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Bayesian Approaches to Assessing Architecture and Stopping Rule

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Bayesian Analyses of the Survivor Interaction Contrast

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Psychological and Brain Sciences
Cognitive Science Program

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Outline

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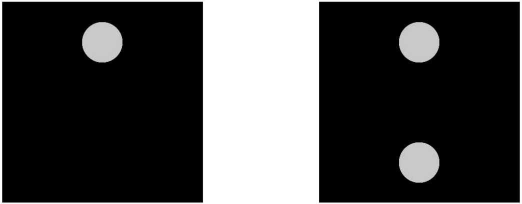
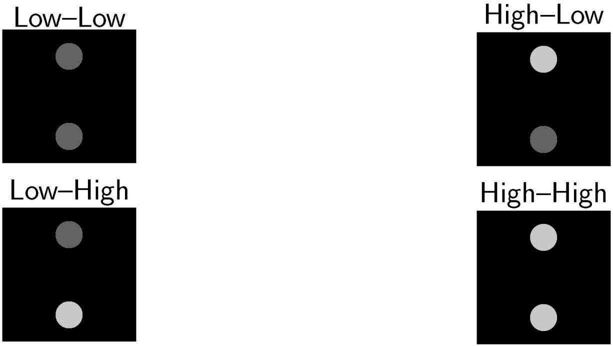


- How do different sources of information combine in mental processing?
    - Are both sources used concurrently, or do we use one at a time?
    - How many sources are enough to respond?

Salience

- To test architecture and stopping rule, without conflating them with workload capacity, factorially speed up and slow down the processing of each source of information.



Survivor Interaction Contrast

- Indicates architecture and stopping rule.

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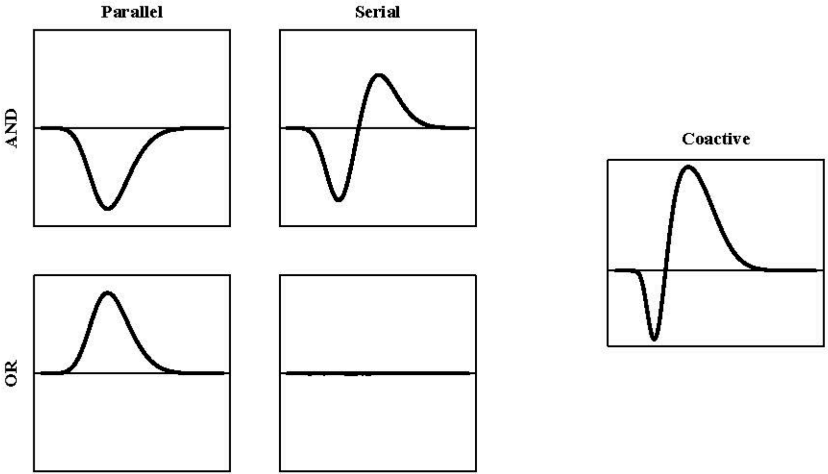
- Indicates architecture and stopping rule.
- The SIC is interaction between the salience manipulations.
  - Instead of just using the mean time, we use the survivor function: $S(t) = \Pr\{T > t\} = 1 - F(t)$.

$$\mathrm{SIC}(t) = [\mathrm{S}_{\mathrm{LL}}(t) - \mathrm{S}_{\mathrm{LH}}(t)] - [\mathrm{S}_{\mathrm{HL}}(t) - \mathrm{S}_{\mathrm{HH}}(t)]$$

Here, the subscripts indicate the salience of each source of information.

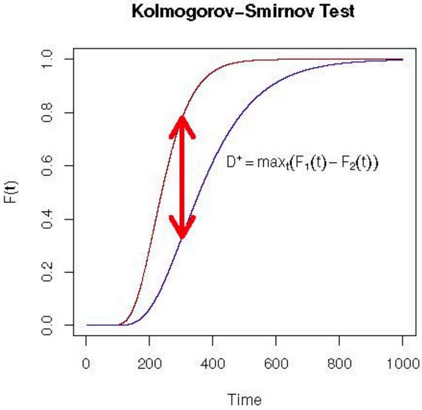
Survivor Interaction Contrast

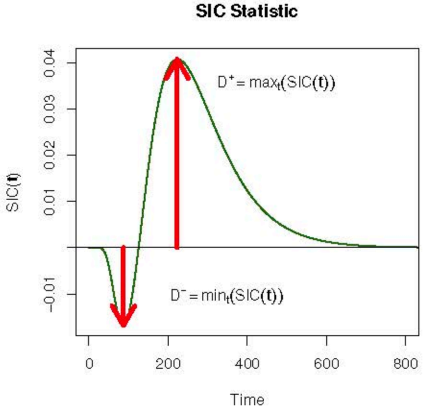


Townsend & Nozawa (1995)
Schweickert, Giorgini & Dzhafarov (2000)

Dzhafarov, Schweickert & Sung (2004)
Houpt & Townsend (2011)

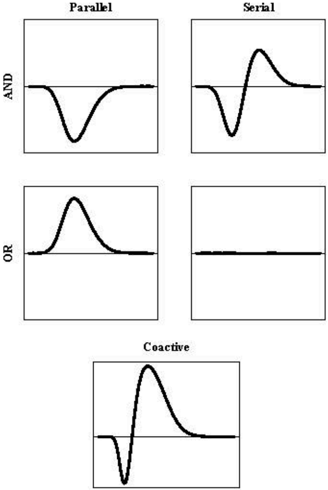
Null Hypothesis Test





$$\lim_{N\to\infty} \Pr\{\sqrt{N}D^{+} \geq x\} = \Pr\{\sqrt{N}D^{-} \geq x\} = e^{-2x^2}$$

$$N_{\mathrm{KS}} = \frac{1}{1/m + 1/n} \qquad N_{\mathrm{SIC}} = \frac{1}{1/k + 1/l + 1/m + 1/n}$$



Model	$\hat{D}^{+}$	$\hat{D}^{-}$	Mean Interaction
Serial-OR	$\emptyset$	$\emptyset$	$\emptyset$
Serial-AND	$\checkmark$	$\checkmark$	$\emptyset$
Parallel-OR	$\checkmark$	$\emptyset$	$\checkmark$
Parallel-AND	$\emptyset$	$\checkmark$	$\checkmark$
Coactive	$\checkmark$	$\checkmark$	$\checkmark$
$\checkmark$: Reject null hypothesis			
$\emptyset$: Fail to reject null hypothesis			

Shortcomings

- Tests positive and negative deflections *not* SIC form.
  - Requires two separate tests.
- Only can gain evidence against a lack of positive or negative deflection.
- Only get a yes/no answer, not relative evidence.

Outline

$f(t)$: Density (PDF) $F(t)$: Cumulative Distribution (CDF)

Parallel-OR $f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$

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Parallel-OR $f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$

Parallel-AND $f_{12}(t) = f_1(t)F_2(t) + f_2(t)F_1(t)$

$f(t)$: Density (PDF) $F(t)$: Cumulative Distribution (CDF)

Parallel-OR	$f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$
Parallel-AND	$f_{12}(t) = f_1(t)F_2(t) + f_2(t)F_1(t)$
Serial-OR	$f_{12}(t) = pf_1(t) + (1 - p)f_2(t)$

$f(t)$: Density (PDF) $\qquad$ $F(t)$: Cumulative Distribution (CDF)

Parallel-OR	$f_{12}(t) = f_1(t)[1 - F_2(t)] + f_2(t)[1 - F_1(t)]$
Parallel-AND	$f_{12}(t) = f_1(t)F_2(t) + f_2(t)F_1(t)$
Serial-OR	$f_{12}(t) = pf_1(t) + (1 - p)f_2(t)$
Serial-AND	$f_{12}(t) = f_1(t) * f_2(t)$

$$T_{i;H} \sim \mathcal{IG}\left(\frac{\alpha}{\nu_H}, \alpha^2\right)$$

$$T_{i;L} \sim \mathcal{IG}\left(\frac{\alpha}{\nu_L}, \alpha^2\right)$$

$$\alpha \sim \Gamma(4, 0.1)$$

$$\eta \sim \mathrm{Exponential}(100)$$

$$\nu_L \sim \Gamma(4, 0.1)$$

$$\nu_H = \nu_l + \eta$$

$$T_{i;H} \sim \mathcal{IG}\left(\frac{\alpha}{\nu_H}, \alpha^2\right) \qquad \eta \sim \text{Exponential}(100)$$

$$T_{i;L} \sim \mathcal{IG}\left(\frac{\alpha}{\nu_L}, \alpha^2\right) \qquad \nu_L \sim \Gamma(4, 0.1)$$

$$\alpha \sim \Gamma(4, 0.1) \qquad \nu_H = \nu_l + \eta$$

$$f_i(t; \nu_i, \alpha) = \sqrt{\frac{\alpha^2}{2\pi t^3}} \exp\left[\frac{-(t\nu_i - \alpha)^2}{2t}\right]$$

$$F_i(t; \nu_i, \alpha) = \Phi\left[\sqrt{\frac{\alpha^2}{t}}\left(\frac{t\nu_i}{\alpha} - 1\right)\right] + \exp\left[2\alpha\nu_i\right]\Phi\left[-\sqrt{\frac{\alpha^2}{t}}\left(\frac{t\nu_i}{\alpha} + 1\right)\right]$$

Simulation Parameters

$$T_i = \inf\{t : X_i(t) \geq \alpha\}$$
$$T_i \sim \mathcal{IG}\left(\frac{\alpha}{\nu_i}, \frac{\alpha}{\sigma^2}\right)$$

$$\alpha = 30 \qquad \nu_H = 0.3$$
$$\sigma^2 = 1 \qquad \nu_L = 0.1$$
$$p = 0.5$$

Simulation Results

	Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial-OR	1.00	0	0	0	0
Serial-AND	0	0.99	0	0.01	0
Parallel-OR	0	0	0.98	0	0.02
Parallel-AND	0	0	0	1.00	0
Coactive	0	0	0	0	1.00

Outline

- Approach: Model the response time *distributions*
  - (as opposed to the RT generating process).
- Assume each RT distribution is an independent sample from a Dirichlet process prior.
- Compare the Bayes factor of each SIC form in the posterior relative to encompassing prior.

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$$\alpha_l \sim \mathcal{DP}(\beta)$$
$$\mathrm{RT}_{l(i)} \sim \alpha_l.$$

Simulation

- Tested on same models as parametric-Bayesian test (but with 1000 rounds rather than 100).
  - Used region of probabilistic equivalence $\pm.1$ for SIC and $\pm.3$ for MIC.

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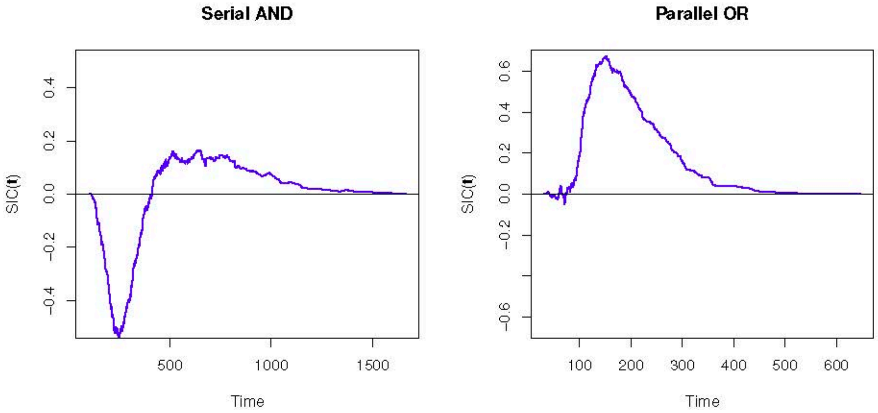
	Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial OR	1.00	0	0	0	0
Serial AND	0	0.79	0	0.21	0
Parallel OR	0	0	0.93	0	0.07
Parallel AND	0	0	0	1.00	0
Coactive	0	0	0	0	1.00

Example SICs

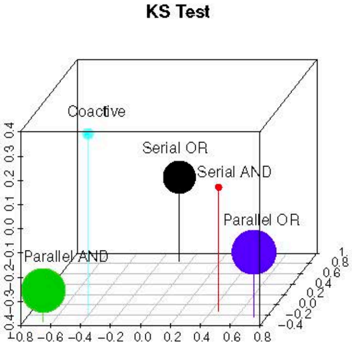


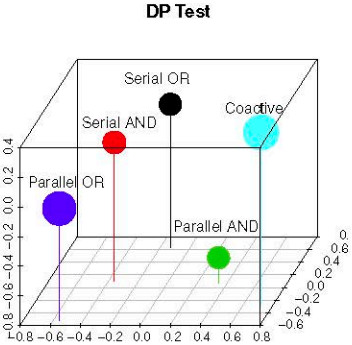
Outline

1. Introduction
2. Parametric Test
   - Model
   - Simulation
3. Nonparametric Test
   - Model
   - Simulation
4. Comparisons Among SIC Tests
   - Simulation
   - Application
5. Conclusion

		Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial OR	KS	0.96	0	0	0.04	0
	DP	1.00	0	0	0	0
	BUGS	1.00	0	0	0	0
Serial AND	KS	0	0.80	0	0.15	0.05
	DP	0	0.79	0	0.21	0
	BUGS	0	0.99	0	0.01	0
Parallel OR	KS	0	0	1.00	0	0
	DP	0	0	0.93	0	0.07
	BUGS	0	0	0.98	0	0.02
Parallel AND	KS	0	0	0	1.00	0
	DP	0	0	0	1.00	0
	BUGS	0	0	0	1.00	0
Coactive	KS	0	0	0.02	0	0.98
	DP	0	0	0	0	1.00
	BUGS	0	0	0	0	1.00

		Serial OR	Serial AND	Parallel OR	Parallel AND	Coactive
Serial OR	KS	0.93	0	0.05	0.02	0
	DP	0.79	0.18	0.02	0.01	0
Serial AND	KS	0	0.41	0	0.56	0.03
	DP	0	0.77	0	0.23	0
Parallel OR	KS	0	0	1.00	0	0
	DP	0	0	0.79	0	0.21
Parallel AND	KS	0	0	0	1.00	0
	DP	0	0.04	0	0.96	0
Coactive	KS	0	0	0.50	0	0.50
	DP	0	0	0	0	1.00


KS Test
Coactive
Serial OR
Serial AND
Parallel OR
Parallel AND
−0.4 −0.3 −0.2 −0.1 0.0 0.1 0.2 0.3 0.4
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4 0.6 0.8
−0.4 −0.2 0.0 0.2 0.4 0.6 0.8 1


DP Test
Serial OR
Serial AND
Coactive
Parallel OR
Parallel AND
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4
−0.8 −0.6 −0.4 −0.2 0.0 0.2 0.4 0.6 0.8
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6 0.

KS Test

Participant	OR Task $\sqrt{N}\hat{D}^{+}$	OR Task $\sqrt{N}\hat{D}^{-}$	AND Task $\sqrt{N}\hat{D}^{+}$	AND Task $\sqrt{N}\hat{D}^{-}$
1	4.86***	0.11	0	4.65***
2	1.11	0.04	0.04	2.73***
3	4.87***	0.14	0	3.61***
4	2.12***	0.77	0.07	3.30***
5	2.59***	0.22	0.21	4.24***
6	3.52***	0.04	0.16	2.79***
7	1.44*	0.11	0.04	2.04***
8	3.64***	0.24	0.11	2.10***
9	3.86***	0.07	0.07	4.98***

Parametric Bayes

	OR Task				
	Serial		Parallel		
	OR	AND	OR	AND	Coactive
1	7991	7985	**7869**	8012	7964
2	8489	8489	**8394**	8486	8488
3	7831	7792	**7623**	7920	7746
4	9480	9504	9530	**9464**	9505
5	9347	9351	**9274**	9352	9335
6	8870	8875	8885	**8830**	8867
7	9210	9216	**9192**	9201	9214
8	8624	8636	**8531**	8638	8620
9	8830	8850	**8828**	8837	8837

Parametric Bayes

	AND Task				
	Serial		Parallel		
	OR	AND	OR	AND	Coactive
1	7861	7863	7872	**7817**	7890
2	7832	7833	**7791**	7871	7836
3	7246	7249	**7242**	7297	7265
4	8883	8880	8922	**8789**	8890
5	9390	9370	**9350**	9360	9380
6	7434	7426	7441	**7374**	7426
7	7853	7857	**7815**	7858	7861
8	8272	8269	**8229**	8250	8273
9	8011	7998	**7968**	8009	8010

Nonparametric Bayes

	OR Task					
	Serial		Parallel			
	OR	AND	OR	AND	Coactive	Np
1	1	0.17	**7.26**	0	0.05	0
2	**160**	2.57	7.24	0.03	0.15	0.02
3	1	0.20	**6.98**	0	0.31	0
4	1	0.12	**3.19**	0	0	0
5	1	0.25	**7.02**	0	0.70	0
6	1	0.25	**7.45**	0	0	0
7	**72**	0.29	7.25	0	0.01	0
8	1	0.25	**7.19**	0	0.13	0
9	1	0.25	**7.22**	0	0.01	0

Nonparametric Bayes

	AND Task					
	Serial		Parallel			
	OR	AND	OR	AND	Coactive	Np
1	1	0.50	0	**7.41**	0	0
2	1	0.25	0	**7.51**	0	0
3	1	0.17	0	**7.69**	0	0
4	1	0.50	0	**7.26**	0	0
5	1	1.00	0	**7.36**	0	0
6	1	0.17	0	**7.37**	0	0.24
7	1	0.50	0	**7.22**	0	0.04
8	1	0.25	0	**7.37**	0	0.48
9	1	0.50	0	**7.31**	0	0

Outline

Overview

- Developed parametric and nonparametic Bayesian tests for architecture and stopping rule.
- Tested each of these approaches on both simulated data and experimental data.
    - Both did quite well on simulated data.
    - Parametric conclusions diverged from NHST and nonparametric tests on human data.

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    - Nonparametric: Continuous (smooth) distributions in the prior.
    - Hierarchical models.

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Thank you.