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Public debt and full employment in a stock-flow consistent model of a corporate economy

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Abstract

This paper examines the fiscal requirements for continuous full employment. We find that (i) changes in the financial behavior of households and firms require adjustments in tax rates and public debt, (ii) the stability of the steady-state solution for public debt depends on the fiscal instrument and the household consumption function, (iii) in stable cases, a fall in government consumption (or a decline in another component of autonomous demand) requires an increase in the steady-state ratio of public debt to capital, and (iv) the steady-state tax rate may be positively or negatively related to the level of debt.

JEL classification: E62, E22

Key words: Public debt, stock-flow consistency, corporate economy, full employment.

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1 Introduction

This paper examines the fiscal requirements of 'full employment growth'. The analysis is motivated by recent policy debates on public debt, and for present purposes we accept some of the premises underlying this debate. Specifically, full employment is taken as a well-defined maximum level of non-inflationary employment, the growth rate of which is exogenously given. Given this (questionable) premise we disregard inflation and assume the full employment path has price stability.

Fiscal policy has received renewed attention since the financial crisis; the recent literature includes, among others, Arestis and Sawyer (2010), Davidson (2010), Kregel (2010), and Nersisyan and Wray (2010). These studies view the adjustment of government spending and taxes as a primary vehicle for maintaining full employment, and the outstanding balance of public debt evolves as a consequence of the chosen fiscal policy.[1] Public debt, however, is not just a consequence of people's spending decisions and government fiscal policy; the debt also influences the spending decisions, and the dynamic interaction between effective demand and public debt implies a particular trajectory of public debt. This paper aims to clarify the mechanisms behind the interaction and the properties of the trajectory of public debt.

Our analysis has affinities with Skott (2001), Schlicht (2006), Nakatani and Skott (2007) and Godley and Lavoie (2007), but in several respects our framework is richer than these previous studies.[2] Godley and Lavoie (2007) assume away accumulation and financial behavior in the firm sector. Their model, moreover, excludes capital gains, and budget deficits represent the only way in which households can save and increase their wealth, as required in full employment growth. Skott (2001), Schlicht (2006) and Nakatani and Skott (2007) include accumulation. Financial aspects, however, are under-developed in these models, and it is assumed that households own fixed capital directly as one of the assets in their portfolio. In this paper, by contrast, firms accumulate and make financial decisions, and households' ownership of firms takes the form of financial assets, rather than a direct ownership of fixed capital.

The key findings are as follows. First, changes in the financial behavior of households and firms must be met by variations in tax rates and associated movements in the level of the public debt in order to maintain full employment growth. Such changes include shifts in household portfolios and changes in the rates of retention or new equity issues by firms. Secondly, the stability of the steady-growth solution for public debt depends on the specifications of the fiscal policy and the household consumption function. In some specifications, the steady-growth solution for public debt is unambiguously stable; other specifications make stability conditional on parameter values and the details of the

[1] Abba Lerner's theory of functional finance (Lerner, 1943) remains influential in the contemporary post-Keynesian work on fiscal policy and public debt.

[2] The role of public debt in addressing demand problems in the presence of deflation is analyzed by Palley (2010). The analysis is short-run, however, and the level of nominal debt is taken as given.

policy regime. In the latter cases, a high tax rate on interest income exerts a stabilizing influence on the trajectory of public debt. These results explain the differences in the conclusions reached by Schlicht (who emphasizes the stability of the debt dynamics) and Nakatani and Skott (who stress the possibility of instability and the possible long-run limitations of fiscal policy). Thirdly, a permanent fall in autonomous demand – a cut in government consumption, for instance, or a reduction in consumer confidence – requires an increase in the steady-growth ratio of public debt to capital when the debt dynamics is stable. This implies, in particular, that there is an inverse relation between government consumption and public debt. Fourthly, for a given level of government spending, the steady-growth tax rate may be positively or negatively related to the level of debt, depending on whether the effective cost of government borrowing is higher or lower than the natural rate of growth.

The paper is structured as follows. Section 2 sets out our model of a corporate economy. Sections 3 analyzes the fiscal requirements for full-employment growth when the tax on distributed income is used as the policy instrument and there is a uniform saving rate out of distributed income. Section 4 examines the sensitivity of the results to changes in the saving and tax assumptions. Section 5 offers a few concluding comments.

2 A model with financial assets, portfolio choice and corporate saving

Extending the approach in Skott (1988, 1989) and Skott and Ryoo (2008), we consider four sets of agents (firms, households, government and banks) and three financial assets (bank deposits, government bonds and stocks).

Banks are passive: they accept deposits from households and give loans to firms. The interest rates on deposits and loans are identical (r_m), and banking involves neither costs nor profits (neither households nor firms hold cash and the amount of loans is equal to the amount of deposits). The interest rate is taken as an exogenous policy variable and since there is no inflation (cf. above) nominal and real interest rates coincide.

The **government** finances current public consumption (G) and interest payments on the public debt through taxes and the issue of short bonds (B). Tax revenue (T) consists of corporate income tax, wage income tax and household property income tax:

$$T = t_f(\pi Y - r_m M) + t_w(1-\pi)Y + t_p[Div + r_m M + r_b B] \tag{1}$$

where t_f, t_w and t_p are the tax rates on corporate earnings, wage income and property income; π is the profit share, Y output, r_b the real interest rate on bonds, and M the amount of deposits (=loans). Div is household dividend income, which is given by

$$Div = (1-s_f)(1-t_f)(\pi Y - r_m M) \tag{2}$$

where s_f is firms' retention rate out of profits net of interest and tax payments.

In section 3, we consider a benchmark case with a uniform proportional tax rate t on household income ($t = t_w = t_p$); the analysis is generalized to include other tax schemes in section 4.

The government budget constraint is given by

$$\dot{B} = r_b B + G - T \tag{3}$$

where a dot over a variable denotes a time derivative ($\dot{B} = dB/dt$). Plugging (1) into (3) and normalizing by the capital stock K, the budget equation can be rewritten,

$$\begin{aligned} \dot{b} = b(\hat{B} - \hat{K}) &= (r_b - \hat{K})b + g - t_f(\pi u\sigma - r_m m) - t_w(1-\pi)u\sigma \\ &- t_p[(1-s_f)(1-t_f)(\pi u\sigma - r_m m) + r_m m + r_b b] \end{aligned} \tag{4}$$

where $b = B/K$, $g = G/K$, $m = M/K$ and a 'hat' over a variable denotes a growth rate ($\hat{B} = \dot{B}/B$); u is the utilization rate and σ the maximum output capital ratio in a fixed-coefficient production function (see equation (10) below).

Households' portfolio contains bank deposits (=money, M), government bonds (B) and stocks (N). The price of money and short bonds is one, and the price of stocks is v. To simplify, we assume that bonds and money are perfect substitutes; hence, they must yield the same return, $r_b = r_m = r$. Household consumption is denoted as C.

The household budget constraint is given by

$$\begin{aligned} C + \dot{M} + \dot{B} + v\dot{N} &= (1-t_w)(1-\pi)Y \\ &+ (1-t_p)[(1-s_f)(1-t_f)(\pi Y - rM) + r(B+M)] \end{aligned} \tag{5}$$

and the portfolio shares are

$$\frac{vN}{W} = \alpha \tag{6}$$

$$\frac{M+B}{W} = 1-\alpha \tag{7}$$

$W = M+B+vN$ is household wealth. Dividing by K, the portfolio assumptions imply

$$m + b = (1-\alpha)\frac{W}{K} \tag{8}$$

The portfolio composition (α) may depend on the expected relative returns on stocks and fixed-interest assets. The return on stocks includes capital gains, and a positive feedback from past gains to expected gains can be a source of bubbles and instability.[3] The resulting dynamics can be very complex and, given the purposes of the present paper, we treat α as an exogenous variable (noting in a couple of places how induced changes in α could affect the conclusions). Perfect substitution between M and B means that the composition

[3] See e.g. Taylor and Rada (2003), Asada et al. (2010), Ryoo (2010) and Skott (2011).

of households' demand for M and B is indeterminate as long as the two assets have the same return.

The level of consumption, finally, is determined by a traditional consumption function:

$$\begin{aligned}\frac{C}{K} &= c_w(1-t_w)(1-\pi)u\sigma \\ &+ c_p(1-t_p)[(1-s_f)(1-t_f)(\pi u\sigma - rm) + r(m+b)] + c_v\frac{W}{K}\end{aligned} \tag{9}$$

The parameters c_w, c_p and c_v are the propensities to consume out of after-tax wage income, after-tax property income, and wealth. Our benchmark model in section 3 assumes that $c_w = c_p$; the general specification with differential rates is analyzed in section 4.

The production function has fixed coefficients and **firms** set prices as a markup on labor cost. Formally,

$$Y = \min\{L, \sigma K\} \tag{10}$$

$$\pi = \frac{\mu}{1+\mu} \tag{11}$$

where μ denotes the markup, L is employment, and labor productivity is normalized to one through the choice of units.

Firms' investment in fixed capital (I) is financed through retained earnings, new issues of equity and bank loans:

$$I = s_f(1-t_f)(\pi Y - rM) + \dot{M} + v\dot{N} \tag{12}$$

New-equity decisions can be specified in terms of the growth of the number of shares ($\hat{N}$) or the share of investment financed by new issues (ψ). The two specifications are related definitionally,

$$vN\hat{N} = \psi I \tag{13}$$

To simplify the analysis, we follow Eichner (1976) and Wood (1975) and take ψ as constant.

By assumption, the production function has fixed coefficients and, disregarding labor hoarding, a full-employment trajectory must satisfy the conditions

$$n = \hat{L} = \hat{Y} = \hat{u} + \hat{K} \tag{14}$$

where n is the growth rate of the labor force. Utilization rates fluctuate over the business cycle but the fluctuations take place around a fairly stable trend; our focus, moreover, is on full-employment. We therefore assume a constant utilization rate,

$$u = u^* \tag{15}$$

Using (14)-(15), we have

$$\hat{K} = n \tag{16}$$

Equations (15)-(16) are consistent with simple versions of a Harrodian model in which u^* is determined by structural factors and (long-run) accumulation is perfectly elastic at $u = u^*$.[4] Appendix A provides an extension of the model which endogenizes u^*.

Using (13), (15) and (16) and dividing through by K in (12), firms' finance constraint can be written

$$n = s_f(1 - t_f)(\pi u^* \sigma - rm) + m\hat{M} + \psi n \tag{17}$$

Using (17), the evolution of m is given by

$$\begin{aligned} \dot{m} &= m(\hat{M} - n) \\ &= (1 - \psi)n - s_f(1 - t_f)\pi u^* \sigma - [n - s_f(1 - t_f)r]m \end{aligned} \tag{18}$$

and a steady growth path satisfies

$$m = m^* \equiv \frac{n(1 - \psi) - s_f(1 - t_f)\pi u^* \sigma}{n - s_f(1 - t_f)r} \tag{19}$$

The parameter s_f represents the ratio of net retained earnings (retained earnings minus depreciation) to net profits, and empirically this ratio is low (about .25 in the US). Thus, for plausible parameter values the coefficient on m in equation (18) will be negative ($-[n - s_f(1 - t_f)r] < 0$) and the stationary solution in (19) is stable. Most of the analysis in the rest of this paper assumes that m has converged to the stationary value; the exceptions are Appendix A (a generalized version of the model in section 3) and Appendix C (in which the corporate tax rate t_f is used as the fiscal instrument).

Using (15) and (16), finally, product market equilibrium requires

$$\frac{C}{K} = u^* \sigma - n - g \tag{20}$$

3 Benchmark dynamics

3.1 Analytical results

Consider a benchmark case with a uniform tax rate on all distributed income and a uniform propensity to consume out of disposable income. Formally, let

$$t_w = t_p \equiv t \tag{21}$$

$$c_w = c_p \equiv c_u \tag{22}$$

The tax rate on corporate earnings (t_f) and government consumption (g) are taken as exogenous in the benchmark case; the government adjusts the tax rate t on household income to achieve full-employment growth.

[4] The long-run sensitivity of investment to persistent changes in utilization rates is contentious among post-Keynesians. Kaleckian models assume a very low sensitivity; Harrodian models reject this restriction. Dutt 1997 and Hein at el. (2012) defend the Kaleckian position; Skott and Zipperer (2010) and Skott (2012) advocate a Harrodian position.

Putting together equations (8), (9), (15), (16) and (19)-(22) and solving for t, we get

$$t^* = 1 - \frac{u^*\sigma - n - g - \frac{m^*+b}{1-\alpha}c_v}{c_u[(1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^*+b)]} \tag{23}$$

Plugging (15), (16), (19), (21) and (23) into (4), straightforward algebra gives the dynamics of b

$$\begin{aligned} \dot{b} &= b(r-n) + g - t_f(\pi u^*\sigma - rm^*) \\ &\quad -t^*[(1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^*+b)] \\ &= \left(\frac{1-c_u}{c_u}\right)(u^*\sigma - n - g) - \psi n - \left[n + \frac{c_v}{c_u(1-\alpha)}\right](b+m^*) \end{aligned} \tag{24}$$

where the last equality uses firms' budget constraint $s_f(1-t_f)(\pi u^*\sigma - rm^*) = (1-\psi-m^*)n$.

Equation (24) implies an important result. There is a negative feedback from the level of debt and the debt dynamics is unambiguously stable:

$$\frac{d\dot{b}}{db} = -n - \frac{c_v}{c_u(1-\alpha)} < 0 \tag{25}$$

The expression for the stationary value of b can be found from (24).

$$b^* = \frac{\left(\frac{1-c_u}{c_u}\right)(u^*\sigma - n - g) - \psi n}{n + \frac{c_v}{c_u(1-\alpha)}} - m^* \tag{26}$$

where m^* is given by (19).[5]

The consumption-wealth ratio also converges to a stationary value:

$$\begin{aligned} c_W^* &= \frac{C}{W} = \frac{C/K}{W/K} = \frac{(1-\alpha)(u^*\sigma - n - g)}{m^* + b^*} \\ &= \frac{[(1-\alpha)nc_u + c_v](u^*\sigma - n - g)}{(1-c_u)(u^*\sigma - n - g) - c_u n\psi} \end{aligned} \tag{27}$$

Equation (27) can be derived from (8), (20) and (26). The stationary ratio, c_W^*, is attained through the adjustment of consumption and wealth; the stability of the debt dynamics in (24) ensures the convergence of this process. The steady-growth tax rate, finally, can be found by using, (23), (26) and (27). The tax rate is below one for all plausible parameter values.[6]

[5] In order to ensure that household total wealth is positive, we need

$$\left(\frac{1-c_u}{c_u}\right)(u^*\sigma - n - g) - \psi n > 0$$

Empirically, this condition is met. The expression $u^*\sigma - n - g$ represents the consumption-capital ratio and ψ is small; in fact, the rate of new issues has been negative in the US since the 1980s.

[6] The steady-growth tax rate can be written

$$t^* = 1 - \frac{(c_W^* - c_v)(m^*+b^*)/(1-\alpha)}{c_u\left[(1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^*+b^*)\right]}$$

3.2 Discussion

The absence of the real interest rate in the stability condition (25) may seem surprising. It is often argued that the debt dynamics will be stable if $n > r$ and unstable if $n < r$.[7] The $n > r$ condition is derived, however, on the assumption that changes in public debt have no effects on the primary budget balance; in general, this premise will be violated if policy is adjusted to maintain full employment.

An increase in government debt raises household wealth and stimulates consumption. The level of consumption normalized by the capital stock, C/K, must be constant, however, in order to maintain full employment (cf. equation (20)). The stimulus therefore has to be offset by a rise in taxes. Formally, using (8), (9), (20) and (22) we have

$$\frac{C}{K} = c_u \left[\frac{Y_h}{K} - \frac{T_h}{K} \right] + c_v \left(\frac{m^* + b}{1 - \alpha} \right) = u^* \sigma - n - g \tag{28}$$

where $Y_h/K \equiv (1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^* + b)$ is household before-tax income and T_h is the tax on household income. The second term in the second expression on the right-hand-side of (28) is increasing in b (the wealth effect), and a rise in debt therefore must reduce household disposable income, $\frac{Y_h}{K} - \frac{T_h}{K}$. But this is just another way of saying that as b increases, taxes must increase by *more* than the rise in interest payments. Thus, there can be no stability problem. Note, however, that the argument depends on a uniform consumption rate out of disposable income; section 4 examines the debt dynamics in a more general setting.

Our second observation follows directly from equation (26): there is an inverse relation between the required debt and government spending.[8] This result may seem paradoxical but the intuition is simple. A high value of g implies that private consumption must fall as a share of output; this fall is induced by a reduction in public debt (which reduces household interest income and wealth). Note that m^* is determined via firms' finance constraint and is not affected by g.

Thirdly, the adjustment of total wealth W to changes in $B + M$ involves endogenous variations in stock prices. Since $\frac{W}{K} = \frac{m+b}{1-\alpha}$, an increase in b raises household wealth for a given m and α. Households will increase their desired stock holdings ($\frac{vN}{K}$) in proportion to the increase in total wealth, and stock prices (v) change to produce the required adjustment. The adjustment mechanism is also reflected in the response of Tobin' q (q) to variations in b. By

The denominator of the second term in the right-hand side is positive, and the numerator will be positive if $c_W^* > c_v$. Using (27), we have:

$$c_W^* - c_v = \frac{c_u[\{(1-\alpha)n + c_v\}(u^*\sigma - n - g) + c_v n\psi]}{(1-c_u)(u^*\sigma - n - g) - c_u n\psi}$$

This expression is positive for all plausible values of ψ. The same condition ensures that c_W^* is increasing in c_u.

[7] E.g. see Arestis and Saywer (2010), Cecchetti et al. (2010) and Galbraith (2011).

[8] Schlicht (2006) obtains a similar result.

definition $q = \frac{M+vN}{K} = m + \frac{\alpha(m+b)}{1-\alpha}$, and a rise in b must raise q if α and m are given.

Fourthly, it is straightforward to derive the effects of some of the behavioral changes that have been associated with financialization and neoliberalism. A reduction in the retention rate (a fall in s_f) increases firms' indebtedness (m^*) by reducing retained earnings, and the rise in m will require a reduction in b.[9] Analogously, an increase in stock buybacks (a fall in ψ) reduces equity finance and raises firms' indebtedness m. Thus b will fall.[10] [11] Distributional changes also affect debt: an increase in the profit share reduces m^* and raises b^*.

Fifthly, household behavior influences the fiscal requirements. Reduced confidence (an increase in pessimism) may be reflected in a decline in the consumption propensities and/or a flight from equity to safe assets (a fall in α). Both of these changes require an increase in b. Intuitively, a reduction in α decreases household wealth for given b and m. This puts downward pressure on consumption (via the wealth effect) and the government must allow its debt to increase to maintain the required level of consumption; the increased public debt raises interest income and wealth and makes up for the fall in α.

For a given b, sixthly, an increase in the interest rate r increases household interest income and wealth, thereby stimulating consumption.[12] (The increase in wealth is due to an induced increase in firm indebtedness m^*) Thus, a reduction in the steady-growth level of public debt is required to keep consumption at the level required by full-employment steady growth. The effect of changes in r for given α could be offset if the increase in r shifts household portfolios away from stocks (a fall in α). The negative effect of changes in r on b dominates the induced effect if the changes in household portfolios are relatively moderate; large changes in the portfolio composition reverse the effect.

Dynamic adjustments of the tax rate, finally, guide the public debt ratio to its new steady state following an exogenous shock, and the short- and long-run effects on taxes can be different, even qualitatively. Consider an increase in household confidence and a portfolio shift towards stocks, that is, an increase in α. Using equation (23) it follows that in the short run this shift should be met by a rise in taxation: an increase in α raises wealth (for given m, b) as the attempts by households to buy up stock leads to an increase in stock prices and capital gains. The increase in wealth stimulates consumption, and disposable income must be reduced in order to keep consumption at the required level. But

[9] Retained earnings will fall as s_f decreases as long as $\pi u^* \sigma - rm^* > 0$; a reversal of this inequality would undermine the long-run viability of a capitalist system.

[10] The reduction in ψ raises both m and $m + b$. The increase in $m + b$, however, is smaller than the increase in m.

[11] Recent attempts by firms to reduce leverage (reduce m^*) conversely have raised the demand for b. An increase in the perceived riskiness of firm debt would work in the same direction. The absence of corporate bonds and the simplifying assumption of perfect substitutability between deposits and government bonds, however, makes the model ill-suited to analyze this aspect.

[12] We have taken 'the real interest rate' as an exogenous policy variable. This is clearly an oversimplification. A distinction should be made between the federal funds rate and other rates, but central banks do have the ability to influence long rates; the experiments with quantitative and qualitative easing represent steps in this direction.

the rise in taxation has dynamic effects. The impact effect of an increase in α will be counteracted by the ensuing decline in government debt and household wealth (where again changes in stock prices play a role, but now producing capital losses). As a result, the steady-growth effects on taxes can go either way depending on the sign of $(1-t^*)r-n$.[13]

To summarize, the details may be messy but changes in private-sector behavior or monetary policy have implications for fiscal policy. Adjustments in taxes and public debt make it possible to maintain full employment growth, and the debt dynamics is stable in the benchmark model.

4 Robustness and qualifications

The benchmark model assumed a uniform household propensity to consume out of wage and property income, and the fiscal instrument was a uniform tax rate on household income. We now relax these assumptions.

Using the general specification in equations (1) and (9), without the restrictions (21) and (22), the equilibrium condition (20) for the product market can be written

$$\begin{aligned} c_w(1-t_w)(1-\pi)u^*\sigma \quad &+ \quad c_p(1-t_p)[(1-s_f)(1-t_f)(\pi u^*\sigma - rm) + rm + rb] \\ &+ \quad \frac{c_v(m+b)}{1-\alpha} + n + g = u^*\sigma \qquad (29) \end{aligned}$$

This condition for full-employment growth can be satisfied by adjusting the tax rates on wage income (t_w), property income (t_p), or corporate income (t_f), or by changing the level of government consumption (g).

4.1 Unstable scenarios

Consider a fiscal regime in which the tax on wage income is adjusted to maintain full employment. Solving (29) for t_w and using (19), we can rewrite the

[13] Consider the government budget constraint in the steady state.

$$b(r-n) + g - t[(1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^*+b)] - t_f(\pi u^*\sigma - rm^*) = 0$$

Taking total derivatives, we have

$$(r-n)db - trdb - [(1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^*+b)]dt = 0$$

or

$$\frac{dt}{db} = \frac{(1-t)r-n}{(1-\pi)u^*\sigma + (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + r(m^*+b)}$$

The denominator is unambiguously positive and the sign of the derivative depends on $(1-t)r-n$. The steady-state tax rate is positively (negatively) related to the debt ratio if $(1-t)r-n>0$ (if $(1-t)r-n<0$). Since an increase in α reduces b^*, t^* is negatively related to α if $(1-t)r-n>0$, and positively related to α if $(1-t^*)r-n<0$.

government budget equation:

$$\begin{aligned}\dot{b} &= \left(\frac{1-c_w}{c_w}\right)(u^*\sigma - n - g) + \left(\frac{c_w - c_p}{c_w}\right)(1-t_p)(1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) \\ &\quad -\psi n - \left[n + \frac{c_v}{c_w(1-\alpha)} - \left(\frac{c_w - c_p}{c_w}\right)(1-t_p)r\right](m^* + b) \qquad (30)\end{aligned}$$

The debt dynamics is stable if

$$\frac{d\dot{b}}{db} = -\left[n - \left(\frac{c_w - c_p}{c_w}\right)(1-t_p)r + \frac{c_v}{c_w(1-\alpha)}\right] < 0 \qquad (31)$$

This condition need not be satisfied. Consider a simple case without saving from wage income ($c_w = 1$), no wealth effect ($c_v = 0$) and no property income tax ($t_p = 0$). Substituting these assumptions in (31), the stability requirement simplifies to

$$n - (1 - c_p)r > 0 \qquad (32)$$

A combination of high saving rates out of property income and a low natural rate of growth reverses the inequality, leading to an unstable trajectory of public debt. This unstable case was examined by Skott (2001) and Nakatani and Skott (2007).

More generally, the condition (31) is met and the debt dynamics will be stable if the household propensity to consume out of property income is greater than or equal to that of wage income ($c_p \geq c_w$). But the system may lose stability if c_w is large compared to c_p, the property income tax rate t_p is low, the real interest rate r is high, the wealth effect c_v is low and the natural rate of growth is low. Low values of c_p and c_v imply that the effect of an increase in government debt on consumption demand will be weak, requiring only a small reduction in consumption demand out of wage income in order to keep C/K constant. If the propensity to consume out of wage income is large, the required reduction in consumption demand can be achieved by a small increase in the tax rate on wage income. Consequently, the increase in tax revenue may be too small to compensate for the increase in interest on government bonds; the debt rises, and in the unstable case it grows faster than the natural rate.

The comparative statics has limited interest in the unstable case but assuming stability, the steady-growth solution for the debt ratio follows from (30):

$$b^* = \frac{\left(\frac{1-c_w}{c_w}\right)(u^*\sigma - n - g) + \left(\frac{c_w - c_p}{c_w}\right)(1-t_p)\frac{Div}{K} - \psi n}{n + \frac{c_v}{c_w(1-\alpha)} - \left(\frac{c_w - c_p}{c_w}\right)(1-t_p)r} - m^* \qquad (33)$$

where $\frac{Div}{K} \equiv (1-s_f)(1-t_f)(\pi u^*\sigma - rm^*)$. Stability implies that b^* is inversely related to c_p, c_v and α. In general, however, the effects on b of changes in parameters become more complicated than in the previous section.

4.2 A general stability result

The change in the debt ratio can be written

$$\dot{b} = (r - n)b + g - \frac{T}{K} \tag{34}$$

Equation (34) implies a sufficient condition for stability: assuming $n > 0$, stability is ensured if

$$\frac{d\frac{T}{K}}{db} \geq r \tag{35}$$

The consumption-capital ratio must be constant in steady growth – equation (20) – and this has implications for the effects of changes in debt on tax revenue. Let total consumption depend on $k+1$ different categories of disposable income, each with a distinct marginal propensity to consume, as well as on total wealth. We take interest income from government bonds as income category $k+1$. Formally,

$$\frac{C}{K} = \phi\left(\frac{Y_1 - T_1}{K}, ..., \frac{Y_k - T_k}{K}, \frac{rB - T_B}{K}, \frac{W}{K}\right) \tag{36}$$

where Y_i and T_i denote income category i and the taxes levied on this income; $T_i = t_i Y_i$ for $i = 1, ..., k$; $\;Y_{k+1} = Y_B = rB, \;\; T_{k+1} = T_B = t_B rB$. Assuming that the full-employment incomes $Y_1/K, ..., Y_k/K$ are independent of the public debt ratio b, the constancy of C/K implies that

$$\sum_{i=1}^{k} -\phi_i \frac{d\frac{T_i}{K}}{db} + \phi_B\left(r - \frac{d\frac{T_B}{K}}{db}\right) + \phi_v \frac{d\frac{W}{K}}{db} = 0 \tag{37}$$

where ϕ_i is the marginal propensity to consume out of income i; ϕ_v is the marginal consumption out of wealth. The changes in tax revenue are given by

$$\frac{d\frac{T_i}{K}}{db} = \frac{Y_i}{K}\frac{dt_i}{db}, \qquad i = 1, ..., k \tag{38}$$

$$\frac{d\frac{T_B}{K}}{db} = rt_B + rb\frac{dt_B}{db} \tag{39}$$

The government adjusts one or more tax rates to maintain full-employment growth. Let us denote as S the set of income categories with adjustments in tax rates; by definition S is a subset of the index set $\{1, 2, ..., k, B\}$. Now consider two cases. In the first case, S contains interest income on bonds ($B \in S$ and $d\frac{T_i}{K}/db = 0$ for $i \notin S$); in the second case, the tax rate on bond income is kept constant ($B \notin S$; $\frac{dt_B}{db} = d\frac{T_i}{K}/db = 0$ for $i \notin S$ and $i \neq B$). In both cases we assume that $dt_i = dt_j$ for all i, j in S.[14]

[14] A generalization to cases with $dt_i \neq dt_j$ is conceptually straightforward but expositionally messy.

Case 1. Variable tax rate on bond interest

In this case $dt_i = dt_B$ for all $i \in S$, and equation (37) implies that

$$\frac{d(T/K)}{db} = r + \frac{\phi_v}{\phi_B(1-\alpha)} + \left(\frac{\phi_B - \bar{\phi}}{\phi_B}\right) \sum_{i \in S \setminus \{B\}} \frac{d(T_i/K)}{db} \tag{40}$$

where $\bar{\phi}$ is the weighted average propensity to consume out of income categories in $S \setminus \{B\}$, i.e. $\bar{\phi} = \sum_{i \in S \setminus \{B\}} \phi_i \frac{Y_i}{K} / \sum_{i \in S \setminus \{B\}} \frac{Y_i}{K}$. See Appendix B for the derivation of (40).

If $\phi_B \geq \bar{\phi}$, the last term in (40) will be non-negative. Thus the increase in tax revenue in response to a marginal increase in b will be greater than r. Our benchmark case in section 3 provides an example of this stable case: the variable tax rate applied to all income categories and the propensity to consume was uniform ($\phi_B = \bar{\phi} = c_u$). The adjustment of the property income tax rate (t_p) under the consumption specification (29) is another example: in this case dividend, interest on deposits and interest on bonds are taxed at the variable tax and subject to the same consumption propensity ($\phi_B = \bar{\phi} = c_p$). In both examples, the general sufficient condition for stability (35) is satisfied. If $\phi_B < \bar{\phi}$, on the other hand, the increase in tax revenue may fall short of r. Thus the debt dynamics may be unstable.

Case 2. Fixed tax rate on bond interest

In this case, we have – see Appendix B –

$$\frac{d(T/K)}{db} = r + \frac{\phi_v}{\tilde{\phi}(1-\alpha)} + \left(\frac{\phi_B - \tilde{\phi}}{\tilde{\phi}}\right)(1-t_B)r \tag{41}$$

where $\tilde{\phi}$ is the weighted average propensity to consume out of income categories in S, i.e. $\tilde{\phi} = \sum_{i \in S} \phi_i \frac{Y_i}{K} / \sum_{i \in S} \frac{Y_i}{K}$.

Thus, if $\phi_B \geq \tilde{\phi}$, the amount of tax revenue generated by a marginal increase in b will be sufficient to cover r; if $\phi_B < \tilde{\phi}$, on the other hand, the increase in tax revenue may be less than r. The potential instability can be tamed by a high tax rate on bond interest income: if the tax rate on interest income t_B is sufficiently high, (41) must be satisfied. The intuition is simple: if a large fraction of interest payment to bondholders returns to the government as tax revenue, the effective cost of government borrowing will decrease. Variable tax rates on wage income in section 3.1 exemplifies this general result.

Putting together these results, stability is ensured if (i) the propensity to consume out of interest on government bonds is greater than or equal to the propensity to consume out of the income that is subject to variable tax rates, or (ii) interest on bonds is taxed at a sufficiently high rate.

These conclusions – which generalize those in sections 3 and 4.1 – are derived for exogenous income-capital ratios: we assumed that the pre-tax distributed incomes $Y_1/K, ..., Y_k/K$ are independent of changes in debt and the associated fiscal response. The assumption is not as restrictive as it may seem, given our focus on growth paths with continuous full-employment. The pre-tax wage-capital ratio, for instance, is determined by the markup and the utilization rate, both of which are constant along the paths that we consider; a disaggregation of the labor force and total wage income into different subcategories based on skill, sector, race, gender or other criteria could also fit the assumption.

Some fiscal policies violate the exogeneity assumption for the pre-tax incomes Y_i, $i = 1, ..., k$. An example is the use of adjustments in corporate taxes to maintain full employment. Corporate taxes influence retained earnings and distributed incomes; in addition there are indirect effects on the corporate debt ratio m. Changes in the corporate tax therefore have repercussions for households' pre-tax incomes. These effects complicate the analysis but as shown in Appendix C, the qualitative conclusions are similar to those in section 3.

4.3 Variable government consumption

So far we have taken government consumption as exogenous. Now consider the case in which g adjusts to maintain full-employment growth, keeping constant all tax rates. The required level of government consumption can be found by solving (29) for g. We have:

$$\begin{aligned} g^* \quad = \quad & u^*\sigma - n - c_w[(1-t_w)(1-\pi)u^*\sigma] \\ & -c_p(1-t_p)[(1-s_f)(1-t_f)(\pi u^*\sigma - rm^*) + rm^* + rb] - \frac{c_v(m^*+b)}{1-\alpha} \end{aligned} \tag{42}$$

The higher is b, the lower is the required government consumption:

$$\frac{dg^*}{db} = -\left[c_p(1-t_p)r + \frac{c_v}{1-\alpha}\right] < 0 \tag{43}$$

This inverse relationship between g^* and b is conducive to the stability of the trajectory of public debt, but stability is not unconditional. To see this, substitute (42) into the government budget equation and differentiate with respect to b:

$$\frac{d\dot{b}}{db} = r - n + \frac{dg^*}{db} - t_p r = -\left[n - (1-c_p)(1-t_p)r + \frac{c_v}{1-\alpha}\right] \tag{44}$$

The stationary point is stable if

$$n - (1-c_p)(1-t_p)r + \frac{c_v}{(1-\alpha)} > 0 \tag{45}$$

An increase in public debt raises household consumption and thereby reduces the required government consumption, but the reduction need not offset the increase in interest payments. High values of c_p, c_v or α are stabilizing; they imply a large consumption effect from any given increase in b and therefore allow a big reduction in government consumption without violating the condition for full employment. The stability criterion (45) also confirms the important role of tax policy in stabilizing the trajectory of the debt ratio: a sufficiently high tax rate on interest income (t_p) ensures stability.[15]

5 Conclusion

The current obsession with public debt is paradoxical from the perspective of mainstream Ramsey models, in which the financing of any given stream of public spending is largely irrelevant. OLG models point to possible long-run effects of public debt on interest rates and the capital stock, but even the saving assumptions that are at the center of these models are questionable: only the young save, leading to an empirically implausible implication that saving rates will be higher out of wage income than out of capital income.[16] The framework in this paper therefore is very different.

The general lessons from our stock-flow consistent model of a corporate economy are quite simple. Fiscal deficits and a rise in public debt are necessary if the government wants to maintain full employment following a decline in demand. This Keynesian insight remains as valid as ever. Looking beyond the short run, one could ask whether current debt levels and fiscal deficits are 'sustainable'. It is not always clear what is meant by sustainability, but the question may be whether the fiscal requirements for full employment growth will generate an ever-increasing debt-GDP ratio. In the models we have considered, the stability of the trajectory of public debt depends on the specifications of fiscal policies and household behavior, and the case of instability cannot be ruled out. The current obsession with debt and austerity is misguided, however. Fluctuations in private sector confidence and financial behavior can and should be offset by variations in public debt. The remedy against instability is not fiscal austerity. Stability can be ensured if the tax on interest income is used as the active fiscal instrument; alternatively, a constant tax rate on interest income guarantees stability as long as the rate is sufficiently high.

The analysis in this paper has many limitations. Most prominently, perhaps, we have assumed a closed economy. Open (and local) economies are in a very different position than sovereign countries that control their own currency; this paper says nothing about the open-economy issues.

Even in a closed-economy context we have focused on a subset of issues, and a large public debt can have negative effects that we have not considered. The size of the public debt influences the effectiveness of monetary policy. A contractionary monetary policy raises interest rates and generates an automatic fiscal expansion unless it is matched by an increase in tax rates. Thus, monetary policy is blunted and this may complicate short-run economic policy.

Secondly, there may be concerns over distribution. The burden of high debt and high interest does not fall on 'future generations'; by construction the

[15] The condition (45) lies behind the stable debt dynamics in Schlicht (2006, section 4) and Godley and Lavoie (2007).

[16] Skott and Ryoo (2011) analyze public debt in a modified OLG setting.

levels of real resources that are passed on to future generations in the form of fixed capital is kept at the full employment path. But the debt may have regressive distributional effects if taxes on wage income are used to finance interest payments to the rich. These distributional effects indirectly played a role in our analysis: the combination of distributional changes and differential saving rates lies behind the potential instability of the debt dynamics. Thirdly, negative incentive effects of taxes may come into play. These incentive effects are, we believe, greatly exaggerated in most economic analysis. Still, incentive issues need to be addressed.

The fiscal requirements of full employment growth, finally, have been derived for given values of a number of variables, including r and π. One may ask whether other policy instruments can be brought to bear on these variables and thereby reduce the required debt levels. If industrial and distributional policies, for example, are used to lower the profit share, then this leads to a reduction in the required level of public debt in our model of a corporate economy. A discussion of these alternative (or complementary) policies is beyond the scope of the present paper.

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Appendix A: Endogenous variations in u and m

The model in the main text took the portfolio composition α as an exogenous variable and employed a simple Harrodian specification of the investment function. This appendix considers an extension in which the desired utilization rate depends on Tobin's q (as in Skott 1989) while the desired portfolio composition is determined by the relative rates of return. Formally,

$$u^* = f(q, x); \quad f_q < 0, f_x < 0 \tag{46}$$
$$\alpha = \alpha(\rho, z); \quad \alpha_\rho > 0, \alpha_z > 0 \tag{47}$$

where ρ and q are the profit rate and Tobin's q; x and z denote the autonomous element of firms' animal spirits and household 'confidence' (an increase in x shifts the investment function upwards and an increase in z shifts the desired portfolio towards stocks for given relative returns). The definition of Tobin's q and the rate of profits ρ are unchanged. Thus,

$$q = \frac{\alpha(m+b)}{1-\alpha} + m \tag{48}$$
$$\rho = \pi u \sigma \tag{49}$$

Plugging (47)-(50) in (46), we have:

$$u^* = f\left(\frac{\alpha(\pi u\sigma, z)(m+b)}{1-\alpha(\pi u\sigma, z)} + m, x\right) \tag{50}$$

In section 3, we assumed that firms' accumulation was kept constant at n and that the debt-capital ratio m had converged to its stationary value. Let us allow the accumulation rate to vary according to a simple investment function:

$$\frac{I}{K} = \phi(u - u^*); \quad \phi' > 0, \ \phi(0) = n \tag{51}$$

After substituting (50) in (51), we can express I/K as a function of u, m, b, z and x.

$$\frac{I}{K} = \gamma(u, m, b, x, z); \quad \gamma_u > 0, \ \gamma_m > 0, \ \gamma_b > 0 \ , \ \gamma_x > 0, \ \gamma_z > 0 \tag{52}$$

Since government fiscal policies maintain the growth rate of output at the natural rate, we have

$$\dot{u} = (\hat{Y} - \hat{K})u = [n - \gamma(u, m, b, x, z)]u \tag{53}$$

Firm's budget constraint implies

$$\dot{m} = \gamma(u, m, b, x, z)(1 - \psi - m) - s_f(1 - t_f)(\pi u\sigma - rm) \tag{54}$$

Product market equilibrium and the government budget equation yield:

$$\begin{aligned} \dot{b} \quad = \quad & g - t_f(\pi u\sigma - rm) - [(1-\pi)u\sigma + (1 - s_f)(1 - t_f)(\pi u\sigma - rm) + rm] \\ & + \frac{u\sigma - \gamma(u, m, b, x, z) - g}{c_u} - \frac{c_v}{c_u[1 - \alpha(\pi u\sigma, z)]} m \\ & - \left[\gamma(u, m, b, x, z) + \frac{c_v}{c_u(1 - \alpha(\pi u\sigma, z))}\right] b \end{aligned} \tag{55}$$

From (53)-(55), the steady-growth solution for b is given by

$$b^* = \frac{\left(\frac{1-c_u}{c_u}\right)(u^*\sigma - n - g) - \psi n}{n + \frac{c_v}{c_u(1-\alpha)}} - m^* \tag{56}$$

where m^* and u^* are given by (19) and (50), respectively. Substituting (56) back into (50), we have:

$$u^* = f\left(\frac{\alpha(\pi u^*\sigma, z)[(1 - c_u)(u^*\sigma - n - g) - c_u\psi n]}{[1 - \alpha(\pi u^*\sigma, z)]c_u n + c_v} + m^*, x\right) \tag{57}$$

If f_q is small, the implicit function theorem ensures the existence of a function that maps parameter values into the utilization rate u^*.

$$u^* = \chi(x, z, g, c_u, c_v, s_f, \psi) \tag{58}$$

Substituting (58) in (49) and then in (47), we obtain the portfolio share as a function of parameters.[17]

$$\alpha^* = \alpha(\pi\chi(x, z, g, c_u, c_v, s_f, \psi)\sigma, z) \tag{59}$$

By plugging (58) and (59) back into (56), we can eliminate u^* and α and express the steady-growth solution for the public debt ratio as a function of the parameters.

The comparative statics is complicated but it can be shown that if f_q is sufficiently small, the effect of g, c_v, and c_u on b remains negative and the effect of s_f and ψ is positive, as in the case of constant α and u^*. It can also be shown that the effect of z on b will be negative; the effect of x on b, on the other hand, is ambiguous.

[17] The existence of an economically meaningful solution for u^* in $(0, 1]$ and for α^* in (0,1) requires restrictions on the related functions.

For the analysis of stability, let us denote as J the Jacobian matrix of the system (53)-(55) evaluated at the stationary point and as J_i a submatrix of J obtained by deleting the ith row and column of J. A sufficiently small value of f_q ensures the local stability of the steady-growth path (if it exists). To see this, note that as $f_q \to 0$, we have:

$$\begin{aligned} Tr(J) &\to -\left[\gamma_u u^* + n - s_f(1-t_f)r + n + \frac{c_v}{c_u(1-\alpha^*)}\right] < 0 \\ \sum_{i=1}^{3} Det(J_i) &\to \frac{c_v}{c_u(1-\alpha^*)}[\gamma_u u^* + n - s_f(1-t_f)r] \\ &\quad +[n - s_f(1-t_f)r](\gamma_u u^* + n) + \gamma_u u^* n > 0 \\ Det(J) &\to -\gamma_u u^*[n - s_f(1-t_f)r]\left[n + \frac{c_v}{c_u(1-\alpha^*)}\right] < 0 \end{aligned}$$

and

$$-Tr(J)\sum_{i=1}^{3} Det(J_i) + Det(J) \to \left[(n - s_f(1-t_f)r) + n + \frac{c_v}{c_u(1-\alpha^*)}\right]$$

$$\times[n - s_f(1-t_f)r + \gamma_u u^*]\left[\gamma_u u^* + n + \frac{c_v}{c_u(1-\alpha^*)}\right] > 0$$

Since the trace and the determinants involved are polynomials of f_q (and therefore continuous functions of f_q), there exists a range of f_q in the neighborhood of zero, where the Routh-Hurwitz condition for local stability will be satisfied. In other words, a sufficiently small value of f_q ensures the local stability of the system (53)-(55)

Appendix B: Derivation of general stability conditions

Case 1. $B \in S$

Equation (37) can be rewritten:

$$\begin{aligned} &\sum_{i \in S\setminus B} -\phi_i \frac{d\frac{T_i}{K}}{db} + \phi_B\left(r - \frac{d\frac{T_B}{K}}{db}\right) + \frac{\phi_v}{1-\alpha} \\ &\quad = \sum_{i \in S\setminus B} (\phi_B - \phi_i)\frac{d\frac{T_i}{K}}{db} + \phi_B\left(r - \frac{d\frac{T}{K}}{db}\right) + \frac{\phi_v}{1-\alpha} = 0 \qquad (60) \end{aligned}$$

Because $\frac{d(T_i/K)}{db} = \frac{Y_i}{K}\frac{dt_B}{db}$ for $i \in S\backslash\{B\}$, we have

$$\begin{aligned}
\sum_{i\in S\backslash B}(\phi_B - \phi_i)\frac{d\frac{T_i}{K}}{db} &= \sum_{i\in S\backslash B}(\phi_B - \phi_i)\frac{Y_i}{K}\frac{dt_B}{db} \\
&= \left(\phi_B - \frac{\sum_{i\in S\backslash B}\phi_i\frac{Y_i}{K}}{\sum_{i\in S\backslash B}\frac{Y_i}{K}}\right)\sum_{i\in S\backslash B}\frac{Y_i}{K}\frac{dt_B}{db} \\
&= (\phi_B - \bar{\phi})\sum_{i\in S\backslash B}\frac{d(T_i/K)}{db} \qquad (61)
\end{aligned}$$

Substituting (61) into (60) and solving for $\frac{d(T/K)}{db}$, we obtain (40).

Case 2. $B \notin S$

From equation (37)-(39) with $dt_B/db = 0$, we have:

$$-\sum_{i\in S}\phi_i\frac{d\frac{T_i}{K}}{db} + \phi_B(1-t_B)r + \frac{\phi_v}{1-\alpha} = 0 \qquad (62)$$

By assumption $\frac{dt_i}{db} = \frac{dt_j}{db}$ for i,j in S. Using $\frac{dt_S}{db}$ to denote this common value, we have

$$\sum_{i\in S}\phi_i\frac{d\frac{T_i}{K}}{db} = \sum_{i\in S}\phi_i\frac{Y_i}{K}\frac{dt_S}{db} \qquad (63)$$

Plugging (63) into (62) and solving for $\frac{dt_S}{db}$,

$$\frac{dt_S}{db} = \left(\sum_{i\in S}\phi_i\frac{Y_i}{K}\right)^{-1}\left[\phi_B(1-t_B)r + \frac{\phi_v}{1-\alpha}\right] \qquad (64)$$

Using (64), we can write

$$\begin{aligned}
\frac{d(T/K)}{db} &= \sum_{i\in S}\frac{d(T_i/K)}{db} + \frac{d(T_B/K)}{db} = \sum_{i\in S}\frac{Y_i}{K}\frac{dt_S}{db} + t_B r \\
&= \left(\frac{\sum_{i\in S}\phi_i\frac{Y_i}{K}}{\sum_{i\in S}\frac{Y_i}{K}}\right)^{-1}\left[\phi_B(1-t_B)r + \frac{\phi_v}{1-\alpha}\right] + t_B r \\
&= \left[\frac{\phi_B}{\tilde{\phi}}(1-t_B)r + \frac{\phi_v}{\tilde{\phi}(1-\alpha)}\right] + t_B r \\
&= r + \frac{\phi_v}{\tilde{\phi}(1-\alpha)} + \left(\frac{\phi_B - \tilde{\phi}}{\tilde{\phi}}\right)(1-t_B)r \qquad (65)
\end{aligned}$$

Appendix C: Corporate taxes as the fiscal instrument

In this scenario the corporate tax rate, t_f, is adjusted to satisfy (29). Firms' budget constraint is given by

$$\dot{m} = (1 - \psi - m)n - s_f(1 - t_f)(\pi u^* \sigma - rm) \tag{66}$$

Thus, the tax rate on corporate income (t_f) affects the dynamics of firm debt m as well as that of public debt b. In addition, changes in t_f have direct effects on household dividend income $(1 - s_f)(1 - t_f)(\pi u^* \sigma - rm)$, thereby affecting consumption demand. Not surprisingly, therefore, the use of the corporate income tax rate as the tax instrument complicates the analysis.

A clearer picture can be obtained by focusing on the sum of private and public debt, $m + b$. Adding (66) to (4) and rearranging terms, we have:

$$\begin{aligned} \dot{m} + \dot{b} &= [n + g] + (1 - t_p)[(1 - s_f)(1 - t_f)(\pi u^* \sigma - rm) + r(m + b)] \\ &- t_w(1 - \pi)u^* \sigma - [\psi n + \pi u^* \sigma] - n(m + b) \end{aligned} \tag{67}$$

The change in the debt of the consolidated firm/public sector depends in their primary spending $n + g$ (the first term on the right-hand side of (67)); the payment of dividend and interest to the household sector net of property income tax revenue (the second term);. taxes on wage income (the third term); and new equity issues and corporate gross profits (the fourth term). The positive growth of capital, finally, helps stabilize the total debt-capital ratio (the fifth term).

The adjustment of t_f by the government is required to keep C/K constant at $u^* \sigma - n - g$. An increase in the total debt ratio $(m + b)$ raises interest income and wealth, which in turn stimulates consumption for a given t_f. To keep C/K constant, the tax rate on corporate income income must be raised to reduce dividend income. We have

$$\frac{d(C/K)}{d(m + b)} = c_p(1 - t_p)\left[\frac{d(Div/K)}{d(m + b)} + r\right] + \frac{c_v}{1 - \alpha} = 0. \tag{68}$$

Thus,

$$\frac{d(Div/K)}{d(m + b)} = -\left[r + \frac{c_v}{c_p(1 - t_p)(1 - \alpha)}\right] < -r \qquad \text{if } c_v > 0 \tag{69}$$

The required reduction in dividend income will more than offset the increase in interest income as long as there is a positive wealth effect ($c_v > 0$); this stabilizes the dynamics of the total debt ratio in (67). The stability of total debt $m + b$, in turn, ensures the convergence of dividend income, $(1 - s_f)(1 - t_f)(\pi u^* \sigma - rm)$, and retained earnings $s_f(1 - t_f)(\pi u^* \sigma - rm)$; this result follows from equation (29) which implies that $(1 - t_f)(\pi u^* \sigma - rm)$ is uniquely determined by $m + b$. From these observations, finally, it follows (using (18)) that firms' debt-capital ratio m must converge; the public debt ratio therefore also converges.

The stationary solutions for m and b are complicated functions of the various parameters. The complications, however, do not invalidate the general point about the role of public debt: in order to maintain a full-employment steady growth path, the adjustment in public debt must keep *total* household wealth at a desired level. Using (66) and (67), the sum of the stationary values – households' total safe assets – is given by

$$b^* + m^* = \frac{\left(\frac{1-c_p}{c_p}\right)(u^*\sigma - n - g) - \left(\frac{c_w - c_p}{c_p}\right)(1-t_w)(1-\pi)u^*\sigma - \psi n}{n + \frac{c_v}{c_p(1-\alpha)}} \tag{70}$$