1 Motivation

There were 35 million people over 65 years of age in the United States as of the Census 2000, a 12% increase since 1990. It is estimated that the number of people over 65 years of age will more than double by 2030, comprising 20% of the population [44]. In the U.S., there is a large network of senior service providers, headed by the U.S. Administration on Aging, which is comprised of state-level Units on Aging, metro-level Area Agencies on Aging (AAA), and local service providers. [43]. Older adults are eligible for senior services, consisting of programs such as home-delivered meals, transit services, and senior centers, which are facilities for older adults who live independently. As focal points for older adults, they serve as sites for maintaining healthy lifestyles including socialization, nutritious meals, exercise, and education.

There is an opportunity to redesign senior center networks as changing demographics of the U.S. older population, increasing suburbanization, and the aging of suburban residents mean that senior centers are often located without reference to where increasing numbers of older adults reside [18, 28]. We propose a two-echelon network of senior centers comprised of *comprehensive* and *satellite* centers. Comprehensive centers offer a wide variety of on-site services, as well as managing one or more satellite centers that provide customized, neighborhood-based services.

The demand for senior centers is not known with certainty. We formulate a two-stage stochastic mixed-integer program for locating senior centers in both levels of the two-echelon network. In the first stage, our model chooses a set of comprehensive and satellite centers to open. In the second stage, after the demand is realized, recourse actions are taken so as to minimize the total distance traveled by the seniors while satisfying network design constraints.

Senior centers provide public services for which equitable delivery is of the utmost importance. We measure equity by distance. Hence, our model allocates older adults only to those centers within a specified distance. These allocation decisions are necessary for the capacity planning of an equitable delivery system. Once having the opportunity to access senior services nearby their neighborhood in a close proximity, older adults are free to patronize any center based on their preferences.

The rest of the paper is organized as follows. Section 2 gives a brief review of the facility location models in the literature, and discusses some of the prior studies about senior services in operations research. Section 3 describes our proposed two-echelon network model of senior centers. We also define roles and properties of comprehensive and satellite centers in this section. Section 4 presents our two-stage stochastic mixed-integer model formulation and its limitations. We propose a Lagrangian scenario decomposition technique described in Section 5 to solve the resulting stochastic program. Section 6 provides a detailed description of the data used for the application of the model to Allegheny County, Pennsylvania. Section 7 presents our computational results and associated policy implications. We conclude and identify future research areas in Section 8. Although we apply our model to the Allegheny County senior center transformation process, this research paper does not represent or state the final design recommendations for Allegheny County regarding the senior center design.

2 Prior Work

Facility location models have been studied since the 1960s; [34] presents a recent and extensive survey. Public and private sector applications of facility location models differ in their optimization criteria. Profit maximization and/or capture of larger market shares are typical objectives in private sector applications, while maximization of social welfare, service accessibility, efficiency or equity may be objectives appropriate for public sector models.

As of the late 1980s, most urban applications of facility location models focused primarily on emergency services [33]. However, many other applications of public sector facility location models have appeared since then, including health centers [7], housing [19], public libraries [23], and schools [8]. [26] and [20] survey recent public sector facility location applications.

Hierarchical models are also widely studied in public facility location [29]. [13] gives a classification of hierarchical facility location models and reviews the applications, and proposed solution methods. Similar to our work, [41] has recently presented a hierarchical location model for a school network planning problem. Their model maximizes the accessibility of facilities at different service levels subject to a nested hierarchy of facilities, maximum and minimum capacity constraints, and user-to-facility assignment constraints.

Stochastic facility location models address future uncertainties that may affect current design considerations. They have a broad range of applications from supply chain network design [39] to telecommunications [21, 37]. We refer the reader to [32] and [38] for detailed reviews of this growing area.

The optimal location of senior centers is a recently emerging area. [18] formulates two different deterministic multi–echelon facility location models for senior centers and applies them to a large urban case study. [3] designs a routing system for daily operations of meals-on-wheels program based on a traveling salesman heuristic. [11] develops an interactive GIS-based heuristic for location of Home-Delivered-Meal kitchens by combining facility location with vehicle routing. An integer programming model of the associated location-routing problem is formulated in [17].

3 Two-Echelon Network of Senior Centers

We propose a two-echelon network of senior centers that can be applied to any metro-level senior service system in the U.S. The first (higher) echelon of this network model is comprehensive centers. They are classical bricks-and-mortar centers to which older adults typically must travel to utilize the available services. Comprehensive centers provide access to community resources and offer services to facilitate older adults living independently. While the type and capacity of activities and services may vary from center to center, each comprehensive center typically offers basic services such as health programs, nutrition services, and advocacy assistance.

The second (lower) echelon is composed of satellite centers whose main function is to serve as outreach posts of the comprehensive centers. Directed by comprehensive centers, satellite centers are designed to be more adaptive to the changing locations and interests of older adults. In addition to providing customized services in local neighborhoods, they also enable increased physical accessibility to services. While a satellite center could be a dedicated, stand-alone bricks-and-mortar site, such locations might be less adaptable and more expensive to maintain. Moreover, the cost, in terms of physical effort, money and/or time spent, of traveling to a regular bricks-and-mortar center may be excessive for many older adults. Therefore, these sites may be best located/co-located within (or with) other existing community assets such as libraries, personal care centers, shopping areas, or any other locations that are attractive to older adults.

Note that there is not necessarily a difference between comprehensive and satellite centers based on their service levels; both can offer similar services if needed. Therefore, our network model is not a hierarchical model. Although being motivated by discussions with the Steering Committee of Allegheny County Senior Center Transformation Project, our proposed two-echelon network design applies to any metro-level senior service system in the U.S.

4 Model Formulation

We formulate a two-stage stochastic facility location/allocation model with mixed-integer recourse for designing a two–echelon network of senior centers in large study areas. Stochastic programs seek to find decisions that hedge against future uncertainties. The first stage of our model decides which comprehensive and satellite centers to open. We denote the set of candidate sites for comprehensive and satellite centers by $\mathcal{I}$ and $\mathcal{J}$, respectively. Binary variable x_i is equal to 1 if comprehensive center i is opened in the first stage, and 0 otherwise. Likewise, binary variable y_j is equal to 1 if satellite center j is opened in the first stage, and 0 otherwise. Each candidate site is located in a service region $n \in \mathcal{N}$. We represent the maximum number of comprehensive centers that is allowed in service region n by κ_n. The fixed costs of opening comprehensive center i and satellite center j are denoted by c_i and f_j, respectively. Parameters w^1 and w^2 represent the minimum number of older adults to be allocated to each opened comprehensive and satellite center, while parameters a^1 and a^2 denote the capacity of comprehensive and satellite centers, respectively.

In the second stage, recourse actions are taken in order to fulfill the demand at each demand site while satisfying network design constraints under various scenarios. A *scenario* s having probability p^s is a realization of the random demand d_k^s at demand site $k \in \mathcal{K}$. We assume that scenario set $\mathcal{S}$ is finite, although this is not a strong assumption [35].

Let $\mathcal{J}(i) \subseteq \mathcal{J}$ be the subset of satellite centers that can be assigned to comprehensive center i based on geographic proximity, and let $\mathcal{I}(j) \subseteq \mathcal{I}$ be the subset of comprehensive centers to which satellite center $j \in \mathcal{J}$ can be assigned. Second-stage binary variable z_{ij}^s is equal to 1 if satellite center $j \in \mathcal{J}(i)$ is directed by comprehensive center i under scenario $s \in \mathcal{S}$, and 0 otherwise. A comprehensive center can direct at most ρ satellite centers due to managerial difficulties and cost-effectiveness concerns.

Let $\mathcal{I}(n) \subseteq \mathcal{I}$ and $\mathcal{K}(n) \subseteq \mathcal{K}$ be the set of comprehensive centers and demand sites that are located in service region $n \in \mathcal{N}$, respectively. Continuous variable v_{ik}^s denotes the fraction of demand at site $k \in \mathcal{K}(n)$ that is allocated to comprehensive center $i \in \mathcal{I}(n)$, and continuous variable u_{ijk}^s represents the fraction of demand at site k that is allocated to satellite center $j \in \mathcal{J}(i)$ of comprehensive center $i \in \mathcal{I}(n)$ in service region $n \in \mathcal{N}$ under scenario $s \in \mathcal{S}$. Finally, parameters q_{ik}^1 and q_{jk}^2 denote the annual per-person costs of traveling from demand site k to comprehensive center i and to satellite center j, respectively.

In our model, allocation of seniors to centers by minimizing total travel cost is necessary to obtain a socially optimal solution while ensuring that each demand point k has an available center in its neighborhood for equitable service delivery. There are some incentives for seniors to patronize their closest facility. For instance, in Pennsylvania, the Department of Aging encourages older adults using state-provided transportation services to patronize the closest facility by charging them less when they travel to the closest senior center in their neighborhood. However, there is evidence that older adults do not always patronize the closest center [42].

Based on the notation defined above, we propose the following two-stage stochastic mixed-integer program for locating the senior centers.

$$\zeta = \min \quad \sum_{i\in\mathcal{I}} c_i x_i + \sum_{j\in\mathcal{J}} f_j y_j + \sum_{s\in\mathcal{S}} p^s Q^s(x,y) \tag{1a}$$

$$\text{subject to} \quad \sum_{i\in\mathcal{I}(n)} x_i \le \kappa_n \qquad n\in\mathcal{N}, \tag{1b}$$

$$x_i, y_j \in \{0,1\} \qquad i\in\mathcal{I}, j\in\mathcal{J}, \tag{1c}$$

where,

$$Q^s(x,y) = \min \quad \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{I}(n)} \sum_{k\in\mathcal{K}(n)} d_k^s \left(q_{ik}^1 v_{ik}^s + \sum_{j\in\mathcal{J}(i)} q_{jk}^2 u_{ijk}^s \right) \tag{2a}$$

$$\text{subject to} \quad \sum_{i\in\mathcal{I}(n)} \left(v_{ik}^s + \sum_{j\in\mathcal{J}(i)} u_{ijk}^s \right) = 1 \qquad k\in\mathcal{K}(n), n\in\mathcal{N}, \tag{2b}$$

$$\sum_{i\in\mathcal{I}(j)} z_{ij}^s = y_j \qquad j\in\mathcal{J}, \tag{2c}$$

$$\sum_{j\in\mathcal{J}(i)} z_{ij}^s \le \rho x_i \qquad i\in\mathcal{I}, \tag{2d}$$

$$w^1 x_i \le \sum_{k\in\mathcal{K}(n)} d_k^s v_{ik}^s \le a^1 x_i \qquad i\in\mathcal{I}(n), n\in\mathcal{N}, \tag{2e}$$

$$\sum_{j\in\mathcal{J}(i)} \sum_{k\in\mathcal{K}(n)} d_k^s u_{ijk}^s \ge w^2 x_i \qquad i\in\mathcal{I}(n), n\in\mathcal{N}, \tag{2f}$$

$$\sum_{k\in\mathcal{K}(n)} d_k^s u_{ijk}^s \le a^2 z_{ij}^s \qquad i\in\mathcal{I}(n), j\in\mathcal{J}(i), n\in\mathcal{N}, \tag{2g}$$

$$z_{ij}^s \in \{0,1\}, \quad v_{ik}^s, u_{ijk}^s \ge 0 \qquad i\in\mathcal{I}, j\in\mathcal{J}, k\in\mathcal{K}. \tag{2h}$$

The objective function (1a) minimizes the total cost of opening senior centers plus the expected traveling cost of allocating older adults to those opened centers. Constraint (1b) restricts the number of comprehensive centers opened in service region $n \in \mathcal{N}$ to be no more than κ_n. Under scenario $s \in \mathcal{S}$, the second-stage objective function (2a) minimizes the total cost of allocating older adults to comprehensive and satellite centers opened in the first stage. This cost is a function of demand weighted distance traveled. Constraint (2b) ensures that the demand of each demand site $k \in \mathcal{K}$ is fulfilled. Constraints (2c) and (2d) together require that every opened satellite center must be directed by an opened comprehensive center and no more than ρ satellite centers can be directed by a single comprehensive center.

Constraints (2e), (2f) and (2g) enforce the minimum and maximum capacity requirements for the opened comprehensive and satellite centers, respectively.

We do not consider per person variable cost of service in our model. This is because constraints (6b) enforce that all demand has to be satisfied and we assumed that comprehensive and satellite centers can offer similar services if needed (e.g. variable costs of services in both type of centers are equivalent). Hence, including the variable cost in our analysis would not affect the network design and demand allocation decisions.

5 Solution Technique

The major difficulty in solving stochastic integer programs is that the function $Q^s(x, y)$ is often nonconvex and discontinuous [40]. Solution algorithms proposed in the literature work for special cases, e.g. simple integer recourse [14], binary first-stage variables and complete mixed-integer recourse [24], mixed-integer first-stage variables, but pure integer second-stage variables [1], pure-integer first-stage and second-stage variables [22] and continuous first-stage variables [31]. Among those, the algorithm proposed by Sen and Sherali [36] that combines decomposition techniques with branch-and-cut method can be used to solve our model. However, a computational study is not included in [36] except small numerical illustrations.

The instances that we deal with is often very large-scale, e.g. on the order of $|\mathcal{I}| \times |\mathcal{J}| \times |\mathcal{K}| \times |\mathcal{S}|$. Therefore, we first decompose model (1) into service regions by relaxing a set of constraints. Then, we propose a Lagrangian scenario decomposition method [6].

5.1 Service Region Decomposition

In model (1) service regions $n \in \mathcal{N}$ are linked through the second-stage constraints (2c) and (2d).

Remark 5.1. *If we relax constraint (2c) by replacing it with:*

$$\sum_{i \in \mathcal{I}(n) \cap \mathcal{I}(j)} z_{ij}^s = y_j \qquad j \in \mathcal{J}, n \in \mathcal{N}, . \tag{3}$$

then constraints (2d) can be decoupled into sets of constraints for each service region n:

$$\sum_{j \in \mathcal{J}(i)} z_{ij}^s \leq \rho x_i \qquad\qquad i \in \mathcal{I}(n), n \in \mathcal{N}. \tag{4}$$

Relaxed constraint (3) allows a satellite center j to be open in more than one service region. Then, model (1) can be decomposed into individual service region problems:

$$
\begin{aligned}
\zeta_n = \min \quad & \sum_{i\in\mathcal{I}(n)} c_i x_i + \sum_{j\in\mathcal{J}(i)} f_j y_j + \sum_{s\in\mathcal{S}} p^s Q_n^s(x,y) && && \text{(5a)}\\
\text{subject to} \quad & \sum_{i\in\mathcal{I}(n)} x_i \le \kappa_n, && && \text{(5b)}\\
& x_i, y_j \in \{0,1\} && i\in\mathcal{I}(n), j\in\mathcal{J}, && \text{(5c)}
\end{aligned}
$$

where,

$$
\begin{aligned}
Q_n^s(x,y) = \min \quad & \sum_{i\in\mathcal{I}(n)} \sum_{k\in\mathcal{K}(n)} d_k^s \left(q_{ik}^1 v_{ik}^s + \sum_{j\in\mathcal{J}(i)} q_{jk}^2 u_{ijk}^s \right) && && \text{(6a)}\\
\text{subject to} \quad & \sum_{i\in\mathcal{I}(n)} \left(v_{ik}^s + \sum_{j\in\mathcal{J}(i)} u_{ijk}^s \right) = 1 && k\in\mathcal{K}(n), && \text{(6b)}\\
& \sum_{i\in\mathcal{I}(n)\cap\mathcal{I}(j)} z_{ij}^s = y_j && j\in\mathcal{J}, && \text{(6c)}\\
& \sum_{j\in\mathcal{J}(i)} z_{ij}^s \le \rho x_i && i\in\mathcal{I}(n), && \text{(6d)}\\
& w^1 x_i \le \sum_{k\in\mathcal{K}(n)} d_k^s v_{ik}^s \le a^1 x_i && i\in\mathcal{I}(n), && \text{(6e)}\\
& \sum_{j\in\mathcal{J}(i)} \sum_{k\in\mathcal{K}(n)} d_k^s u_{ijk}^s \ge w^2 x_i && i\in\mathcal{I}(n), && \text{(6f)}\\
& \sum_{k\in\mathcal{K}(n)} d_k^s u_{ijk}^s \le a^2 z_{ij}^s && i\in\mathcal{I}(n), j\in\mathcal{J}(i), && \text{(6g)}\\
& z_{ij}^s \in \{0,1\}, \quad v_{ik}^s, u_{ijk}^s \ge 0 && i\in\mathcal{I}(n), j\in\mathcal{J}, k\in\mathcal{K}(n). && \text{(6h)}
\end{aligned}
$$

We solve each service region problem (5) separately using a Lagrangian dual-based method and then combine their solutions. If constraint (2c) is violated in the overall solution, we apply a heuristic algorithm presented in Section 5.3 to restore feasibility.

Proposition 5.1. *For any service region $n \in \mathcal{N}$,*

$$\sum_{i\in\mathcal{I}(n)} a^1 x_i + \sum_{j\in\mathcal{J}} a^2 y_j \ge \max_{s\in\mathcal{S}} \left\{ \sum_{k\in\mathcal{K}(n)} d_k^s \right\}$$

is a valid inequality.

Proposition 5.1 directly follows from the capacity required to satisfy demand under all scenarios.

5.2 Scenario Decomposition

We propose a Lagrangian scenario decomposition method [6] to solve service region problems (5). We denote second-stage variable vector (z^s, v^s, u^s) by γ^s. For each scenario s, define the set:

$$F_n^s(x,y) := \left\{ (x,y,\gamma^s) : \sum_{i\in\mathcal{I}(n)} x_i \le \kappa_n, \quad x,y \in \{0,1\}, \quad (6b)-(6h) \right\}. \tag{7}$$

Then, problem (5) is equivalent to:

$$\zeta_n = \min \left\{ \sum_{i\in\mathcal{I}(n)} c_i x_i + \sum_{j\in\mathcal{J}} f_j y_j + \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}(n)} \sum_{k\in\mathcal{K}(n)} p^s d_k^s \left(q_{ik}^1 v_{ik}^s + \sum_{j\in\mathcal{J}(i)} q_{jk}^2 u_{ijk}^s \right) \right.$$
$$\left. : (x,y,\gamma^s) \in F_n^s, \quad s \in \mathcal{S} \right\}. \tag{8}$$

We split problem (8) into scenario subproblems by introducing copies x^s, y^s of the first-stage variables for each scenario $s \in \mathcal{S}$ and reformulate (8) as:

$$\zeta_n = \min \left\{ \sum_{s\in\mathcal{S}} p^s \left[\sum_{i\in\mathcal{I}(n)} c_i x_i^s + \sum_{j\in\mathcal{J}} f_j y_j^s + \sum_{i\in\mathcal{I}(n)} \sum_{k\in\mathcal{K}(n)} d_k^s \left(q_{ik}^1 v_{ik}^s + \sum_{j\in\mathcal{J}(i)} q_{jk}^2 u_{ijk}^s \right) \right] \right.$$
$$\left. : (x^s, y^s, \gamma^s) \in F_n^s, x^s = x^{s'}, y^s = y^{s'}, \quad \forall s' \ne s, \quad s', s \in \mathcal{S} \right\}. \tag{9}$$

The constraints $x^s = x^{s'}, y^s = y^{s'}, \forall s' \ne s$ require that first-stage decisions are only based on the currently available information about the system. In other words, they are *non-anticipative* of the future information that will be gained after the demand uncertainty reveals in the second stage. We introduce the Lagrangian coefficients $\lambda^{ss'}$ and $\beta^{ss'}$ for those non-anticipativity constraints and relax them by penalizing their violation in the objective function. Then, the Lagrangian relaxation of problem (9) with respect to the non-anticipativity constraints is defined by:

$$D_n(\lambda,\beta) = \min \left\{ \sum_{s\in\mathcal{S}} L_n^s(\lambda,\beta) + p^s \left[\sum_{i\in\mathcal{I}(n)} c_i x_i^s + \sum_{j\in\mathcal{J}} f_j y_j^s + \right. \right.$$
$$\left. \left. \sum_{i\in\mathcal{I}(n)} \sum_{k\in\mathcal{K}(n)} d_k^s \left(q_{ik}^1 v_{ik}^s + \sum_{j\in\mathcal{J}(i)} q_{jk}^2 u_{ijk}^s \right) \right] : (x^s, y^s, \gamma^s) \in F_n^s, \quad s \in \mathcal{S} \right\}, \tag{10}$$

where,

$$L_n^s(\lambda, \beta) = \sum_{s' \in \mathcal{S}, s' \neq s} \left[\sum_{i \in \mathcal{I}(n)} (\lambda_i^{ss'} - \lambda_i^{s's}) x_i^s + \sum_{j \in \mathcal{J}} (\beta_j^{ss'} - \beta_j^{s's}) y_j^s \right] \qquad \forall s \in \mathcal{S}. \tag{11}$$

As a result, the Lagrangian dual of problem (9) is defined by:

$$\theta_n = \max_{\lambda, \beta} D_n(\lambda, \beta), \tag{12}$$

which is separable into scenario subproblems;

$$\max_{\lambda, \beta} D_n(\lambda, \beta) = \max_{\lambda, \beta} \sum_{s \in \mathcal{S}} D_n^s(\lambda, \beta). \tag{13}$$

The Lagrangian dual (13) is a concave non-differentiable problem which is solved by subgradient algorithm [10]. The optimal objective value of the Lagrangian dual θ_n is a lower bound on the optimal objective value of the original problem ζ_n due to integer requirements in (7). This lower bound is not smaller than the LP relaxation bound [30]. We use a branch-and-bound algorithm to close the gap between θ_n and ζ_n.

5.3 Feasibility Heuristic

We apply a heuristic algorithm to restore feasibility if constraint (2c) is violated in the overall solution $(\hat{x}, \hat{y}, \hat{\gamma})$ obtained by combining the solutions of individual service region problems. When a satellite center j has violations (e.g. it is opened in more than one service region), the main idea of the algorithm is to reallocate older adults to other available satellite centers so that only older adults from a single service region remain to patronize that satellite center.

Let $\hat{\mathcal{N}}(j) \subseteq \mathcal{N}$ be the subset of regions in which satellite center j is opened in the current solution. Suppose that $\left|\hat{\mathcal{N}}(j)\right| > 1$, since otherwise constraint (2c) is not violated. Let $\vartheta_n^s(j)$ be the comprehensive center in region $n \in \hat{\mathcal{N}}(j)$ that directs satellite center j under scenario s. First, we identify an unopened satellite center, which is closest to satellite center j and available (e.g., close enough) to comprehensive center $\vartheta_n^s(j)$. We denote such a center by $\omega_n^s(j)$ and calculate the expected cost of reallocating older adults from satellite center j to the alternative satellite center $\omega_n^s(j)$ by the following formula:

$$\mathcal{V}(n, j) = \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} p^s \hat{u}_{\vartheta jk}^s d_k^s \left(q_{\omega k}^2 - q_{jk}^2 \right). \tag{14}$$

In (14), some of the indices are omitted when it is clear from the context. We sort $\mathcal{V}(n, j)$ values in ascending order for all $n \in \hat{\mathcal{N}}(j)$ and reallocate the older adults from the first

$\left|\hat{\mathcal{N}}(j)\right| - 1$ regions to the alternative satellite centers. Finally, we set $y_\omega = 1$, $u^s_{\vartheta\omega k} = \hat{u}^s_{\vartheta jk}$, $u^s_{\vartheta jk} = 0$, $z^s_{\vartheta\omega} = 1$, and $z^s_{\vartheta j} = 0$ under each scenario $s \in \mathcal{S}$ for all satellite centers $j \in \mathcal{J}$ from which the older adults are reallocated to other available satellite centers.

6 Model Calibration

We apply our model and proposed solution technique to Allegheny County, Pennsylvania, which contains Pittsburgh, and is a region with high demand for senior services, including senior centers. The Area Agency on Aging in Allegheny County has initiated the Senior Center Transformation Project in order to redesign County's senior center network. We worked together with the Steering Committee of this project (the *Steering Committee*) to calibrate our model and validate our modeling assumptions. However, it should be noted that our model was developed prior to the final Steering Committee report, which is not in the public domain, and before completion of the County's plan, which is currently under development. Therefore, we base our calibration on some preliminary findings outlined in a progress report [28].

As of 2009, there were 61 senior centers managed by 17 service provider agencies in Allegheny County. These centers vary in size and complexity, and do not share a well defined service-level hierarchy. Allegheny County's size and diversity according to community type (urban, suburban, rural), transportation accessibility, and population density of older adults makes policy planning and monitoring difficult. Therefore, the Steering Committee proposed to divide the County into service regions based on topography (e.g., rivers), distance, and townships that have common linkages [28].

Older adults in a particular service region would be served through the comprehensive centers in that region. Figure 1 shows an example of our study area divided into seven regions, as well as locations of existing senior centers. The number of senior centers to be located in a service region could depend on the current profile and the future projection of older adult population in that region. Table 1 provides a summary of the population statistics of Allegheny County and the maximum number of comprehensive centers that could be located in each one of the seven service region proposed by the Steering committee.

A service region consists of multiple townships and boroughs, which we refer to as *municipalities*. We consider the geographical centroids of the first five municipalities that have the

Figure 1: Service regions and existing senior centers [28].

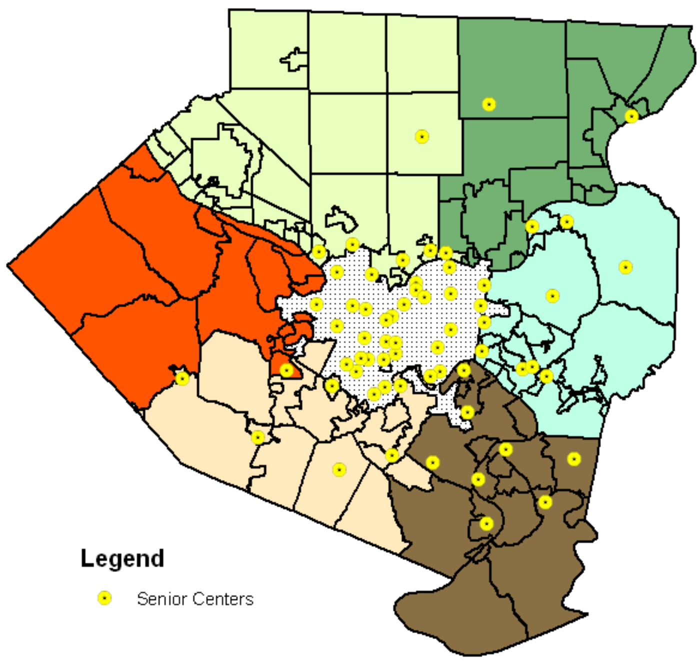


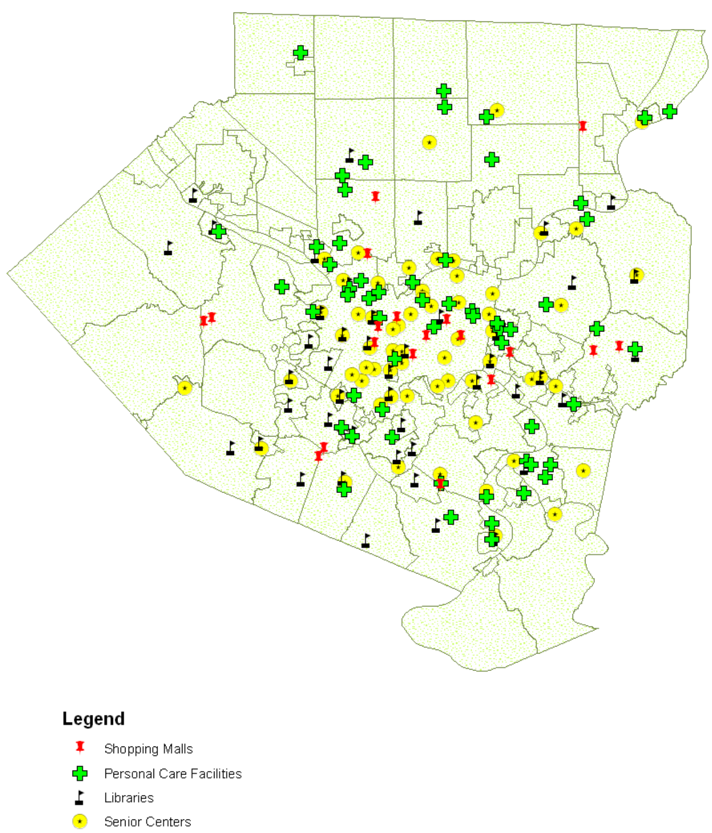
Table 1: Distribution of older adults among seven regions

	West	Northwest	Northeast	East	Southeast	South	Pittsburgh
Total Older Adults, 2000 [44]	7%	15%	6%	17%	13%	18%	24%
Total Poor Older Adults, 2000 [44]	6%	13%	5%	15%	14.5%	10.5%	36%
Total Older Adults, 2019 [5]	9%	16%	6%	16%	12%	15%	26%
Total Consumers, 2006 [2]	4%	8.5%	4%	27%	6%	17%	33.5%
Max Number of Comp. Centers	1	1	1	2	1	2	3

largest older adult population as candidate locations for comprehensive centers in a service region. As seen in Figure 1, there is a small municipality embedded in the City of Pittsburgh. Therefore, there are two municipality centroid within the City. Due to a high density of low-income older adults in the City, the Steering committee proposed opening up to three comprehensive centers in this region. Therefore, we consider the centroid of the first three census tracts located in the City that have the largest older adult population as candidate locations for comprehensive centers as well.

For candidate locations of satellite centers, we use *community assets*, such as libraries, personal care centers and shopping areas that are frequently visited by older adults. Figure 2 depicts the locations of 125 community assets that we consider.

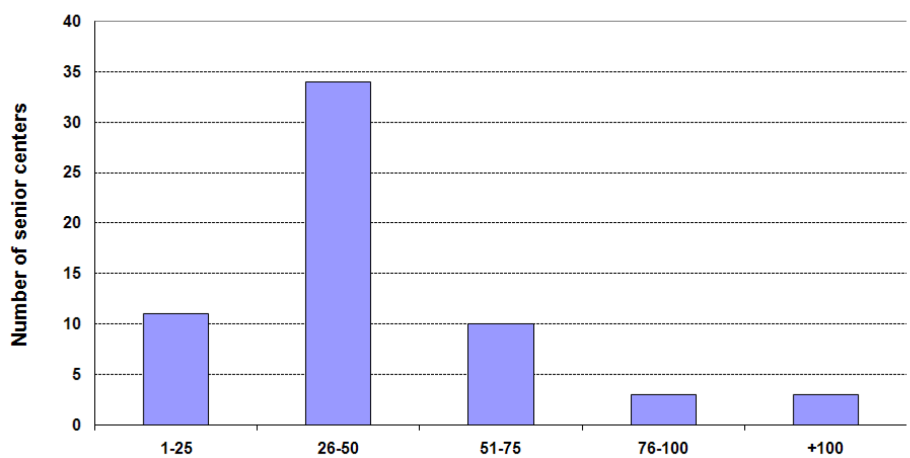
Figure 2: Map of facilities and services that are highly related to senior population needs [42].



We assume that older adults are concentrated at the centroid of municipalities and construct the set of demand sites accordingly. Note that a demand site will be served through the comprehensive centers located in its service region. We use a GIS software ArcGIS [9] to calculate the distances between the candidate senior center locations and demand sites.

[42] reports that in Allegheny County participation rate of senior centers has been steadily declining. In the current system attendance patterns show a few centers with robust attendance, and a significant number of centers with average daily attendance of 30 or fewer consumers. Figure 3 depicts the distribution of the daily attendance to the 61 senior centers in Allegheny County in 2007.

Figure 3: Daily Attendance of Senior Centers in Allegheny County, 2006-2007 [2].

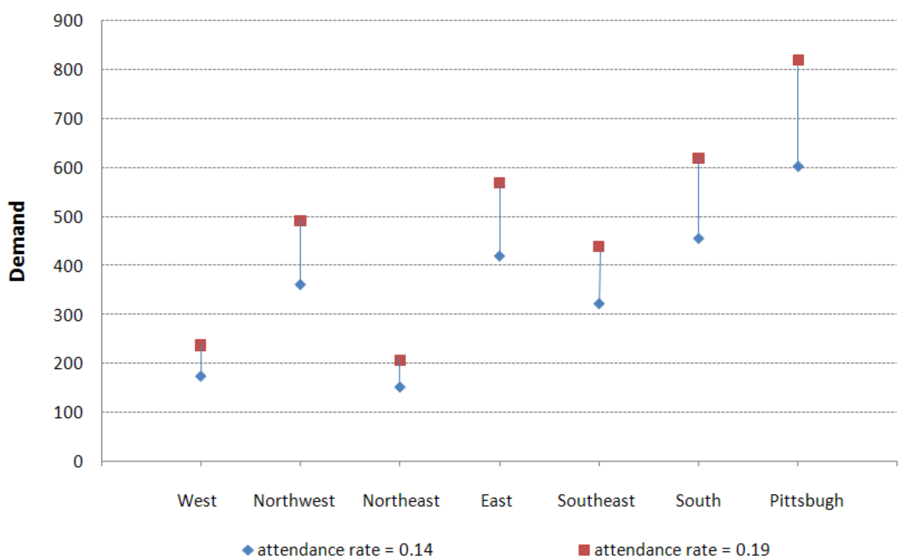


A recent study showed that the size of a center could contribute to the quality and extent of programming [27]. First, larger sites, with larger budgets and revenue sources, can attract more-qualified staff. Second, larger numbers of consumers provide a justification for more varied programming. Therefore, the Steering committee proposed that at a minimum, each comprehensive center should be capable of serving 125 consumers/day in its primary facility-based programming and an additional 50 consumers/day within programs offered in its satellite centers [28].

These minimum numbers of consumers are based on historical data. In 2006 - 2007, the Allegheny County senior service system served about 2,800 individual consumers each day [2]. Half of these consumers would patronize comprehensive centers [28]. Since up to 11 comprehensive centers could be opened across the County (see Table 1), each comprehensive center should cover about 125 consumers per day. The remaining consumers and potential future consumers would be served via programming within the satellite centers.

There were 177,280 people over 65 years of age in Allegheny County as of the Census 2000 [44] and as stated before around 2,800 individuals are served through Allegheny County Senior Center Service System each day in 2006-2007. That is, 16 out of each 1000 older adults attended senior centers on each day in 2006-2007. Within this context, we estimate the demand for senior services under pessimistic, neutral, and optimistic scenarios. The pessimistic scenario assumes that in the future $1.4\% - 1.5\%$ of the over 65 population would patronize a senior center on daily basis; the neutral scenario assumes $1.6\% - 1.7\%$ patronage; and the optimistic scenario assumes $1.8\% - 1.9\%$ patronage. We present sensitivity analysis results on the attendance rates in Section 7. We set the probabilities of the pessimistic and optimistic scenarios to 0.25 whereas the probability of the neutral scenario is set to be 0.5. Figure 4 depicts the demand range of each region under these settings.

Figure 4: Expected demand in each region [44].



To open a comprehensive center in a service region, at least 175 older adults patronizing the primary facility and its satellite centers are required under each scenario. As seen in Figure 4, this condition holds for the Northeast region only under the optimistic scenario, and it barely holds for the West region. We incorporate these concerns into our model solution as follows. Although the West region has a low population of older adults and a low poverty rate, the number of older adults is expected to grow in the future (see Table 1); hence, locating a comprehensive center in the West region would be prudent. However, though the Northeast region also has a low population of older adults and a low poverty rate, its older adult population is not expected to increase. Thus, we assume that the County would choose not to enforce minimum capacity requirements on satellite centers opened in this region. Consequently, in our computations we remove constraint (6f) for the Northeast region under the pessimistic scenario to ensure feasibility of all problem instances.

When older adults in Allegheny County were asked how far they would travel to a senior center, 77% of those who expressed an interest in attending to a senior center were willing to travel up to 10 miles from their residence to do so [12]. Therefore, in our model, we allow satellite centers that are no more than 10 miles apart from a comprehensive center to be assigned to that comprehensive center. Moreover, we assume that at most 7 satellite centers can be assigned to a comprehensive center for eliminating impractical solutions in which excessive number of satellite centers are assigned to a single comprehensive center.

The fixed cost parameters c_i and f_j include both capital investment cost (for new facilities) and business overhead costs (for all facilities). We parameterize these costs using values established in the literature [18]. The annual capital cost is set to zero for satellite centers because they already exist (as community assets) whereas we assume a capital investment cost of \$100,000 for comprehensive center facilities. Moreover, we assume that annual overhead cost is \$30,000 for satellite centers and \$50,000 for comprehensive centers. As a result, we set c_i for comprehensive centers to \$150,000/year and f_j for satellite centers to \$30,000/year.

We estimate the annual per person traveling costs q^1_{ik} and q^2_{jk} based on actual Euclidian distance. We assume that the unit travel cost of an older adult reflects not only the out-of-pocket cost of traveling but opportunity costs and personal discomfort as well; this cost is assumed to be \$0.5 per mile, roughly consistent with government reimbursement rates [16]. In addition, we assume that an individual patronizes a senior center twice a week, or about 100 times per year. Hence, we set the annual travel costs by multiplying the actual distance between center locations and demand sites by \$50.

A senior service professional will be responsible for providing programming at a satellite center. Senior service providers in the Steering committee stated that service professionals can not serve more than 50 people while keeping acceptable service quality. Therefore, we set the capacity of satellite centers to 50 people/day. If 1.9% of the older adults patronize centers in the optimistic scenario, then, according to the data from Census 2000 [44], a total of about 3,400 older adults would patronize senior centers throughout Allegheny County each day. Based on our assumptions that approximately half of this demand will be satisfied by satellite centers and a maximum of 11 comprehensive centers are allowed, each comprehensive center should accommodate around 150 consumers per day in the optimistic scenario. Allowing some flexibility for demand variations, we set the capacity of comprehensive centers to 200 people/day.

7 Computational Results

The extensive form instance of our senior center location problem (1) is computationally challenging as it has around 20,000 constraints and over 60,000 binary and continuous variables. Standard integer programming solvers are not capable of handling our instances. Therefore, we first decompose problem (1) into service regions, and then solve each region's problem using the Lagrangian dual-based branch-and-bound algorithm. We use a subgradient algorithm [10] for the optimization of Lagrangian dual problems (12), and CPLEX 11 callable library [15] for the optimization of mixed-integer subproblems in the branch-and-bound tree. We ran our algorithm on a computer with AMD Opteron 240 processor and 3.6 GB memory for 10 hours for each service region problem. As this redesign would happen infrequently, such a time limit is reasonable.

The Lagrangian scenario decomposition approach does not scale well with the increase in the number of scenarios. Therefore, we consider three scenarios for the attendance rates: pessimistic, neutral and optimistic. Under each scenario, we consider two different values of the attendance rates with discrete intervals of 0.1% and rounded up all fractional demand values. We perform our numerical tests for all 8 different combinations of these attendance rates. The value of the best solution returned by the Lagrangian dual-based branch-and-bound algorithm after 10 hours of computation time is reported in Table 2 for each service region and for each attendance rate combination.

As seen in Table 2, the optimality gap is at most 6% in all cases. Shorter maximum running times are likely to return similar results since the improvement in the last three hours was small. We refer to the county-wide solution formed by combining the solutions of all service regions reported in Table 2 as the "initial solution." Once we get this solution, we check whether it violates constraint (2c). Note that for a particular satellite center, constraint (2c) could be violated by at most six comprehensive centers in the initial solution since intra-regional violations are not allowed by constraint (3).

We run the feasibility heuristic described in Section 5.3 for those satellite centers that

Table 2: Bounds obtained after 10 hours

Attendance rate	Pessimistic	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
	Neutral	0.016	0.016	0.017	0.017	0.016	0.016	0.017	0.017
	Optimistic	0.018	0.019	0.018	0.019	0.018	0.019	0.018	0.019
West	Gap (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Objective ($)	207,024	207,581	208,184	208,741	207,616	208,172	208,776	209,333
Northwest	Gap (%)	1.71	5.56	1.51	5.48	1.47	5.41	1.40	5.35
	Objective ($)	360,132	386,552	361,440	387,602	360,897	386,963	362,205	388,014
Northeast	Gap (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Objective ($)	214,252	214,946	215,809	216,502	214,278	214,971	215,835	216,528
East	Gap (%)	0.00	1.86	0.00	2.05	0.00	1.85	0.00	1.94
	Objective ($)	381,684	446,577	382,863	447,782	382,112	446,759	383,291	448,154
Southeast	Gap (%)	2.76	0.00	2.77	0.00	2.80	0.00	2.67	0.00
	Objective ($)	330,327	330,976	331,872	332,520	330,770	331,419	332,315	332,964
South	Gap (%)	0.01	1.97	0.01	2.22	0.00	1.89	0.00	1.78
	Objective ($)	452,410	478,644	453,893	479,725	453,075	479,204	454,557	480,285
Pittsburgh	Gap (%)	2.11	2.77	2.02	2.75	2.14	2.76	2.04	2.72
	Objective ($)	550,403	580,477	550,842	580,879	550,583	580,657	551,022	581,059

violate constraint (2c). Provided the initial solution, the heuristic algorithm opens additional satellite centers to obtain a good feasible solution. We refer to the solution obtained after running the heuristic algorithm as the "proposed solution."

We consider cost elements related to network design: travel cost, fixed cost and overhead cost. The numbers of comprehensive and satellite centers that were opened in each region as well as the estimated travel and total costs (e.g. sum of travel, fixed, and operating costs) of our proposed solution for each region and for each attendance rate combination are summarized in Table 3.

Total costs reported in Table 3 should not be compared against the available budget for running the senior service system because they do not include the variable cost of service. Moreover, our total cost values include the travel cost of older adults which is generally not an element of the budget.

Note that if the opened centers (e.g. first-stage solution) stay the same for increased number of expected demand, then intuitively both the travel and total costs increase. However, if more centers are opened when the expected demand increases, then the expected travel cost may decrease. For instance, in Table 3, consider the Northwest region with attendance rate combinations $[0.014, 0.016, 0.018]$ and $[0.014, 0.016, 0.019]$. Expected demand is clearly higher in the latter combination but the expected travel cost is higher for the former one. This is because travel cost savings achieved by opening an additional satellite center exceeds the travel costs of new consumers.

Table 3: Number of senior centers and cost estimates of the proposed solution

Attendance rate	Pessimistic	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
	Neutral	0.016	0.016	0.017	0.017	0.016	0.016	0.017	0.017
	Optimistic	0.018	0.019	0.018	0.019	0.018	0.019	0.018	0.019
West	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	1	1	1	1	1	1	1	1
	Travel Cost ($)	27,024	27,581	28,184	28,741	27,636	28,193	28,796	29,353
	Total Cost ($)	207,024	207,581	208,184	208,741	207,636	208,193	208,796	209,353
Northwest	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	6	7	6	7	6	7	6	7
	Travel Cost ($)	30,132	26,552	31,441	27,602	30,897	26,963	32,205	28,014
	Total Cost ($)	360,132	386,552	361,440	387,602	360,897	386,963	362,205	388,014
Northeast	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	1	1	1	1	1	1	1	1
	Travel Cost ($)	34,252	34,946	35,809	36,503	34,353	35,047	35,910	36,604
	Total Cost ($)	214,252	214,946	215,809	216,502	214,353	215,047	215,910	216,604
East	Comprehensive	1	2	1	2	1	2	1	2
	Satellite	7	4	7	4	7	4	7	4
	Travel Cost ($)	21,684	26,577	22,863	27,782	22,112	26,759	23,291	28,154
	Total Cost ($)	381,684	446,577	382,863	447,782	382,112	446,759	383,291	448,154
Southeast	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	5	5	5	5	5	5	5	5
	Travel Cost ($)	30,327	30,976	31,872	32,521	30,805	31,454	32,350	32,999
	Total Cost ($)	330,327	330,976	331,872	332,520	330,805	331,454	332,350	332,999
South	Comprehensive	2	2	2	2	2	2	2	2
	Satellite	4	5	4	5	4	5	4	5
	Travel Cost ($)	32,411	28,644	33,893	29,725	33,075	29,204	34,557	30,285
	Total Cost ($)	452,410	478,644	453,893	479,725	453,075	479,204	454,557	480,285
Pittsburgh	Comprehensive	2	2	2	2	2	2	2	2
	Satellite	8	9	8	9	8	9	8	9
	Travel Cost ($)	10,403	10,477	10,842	10,879	10,583	10,657	11,022	11,059
	Total Cost ($)	550,403	580,477	550,842	580,879	550,583	580,657	551,022	581,059
Total	Comprehensive	9	10	9	10	9	10	9	10
	Satellite	32	32	32	32	32	32	32	32
	Travel Cost ($)	186,232	185,752	194,903	193,752	189,461	188,277	198,132	196,468
	Total Cost ($)	2,496,232	2,645,753	2,504,903	2,653,751	2,499,461	2,648,277	2,508,131	2,656,468

Expected occupancy of the comprehensive and satellite centers opened in the proposed solution in each region and for each attendance rate combination is presented in Table 4.

Table 4: Expected occupancy (older adult/day) of senior centers in the proposed solution

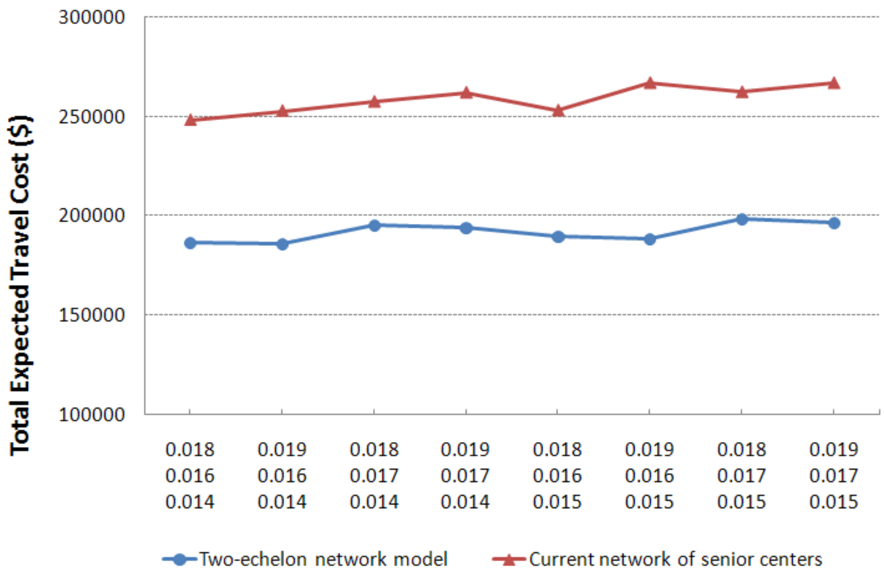
Attendance rate	Pessimistic	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
	Neutral	0.016	0.016	0.017	0.017	0.016	0.016	0.017	0.017
	Optimistic	0.018	0.019	0.018	0.019	0.018	0.019	0.018	0.019
West	Comprehensive	152	155	158	161	155	158	161	164
	Satellite	49	49	49	49	49	49	49	49
Northwest	Comprehensive	192	193	192	193	191	192	191	192
	Satellite	37	33	40	35	39	34	41	36
Northeast	Comprehensive	131	134	137	139	133	136	139	142
	Satellite	44	44	44	44	44	45	44	44
East	Comprehensive	185	185	188	188	189	189	192	192
	Satellite	28	24	40	26	37	30	40	26
Southeast	Comprehensive	194	195	150	156	194	195	194	158
	Satellite	36	37	47	47	37	38	39	47
South	Comprehensive	193	193	193	193	197	197	197	197
	Satellite	35	29	39	33	35	29	39	33
Pittsburgh	Comprehensive	200	200	199	199	200	200	200	200
	Satellite	37	31	35	36	38	35	41	34
Average	Comprehensive	179	180	174	176	180	181	182	178
	Satellite	38	36	42	39	40	38	42	39

Recall that comprehensive centers are meant to be community focal points attracting as many older adults as possible in their region, while satellite centers are designed to ease the physical access and increase flexibility of programming to better meet the demands of older adults. Therefore, we do not expect or require high occupancy of satellite centers and the occupancy profile of senior centers given in Table 4 matches well with the purpose of the proposed two-echelon network model.

Along with the occupancy of centers, the change in travel cost should also be considered. We calculated the total expected travel cost of the current senior center network by setting the capacity of each existing center to 100 customer/day (note that the current capacity of most existing centers is less than 100, see Figure 3), and again allocating older adults to centers to minimize total travel cost. Figure 5 plots the total expected travel costs calculated in the current setting and the total expected travel costs of the proposed network model reported in Table 3.

As seen in Figure 5, the travel cost saving of the proposed network model as compared to the current one is around 25% on average for all attendance rate combinations. Our two-echelon network model increases the occupancy of comprehensive centers by bringing together older adults from various demand sites. In Table 3, the model proposes opening $9-10$ comprehensive centers and 32 satellite centers as opposed to 61 senior centers in

Figure 5: Expected total travel cost in the current network and in the proposed two-echelon network.



the current setting. Therefore, it might seem counterintuitive that the proposed two-echelon model, with fewer senior centers than the current network, would result in travel cost savings. Note, however, that in the current network senior centers are often located without any reference to where older adults reside, while the proposed model minimizes total travel cost.

Satellite centers are more desirable than comprehensive centers for our problem instance because; (i) per person fixed cost of satellite centers are lower than that of comprehensive centers, e.g. $\$30,000/50 < \$150,000/200$, and; (ii) we assume that similar services can be provided in both type of centers without any difference in variable service cost.

The desirability of satellite centers are balanced through the limit on the maximum number of satellite centers allowed to be assigned to a comprehensive center, which is 7 in our model. That is, if we do not enforce this limit, the model would open only one comprehensive center in each service region, just for managing the satellite centers, and the majority of the region's demand would be served via programming in satellite centers. To verify this, we rerun our model by changing constraint (2d) with:

$$z_{ij}^s \leq x_i \qquad j \in \mathcal{J}(i), i \in \mathcal{I}. \tag{15}$$

Constraint (15) does not restrict the number of satellite centers that can be directed by a comprehensive center. In this setting, the numbers of comprehensive and satellite centers opened in each service region are given in Table 5.

Table 5: Number of senior centers in each region without restricting the number of satellite centers that can be directed by a comprehensive center

Attendance rate	Pessimistic	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.015
	Neutral	0.016	0.016	0.017	0.017	0.016	0.016	0.017	0.017
	Optimistic	0.018	0.019	0.018	0.019	0.018	0.019	0.018	0.019
West	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	1	1	1	1	1	1	1	1
Northwest	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	6	7	6	7	6	7	6	7
Northeast	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	1	1	1	1	1	1	1	1
East	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	7	8	7	8	7	8	7	8
Southeast	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	5	5	5	5	5	5	5	5
South	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	8	9	8	9	8	9	8	9
Pittsburgh	Comprehensive	1	1	1	1	1	1	1	1
	Satellite	12	13	12	13	12	13	12	13
Total	Comprehensive	7	7	7	7	7	7	7	7
	Satellite	40	44	40	44	40	44	40	44

As expected, only one comprehensive center is located in each region, and the number of satellite centers increases as compared to the proposed solution in Table 3. The average travel and total costs of the solution presented in Table 5 over all attendance rate combinations are \$191,343 and \$2,501,344, respectively. That is, restricting the number of satellite centers that can be assigned to a comprehensive center in our proposed solution does not cause significant cost changes.

One of the metrics that measure the quality of the stochastic solution over the ones that are obtained by solving simpler deterministic problems is the *expected value of perfect information* [4].

The expected value of perfect information ($EVPI$) is the maximum amount a decision maker would pay for exactly knowing the future. Let $\zeta(s)$ denote the optimal value of problem (1) for a particular scenario s. Then, in our problem

$$EVPI = \zeta - \sum_{s \in \mathcal{S}} p^s \zeta(s). \tag{16}$$

In the literature, ζ is known as the *recourse problem solution* [4] and $\sum_{s \in \mathcal{S}} p^s \zeta(s)$ is known as the *wait-and-see solution* [25]. We calculate the $EVPI$ for our problem under each attendance rate combination. The results presented in Table 6 indicate that average EVPI across various attendance rate combinations is \$193,132 per year which is equal to 7.44% of

Table 6: $EVPI$ under each attendance rate combination

Attendance rate	Recourse problem ($)	Wait-and-see ($)	EVPI ($)	EVPI/RP
0.014 0.016 0.018	2,496,232	2,314,542	181,690	7.28%
0.014 0.016 0.019	2,645,753	2,349,174	296,579	11.21%
0.014 0.017 0.018	2,504,903	2,405,173	99,730	3.98%
0.014 0.017 0.019	2,653,751	2,439,804	213,947	8.06%
0.015 0.016 0.018	2,499,461	2,339,808	159,654	6.39%
0.015 0.016 0.019	2,648,277	2,374,439	273,838	10.34%
0.015 0.017 0.018	2,508,131	2,405,173	102,958	4.10%
0.015 0.017 0.019	2,656,468	2,439,804	216,664	8.16%
Average	2,576,622	2,383,490	193,132	7.44%

the total cost of the proposed solution. As a result, benefit of knowing the future demand with certainty is high enough justifying the use of a stochastic programming model.

8 Conclusions

We formulate a two-stage stochastic facility location/allocation model with integer recourse for senior center network redesign problem. We apply our model to Allegheny County, Pennsylvania, a region with high demand for senior services, including senior centers. We calibrate the model using census and GIS data as well as the expert opinions gleaned from the County's plans for redesigning its current senior center service system. To mitigate the expenditures of building new facilities while redesigning the network, we consider community assets such as libraries, personal care centers, and shopping areas as candidate locations for the satellite centers. This innovative idea is motivated by the fact that older adults are already visiting such places frequently.

We decompose our network redesign problem into service region problems by relaxing a set of coupling constraints. Decomposition is especially useful for large study areas for which the overall model is very hard to solve due to it excessive size. We solve each service region problem using a Lagrangian scenario decomposition approach, which returns small optimality gaps after 10 hours of solution time. We also propose a heuristic algorithm to remove the violations of relaxed constraints.

We calculate the expected value of perfect information in order to measure the benefit of our stochastic programming model. This value is found to be around $200,000 per year, which means that using a stochastic programming model is well justified.

Our results indicate that a two-echelon network of comprehensive and satellite centers is appropriate for increasing the occupancy of senior centers as community focal points while maintaining customized and accessible programming in small neighborhood areas. Specifically, occupancy of comprehensive centers are found to be above 150 older adults per day in all service regions except one. As a result, comprehensive centers take advantage of large scale programs while assuring highly customized, accessible services via satellite centers. This justifies the benefit of using a two-echelon network design.

We do not consider specific services provided in senior centers in this paper since both comprehensive and satellite centers can offer similar services if needed. Senior centers provide public services for which equitable delivery is essential. We measure equity by distance and allocate older adults only to those centers within a specified distance. These allocation decisions are necessary for the capacity planning of an equitable delivery system. Our model ensures access to senior services in close proximities. Once having this opportunity, older adults are free to patronize any center based on their preferences.

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