Available online at www.sciencedirect.com

SCIENCE DIRECT®

Chaos, Solitons and Fractals 28 (2006) 285–289

CHAOS
SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Application of *E*-infinity theory to biology

Ji-Huan He

College of Science, Donghua University, 1882 Yan-an Xilu Road, Shanghai 200051, China

Accepted 8 August 2005

Abstract

Albert Einstein combined continuous space and time into his special relativity, El-Naschie discovered the transfinite discontinuity of space–time in his *E*-infinity theory where infinity of dimensions was created. We find a partner of both space–time and *E*-infinity in biology. In our theory, the number of cells in an organism endows an additional dimension in biology, leading to explanation of many complex phenomena.

In recent years there has been a flurry of original papers published on the foundation and application of El-Naschie's *E*-infinity Cantorian spacetime theory [1–9]. The main application of *E*-infinity theory shows miraculous exactness compared to experimental measurements, especially in determining theoretically coupling constants and the mass spectrum of the standard model of elementary particles [5,7–9]. In this letter we will apply *E*-infinity theory to biology for the first time.

In the theory of *n*-dimensional spaces, what we mean by *n*-dimensional is simply that we need *n* numbers representing *n* coordinates to fix the position of a point in this space. In our classical space time, these are the familiar triple x, y and z; while in relativity we have a fourth coordinate or dimension, namely the time t. The formal dimension in *E*-infinity theory, however, is

$$D_F = \infty. \quad (1)$$

The physical understanding of Eq. (1) in biology will be discussed later. The topological dimension in *E*-infinity theory reads

$$D_T = 4. \quad (2)$$

The dimension $3 + 1 = 4$ means that at our observation level, the biological world appears to us as if it were four-dimensional, where the number of cells in an organ is endowed with an additional dimension [10–13]

$$n \propto r. \quad (3)$$

Here n is the number of the organ; and r is the average radius of the cells in the organ. We call this dimension *life dimension* [12]. This is very similar to Einstein's theory where time is considered a dimension. It is a well-known fact that time

E-mail address: jhhe@dhu.edu.cn

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2005.08.001

enters into the metric with a negative sign. In our theory, the dimension of life enters into the manifold with random cell construction like that of El Naschie's Chaos-like quantum world. Life characteristic appears only in $3+1$ dimensional space-number manifold. If a cell is isolated from the heart of a rat, or heart cells are cultivated on a plane ($D=2$), no life dimension is involved. Therefore, these cells have no life functional characteristic. However, if sufficient number of heart cells are cultivated and accumulated together in three dimensions, the isolated cardiac cell begins to beat [14], and life dimension appears.

One of the miraculous hallmarks of the introduced space-number manifold is that it provides us with a powerful mathematical tool in dealing with complex phenomena in biology. As an intuitive and heuristical example, we consider the metabolic rate B of an organ, which scales linearly with respect to its total surface of the cells in the organ

$$B \propto nr^2 \propto r^3. \tag{4}$$

And the mass M of the organ scales linearly with respect to its total volume of cells

$$M \propto nr^3 \propto r^4. \tag{5}$$

From the scaling relations (4) and (5), we obtain immediately the well-known Kleiber's 3/4 allometric scaling law [15–19], which reads

$$B \propto M^{3/4}. \tag{6}$$

In 1977, Blum [20] suggested that the 3/4-law can be understood by a four-dimensional approach. In $D+1$ dimensions, the "area" A of the hypersurface enclosing a $D+1$ dimensional hypervolume scales like $A \propto V^{D/(D+1)}$, where D is the spatial dimension of the organism. When $D=3$, we have $A \propto V^{3/4}$, a four-dimensional construction.

Now we consider a leaf of a plant, a two-dimensional construction. We have proved that [10,12]

$$n \propto r^0. \tag{7}$$

That means that life dimension is not a necessity in the leaf, just like time dimension is not required in Newton's three-dimensional Universal where time is a mere flying parameter. Similarly one of the hallmarks of the two-dimensional organs is that number of cells is mere parameter, not a dimension. The output of photosynthesis of a leaf depends upon its total number of cells, but the life characteristic of the two-dimensional organs does not seriously depend upon the number of cells involved. This is the reason why a destroyed leaf can still make photosynthesis, and can replace the dead cells, whereas the adult human brain cannot replace lost neurons [21].

In view of the scaling relation (7), the metabolic rate of a leaf and its mass scale as

$$B_{\text{leaf}} \propto nr^2 \propto r^2, \tag{8}$$

$$M \propto nr^3 \propto r^3. \tag{9}$$

We, therefore, obtain the well-known Rubner's 2/3 allometric scaling law [10,12,22], which reads

$$B_{\text{leaf}} \propto M^{2/3}. \tag{10}$$

Generally for D dimensional organs, it follows [10] the scaling

$$B \propto M^{D/(D+1)}. \tag{11}$$

It is a mystery that a liver in mammals acts like a leaf of plant which scales like $B_{\text{liver}} \propto M^{2/3}$. The liver contains 50,000–100,000 individual lobules. The liver lobule is constructed around a central vein. The lobule itself is composed of many ***slice***-like hepatic cellular plates like many leaves piled up, each hepatic plate being one- or two-cell thick. Therefore, the liver obeys the same scaling law as a leaf of plant (see Fig. 1), and it is of two-dimensional construction ($D=2$). Accordingly the liver can be transplanted like a plant [12].

In E-infinity view [1–3,7–9], space–time is an infinite dimensional fractal that happens to have $D=4$ as the expectation value for the topological dimension. Similarly in biology, $3+1$ dimension means that the prediction (6) is valid macroscopically. $D=4$ is expected to be the expectation value for the topological dimension of the discussed organ. Consequently the number of dimension in biology may appear to be 4, 5, 10 or 26 dimensions and that is not a contradiction—it all depends on the scale with which we are making our observations. This innocent sounding message has major repercussions and consequences for understanding physics and biology in general and memory in particle which we will be discussing in this letter.

Now we write down the Hausdorff dimension in E-infinity theory [1–3,7–9]:

$$D_H = \left\langle \operatorname*{Dim}_H \varepsilon^{(\infty)} \right\rangle = 4 + \phi^3 = 4 + \cfrac{1}{4 + \cfrac{1}{4 + \ddots}} = 4.23606\cdots, \tag{12}$$

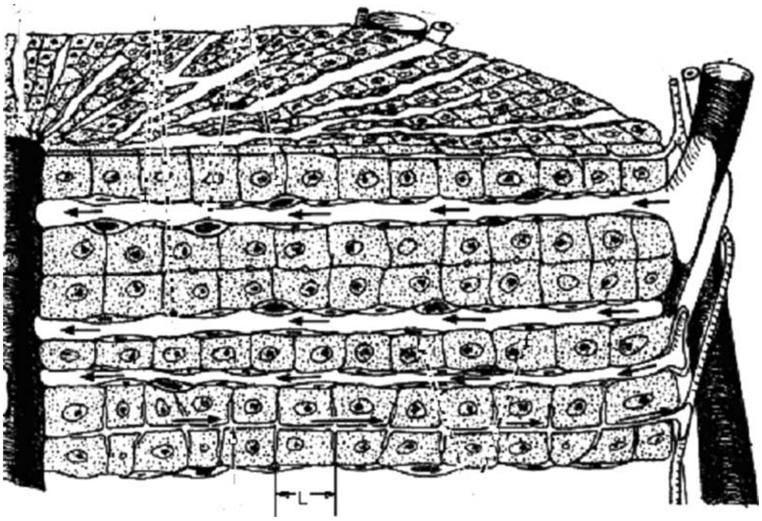

Fig. 1. Diagram showing the structure of liver lobule, which is composed of many ***slice***-like hepatic cellular plates, each hepatic plate being one- to two-cell thick, so the liver of two-dimensional construction.

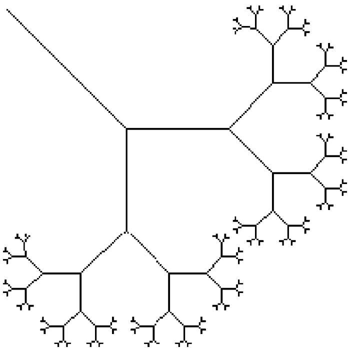

Fig. 2. Fractal geometry of the blood-vessel system.

where $\phi = (\sqrt{5}-1)/2$ is the golden mean. Like pi [23], ϕ is a very particular number, and it plays an important role in E-infinity theory.

Recently fractal analysis of blood vessel has been described [16,24,25]. Geometry of biological structure can be found everywhere in biology. Mammalian circulatory and respiratory systems are composed of branching tubes, as illustrated in Fig. 2. We sum an infinite number of Hausdorff-fractal dimension of one dimensional Cantor sets, with Hausdorff dimension $(d_c^{(0)})^n$, where $n = 0, 1, 2, \ldots$

Consequently we have

$$\sum_{i=0}^{\infty} (d_c^{(0)})^n = 1 + (d_c^{(0)})^1 + (d_c^{(0)})^2 + \cdots. \tag{13}$$

Here we assume that the length of mother vessel is equal to 1, and $(d_c^{(0)})^n$ is the length of nth children vessel. The expectation value for this sum in terms of $d_c^{(0)}$ is

$$\langle d_c \rangle = \sum_{i=0}^{\infty} (d_c^{(0)})^n / d_c^{(0)} = \frac{1}{d_c^{(0)}(1-d_c^{(0)})}. \tag{14}$$

E-infinity theory shows that a randomly constructed triadic Cantor set has a Hausdorff-fractal dimension equal to $d_c^{(0)} = \phi$. Inserting the result in (14), we have

$$\langle d_c \rangle = \frac{1}{\phi(1-\phi)} = \frac{1}{\phi^3} = 4 + \phi^3, \quad (15)$$

exactly as anticipated by E-infinity theory [8]. This formula already implies that $n_F = \infty$ in biology. This is because we have been summing over $n = 0, \ldots, \infty$ and this is the formal dimension of E-infinity.

Consequently we modify (8) to the form

$$B \propto M^{(D_H-1)/D_H}. \quad (16)$$

The 3/4-Kleiber law, therefore, becomes

$$B \propto M^{0.76}. \quad (17)$$

E-infinity allows us to use different dimensions at different levels to optimally describe intractable problems in biology. One of the hallmarks of cortical functional architecture is that maps vary continuously across the cortical surface, but are punctuated by occasional jumps or discontinuities [26]. According to E-infinity, we need infinite dimensions to completely describe the intricate activities in human brain. However, to model pain or memory, we should study in dendrites level, leading to five dimensions of human brain [12]:

$$B_{\text{brain}} \propto M_{\text{brain}}^{4/5}. \quad (18)$$

Noticing that Einstein's special relativity forbids traveling faster than the speed of light, so Newton's law becomes invalid when speed is near to the speed of light. In biology, animals cannot exceed their threshold speeds, though the values of the threshold speeds might be different from species to species. Our previous predictions are actually valid only for the case when an animal is in rest or moves at a lower speed than its threshold speed. When an animal exercises intensely, "time dimension" should be introduced like in Einstein's theory. Combining the time dimension into our space-number manifold, we can modify our scaling laws (11) and (16) in the forms

$$B_{\text{motion}} \propto M^{(D+1)/(D+2)}, \quad (19)$$

or

$$B_{\text{motion}} \propto M^{D_H/(D_H+1)}. \quad (20)$$

Our prediction shows that $B_{\text{motion}} \propto M^{0.8}$ or $B_{\text{motion}} \propto M^{0.81}$, which is very close to Taylor's data [27]: $B_{\text{motion}} \propto M^{0.86}$. Space–time-number manifold is also suitable for the heart. Our prediction for heart is $B_{\text{heart}} \propto M^{0.81}$, while the experimental observation [28] shows $B_{\text{heart}} \propto M^{0.87}$.

Considering the case $D = \infty$, from (11) we have $B \propto M$. Such a case arises in body expansion [29] and for all trophic groups [30].

We like to stress that E-infinity theory is more of a framework for understanding complex phenomena in biology than just a new equation. Of course we have obtained some totally novel equations and results but this is not the main point. The main point is stressing the fact that everything we see or measure in biology is resolution dependent. El-Naschie's E-infinity theory may turn out to have considerable implications in biology in the near future.

References

[1] El Naschie MS. On a fuzzy Kähler-like manifold which is consistent with the two slit experiment. Int J Nonlinear Sci Numer Simul 2005;6(2):95–8.

[2] El Naschie MS. Transfinite electrical networks, spinoral varieties and gravity Q bits. Int J Nonlinear Sci Numer Simul 2004;5(3):191–7.

[3] El Naschie MS. Einstein in a complex time—some very personal thoughts about E-infinity theory and modern physics. Int J Nonlinear Sci Numer Simul 2005;6(3):331–3.

[4] Goldfain E. Complex dynamics and the high-energy regime of quantum field theory. Int J Nonlinear Sci Numer Simul 2005;6(3):223–34.

[5] He JH. In Search of 9 hidden particles. Int J Nonlinear Sci Numer Simul 2005;6(2):93–4.

[6] He JH. Space, time and beyond. Int J Nonlinear Sci Numer Simul 2005;6(4):343–6.

[7] El Naschie MS. A review of E infinity theory and the mass spectrum of high energy particle physics. Chaos, Solitons & Fractals 2004;19(1):209–36.

[8] El Naschie MS. A guide to the mathematics of E-infinity Cantorian spacetime theory. Chaos, Soliton & Fractals 2005;25:955–64.

[9] El Naschie MS. The VAK of vacuum fluctuation, Spontaneous self-organization and complexity theory interpretation of high energy particle physics and the mass spectrum. Chaos, Solitons & Fractals 2003;18(2):401–20.

[10] He JH, Chen H. Effects of size and Ph on metabolic rate. Int J Nonlinear Sci Numer Simul 2003;4(4):429–32.
[11] He JH, Zhang J. Fifth dimension of life and the 4/5 allometric scaling law for human brain. Cell Biol Int 2004;28(11):809–15.
[12] He JH, Huang Z. A novel model for allometric scaling laws for different organs. Chaos, Solitons & Fractals, in press. Corrected Proof, Available from: doi:10.1016/j.chaos.2005.04.082.
[13] He JH. An allometric scaling law between gray matter and white matter of cerebral cortex. Chaos, Solitons & Fractals, in press. Corrected Proof, Available from: doi:10.1016/j.chaos.2005.04.081.
[14] NASA microgravity research highlights, Advancing Heart Research, Reprinted from the Spring 2000 issue of Microgravity News. Detailed information is published in Amer J Physiol 277 (Heart Circ Physiol 46), H433–44 (1999) and Biotech Bioeng 1999;64:580–9.
[15] Kleiber M. Body size and metabolism. Hilgardia 1932;6:315–53.
[16] West GB, Brown JH, Enquist BJ. A general model for origin of allometric scaling laws in biology. Science 1997;276:122–6.
[17] Darveau CA, Suarez RK, Andrews RD, Hochachka PW. Allometric cascade as a unifying principle of body mass effects on metabolism. Nature 2002;417:166–70.
[18] Kuikka JT. Scaling laws in physiology: relationships between size, function, metabolism and life expectancy. Int J Nonlinear Sci Numer Simul 2003;4:317–28.
[19] Kuikka JT. Fractal analysis of day-to-day variation of heart rate and blood pressure: a case report. Int J Nonlinear Sci Numer Simul 2005;6(2):107–12.
[20] Blum JJ. On the geometry of four-dimensions and the relationship between metabolism and body mass. J Theor Biol 1997;64:599–601.
[21] Rakic P. Nature 2004;427:685–6.
[22] Rubner M. Z Biol 1883;19:536–62.
[23] He JH. Mysterious pi and a possible link to DNA sequencing. Int J Nonlinear Sci Numer Simul 2004;5(3):263–74.
[24] Gabrys E, Rybaczuk M, Kedzia A. Fractal models of circulatory system. Symmetrical and asymmetrical approach comparison. Chaos, Solitons & Fractals 2005;24:707–15.
[25] Kalda J. The fractal model of the blood-vessel system, available from the web: http://ioc.ee/res/fractal.html.
[26] Ohki K, Chung S, Ch'ng YH, Kara P, Reid R. Functional imaging with cellular resolution reveals precise micro-architecture in visual cortex. Nature 2005;433:597–603.
[27] Taylor CR et al. Respir Physiol 1981;44:25–37.
[28] Wang ZM, O'Connort TP, Heshka S, Heymsfield SB. The reconstruction of Kleiber's law at organ-tissue level. J Nutr 2001;131:2967–70.
[29] Kozlowski J, Konarzewski M, Gawelczyk AT. Cell size as a link between noncoding DNA and metabolic rate scaling. PNAS 2003;100:14080–5.
[30] Haskell JP, Ritchie ME, Olff H. Fractal geometry predicts varying body size scaling relationships for mammal and bird home ranges. Nature 2002;418:527–30.