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Solitons and Compactons

Ji-HUAN HE¹, SHUN-DONG ZHU²

¹ Modern Textile Institute, Donghua University, Shanghai, China

² Department of Science, Zhejiang Lishui University, Lishui, China

Article Outline

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Glossary

Soliton A soliton is a stable pulse-like wave that can exist in some nonlinear systems. The soliton, after a collision with another soliton, eventually emerges unscathed.

Compacton A compacton is a special solitary traveling wave that, unlike a soliton, does not have exponential tails.

Generalized soliton A generalized soliton is a soliton with some free parameters. Generally a generalized soliton can be expressed by exponential functions.

Compacton-like solution A compacton-like solution is a special wave solution which can be expressed by the squares of sinusoidal or cosinoidal functions.

Definition of the Subject

Soliton and compacton are two kinds of nonlinear waves. They play an indispensable and vital role in all ramifications of science and technology, and are used as constructive elements to formulate the complex dynamical behavior of wave systems throughout science: from hydrodynamics to nonlinear optics, from plasmas to shock waves, from tornados to the Great Red Spot of Jupiter, from traffic flow to Internet, from Tsunamis to turbulence. More recently, soliton and compacton are of key importance in the quantum fields and nanotechnology especially in nanohydrodynamics.

Introduction

Solitary waves were first observed by John Scott Russell in 1895, and were studied by D. J. Korteweg and H. de Vries in 1895. Compactons are special solitons with finite wavelength. It was Philip Rosenau and his colleagues who first found compactons in 1993. Please refer to “► [Soliton Perturbation](#)” for detailed information.

Solitons

A soliton is a special solitary traveling wave that after a collision with another soliton eventually emerges unscathed. Solitons are solutions of partial differential equations that model phenomena like water waves or waves along a weakly anharmonic mass-spring chain.

The Korteweg–de Vries (KdV) equation is the generic model for the study of nonlinear waves in fluid dynamics, plasma and elastic media. KdV equation is one of the most fundamental equations in nature and plays a pivotal role in nonlinear phenomena. We consider the KdV equation in the form

$$\frac{\partial u}{\partial t} + 6u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0. \quad (1)$$

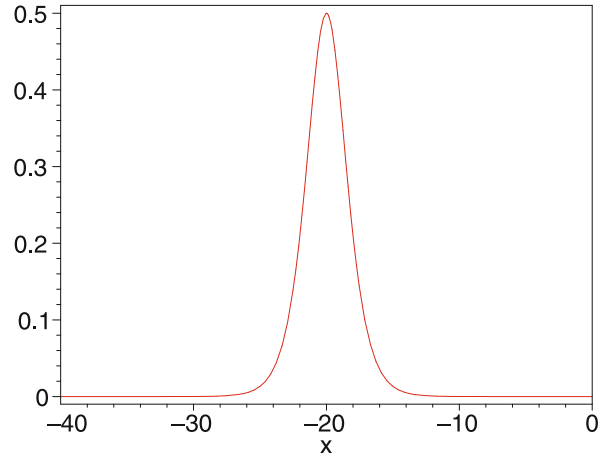
Its solitary traveling wave solution can be solved as

$$u(x, t) = \frac{1}{2} c \operatorname{sech}^2 \left\{ \frac{1}{2} c^{1/2} (x - ct) \right\}. \quad (2)$$

The bell-like solution as illustrated in Fig. 1 is called a soliton.

We re-write Eq. (2) in an equivalently form:

$$u(\xi) = p \operatorname{sech}^2(q\xi) = \frac{4p}{e^{2q\xi} + e^{-2q\xi} + 2}. \quad (3)$$



Solitons and Compactons, Figure 1
Bell-like solitary wave

where $u(x, t) = u(\xi)$, $\xi = x - ct$, c is the wave velocity.

It is obvious that

$$\lim_{\xi \rightarrow \infty} u(\xi) = 0 \quad \text{and} \quad \lim_{\xi \rightarrow -\infty} u(\xi) = 0. \quad (4)$$

The soliton has exponential tails, which are the basic character of solitary waves. The soliton obeys a superposition-like principle: solitons passing through one another emerge unmodified, see Fig. 2.

Compactons

Now consider a modified version of KdV equation in the form

$$u_t + (u^2)_x + (u^2)_{xxx} = 0. \quad (5)$$

Introducing a complex variable ξ defined as $\xi = x - ct$, where c is the velocity of traveling wave, integrating once, we have

$$-cu + u^2 + (u^2)_{\xi\xi} = D, \quad (6)$$

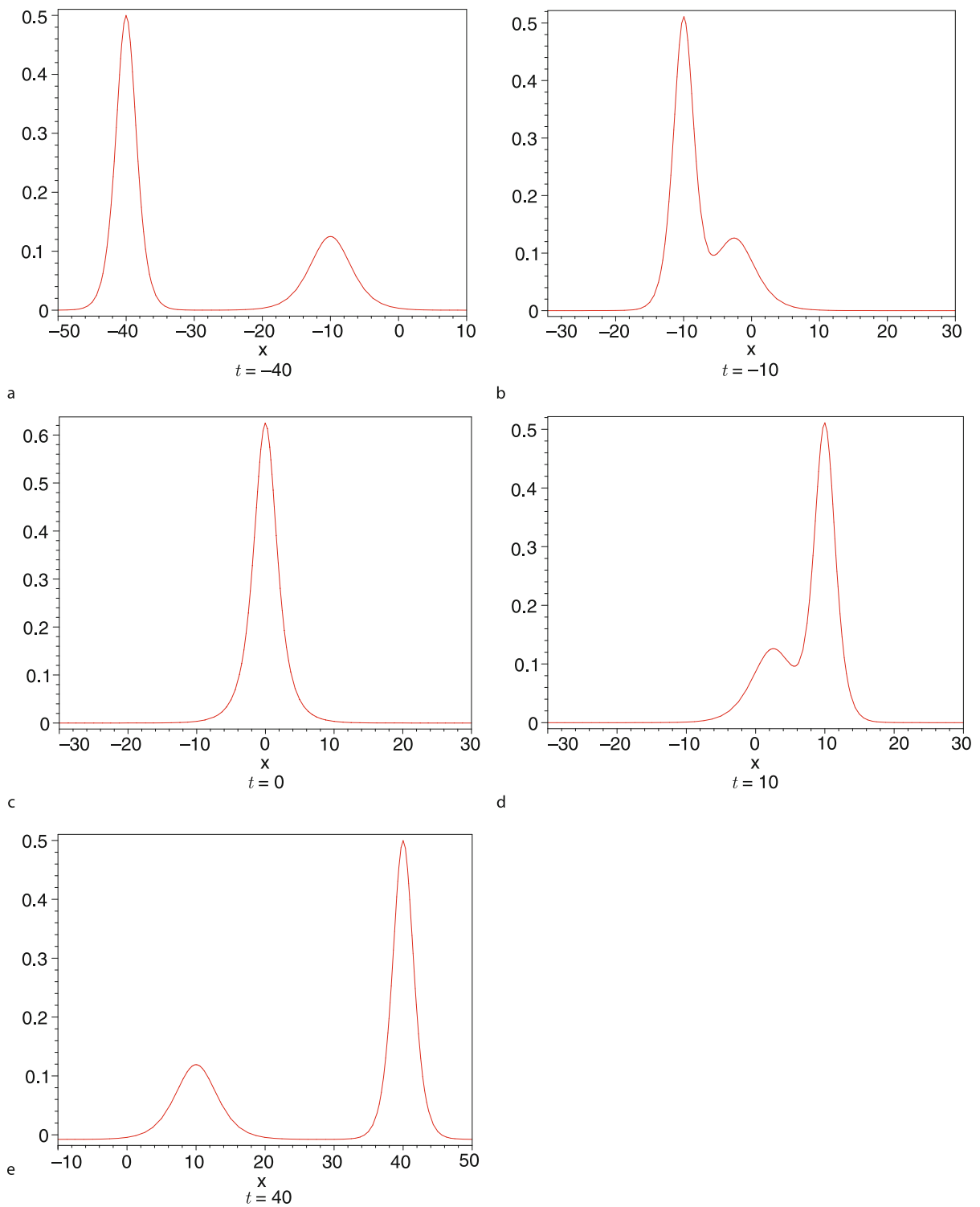
where D is an integral constant.

To avoid singular solutions, we set $D = 0$. We re-write Eq. (6) in the form

$$v_{\xi\xi} + v - cv^{1/2} = 0, \quad (7)$$

where $u^2 = v$.

In case $c = 0$, we have periodic solution: $v(\xi) = A \cos \xi + B \sin \xi$. Periodic solution of nonlinear oscillators can be approximated by sinusoidal function. It helps understanding if an equation can be classified as oscillatory by direct inspection of its terms.



Solitons and Compactons, Figure 2
Collision of two solitary waves

We consider two common order differential equations whose exact solutions are important for physical understanding:

$$u'' - k^2 u = 0, \quad (8)$$

and

$$u'' + \omega^2 u = 0. \quad (9)$$

Both equations have linear terms with constant coefficients.

The crucial difference between these two very simple equations is the sign of the coefficient of u in the second term. This determines whether the solutions are exponential or oscillatory. The general solution of Eq. (8) is

$$u = Ae^{kt} + Be^{-kt}. \quad (10)$$

The second Eq. (9) has a positive coefficient of u , and in this case the general solution reads

$$u = A \cos \omega t + B \sin \omega t. \quad (11)$$

This solution describes an oscillation at the angular velocity ω .

Equation (7) behaves sometimes like an oscillator when $1 - cv^{-1/2} > 0$, i. e., $u = v^{1/2}$ has a periodic solution, we assume v can be expressed in the form

$$v = u^2 = A^2 \cos^4 \omega \xi. \quad (12)$$

Substituting Eq. (12) into Eq. (7) results in

$$12A^2\omega^2 \cos^2 \omega \xi - 16A^2\omega^2 \cos^4 \omega \xi + A^2 \cos^4 \omega \xi - cA \cos^2 \omega \xi = 0. \quad (13)$$

We, therefore, have

$$\begin{aligned} 12A^2\omega^2 - cA &= 0 \\ -16A^2\omega^2 + A^2 &= 0. \end{aligned} \quad (14)$$

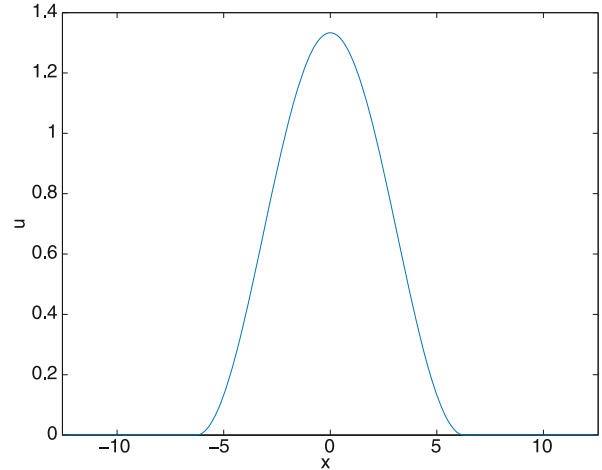
Solving the above system, Eq. (14), yields

$$\omega = \frac{1}{4}, \quad A = \frac{4}{3}c. \quad (15)$$

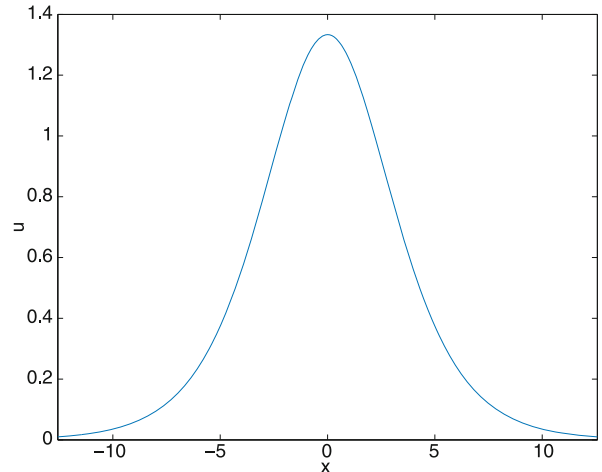
We obtain the solution in the form

$$u = v^{1/2} = \frac{4c}{3} \cos^2 \left[\frac{1}{4}(x - ct) \right]. \quad (16)$$

By a careful inspection, v can tend to a very small value or even zero, as a result, $1 - cv^{-1/2}$ tends to negative infinite,



Solitons and Compactons, Figure 3
Compacton wave without tails



Solitons and Compactons, Figure 4
Solitary wave with two tails

and Eq. (7) behaves like Eq. (8) with $k \rightarrow \infty$, the exponential tails vanish completely at the edge of the bell-shape (see Fig. 3):

$$u = \begin{cases} \frac{4c}{3} \cos^2 \left[\frac{1}{4}(x - ct) \right], & |x - ct| \leq 2\pi \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

This is a compact wave. Unlike solitons (Fig. 4), compacton does not have exponential tails (Fig. 3).

Generalized Solitons and Compacton-like Solutions

Solitary solutions have tails, which can be best expressed by exponential functions. We can assume that a solitary

solution can be expressed in the following general form

$$u(\eta) = \frac{\sum_{n=-c}^d a_n \exp(n\eta)}{\sum_{m=-p}^q b_m \exp(m\eta)}, \tag{18}$$

where $c, d, p,$ and q are positive integers which are unknown to be further determined, a_n and b_m are unknown constants. The unknown constants can be easily determined using Matlab, the method is called the Exp-function method.

We consider the modified KdV equation in the form:

$$u_t + u^2 u_x + u_{xxx} = 0. \tag{19}$$

Using a transformation: $u(x, t) = u(\xi), \xi = kx + \omega t,$ we have

$$\omega u' + ku^2 u' + k^3 u''' = 0, \tag{20}$$

where prime denotes the differential with respect to $\xi.$

We suppose that the solution of Eq. (20) can be expressed as

$$u(\xi) = \frac{a_c \exp(c\xi) + \dots + a_{-d} \exp(-d\xi)}{b_p \exp(p\xi) + \dots + b_{-q} \exp(-q\xi)}. \tag{21}$$

To determine values of c, d, p and $q,$ we balance the linear term of highest order in Eq. (20) with the highest order nonlinear term. According to the homogeneous balance principle, we obtain the result $c = p$ and $d = q.$ For simplicity, we set $c = p = 1$ and $d = q = 1,$ so Eq. (21) reduces to

$$u(\xi) = \frac{a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{\exp(p\xi) + b_0 + b_{-1} \exp(-\xi)}. \tag{22}$$

Substituting Eq. (22) into Eq. (20), and by the help of Matlab, clearing the denominator and setting the coefficients of power terms like $\exp(j\xi), j = 1, 2, \dots$ to zero yield a system of algebraic equations, solving the obtained system, we obtain the following exact solutions:

$$\begin{cases} a_0 = a_1 b_0 + \frac{3k^2 b_0}{a_1}, & a_{-1} = \frac{b_0^2 (3k^2 + 2a_1^2)}{8a_1}, \\ b_{-1} = \frac{b_0^2 (3k^2 + 2a_1^2)}{8a_1^2}, & \omega = -ka_1^2 - k^3, \end{cases} \tag{23}$$

where a_1 and b_0 are free parameters, which depends upon the initial conditions and/or boundary conditions. The

property that stability may depend on initial/boundary conditions is characteristic only for nonlinear systems.

The relationship between wave speed and frequency is

$$\omega = -ka_1^2 - k^3. \tag{24}$$

Note that the value of a_1 is determined from the initial/boundary conditions, so frequency or wave speed may not independent of initial/boundary conditions.

Then, the closed form solution of Eq. (19) reads

$$\begin{aligned} u(x, t) &= \frac{\left(a_1 \exp[kx - (ka_1^2 + k^3)t] + a_1 b_0 + \frac{3k^2 b_0}{a_1} + \frac{b_0^2(3k^2 + 2a_1^2)}{8a_1} \exp[-kx + (ka_1^2 + k^3)t] \right)}{\left(\exp[kx - (ka_1^2 + k^3)t] + b_0 + \frac{b_0^2(3k^2 + 2a_1^2)}{8a_1^2} \exp[-kx + (ka_1^2 + k^3)t] \right)} \\ &= a_1 + \frac{\frac{3k^2 b_0}{8}}{\left(\exp[kx - (ka_1^2 + k^3)t] + b_0 + \frac{b_0^2(3k^2 + 2a_1^2)}{8a_1^2} \exp[-kx + (ka_1^2 + k^3)t] \right)}. \end{aligned} \tag{25}$$

Generally a_1, b_0 and k are real numbers, and the obtained solution, Eq. (25), is a generalized soliton solution.

If we choose $k = 1, a_1 = 1, b_0 = \sqrt{8/5},$ Eq. (25) becomes

$$u(x, t) = 1 + \frac{3\sqrt{1/40}}{\exp[x - 2t] + \sqrt{8/5} + \exp[-x + 2t]}. \tag{26}$$

The bell-like solution is illustrated in Fig. 5.

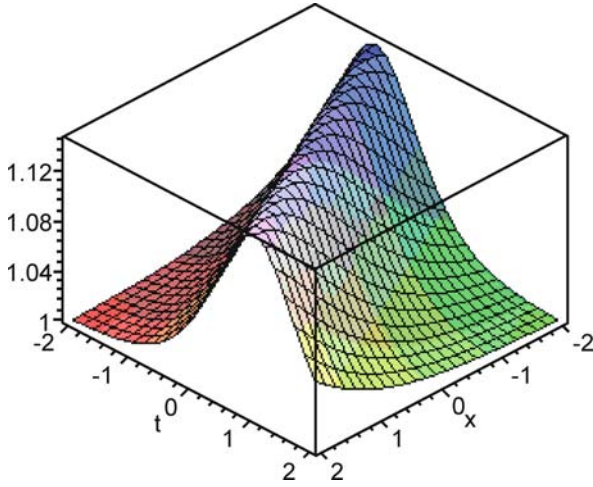
In case k is an imaginary number, the obtained solitary solution can be converted into periodic solution or compact-like solution. We write $k = iK,$ Eq. (25) becomes

$$\begin{aligned} u(x, t) &= a_1 + \frac{-\frac{3K^2 b_0}{8}}{\left((1 + p) \exp[Kx - (Ka_1^2 - K^3)t] + b_0 + i(1 - p) \frac{b_0^2(3k^2 + 2a_1^2)}{8a_1^2} \exp[-Kx + (Ka_1^2 - K^3)t] \right)}, \end{aligned} \tag{27}$$

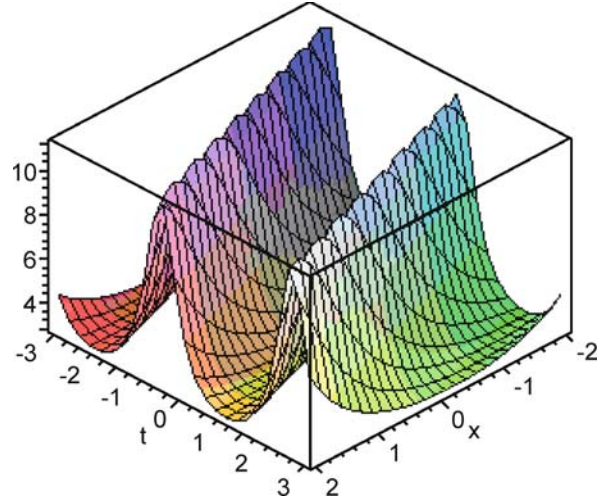
where $p = \frac{b_0^2(-3K^2 + 2a_1^2)}{8a_1^2}.$

If we search for a periodic solution or compact-like solution, the imaginary part in the denominator of Eq. (27) must be zero, that requires that

$$1 - p = 1 - \frac{b_0^2(-3K^2 + 2a_1^2)}{8a_1^2} = 0. \tag{28}$$



Solitons and Compactons, Figure 5
Propagation of a solution with respect to time



Solitons and Compactons, Figure 6
Periodic solution

Solving b_0 from Eq. (28), we obtain

$$b_0 = \pm \sqrt{\frac{8}{-3K^2 + 2a_1^2}}. \tag{29}$$

Substituting Eq. (29) into Eq. (27) results in a periodic solution, which reads

$$u(x, t) = a_1 + \frac{\pm 3K^2 \sqrt{\frac{2}{-3K^2 + 2a_1^2}}}{\cos[Kx - (Ka_1^2 - K^3)t] \pm \sqrt{\frac{2}{-3K^2 + 2a_1^2}}}, \tag{30}$$

or a generalized compact-like solution:

$$u(x, t) = \begin{cases} a_1 + \frac{\pm 3K^2 \sqrt{\frac{2}{-3K^2 + 2a_1^2}}}{\left(\cos[Kx - (Ka_1^2 - K^3)t] \pm \sqrt{\frac{2}{-3K^2 + 2a_1^2}}\right)}, \\ a_1 + 3K^2, & \text{otherwise} \end{cases}, \tag{31}$$

$$|Kx - (Ka_1^2 - K^3)t| \leq \frac{\pi}{2}$$

where a_1 and K are free parameters, and it requires that $2a_1^2 > 3K^2$. If we choose $k = 1, a_1 = 1, b_0 = \sqrt{8/5}$, Eq. (30) becomes

$$u(x, t) = 1 + \frac{3\sqrt{2}}{\cos[x - 3t] + \sqrt{2}}. \tag{32}$$

The periodic solution is illustrated in Fig. 6.

Now we give an heuristical explanation of why Eq. (19) behaves sometimes periodically and sometimes compacton-like.

We re-written Eq. (20) in form

$$u'' + \frac{\omega}{k^3}u + \frac{1}{3k^2}u^3 = 0. \tag{33}$$

It is a well-known Duffing equation with a periodic solution for all $\omega > 0$ and $k > 0$.

Actually in our study ω can be negative, we re-write Eq. (33) in the form

$$u'' - \frac{\omega}{k^3}u + \frac{1}{3k^2}u^3 = 0, \quad \omega > 0. \tag{34}$$

This equation, however, has not always a periodic solution. We use the parameter-expansion method to find its period and the condition to be an oscillator. In order to carry out a straightforward expansion like that in the classical perturbation method, we need to introduce a parameter, λ , because none appear explicitly in this equation. To this end, we seek an expansion in the form

$$u = u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3 + \dots. \tag{35}$$

The parameter λ is used as a bookkeeping device and is set equal to unity.

The coefficients of the linear term and nonlinear term can be, respectively, expanded in a similar way:

$$-\frac{\omega}{k^3} = \Omega^2 + m_1\lambda + m_2\lambda^2 + \dots \tag{36}$$

$$\frac{1}{3k^2} = n_1\lambda + n_2\lambda^2 + \dots, \tag{37}$$

where m_i and n_i are unknown constants to be further determined.

Interpretation of why such expansions work well is given by [1].

Substituting Eqs.(35)–(37) to (34), we have

$$\begin{aligned} & (u_0 + \lambda u_1 + \lambda^2 u_2 + \dots)'' \\ & + (\Omega^2 + m_1 \lambda + m_2 \lambda^2 + \dots) (u_0 + \lambda u_1 + \lambda^2 u_2 + \dots) \\ & + (n_1 \lambda + n_2 \lambda^2 + \dots) \cdot (u_0 + \lambda u_1 + \lambda^2 u_2 + \dots)^3 = 0 \end{aligned} \quad (38)$$

and equating coefficients of like powers of λ , we obtain

Coefficient of λ^0

$$u_0'' + \Omega^2 u_0 = 0. \quad (39)$$

Coefficient of λ^1

$$u_1'' + \Omega^2 u_1 + m_1 u_0 + n_0 u_0^3 = 0. \quad (40)$$

The solution of Eq. (39) is

$$u_0 = A \cos \Omega t. \quad (41)$$

Substituting u_0 into (40) gives

$$\begin{aligned} u_1'' + \Omega^2 u_1 + A(m_1 + \frac{3}{4} n_0 A^2) \cos \Omega t \\ + \frac{1}{4} n_0 A^3 \cos 3\Omega t = 0. \end{aligned} \quad (42)$$

No secular term in u_1 requires that

$$m_1 + \frac{3}{4} n_0 A^2 = 0 \quad \text{or} \quad A = 0. \quad (43)$$

If the first-order approximate solution is searched for, then we have

$$-\frac{\omega}{k^3} = \Omega^2 + m_1 \quad (44)$$

$$\frac{1}{3k^2} = n_1. \quad (45)$$

We finally obtain the following relationship

$$\Omega^2 = -\frac{\omega}{k^3} + \frac{1}{4k^2} A^2. \quad (46)$$

To behave like an oscillator requires that

$$-\frac{\omega}{k^3} + \frac{1}{4k^2} A^2 > 0 \quad (47)$$

or

$$\frac{\omega}{k} < \frac{1}{4} A^2. \quad (48)$$

The amplitude A may strongly depend upon initial/boundary conditions which may determine the wave type of a nonlinear equation.

Now we approximate Eq. (34) in the form

$$u'' + \frac{1}{k^2} \left(-\frac{\omega}{k} + \frac{A^2 \cos^2 \Omega t}{3} \right) u = 0. \quad (49)$$

In case $|\Omega t| \rightarrow \pi/2$, the above equation behaves exponentially, resulting in a compact-like wave as discussed above.

Future Directions

It is interesting to identify the conditions for a nonlinear equation to have solitary, or periodic, or compacton-like solutions. In most open literature, many papers on soliton and compacton are focused themselves on a special solution with either a soliton or a compacton without considering the initial/boundary conditions, which might be vital important for its actual wave type.

Solitons and compactons for difference-differential equations (e.g. Lotka–Volterra-like problems) have been caught much attention due to the fact that discrete space-time may be the most radical and logical viewpoint of reality (refer to E-infinity theory detailed concept). For small scales, e.g., nano scales, the continuum assumption becomes invalid, and difference equations have to be used for space variables.

Fractional differential model is another compromise between the discrete and the continuum, and can best describe solitons and compactons.

Many interesting phenomena arise in nanohydrodynamics recently, such as remarkably excellent thermal and electric conductivity, and extremely extraordinary fast flow in nanotubes. Consider a single compacton wave along a nanotube, and its wavelength is as same as the diameter of the nanotubes, under such a case, almost no energy is lost during the transportation, resulting in extremely extraordinary fast flow in the nanotubes.

The physical understanding of the transformation $k = iK$ is also worth further studying.

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► [Soliton Perturbation](#)

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FRANÇOIS MARIN

Laboratoire Ondes et Milieux Complexes, Fre CNRS
3102, Le Havre Cedex, France

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