Fall November 16, 2012

General Recognition Theory Extended to Include Response Times: Predictions for a Class of Parallel Systems

Joseph W Houpt, *Wright State University - Main Campus*
James T Townsend, *Indiana University - Bloomington*
Noah H Silbert

General Recognition Theory Extended to Include Response Times: Predictions for a Class of Parallel Systems

James T. Townsend[1]
Joseph W. Houpt[2] Noah H. Silbert[3]

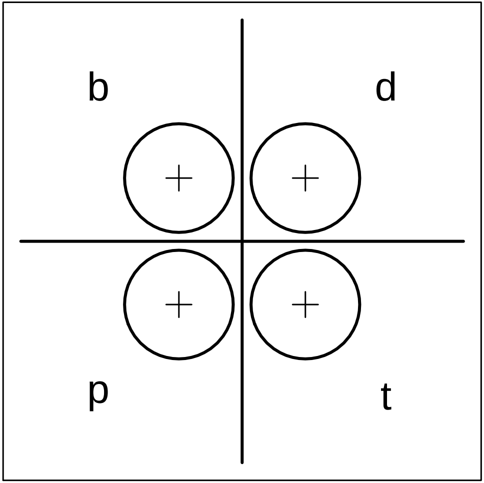
[1]Indiana University [2]Wright State University [3]Center for Advanced Study of Language

Psychonomic Society
Annual Meeting
November 16, 2012

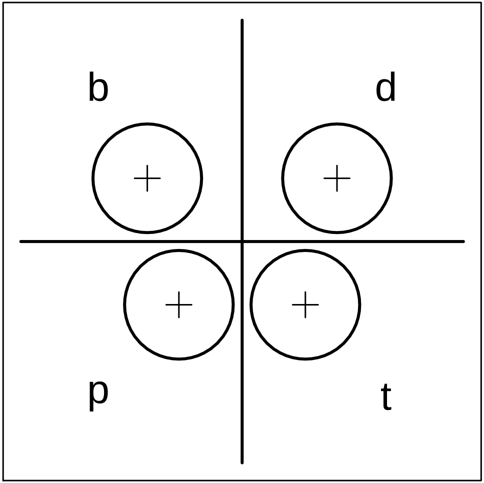
- We are interested in the nature of dependencies in perception.

green /ba/

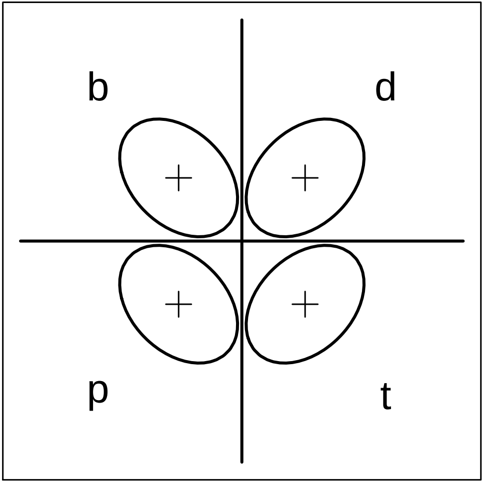
Traditional (Static) GRT



Failure of Static Perceptual Separability



Failure of Static Perceptual Independence



Overview

- We seek, in the present work, to extend GRT by combining patterns of accuracy with response times (RTs).
  - There are many models that use both response time and accuracy to understand cognitive processes, but they are rarely used to explore perceptual dependencies.
- We extend perceptual independence, perceptual separability, and decisional separability to a general stochastic-dynamic model.
- We establish extensions of report independence and marginal response invariance that account for both RT and accuracy

- Two channels operating in parallel $(X(t), Y(t))$
  - Each is composed of two subchannels (e.g., $X = \big(X_{brown}(t), X_{blue}(t)\big)$)
  - A decision is made on a channel when either of the subchannels reaches its bound
  - The decision of the system is the combination of the first subchannel to reach its bound in each channel
- For example
  - $X(t)$ is the process representing color while $Y(t)$ represents shape
  - $X_{brown}(t)$ is evidence accumulated for brown; $X_{blue}(t)$ is evidence accumulated for blue
  - $Y_{\blacksquare}(t)$ is evidence accumulated for square; $Y_{\blacktriangle}(t)$ is evidence accumulated for triangle

Decisional Separability (DS)

Definition

Decisional separability holds on dimension X if and only if $\mathrm{P}\{C_{\mathrm{color}} = \gamma | Y(t), C_{\blacksquare}, C_{\blacktriangle}\} = \mathrm{P}\{C_{\mathrm{color}} = \gamma\}$ for $\mathrm{color} \in \{\mathit{brown}, \mathit{blue}\}$ and all t. Similarly, decisional separability holds on dimension Y if and only if $\mathrm{P}\{C_{\mathrm{shape}} = \gamma | X(t), C_{\mathit{brown}}, C_{\mathit{blue}}\} = \mathrm{P}\{C_{\mathrm{shape}} = \gamma\}$ for $\mathrm{shape} \in \{\blacksquare, \blacktriangle\}$ and all t.

Decisional Separability (DS)

Definition

Decisional separability holds on dimension X if and only if $\mathrm{P}\{C_{\text{color}} = \gamma | Y(t), C_{\blacksquare}, C_{\blacktriangle}\} = \mathrm{P}\{C_{\text{color}} = \gamma\}$ for $\text{color} \in \{\textit{brown}, \textit{blue}\}$ and all t. Similarly, decisional separability holds on dimension Y if and only if $\mathrm{P}\{C_{\text{shape}} = \gamma | X(t), C_{\textit{brown}}, C_{\textit{blue}}\} = \mathrm{P}\{C_{\text{shape}} = \gamma\}$ for $\text{shape} \in \{\blacksquare, \blacktriangle\}$ and all t.

- Although there are possibly some very interesting effects of failures of decisional separability on these models, we do not explore these effects in this paper.

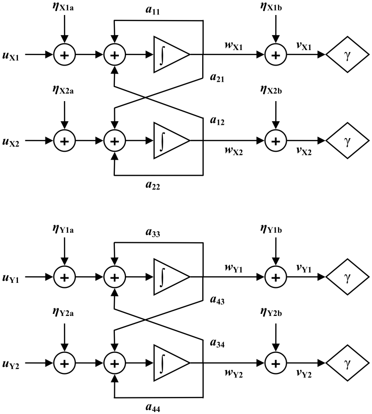
Perceptual Separability (PS)

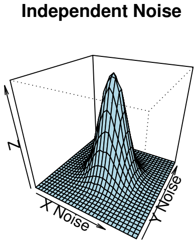
Definition

Perceptual separability of one channel at a particular stimulus level is defined as invariance of the marginal processes of that channel over changes in the stimulus level of the other channel. Thus, for perceptual separability to hold on X at $s_x = brown$, then for all x, t,

$$\mathrm{P}\{X_{brown}(t) \leq x; s = \blacksquare\} = \mathrm{P}\{X_{brown}(t) \leq x; s = \blacktriangle\}$$
$$\mathrm{P}\{X_{blue}(t) \leq x; s = \blacksquare\} = \mathrm{P}\{X_{blue}(t) \leq x; s = \blacktriangle\}.$$

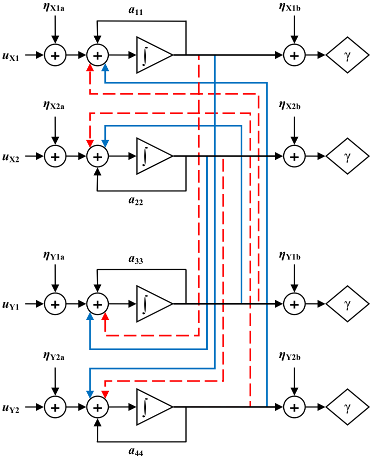
Example model with perceptual separability



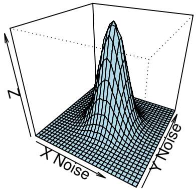


Within channel cross-talk does not affect perceptual separability or independence.

Example model without perceptual separability



Independent Noise



Marginal Response Invariance (MRI)

Definition

Marginal response invariance on a channel holds if and only if the marginal probability of a particular response on that dimension is invariant across the level of the other stimulus dimension. For marginal response invariance to hold on channel X,

$$\begin{aligned}
&\mathrm{P}\{R_{\text{color}} = \textit{brown}; s = \blacksquare\} \\
&\quad = \mathrm{P}\{R = \blacksquare; s = \blacksquare\} + \mathrm{P}\{R = \blacktriangle; s = \blacksquare\} \\
&\quad = \mathrm{P}\{R = \blacksquare; s = \blacktriangle\} + \mathrm{P}\{R = \blacktriangle; s = \blacktriangle\} \\
&\quad = \mathrm{P}\{R_{\text{color}} = \textit{brown}; s = \blacktriangle\}.
\end{aligned}$$

Timed Marginal Response Invariance (tMRI)

Definition

Timed marginal response invariance is defined by satisfaction of the condition for all $t > 0$,

$$
\begin{aligned}
&\mathrm{P}\{R_{\mathrm{color}} = \mathit{brown}, T \le t; s = \blacksquare\} \\
&\quad = \mathrm{P}\{R = \blacksquare, T \le t; s = \blacksquare\} + \mathrm{P}\{R = \blacktriangle, T \le t; s = \blacksquare\} \\
&\quad = \mathrm{P}\{R = \blacksquare, T \le t; s = \blacktriangle\} + \mathrm{P}\{R = \blacktriangle, T \le t; s = \blacktriangle\} \\
&\quad = \mathrm{P}\{R_{\mathrm{color}} = \mathit{brown}, T \le t; s = \blacktriangle\}.
\end{aligned}
$$

MRI Theorems

Proposition

Perceptual separability and decisional separability imply marginal response invariance in accrual halting parallel models.

Proposition

Timed marginal response invariance implies static marginal response invariance but not conversely.

Proposition

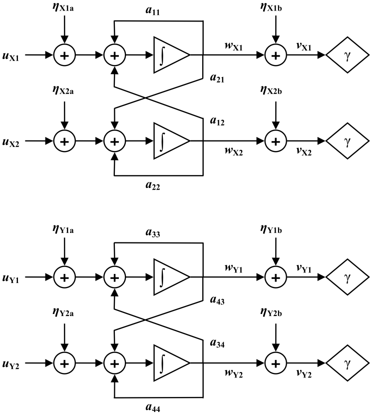
Perceptual separability, decisional separability and speed invariance imply timed marginal response invariance in accrual halting parallel models.

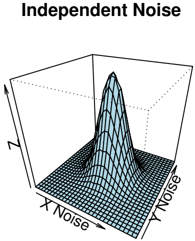
Perceptual Independence (PI)

Definition

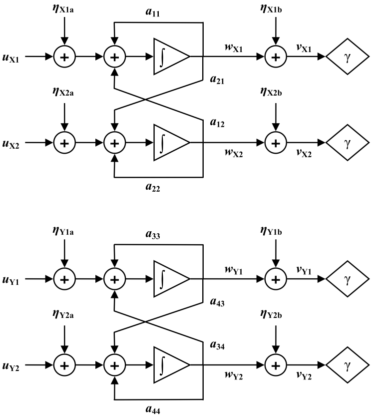
Two channels are said to be perceptually independent if $\{X(t); t \geq 0\}$ and $\{Y(t); t \geq 0\}$ are independent.

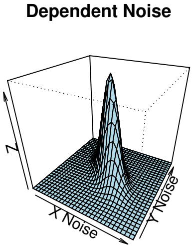
Example model with perceptual independence





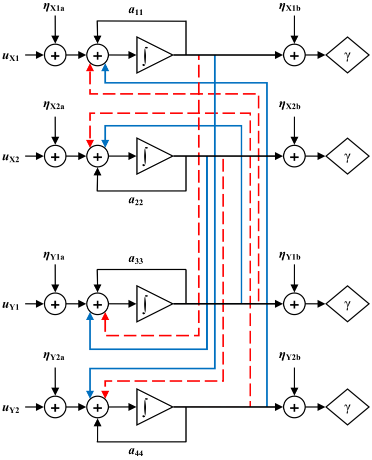
Example model without perceptual independence

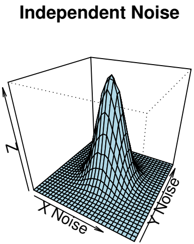




(Perceptual separability holds if noise is only dependent at B).

Example model without perceptual independence





(If noise is at A).

Report Independence (RI)

Definition

We say that report independence holds for a particular stimulus–response combination if the probability of that response is equal to the product of the marginal probability of each of the response dimensions. Formally, report independence holds for $R = \blacksquare$ with stimulus $R = \blacksquare$ if,

$$\begin{aligned}
&\mathrm{P}\{R = \blacksquare; s = \blacksquare\} = \\
&\qquad \left[\mathrm{P}\{R = \blacksquare; s = \blacksquare\} + \mathrm{P}\{R = \blacktriangle; s = \blacksquare\}\right] \\
&\qquad \times \left[\mathrm{P}\{R = \blacksquare; s = \blacksquare\} + \mathrm{P}\{R = {\color{blue}\blacksquare}; s = \blacksquare\}\right].
\end{aligned}$$

Timed Report Independence (tRI)

Definition

We say that timed report independence holds for a particular stimulus–response combination if the probability of a particular response, given that the response was made by t, is equal to the product of the marginal probabilities on each response dimension given the response was made by t, for all $t > 0$. Formally, report independence holds for $R = \blacksquare$ with stimulus $R = \blacksquare$ if for all $t > 0$,

$$
\begin{aligned}
&\mathrm{P}\{R = \blacksquare | \mathrm{RT} \leq t; s = \blacksquare\} \\
&\quad = \mathrm{P}\{R_{\mathrm{color}} = \mathit{brown} | \mathrm{RT} \leq t; s = \blacksquare\}\mathrm{P}\{R_{\mathrm{shape}} = \blacksquare | \mathrm{RT} \leq t; s = \blacksquare\}.
\end{aligned}
$$

Equivalently,

$$
\begin{aligned}
&\mathrm{P}\{R = \blacksquare, \mathrm{RT} \leq t; s = \blacksquare\}\mathrm{P}\{\mathrm{RT} \leq t; s = \blacksquare\} \\
&\quad = \mathrm{P}\{R_{\mathrm{color}} = \mathit{brown}, \mathrm{RT} \leq t; s = \blacksquare\}\mathrm{P}\{R_{\mathrm{shape}} = \blacksquare, \mathrm{RT} \leq t; s = \blacksquare\}.
\end{aligned}
$$

tRI Theorems

Proposition

Decisional separability and perceptual independence imply report independence.

Proposition

Timed report independence implies ordinary report independence but not conversely.

Proposition

Perceptual independence and decisional separability imply timed report independence in accrual halting parallel models.

Summary

- Described a general extension of static GRT to the time domain.
  - A generalization of earlier efforts (e.g., Ashby, 1989; Ashby, 2000).

Summary

- Described a general extension of static GRT to the time domain.
  - A generalization of earlier efforts (e.g., Ashby, 1989; Ashby, 2000).
- Timed MRI is stronger than static MRI.
- Timed RI is stronger than static RI.

Summary

- Described a general extension of static GRT to the time domain.
  - A generalization of earlier efforts (e.g., Ashby, 1989; Ashby, 2000).
- Timed MRI is stronger than static MRI.
- Timed RI is stronger than static RI.
- Perceptual separability, decisional separability and speed invariance imply timed MRI.
- Perceptual independence and decisional separability imply timed RI.

Summary

- Described a general extension of static GRT to the time domain.
  - A generalization of earlier efforts (e.g., Ashby, 1989; Ashby, 2000).
- Timed MRI is stronger than static MRI.
- Timed RI is stronger than static RI.
- Perceptual separability, decisional separability and speed invariance imply timed MRI.
- Perceptual independence and decisional separability imply timed RI.
- Thus, the inclusion of response time provides more rigorous tests of the underlying psychological mechanisms.

Summary

- Described a general extension of static GRT to the time domain.
  - A generalization of earlier efforts (e.g., Ashby, 1989; Ashby, 2000).
- Timed MRI is stronger than static MRI.
- Timed RI is stronger than static RI.
- Perceptual separability, decisional separability and speed invariance imply timed MRI.
- Perceptual independence and decisional separability imply timed RI.
- Thus, the inclusion of response time provides more rigorous tests of the underlying psychological mechanisms.

Thank you.

Supported by: NIH-NIMH MH 057717-10 and AFOSR FA 9550-07-1-0078