October, 2010

Continuing discussion of mathematical habits of mind

Annie Selden, *New Mexico State University*
Kien H Lim, *University of Texas at El Paso*

CONTINUING DISCUSSION OF MATHEMATICAL HABITS OF MIND

Annie Selden
New Mexico State University
js9484@usit.net

Kien Lim
University of Texas at El Paso
kienlim@utep.edu

The idea of "mathematical habits of mind" has been introduced to emphasize the need to help students think about mathematics "the way mathematicians do." There seems to be considerable interest among mathematics educators and mathematicians in helping students develop mathematical habits of mind. The objectives of this working group are: (a) to continue the discussion of various views and aspects of mathematical habits of mind begun at PME-NA 31, (b) to explore avenues for research, (c) to encourage research collaborations, and (d) to interest doctoral students in this topic. In the Proceedings of PME-NA 31, we provided an overview of mathematical habits of mind, including concepts that are closely related to habits of mind—ways of thinking, mathematical practices, knowing-to act in the moment, cognitive disposition, and behavioral schemas. Below we provide a summary of the discussions held at PME-NA 31. We invite returning participants, as well as other mathematics educators who are interested in mathematical habits of mind, especially those who have conducted research related to habits of mind, to our discussions.

An Overview of Mathematical Habits of Mind

The term *habits of mind* was introduced by Cuoco, Goldenberg, and Mark (1996) as an organizing principle for mathematics curricula in which high-school students and college students think about mathematics the way mathematicians do. Lim and Selden (2009) highlighted two key attributes of habits of mind: the *habitual* characteristic and the *thinking* characteristic. The habitual nature of habits of mind was underscored in Goldenberg's (1996) description of habits of mind, which "one acquires so well, makes so natural, and incorporates so fully into one's repertoire, that they become mental habits—one not only can draw upon them easily, but one is likely to do so" (p. 13). Mason and Spence's (1999) notion of *knowing-to act in the moment* accentuates this habitual character. They have differentiated between two types of knowledge. The first type, referred to as *knowing-about*, consists of Ryle's (1949, cited in Mason & Spence) three classes of knowledge: *knowing-that* (factual knowledge), *knowing-how* (procedural skills), and *knowing-why* (personal stories to account for phenomena). The second type, referred to as *knowing-to*, is tacit knowledge that is context/situation dependent and becomes present in the moment when it is required. This distinction is important because "knowing to act when the moment comes requires more than having accumulated knowledge-about . . ." (Mason & Spence, 1999, p. 135).

> *Knowing-about … forms the heart of institutionalized education: students can learn and be tested on it. But success in examinations gives little indication of whether that knowledge can be used or called upon when required, which is the essence of knowing-to* (Mason & Spence, p. 138).

Mason and Spence advocate the practice of reflection as a means to help students improve their knowing-to act in the moment. Students should be encouraged to reflect on (a) what they have done after an action, and (b) what they are doing while enacting it, which were termed by

Schön (1983) *reflection-on-action* and *reflection-in-action* respectively. With respect to reflection-in-action, students should routinely ask themselves “What do I know?” and “What do I want?” (Mason & Spence, p. 154).

The thinking characteristic differentiates habits of mind from behavioral habits such as knuckle cracking and nail biting. Costa and Kallick (2000) identified sixteen habits of mind that can and should be cultivated in schools. Habits of minds in Costa and Kallick’s list that are related to mathematical thinking and learning include persisting, managing impulsivity, thinking flexibly, metacognition, striving for accuracy, and thinking and communicating with clarity and precision. Cuoco (1996) differentiates mathematical habits of mind, such as talking big thinking small, talking small thinking big, thinking in terms of functions, and mixing deduction and experiment, from more general ones like pattern-sniffing, experimenting, formulating, tinkering, inventing, visualizing, and conjecturing. Harel’s (2008) distinction between ways of thinking and ways of understanding highlights two complementary subsets of mathematics: the former refers to conceptual tools or mathematical habits of mind that are used for creating the latter, which refers to collections of institutionalized definitions, theorems, proofs, problems, and solutions. These two aspects of mathematics are analogous to the two types of standards—process and content—outlined in the NCTM’s (2000) *Principles and Standards for School Mathematics*.

According to Levasseur and Cuoco (2003), mathematical habits of mind should not “be the explicit objects of our teaching, rather, each student should internalize them as they do math” (p. 34). Mathematical habits of mind can be fostered by providing students opportunities to engage in authentic mathematical activities such as modeling and realistic problem solving (see Lesh & Doerr, 2003; Schoenfeld, 1985). The National Council of Teachers of Mathematics published two books on *Teaching Mathematics through Problem Solving*; one for Grades PreK-6 and the other for Grades 6-12. There is a chapter on mathematical habits of mind in each book: (a) Goldenberg, Shteingold, and Feurzeig (2003) discuss five habits of mind that are particularly relevant to elementary grade levels; they include thinking about word meaning, justifying claims and proving conjectures, distinguishing between agreement and logical necessity, analyzing answers, problems, and methods, and seeking and using heuristics to solve problems; and (b) Levasseur and Cuoco (2003) discuss how secondary school students can acquire habits of mind such as guessing, challenging solutions, looking for patterns, conserving memory, using alternative representations, thinking algebraically, and classifying carefully. Driscoll and colleagues (1999, 2007) have created professional development materials to foster algebraic thinking and geometric thinking. Their goal is to help teachers and their students develop algebraic habits of mind such as doing/undoing, building rules to represent functions, abstracting from computation, reasoning with relationships, generalizing geometric ideas, investigating invariants, and sustaining reasoned exploration by trying different approaches and stepping back to reflect. Lewis (2008, January) commented that their professional development project was designed to help teachers develop “habits of mind of a mathematical thinker” and in turn foster these habits in their students. Rasmussen (2009, January) emphasized the need for teachers to be deliberate about initiating and sustaining particular classroom norms so as to promote certain desirable habits of mind and effect students’ beliefs and values.

The *Standards for Mathematical Practice* in the Common Core State Standards in Mathematics (CCSSI, 2010) highlight the following habits of minds that mathematics educators at all levels should seek to develop in their students:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

A collection of lists of habits of mind from various sources, including CCSSI (2010), can be downloaded from http://works.bepress.com/kien_lim/19/. More information on mathematical habits of mind can be found in the Mathematical Habits of Mind Working Group paper in the *Proceedings of PME-NA 31* (Lim & Selden, 2009) and on the website: http://www.math.utep.edu/Faculty/kienlim/mhom.

The working group on mathematical habits of mind met for the first time in September 2009 at PME-NA 31. There were three meetings. We began with individual presentations on research related to habits of mind. We then had an open forum to discuss theoretical and pedagogical issues related to this topic followed by small subgroup breakout sessions. We concluded the meeting by having reports from the various subgroups. To facilitate the continuation of this working group at PME-NA 32, we summarize below the information gathered during the previous working group sessions.

Individual Presentations at the PME-NA 31 Working Group

The working group began with six 10-minute presentations. Below are the title, name of the presenter(s), and a short summary for each presentation, listed in the order they were presented.

Mathematical Habits of Mind for Preservice Elementary Teachers
Richard S. Millman, Georgia Institute of Technology

The concept of mathematical habits of mind urges preservice teachers to use, in their teaching, ideas such as posing questions (e.g. "Is there a different way to think about this problem?" and "What is there that I am not seeing?"), seeking possible generalizations, considering the necessity and use of careful definitions, and dealing with the vagueness of open-ended questions. These ideas are aimed at having preservice elementary teachers understand how mathematicians might think. In the presentation, Millman described how mathematical habits of mind could be introduced in a mathematics content course for future elementary teachers through their inclusion in the textbook, *Mathematical Reasoning for Elementary Teachers,5th Edition* (Long, DeTemple, & Millman, 2009). He provided some reactions of pre-service teachers, as well as those of eleven reviewers of the text (six of whom used the textbook and five of whom didn't), concerning the inclusion and emphasis of mathematical habits of mind in the textbook.

Undesirable Habits of Mind of Preservice Teachers
Kien H. Lim, University of Texas at El Paso

Many preservice K-8 teachers enter college with undesirable habits of mind such as (a) spontaneously proceeding with the first action that comes to mind without analyzing the problem situation, and (b) not attending to meaning of numbers and symbols. Such habits of mind can negatively impact what and how they learn mathematics. For example, students often tend to focus on procedures for solving problems rather than on underlying mathematical structures. In the presentation, Lim offered several strategies to help prospective teachers address their undesirable habits of mind. One strategy is to pose problems for which a recently learned idea will not work, and thereby present students with a need to investigate the principles that underlie that idea. Another strategy is to emphasize the need for understanding the quantities embedded in a problem situation and how these quantities are related. A study was conducted to investigate the viability of using nonproportional missing-value problems to address students' tendency to overgeneralize proportional approaches (see Lim & Morera, 2010). An instrument, called the likelihood-to-act survey, has been developed to assess students' problem-solving disposition along the impulsive-analytic disposition (see Lim, Morera, & Tchoshanov, 2009).

Transforming pre-service teachers' dispositions towards mathematics through reflection in-activity and post-activity

Dionne I. Cross, Indiana University-Bloomington

Mathematics education is still undergoing a transition from a transmission view of instruction to one that involves students actively engaging in "doing" mathematics. One way to address this problem is by working with pre-service teachers to begin transforming their ideas about mathematics and mathematics learning. Using examples, the transformation of the teachers' approaches to non-traditional problems were described—*from* initially being unable to solve problems that do not have a singular solution or problems that prioritized thinking and reasoning *to* making generalizations and aligning algebraic notation with specific aspects of a problem. Students' development of these habits of mind (Driscoll, 1999) were attributed to focused efforts to teach students (a) how to engage in reflection both during an activity and following the activity (Schön, 1983), and (b) how to articulate their thoughts both verbally and in writing.

Habits of Mind in the Proving Process

Annie Selden & John Selden, New Mexico State University

The Seldens view the proving process as a sequence of mental or physical actions that cannot be fully reconstructed from the final written proof. Such actions often appear to be due to the enactment of small, automated situation-action pairs that they call *behavioral schemas* (Selden & Selden, 2008; Selden, McKee, & Selden, 2010). A common beneficial behavioral schema consists of a situation where one has to prove a universally quantified statement like, "For all real numbers x, $P(x)$" and the action is writing into the proof something like, "Let x be a real number," meaning x is arbitrary but fixed. Focusing on such behavioral schemas, that is., small habits of mind, has two advantages. First, the uses, interactions, and origins of behavioral schemas are relatively easy to examine. Second, this perspective is not only explanatory but also suggests concrete teaching actions, such as the use of practice to encourage the formation of beneficial schemas and the elimination of detrimental ones.

Mathematics Immersion and Educators' Habits of Mind: Preliminary Results from Two Programs

Karen Graham & Todd Abel, University of New Hampshire

Mathematics immersion is a form professional development where educators are encouraged to work through unfamiliar mathematical content in ways that simulate the activities and practices of mathematicians. Graham and Abel briefly described two such programs that they have examined. The first program was a one-week summer institute, participated in by 18 faculty members and 7 graduate students. Analysis of participant journals and follow-up surveys uncovered the following themes: (a) freedom to experiment, conjecture, and guess, (b) value in using multiple points of view, and (c) joy in doing mathematics. The second was a two-summer professional development program participated in by 50 high-school and middle-school teachers. Analysis of pre-and-post interviews with six participants revealed the following themes: (a) the motivational role of pattern-sniffing, (b) the importance of mixing deduction and experiment, and (c) the importance of classroom practice (e.g., the usefulness of activities for classroom work). Mathematics immersion increased participants' awareness of, but did seem to impact participants' use of, habits of mind.

Richard Lesh, Indiana University-Bloomington

Do students develop rigid and unchanging profiles of habits, dispositions, and attitudes? Or, do productive problem solvers manipulate their own profiles to suit circumstances? Evidence was presented to show that (a) productive-but-implicitly-functioning habits of mind can be developed using reflection activities similar to those used by athletes and performing artists; (b) students can develop more powerful ways of seeing (or interpreting) their own problem solving experiences; (c) both learning and application of ideas and processes develop synchronously during mathematical model-development activities; and (d) the productivity of relevant processes, beliefs, dispositions, and habits of mind vary across time. Productive students can learn to manipulate their own profiles to suite circumstances. This research is based on models and modeling perspectives of mathematical problem solving, learning, and teaching

PowerPoint slides or write-ups for the above presentations can be downloaded at http://www.math.utep.edu/Faculty/kienlim/mhom. In addition, other presentations on this topic at the 2008 and the 2009 Joint Mathematics Meetings are posted at http://www.math.utep.edu/Faculty/kienlim/hom. Presenters at those sessions included Hyman Bass, Al Cuoco, Paul Goldenberg, Guershon Harel, Kien Lim, Chris Rasmussen, Annie Selden and John Selden.

Questions Raised During the Brainstorming Session

A number of ideas and questions emerged during a brainstorming session by the working group last year. These ideas and questions can be grouped into four broad categories: epistemology, cognition, pedagogy, and research.

Epistemology-related questions. What do we, or should we, mean by mathematical habits of mind? What is meant by a mathematical disposition? Can we get to a common understanding of what we mean by mathematical habits of mind? What does it mean to think mathematically? How do mathematicians' (whether pure, applied, or statisticians) views of mathematical habits of mind differ from those of mathematics teachers? What are engineers' views of mathematical habits of mind? With the introduction of technology (e.g., computers and calculators), are there new mathematical habits of mind that it would be beneficial to acquire? Are there special mathematical habits of mind that are useful when using internet resources?

Cognition-related questions. How are mathematical habits of mind different from beliefs, metacognitive strategies, or problem-solving strategies? Is curiosity a habit of mind? Is being meticulous a habit of mind? What is not a habit of mind? What are some tacitly functioning mathematical habits of mind? How have mathematicians acquired mathematical habits of mind? Is it adaptive or useful for students to think like mathematicians? Are there negative mathematical habits of mind? What are they? What impact do positive (or negative) mathematical habits of mind have on students' problem solving ability?

Pedagogy-related questions. Are there progressive stages to developing mathematical habits of mind? What are some impediments or obstacles to acquiring positive mathematical habits of mind? Can you teach positive mathematical habits of mind to students? If so, how? Is there teacher-to-student transfer or professor-to-teacher-to-student transfer of mathematical habits of mind? How do teachers' mathematical habits of mind affect their teaching? What are some ways of assessing the acquisition of positive mathematical habits of mind?

Research-related questions. What sorts of theoretical frameworks could one use in researching mathematical habits of mind? Are mathematical habits of mind culturally influenced? Is their acquisition a matter of enculturation into the larger mathematical culture? What is the role of language in acquiring mathematical habits of mind? In general, how would one conduct research on mathematical habits of mind?

PME-NA 31 Working Subgroup Discussions

The larger working group at PME-NA 31 broke-out into four subgroups to discuss the following topics: (a) Defining habits of mind and the role of language in developing mathematical habits of mind; (b) How does one help teachers or students to develop mathematical habits of mind? (c) Small mathematical habits of mind (behavioral schemas) in proving and problem-solving, including tacit habits of mind; and (d) Negative mathematical habits of mind.

Defining mathematical habits of mind. The first subgroup came up with the following definitions and characteristics of habits of mind which they summarized for the entire working group. Habits of mind are automatic mental processes in response to stimuli that can produce behavior. There is a hierarchy, or spectrum, of habits of mind from low level to high level. An example of a low-level of habit of mind is helplessness (e.g., the response of "show me how to do it") often exhibited by novices. A middle-level habit of mind might be the ability to go through several problem-solving strategies, often exhibited by those developing expert habits of mind. A high-level of habit of mind includes reflecting, monitoring, and questioning, as is often exhibited by experts.

Encouraging mathematical habits of mind in students. The second subgroup took as their starting point Cuoco, Goldenberg, and Mark's (1996) view that "Much more important than specific mathematical results are the habits of mind used by the people who created those results." The subgroup also considered mathematical habits of mind to be "the methods by which mathematics is created, and the techniques used by researchers." They considered the following aspects of, and questions related to, mathematical habits of mind: Justification and defense of results to others in the community. Are there mathematical versus non-mathematical habits of mind? Are mathematical habits of mind dependent on personal experience? Is there some aspect of enculturation? How do the habits of mind used by researchers in the creation of new mathematical results develop? For possible research questions, they suggested: (a) By what processes does a person develop the habits of mind that are typical of mathematicians? (b) How does a person become enculturated into mathematical practices? (c) How do young children develop early mathematical habits of mind?

Small, possibly tacit, mathematical habits of mind. The third subgroup considered the question: Why does it matter to have a small, possibly tacit, habit of mind? They viewed a small, possibly tacit, habit of mind as having two components: (a) the interpretation component, of which one is aware, and (b) the execution, or doing, component, which is often automatic and of which one is usually not aware. The interpretation of the situation is the key, or important, part of the habit. Having an automated habit doesn't take up much working memory, so one can concentrate on other things.

This subgroup made some basic assumptions: (a) Habits of mind develop over time (as there must be a time when one did not have a particular habit); and (b) People have profiles, or constellations of related habits of mind. The following research question was proposed: How do such small, possibly tacit, habits of mind, or other habits of mind, develop?

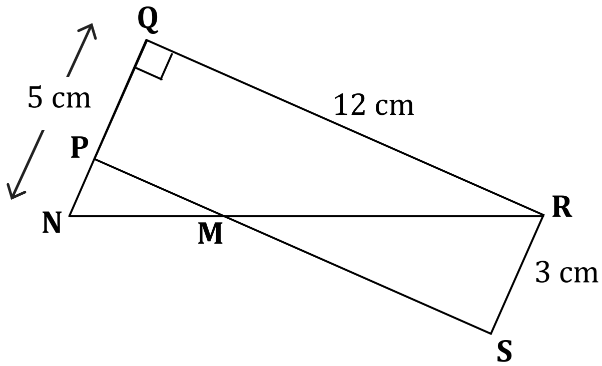
The subgroup also considered whether there were habits of mind with, and without, understanding the underlying habit. Is doing so a mathematical habit of mind? It was observed by one of the group members that he had not found a habit of mind that was not sometimes counterproductive.

The group made the following observations: (a) The tacit part of some small habits of mind are absolutely critical; (b) People can have habits of mind, and other habits, and not know they have them. For example, some people walk in a certain way. Other people know this and can recognize them from their walk, but they aren't aware of the way they walk; and (c) From problem solving studies, it has been observed, for example, that the observer can say a student is drawing a picture, but that's what the observer thinks is going on. However, the student may say that's not what he/she was doing. If one questions the student, he/she may say, "I was trying to figure out what was going on [not drawing a picture]."

Dick Lesh reported a recent study conducted with two groups of students. One group watched a PBS program, *Cyberquest*, about problem-solving teams. Both groups of students were given two problems to solve at the start and two problems to solve at the end. The researchers were given a list of things to notice, such as the role of individuals (the leader, etc.); group functioning, data gathering and data processing. Most of what they observed could be considered habits of mind. At the start, both groups of children had the same number of habits of mind. At the end, both groups invoked the same habits of mind, but the students who watched the program got the problems correct. Further, there was no correlation between the number of processes (habits) and whether the students got the problems right. So what was the difference between the two groups? The group that got the problems correct did them (the habits of mind) at the "right time" and for the "right reasons."

Negative habits of mind. This subgroup considered mainly negative instances of spontaneous, or impulsive, disposition or tendency to act. For example, when a student is asked to solve $(x+3)(x-4)=0$ and automatically multiplies the left hand side. It seems that students are doing what is familiar to them, that they are programmed to react to certain patterns in certain manners, and that this might be the result of certain instructional environments. Such spontaneous dispositions may be limiting to, or block, cognitive growth.

Another example considered was geometrical. The figure below shows a right triangle NQR and a rectangle PQRS; NQ = 5 cm, QR = 12 cm, and RS = 3 cm. When asked to find the length PM, a student may begin by using the Pythagorean theorem to find the length NR. Such a student is said to be impulsive because he or she spontaneously applied the first idea that came to mind without checking its appropriateness.



This subgroup generated the following research questions. (a) How can we detect spontaneous dispositions in situations when the responses are correct responses? (b) What situations/tasks elicit a spontaneous disposition? (c) Is a spontaneous disposition visually influenced? What is the case for blind people? And (d) Can eye tracking collect useful information that can help us understand a student's impulsive disposition?

Plans for the Working Group at PME-NA 32

At the request of the 2009 participants of the Working Group on Mathematical Habits of Mind, we do not intend to have formal presentations. Instead, during the first session we will determine the interests of the participants and formulate an agenda for subsequent sessions. We hope to make progress in this second working group by accomplishing the following:

- Come up with an operational definition that is useful for research purposes.
- Identify important issues/questions and develop research agendas to address/answer them.
- Form collaborations to conduct research.
- Discuss the usefulness and viability of having a support group for those who are conducting research on mathematical habits of mind, and explore the interest in and feasibility of having a special issue of a research journal dedicated to mathematical habits of mind.

Participants with similar interests will team up to discuss theoretical, pedagogical, and/or research issues related to mathematical habits of mind. These teams will report back their discussions and findings to the entire working group at the final working group session. We will conclude the working group with a discussion of next steps.

To facilitate communication and discussion among individuals who are interested in mathematical habits of mind, we have created a site at http://habitsofmind.ning.com/. Anyone can register and be a member of this professional network.

References

Costa, A. L., & Kallick, B. (2000). *Discovering and exploring habits of mind*. Alexandria, VA: Association for Supervision & Currculum Development.

Common Core State Standards Initiative (CCSSI) (2010). *Common Core State Standards in Mathematics*. Retrieved June 2, 2010, from http://www.corestandards.org/the-standards/mathematics

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for a mathematics curriculum. *Journal of Mathematical Behavior, 15*(4), 375-402.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, Grades 6-10.* Portsmouth, NH Heinemann.

Driscoll, M., DiMatteo, R. W., Nikula, J. E., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers grades 5-10*. Portsmouth, NH: Heinemann.

Goldenberg, P. (1996). Habits of mind as an organizer for the curriculum. *Journal of Education, 178* (1), 13-34.

Goldenberg, E. P., Shteingold, N., & Feurzeig, N. (2003). Mathematical habits of mind for young children. In F. K. Lester, Jr. & Charles, R. I. (Eds.), *Teaching mathematics through problem solving: Prekindergarten-Grade 6* (pp. 15-29). Reston, VA: National Council of Teachers of Mathematics.

Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. In B. Gold & Simons, R. (Eds.), *Current issues in the philosophy of mathematics from the perspective of mathematicians*. Washington, DC: Mathematical American Association.

Lesh, R., & Doeer, H. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh, & Doerr, H. (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 3-34). Mahwah, NJ: Earlbaum.

Levasseur, K., & Cuoco, A. (2003). Mathematical habits of mind. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving: Grades 6-12* (pp. 23-37). Reston, VA: National Council of Teachers of Mathematics.

Lim, K. H., & Morera, O. (2010). Addressing impulsive disposition: Using non-proportional problems to overcome overgeneralization of proportionality. *Proceedings of the 13th Annual Conference Research on Undergraduate Mathematics Education. Raleigh, NC.*

Lim, K. H., Morera, O., & Tchoshanov, M. (2009). Assessing problem-solving dispositions: Likelihood-to-act survey. In S. L. Swars, Stinson, D. W. & Lemons-Smith, S. (Eds.), *Proceedings of the Thirty-first Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 700-708). Atlanta: Georgia State University.

Lim, K. H., & Selden, A. (2009). Mathematical habits of mind. In S. L. Swars, Stinson, D. W., & Lemons-Smith, S. (Eds.), *Proceedings of the Thirty-first Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1576-1583). Atlanta: Georgia State University.

Long, C. T., DeTemple, D. W., & Millman, R. (2009). *Mathematical reasoning for elementary teachers,* 5th Ed. Upper Saddle River, NJ: Pearson Education.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38*, 135-161.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Rasmussen, C. (2009, January). *Mathematical proficiency: An example from the inquiry oriented differential equations project*. Paper presented at an AMS-MAA-MER Special Session on Mathematics and Education Reform, Joint Mathematics Meetings, Washington, DC.
Ryle, G. (1949). *The concept of mind*. London: Hutchison.
Selden, A., McKee, K., & Selden, J. (2010). Affect, behavioural schemas and the proving process. *International Journal of Mathematical Education in Science and Technology, 41*(2), 199-215.
Selden, J, & Selden, A. (2008). Consciousness in enacting procedural knowledge. *Proceedings of the 11th Annual Conference on Research on Mathematics Undergraduate Mathematics Education.* Retrieved from http://rume.org/crume2008/Proceedings/Selden_Procedural%20LONG.pdf.
Schoenfeld, A. (1985). *Mathematical problem solving*. New York, NY: Academic Press.
Schön, D. (1983). *The reflective practitioner: How professionals think in action*. London: Temple Smith.