The option to wait in collective decisions*

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Abstract

We consider a model in which voters over time receive more information about their preferences concerning an irreversible social decision. Voters can either implement the project in the first period, or they can postpone the decision to the second period. We analyze the effects of different majority rules. Individual first period voting behavior may become "less conservative" under supermajority rules, and it is even possible that a project is implemented in the first period under a supermajority rule that would not be implemented under simple majority rule.

We characterize the optimal majority rule, which is a supermajority rule. In contrast to individual investment problems, society may be better off if the option to postpone the decision did not exist. These results are qualitatively robust to natural generalizations of our model.

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1 Introduction

In most political economy models, individuals know their preferences over candidates or social actions. In another branch of the literature, individuals know their fundamental preferences, but which action is best suited to implement them depends on an unknown state of the world. The main objective of this type of models is to analyze how individuals can aggregate dispersed information through strategic voting.[1]

In the present paper we focus on a third case that has received little attention so far: Collective decisions under uncertainty when individuals discover their own preferences over time. In our model, individuals get additional information over time about their heterogeneous preferences regarding an investment project, and have to choose whether to implement it immediately, or delay the decision. In the latter case, they can either implement it after receiving additional information, or pass on it completely. While investment problems under uncertainty have been analyzed extensively for single decision makers,[2] we analyze such problems when the investment decision is made by a society through voting. Our main focus is twofold: Firstly, we examine the effect of the majority rule on individual voting behavior and social decisions. We show that a higher majority rule makes individual voters in the first period more conservative relative to projects whose expected payoffs in the future are low, and less conservative relative to projects whose expected payoffs in the future are high. From an ex-ante point of view, this change of individual voting behavior is desirable and has the effect that the *optimal* majority rule is larger when society has the option to wait than when voters are forced to make a final up-or-down decision in the first period. In particular, we show in a symmetric setting, where simple majority rule is optimal without the option to wait, a supermajority rule becomes optimal with the option to wait.[3] Secondly, we show that a society is often worse off (from an ex-ante point of view) if voters have the option to wait, rather than being forced to decide once and for all. This result holds even if society adopts the optimal majority rule in both cases.

Specifically, we consider the following dynamic social investment problem. In the first period, each voter knows whether he would be a winner or a loser in the first period, but his second period type is random. If the project is implemented in the first period, it is irreversible and payoffs to voters accrue in both periods according to their type realizations. Alternatively, if the project is not implemented in the first period, voters find out their respective second period types, and vote on whether to implement the project for the second period. We parameterize projects according to the relative size of the gain of winners to the loss of losers. A "good" project is one where this ratio is large (i.e., the ex-ante expected average payoff is positive), and vice versa.

A possible advantage of delaying investment in the first period is that agents receive information about their payoffs in the next period: There is an "option value of waiting". We analyze how the type

[1] See, e.g., Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996), Feddersen and Pesendorfer (1998).

[2] See Dixit and Pindyck (1994) for a review of this literature.

[3] By a supermajority rule, we mean a voting rule that specifies that the status quo is only to be changed if a certain proportion of the electorate (greater than the 50%, the "simple majority") votes in favor of change.

of majority rule influences the value of waiting, and thus, the voting behavior of individuals and the first period implementation decision. The expected second period payoff for a voter, if the project is delayed in the first period, may go in either direction as the majority rule changes: A higher majority rule may increase the risk that a "good" project with a positive expected value (i.e., one in which winners gain more than losers lose) is not implemented in the second period, thus diminishing the value of waiting and inducing voters to implement the project already in the first period. In contrast, a higher majority rule decreases the risk that a "bad" project is implemented in period 2, thus increasing the value of waiting.

A higher value of waiting makes voters more reluctant to implement the project already in the first period. Thus, a higher majority rule makes each voter more willing to agree to good projects, even if he is a loser today, and less willing to agree to bad projects, even if he is a winner today. There is also a second, direct, effect of a higher majority rule: More voters have to agree, making first-period implementation less likely. For bad projects, both effects go in the same direction, making implementation less likely for higher majority rules. In contrast, for good projects, the first effect may outweigh the second one, leading to more projects being implemented in period 1 under a higher majority rule.

On the normative side, we focus on an ex-ante point of view, that is, taking expectation over both voter type realizations and project types. We show that, relative to a situation where all decisions have to be made in the first period, the option to wait (weakly) increases the optimal majority rule in large electorates. Intuitively, higher majority rules have the advantage that, for socially bad projects, voters become more conservative and thus fewer of these projects are implemented, while for good projects, voters become more willing to implement in the first period. Moreover, since the best projects are already implemented in period 1, those projects that are reconsidered in period 2 form a negative selection from the set of all projects, and a higher majority rule is socially beneficial for these cases as well.[4]

We also characterize the ex-ante optimal supermajority rule explicitly under the additional assumptions that each voter has a 50 percent chance of being a high type, and that project types are uniformly distributed at the constitutional stage. The optimal supermajority rule in this case is approximately (i.e., up to integer constraints) between $7/11 \approx 63.6\%$ and $2/3$, for any number of voters.

It is also interesting to analyze the social ex-ante value of the option to wait. In unilateral investment problems, this value is always nonnegative, and often positive, as individuals may strictly benefit from postponing the decision. In contrast, a *society* may be better off if it is forced to invest either immediately or not at all, rather than having the option of postponing this decision. Indeed, we show that, from an ex-ante point of view (and with uniformly distributed project costs), this is the case even if society chooses the *optimal* majority rule for the case when waiting is possible.

Our results shed light on an important question in the endogenous determination of institutions: Why

[4]Even at the interim stage (i.e., in the first period when voters know the project type and their own first-period type), simple majority rule may be Pareto inefficient for some bad projects. This is the case if, under simple majority rule, there is a majority of voters who approve immediate implementation; but even those voters would prefer to postpone implementation, if the majority rule is changed to unanimity rule. In contrast, transition from unanimity rule to simple majority rule cannot yield an ex-post Pareto improvement.

do some organizations choose supermajority rules, and which features of decision problems influence this choice? Majority rules within organizations vary considerably, from simple majority to unanimity. Often, the choice of the majority rule that is to govern future decision making is a contentious issue itself, such as in the recent EU summit, which eventually adopted a supermajority rule. Most countries use supermajority rules for a change of the constitution, and, often implicitly, for "normal" legislation.[5] This paper contributes to the literature on the relative advantages of different majority rules (discussed in more detail in Section 5), by providing a fundamentally new rationale for supermajority rules, which relies on voters' uncertainty over the consequences of project implementation, and the option value of waiting until new information is available. Thus, our model is most relevant for societies that frequently face decision problems with such characteristics.

Several previous papers have analyzed supermajority rules from an economic point of view. Buchanan and Tullock (1962) argue that, under a simple majority rule, a majority of people may implement socially bad projects because they can externalize a part of the associated cost to the losing minority, while under unanimity rule, only Pareto improving projects are implemented. However, Guttman (1998) shows that unanimity rule leads to a rejection of many projects that are not Pareto improvements, but nevertheless worthwhile from a reasonable social point of view. Assuming that the social goal is to minimize the sum of both types of mistakes, he shows that, in a symmetric setting, simple majority rule is optimal. In a symmetric setup in our model, the same result obtains if voters have to make a once-and-for-all decision about the project in the first period. However, with the option to postpone a decision to the second period, we show that a supermajority rule is optimal.[6]

Our model is most closely related to a small literature in which voters learn about their preferences over time. Compte and Jehiel (2008) develop an infinite-period search model in which the decision when to stop the search is taken by a committee. They analyze the implications of different majority rules on the voters' acceptance thresholds and the implied search duration. If voters are sufficiently patient then higher majority rules imply that voters become more picky and average welfare increases. Albrecht, Anderson, and Vroman (2008) consider a simplified version of this framework in which all voters draw valuations from the same distribution and obtain results also for the case of intermediate and low patience levels. In particular, they show that the optimal majority rule is monotonically increasing in voters' discount rate. Moreover, if voters are sufficiently impatient, their expected equilibrium payoff increases with the size of the committee.

Both of these papers focus on the analysis of individual voting behavior and welfare under different

[5]For example, in parliamentary systems with a strong committee organization, a legislative proposal usually needs the support of *both* the respective committee and the house. In parliamentary systems with two chambers, certain legislative proposals need the support of both chambers. Tullock (1998), p. 216, estimates that legislative rules in the US for changing the status quo are "roughly equivalent to requiring a 60% majority in a single house elected by proportional representation."

[6]Other rationales for supermajority rules are discussed in Section 5.2 and include the problem of time inconsistency of optimal policies under simple majority rule (Gradstein (1999), Dal Bo (2006)), the possibility of electoral cycles under simple majority rule (Caplin and Nalebuff (1988)), and a strategic use of supermajority rules when preferences change deterministically (Messner and Polborn (2004)).

(exogenous) majority rules in a collective decision problem which is naturally framed as a stopping problem. While our setup differs somewhat, the main difference is in the questions we analyze. We are mostly interested in the optimal majority rule under the option to wait, and, in particular, how the optimal majority rule and voter welfare compare in the cases with and without the option to wait.

Strulovici (2007) analyzes a model in which a society has to choose in continuous time between a risky and a secure project. Ex-ante, all individuals are identical; over time, some individuals discover that they are winners and then receive a payoff forever after. The arrival rate is unknown, and voters continuously update their beliefs as long as the risky action is played. In contrast to our model, information arrives only as long as the risky action is played, and the project is reversible. Voters decide under simple majority rule or unanimity rule when (and if at all) to stop experimentation with the uncertain action. Strulovici (2007) finds that society always stops experimentation too early compared with a utilitarian optimum, and that unanimity rule may lead to more or less experimentation than simple majority rule.

Gersbach (1993b), in a framework generalizing Glazer (1989), considers a model where voters in period 1 choose between implementing an irreversible long run project that delivers benefits both in period one and in period two, and sticking with the status quo; in the latter case, they reconvene in period 2 and decide whether to implement a short-run project then. The voters' period-2 valuations are unknown at the time of the first election. Our model shares the temporal setting of Gersbach (1993b), but differs in several aspects and mainly in our focus of analysis, which is the determination the optimal institutional response of a society that repeatedly faces such decision problems under uncertainty.

Another social learning paper in which new information arrives only as long as society is experimenting is Callander (2008). Citizens know how the status quo policy translates into outcomes, but the farther a policy is away from the status quo policy, the less certain are its consequences. Callander shows that an initial phase of experimentation and learning is eventually terminated if a policy achieves an outcome that is sufficiently close to the ideal outcome of the median voter.

Fernandez and Rodrik (1991) analyze a model of voting on reform projects that generate winners and losers. They show that a project that ex-post benefits the majority of the population need not be implemented, if the ex-ante expected benefit is negative for a majority of the population. If, instead, a majority of the population has positive ex-ante expected benefits, but ex-post, payoffs are negative for a majority, then a reform may be implemented initially, but would be reversed after payoff information becomes known. Thus, there is a bias in favor of the status quo. In contrast to Fernandez and Rodrik (1991), we analyze a setting in which reforms are not reversed, so that there is no status quo bias in our setup. Also, our focus is on comparing different majority rules and how they influence voting behavior and implementation decisions, while Fernandez and Rodrik (1991) only consider simple majority rule.

The paper proceeds as follows. The next section presents the model. Our main results follow in Section 3. We analyze the robustness of the model and several extensions in Section 4. In Section 5, we discuss the relevance of our results for the endogenous determination of majority rules, as well as the relation to previous literature on this subject. Section 6 concludes.

2 The model

2.1 Description

A group of N (odd) risk neutral individuals has to decide whether to undertake an investment project that creates costs and benefits (described in more detail below) for all group members. The decision about the project has two stages. At the beginning of period 1, the group chooses between implementing the project right away and postponing the decision to the beginning of period two. In the latter case, the group makes the final decision on whether or not to implement the project at the beginning of period 2. In both periods, the decision is governed by a *voting rule* indexed by m. The project is implemented if and only if at least m individuals approve. The majority rule may range from simple majority to unanimity, i.e. $m \in \{(N+1)/2, \ldots, N\}$.

In each period that the project is not implemented, all voters receive a net payoff normalized to 0. If the project is implemented in or before period t, then individual i receives a payoff of $V_t^i - c$ in period t. We refer to V_t^i as voter i's *type* in period t. Types of different voters, as well as first period and second period types, are identically and independently distributed.[7] Specifically, we assume that V_t^i is equal to 1 with probability θ, and 0 otherwise. The *project type* $c \in (0, 1)$ is the same for both periods, and can either be interpreted as the per-capita "cost" of the project (to be shared equally by all voters), or as a utility index that captures the size of the gains of those people who are better off than in the status quo, relative to the losses of those who would prefer the status quo.[8] For simplicity, we assume that individuals do not discount the future, so that they value future and current payoffs equally.

At the time of the election in period 1, individuals only know their own period 1 type, but not their period 2 type. In the period 2 election (if any), individuals know also their period 2 type. In elections, each individual votes for the option that would provide him with the higher expected utility: In period 2, voter i votes for the project if and only if $V_2^i = 1$. In period 1, voter i votes for project implementation if and only if he weakly prefers immediate implementation to the expected payoff from postponing (given that all voters behave in period 2 as described above). Formally, we use iterated elimination of weakly dominated strategies, a standard refinement in voting games.[9]

Eventually, we are interested in the endogenous determination of the voting rule. We envision that this choice occurs at an initial stage, before type realizations for the project are known. Thus, all voters are identical and agree to choose the majority rule that maximizes their ex-ante expected payoffs. Note that we can also interpret such a constitution normatively as the one that maximizes ex-ante utilitarian welfare.

[7] We relax these independence assumptions in Section 4.

[8] Clearly, we could just specify the net payoff of each individual through one variable, but our approach allows us to use c in order to easily distinguish projects with a high expected average payoff (i.e., low c) from those with a low expected average payoff.

[9] This refinement, for example, eliminates (rather strange) Nash equilibria of the voting game in which everybody opposes investment, even if he would benefit from implementation.

Ex-ante payoffs, and thus the optimal majority rule, also depend on the project type c. Typically, however, it is not feasible to construct a constitution where the applicable majority rule depends on the type c of the project under consideration, since there would be verifiability problems, and such a rule would unavoidably lead to conflicts of interpretation. Thus, we focus on the majority rule that is optimal *in expectation*, when c is drawn from some distribution with cumulative distribution $F(c)$.

The model is a relatively tractable framework for the analysis of intertemporal information arrival in social decision problems. It is deliberately simple in some aspects, and we discuss the robustness of the model to several extensions in Section 4 and show that the main results do not depend critically on the particular modeling choices.

2.2 Application – Voting on hiring

There are many applications in which societies have to decide on investment projects with uncertain returns. Here, we just present an example that illustrates some of the main features and questions of the model. The reader who is more interested to proceed to the theoretical results can, without problems, skip the remainder of this section.

Late in the junior job market, an economics department can choose to offer an assistant professorship to Candidate A, who would accept such an offer. A decision to hire A is irreversible in the next years (i.e., the line is filled, and A can be fired only after considerable delay at tenure time). Payoffs accrue to department members, both now and in the future, depending on how good a researcher and colleague Professor A turns out to be. From today's perspective, this is a random event, and also a question in which individual tastes of existing department members may differ, so department members may disagree even ex-post whether hiring A was a good decision.

Alternatively, the department can choose to leave the position unfilled and wait till the following year, when there is a new draw of an available candidate, say, B. Again, the department can decide whether to hire B or not. In principle, the department's hiring problem is an infinite period problem, assuming that their dean would always renew searches for lines that were not filled in the previous year. In the interest of tractability, we simplify the infinite period world to a two-period one in our model.[10]

A central result of our model is that supermajority rules outperform simple majority rule with respect to voters' ex-ante expected utility. While we have only anecdotal evidence, supermajority rules appear also prevalent in groups that decide on hiring and/or promotion through voting.[11] We also show that – in contrast to individual investment problems – the ability to postpone the decision can hurt a society.

[10]The reader may notice that there is a small, but largely irrelevant difference between individuals' payoffs in our model and in this application. In our model, an individual's second period payoff is the same, whether the project was implemented in the first period or only in the second period. In contrast, in this application, an individual voter's realized second period payoff from A (if he was hired in the first period) may very well differ from that individual's payoff from B (in the same period). However, all that matters for individual voters when they decide how to vote in the first period is the *expectation* of their second period payoff, so the difference between realized payoffs in the model and in the application is immaterial for our main results.

[11]This is certainly true, as a practical matter, for promotion votes in universities. A candidate who receives a bare majority

Thus, groups have an incentive to construct rigid rules in an attempt to commit against reconsideration, if possible. For example, consider a tenure decision for a marginal (i.e., neither awful nor great) candidate. If there is uncertainty about the quality of the candidate's unpublished work, it would appear wise to postpone the decision on whether to grant tenure by an additional year or two. However, university regulations usually preclude such a course of action and force an immediate decision. While such a rigid rule would often lower and never increase the utility of a *single* decision maker, our model shows that it may be strictly welfare increasing in a group decision problem.

3 Results

3.1 The benchmark case: No option to wait

We first analyze the benchmark case in which the electorate has to take the decision about the project once and for all in period 1. That is, a first period rejection of the project is final. Voter i's expected total payoff from immediate implementation is

$$U_I^i(V_1^i, c) = V_1^i + E(V_2^i) - 2c = V_1^i + \theta - 2c. \tag{1}$$

Each voter approves the project if and only if its net present value is nonnegative.[12] Thus, a voter with first period type $V_1^i = 1$ (a *high type*) votes in favor if and only if $1 + E(V_2^i) - 2c = 1 + \theta - 2c \geq 0$, hence if $c \leq \frac{1+\theta}{2}$. Similarly, a *low type* voter ($V_1^i = 0$) casts a favorable ballot if and only if $E(V_2^i) - 2c = \theta - 2c \geq 0$, or $c \leq \theta/2$.

Thus, projects with type $c \leq \theta/2$ are unanimously approved, and those with $c > \frac{1+\theta}{2}$ are unanimously rejected. If, instead, $c \in \left(\frac{\theta}{2}, \frac{1+\theta}{2}\right]$ then the realization of types matters. It is useful to define $p(m, N, \theta) = \binom{N-1}{m-1}\theta^{m-1}(1-\theta)^{N-m}$ as the probability that there are exactly $m-1$ high types among the other $N-1$ voters. We can think of p as the probability of voter i being pivotal, if the majority rule is m. Also, let $q(m, N, \theta) = \sum_{l=m}^{N-1} \binom{N-1}{l}\theta^l(1-\theta)^{N-1-l}$ be the probability that there are m or more high types among the other $N-1$ voters. From the point of view of an individual voter i, q is the probability that the project is implemented through the votes of the other voters, independently of voter i's preference on the project.

To determine the majority rule that maximizes the ex-ante expected payoff of voters, denote a single voter i's ex-ante expected payoff under majority rule m with N voters by $\tilde{\pi}_i(m, N, c)$. In case that voting is type dependent (i.e., $c \in \left(\frac{\theta}{2}, \frac{1+\theta}{2}\right]$), there are two cases with different conditional expected implementation payoffs for voter i. With probability $q(m, N, \theta)$, there are m or more high types among the other $N-1$ voters, so that voter i's expected payoff is simply the ex-ante expected implementation payoff, $2(\theta - c)$. In contrast, with probability $p(m, N, \theta)$, there are exactly $m-1$ high types among the other $N-1$ voters,

of favorable votes in his own department usually is in severe problems at the college or university level.

[12] As a tie-breaking assumption, we assume that voters who are indifferent always approve the project. No results of our model qualitatively depend on this assumption.

then the project is implemented with probability θ (namely, if and only if i's type is $V_1^i = 1$), in which case voter i's implementation payoff over both periods is $1 + E(V_2^i) - 2c = 1 + \theta - 2c$. Thus, we have

$$\tilde{\pi}(m, N, c, \theta) = \begin{cases} 2(\theta - c) & \text{if } c \leq \theta/2 \\ q(m, N, \theta) \cdot 2(\theta - c) + p(m, N, \theta)\theta(1 + \theta - 2c) & \text{if } c \in \left(\frac{\theta}{2}, \frac{1+\theta}{2}\right] \\ 0 & \text{if } c > (1 + \theta)/2 \end{cases} \tag{2}$$

where we have dropped the index i, since this payoff is identical for all individuals. To save on notation, we will also suppress the arguments N and θ in functions like $\tilde{\pi}$, p or q, when no confusion can arise (i.e., when we consider a situation in which N and θ are fixed).

Clearly, $\tilde{\pi}(m, c)$ is a piecewise linear function of c. Moreover, $\tilde{\pi}(m, c)$ jumps downward at $c = \theta/2$, and upward at $c = (1 + \theta)/2$ for all majority rules except unanimity rule. To see this, note that $\lim_{c\downarrow\theta/2} \tilde{\pi}(m, c) = [q(m) + p(m)]\theta < \tilde{\pi}(m, \theta/2) = \theta$, and $\lim_{c\uparrow\frac{1+\theta}{2}} \tilde{\pi}(m, c) = -(1 - \theta)q(m)$, while $\lim_{c\downarrow(1+\theta)/2} \tilde{\pi}(m, c) = 0$. For unanimity rule, $q(N) = 0$, so that $\tilde{\pi}(N, \cdot)$ is discontinuous only at $c = \theta/2$, but not at $c = (1 + \theta)/2$.

Intuitively, at $c = \theta/2$, high types strictly benefit from implementation, while low types are just indifferent. Hence, from an ex-ante perspective, voters strictly benefit if the project is implemented. Implementation always occurs for $c \leq \theta/2$, while for $c > \theta/2$, implementation depends on the realization of preference types and is thus not guaranteed. Hence, $\tilde{\pi}(m, c)$ drops at $c = \theta/2$. Similarly, for $c = (1 + \theta)/2$, high types are just indifferent towards implementation, while low types strictly suffer. Thus, voters suffer from an ex-ante perspective if the project is implemented. Implementation never occurs for $c > (1 + \theta)/2$ (so that $\tilde{\pi}(m, c) = 0$ for all $c > (1 + \theta)/2$), while for $c \in \left(\frac{\theta}{2}, \frac{1+\theta}{2}\right]$, implementation depends on the realization of preference types. Figure 1 shows the ex-ante payoff $\tilde{\pi}$ for the case $N = 15$, $\theta = 1/2$ and $m = 8$ and $m = 9$.

We now analyze the optimal voting rule for different levels of c. The expected payoff consists of two parts: Conditional on being pivotal, a voter always has a positive payoff, while conditional on not being pivotal, the payoff can be positive or negative. As for the probabilities of these events, it is clear that q is a decreasing function of m. In contrast, the most likely number of other voters who are high types is $\theta(N - 1)$ (rounded up or down, since voter numbers are integers), and so p is increasing in m if $m - 1$ is lower than this value, and decreasing afterwards. The resulting tradeoffs determine the optimal majority rule as follows.

Proposition 1. *Suppose that society can either implement the investment project in period* 1*, or not at all.*

1. *If $c \leq \theta/2$, or $c > (1 + \theta)/2$, all majority rules yield the same expected payoff $\tilde{\pi}(\cdot, c)$.*

2. *For $c \in [\theta/2, (1 + \theta)/2)$ the expected payoff is single peaked in the majority rule m, unless $N(2c - \theta)$ is an integer; the majority rule that maximizes the expected payoff is given by $m^* = \max\left(\frac{N+1}{2}, \lceil N(2c - \theta)\rceil\right)$. In particular, for c close to $(1 + \theta)/2$, the unique optimal majority rule*

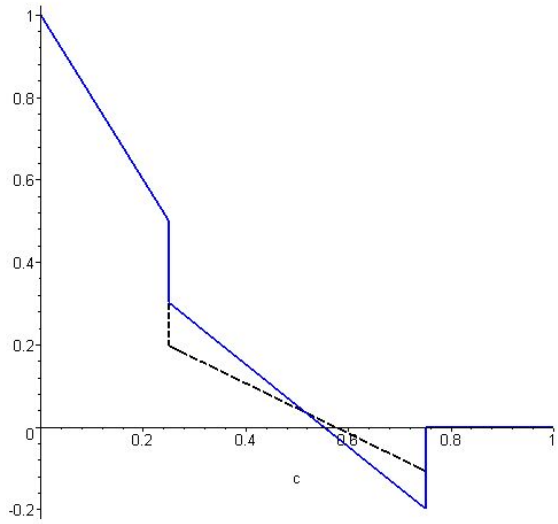


Figure 1: The function $\tilde{\pi}$ for $N = 15$, $\theta = 1/2$ and $m = 8$ (solid) and $m = 9$ (dashed)

is unanimity rule. If $N(2c - \theta)$ is an integer greater than $N/2$, then $\tilde{\pi}$ is maximized by both $m^* = N(2c - \theta)$ and $m^{**} = N(2c - \theta) + 1$.

Proof. The first part is obvious since all voters agree when $c \leq \theta/2$, or $c > (1 + \theta)/2$. For part 2, see Appendix. □

If $(2c - \theta) < 1/2$, society is constrained by our assumption that m must be greater or equal to $(N+1)/2$ (because there are clearly stability problems with submajority rules).[13] If this constraint is not binding, then the optimal majority rule $m^*/N \approx (2c - \theta)$ has an intuitive interpretation as the one that maximizes utilitarian welfare: If there are $\hat{M}$ high types in the first period, then the per-capita expected utility from implementation is $\hat{m} - c + \theta - c$ (where $\hat{m} = \hat{M}/N$). A social planner would like to implement the project if and only if this expected utility is positive, and setting $m^* = \lceil (2c - \theta) N \rceil$ guarantees just that.

Now consider the problem of choosing an optimal majority rule when the constitution cannot condition the majority rule on the project type c. From an ex-ante perspective, c is distributed according to some (rather arbitrary) distribution F.

Proposition 2. *Suppose that society can either implement the investment project in period* 1*, or not at all. Furthermore, suppose that the constitution cannot condition the majority rule on c and that c is drawn from a distribution with cumulative distribution function F that satisfies $F((1 + \theta)/2) - F(\theta/2) > 0$.*[14]

[13]If, for some reason, society were able to use submajority rules, then $\lceil N(2c - \theta) \rceil$ is always an optimal majority rule. This would not affect our qualitative results in the following.

[14]If $F((1 + \theta)/2) = F(\theta/2)$, then all decisions are unanimous and thus, the majority rule never matters.

Then, $\tilde{\Pi}(\cdot, N, \theta)$ *is maximized by*

$$m^* = \max\left\{(N+1)/2, \left\lceil\left(\frac{1}{2} + 2\frac{x}{y}\right)N\right\rceil\right\},$$

where $x = \int_{\theta/2}^{(1+\theta)/2}[c - (1+2\theta)/4]dF(c)$ *and* $y = F((1+\theta)/2) - F(\theta/2)$.[15] *In particular, if* F *is symmetric on* $[\theta/2, (1+\theta)/2]$ *(around the midpoint* $(1+2\theta)/4$*), then* $x = 0$*, so that simple majority is optimal.*

Proof. See Appendix. □

Note that x/y is the expected deviation of c from the midpoint of the interval in which majority rules matter (conditional on c being in that interval). If the expected value of c coincides with the midpoint (which is the case, for example, for the uniform distribution), then simple majority rule is optimal.

Note that this is true no matter what the value of θ is. To get an intuition for this result, observe that the average payoff of a first period high type is $1 + \theta - 2E(c)$,[16] while the expected payoff of a first period low type is $\theta - 2E(c)$. If $E(c) = (1+2\theta)/4$, then the expected payoffs of $(1-\theta)/2$ for a high type, and $-(1-\theta)/2$ for a low type are symmetric to each other. Simple majority rule maximizes the expected payoff in a situation where the payoffs of winners and losers are symmetric around 0. If $E(c) < (1+2\theta)/4$,[17] then winners gain more than losers lose, so that a social planner would like to encourage implementation even more; in this case, simple majority rule is the (constrained) optimal rule. Finally, if $E(c) > (1+2\theta)/4$, then winners gain on average less than losers lose, so that a supermajority rule is optimal.

It is interesting to note that Proposition 2 implies that, if c is distributed uniformly, then simple majority rule is optimal, independent of the probability θ of being a winner. As explained above, while θ affects the interval in which individuals disagree with each other, conditional on c being in this interval, first period high and low types are, on average, symmetric. Thus, for example, even if θ is quite low, it is not optimal to change to a supermajority rule.

3.2 Individual voting behavior and the option to wait

We are now ready to analyze the implications of the option to delay the decision on the implementation of the public project. Obviously, in the second period, player i votes in favor of the project if and only if $V_2^i = 1$, and the project is implemented if and only if there are at least m players with a high second period type. Let $I_2(m, \theta)$ denote the event that the project is implemented in the second period, given that the majority rule is m; and let $P(I_2(m, \theta))$ denote the probability of this event.

[15] If $(y - 2x)N/2y$ is an integer, $\tilde{\Pi}$ is also maximized by $\left\lceil\left(\frac{1}{2} + 2\frac{x}{y}\right) + 1\right\rceil N$.

[16] Note that $E(c)$ is, of course, the average c *conditional* on c being in the interval where decisions may be non-unanimous, that is, where majority rules matter at all.

[17] This case arises, for example, for all θ if the density $f(c)$ is a decreasing function.

Consider now the first period decision. If the project is not implemented in the first period, then player i can expect to obtain the payoff $\left(E[V_2^i|I_2(m,\theta)] - c\right) P(I_2(m,\theta))$. It is useful to write this expected continuation utility, the expected *value of waiting*, as

$$\begin{aligned} U_W(c,m,N,\theta) = p(m,N,\theta)E[\max\{V_2^i - c, 0\}] + q(m,N,\theta)(E(V_2^i) - c) = \\ \theta p(m,N,\theta)(1-c) + q(m,N,\theta)(\theta - c), \end{aligned} \tag{3}$$

using the definitions of $p(\cdot)$ and $q(\cdot)$ from the last section. Since voter i's payoff from implementing the project immediately is $U_I^i(V_1^i, c) = V_1^i + \theta - 2c$, he will approve immediate implementation in period 1 if and only if

$$V_1^i + \theta - 2c \geq \theta p(m,N,\theta)(1-c) + q(m,N,\theta)(\theta - c). \tag{4}$$

Note an important difference to the benchmark case without the option to wait: An individual voter's first period behavior as characterized by (4) depends on the majority rule m, because that rule determines the expected value of waiting.

If $c \leq \theta$, then both terms on the right-hand side of (4) are positive, so that the option to wait induces voters to behave more conservatively than in situations where the decision may not be delayed. Moreover, $p(\cdot)$ is a decreasing function of m if $m \geq \theta(N-1)+1$, and $q(\cdot)$ is always a decreasing function of m. Thus, at least for all $\theta \leq 1/2$, the tendency to behave more conservatively is the stronger the lower the majority rule m. In this case, the cost threshold below which a low voter type approves of a project shifts to the left as the option to delay the decision is introduced, and this shift is the stronger, the lower the majority rule.[18]

If $c > \theta$, then the value of waiting is neither necessarily positive, nor is it necessarily decreasing in m. In contrast to private decisions, where the value of waiting is always positive, society sometimes implements projects that are not socially beneficial. If the right-hand side of (4) is negative, then it is possible that a high type voter votes for immediate implementation of an investment project even though his expected implementation payoff is negative. The reason for this (seemingly strange) behavior is that the voter's payoff from immediate implementation is at least better than his expected payoff if he forgoes immediate implementation and is then (perhaps) hit by implementation in the second period, when his type may be low. In this case, a higher majority rule may increase the value of waiting, as it increases the voters' protection in the next period against the implementation of a project that they oppose.

We now proceed to a more formal analysis of the value of waiting and its implications for individual voting behavior. Lemmas 1 and 2 are used repeatedly in the proofs of the following propositions, and are presented here in the text, because they are of independent interest and provide an intuition for the economic effects in our model.

Lemma 1 shows that a higher majority rule increases the probability of voter i being pivotal, relative to the probability that the project is implemented independent of voter i's preferences. This effect underlies a benefit of supermajority rules, because a voter always gets a nonnegative payoff if he is pivotal, but

[18]For $c \leq \theta$, (4) implies that high types always favor implementation, so that their behavior does not change relative to the case that waiting is not possible.

may receive a negative expected payoff if a project with high c is implemented independently of voter i's will.

Lemma 1. *The ratio* $\dfrac{p(m,N,\theta)}{q(m,N,\theta)}$ *is increasing in m.*

Proof. See Appendix. □

Lemma 2 shows that the value of waiting $U_W(c,m,N,\theta)$, defined in equation (3), is increasing in m if $m < Nc$ and decreasing in m if $m > cN$. Thus, for any $c \in [0,1]$ the value of waiting is single-peaked in m.

Lemma 2. *If* $m < cN$ $(m > cN)$*, then* $U_W(c,m,N,\theta) < U_W(c,m+1,N,\theta)$ $(U_W(c,m,N,\theta) < U_W(c,m+1,N,\theta))$.

Proof. See Appendix. □

If $c \leq 1/2$, the condition $m > cN$ is satisfied for all admissible majority rules. Intuitively, if $m/N > c$, the project is implemented in period 2 only if the per-capita benefit m/N exceeds the per-capita cost c. A further increase of the majority rule then means that the project is not implemented in some cases where the project's average payoff is positive. Thus, the value of waiting in period 1 decreases. Conversely, the value of waiting increases in m if $m < cN$, as the project is implemented less often when the average payoff is negative.

Note that the single-peakedness of U_W in m implies that, if for some given c the value of waiting is negative for majority rule m^0, then the same must hold for any majority rule $m < m^0$ (i.e. $U_W(m^0,c) < 0$ implies $U_W(m,c) < 0$ for all $m < m^0$).

Proposition 3 below characterizes first-period voting behavior with the option to wait. For each majority rule m there are two cutoffs $\underline{c}$ and $\bar{c}$ such that low types vote for implementation if and only if $c \leq \underline{c}$ and high types vote for implementation if and only if $c \leq \bar{c}$. Again, there are three different regimes: If $c \leq \underline{c}$ or $c > \bar{c}$, all voters agree to implement or reject, respectively. If $c \in (\underline{c}, \bar{c}]$, implementation depends on the number of first-period high types.

Proposition 3 also characterizes the range in which $\underline{c}$ and $\bar{c}$ lie, and how they change with m. Since $\underline{c} < 1/2$, the value of waiting for c close to $\underline{c}$ is positive and decreases in m. A higher majority rule increases the willingness of low types to implement in the first period, as second period implementation becomes less likely. Thus, $\underline{c}$ increases in m. In contrast, $\bar{c} > 1/2$, and the value of waiting is non-monotonic in m in that region. For low majority rules, the value of waiting is negative for c close to $\bar{c}$, and increases with m. Thus, high types become more conservative as m increases, so that $\bar{c}$ decreases. In contrast, for high majority rules, the value of waiting is positive and decreases with a further increase in m, thus making high types less conservative, so that $\bar{c}$ increases in m for high levels of m. Thus, $\bar{c}$ is a U-shaped function of m.

Proposition 3. *For any majority rule m, there exist threshold values $\underline{c}(m,N,\theta)$ and $\bar{c}(m,N,\theta)$, with $\underline{c}(m,N,\theta) < \bar{c}(m,N,\theta)$, such that low types (high types) vote for first period implementation of a project if and only if $c \le \underline{c}(m,N,\theta)$ ($c \le \bar{c}(m,N,\theta)$).*

Moreover, $\underline{c}(\cdot,N,\theta)$ is an increasing function and satisfies $0 \le \underline{c}(\cdot,N,\theta) < \theta/2$. In contrast, $\bar{c}(\cdot,N,\theta)$ is U-shaped, assumes its minimum for some $m \in \{N/(2-\theta), \lceil(1+\theta)N/2\rceil\}$ and satisfies $1/(2-\theta) < \bar{c}(\cdot,N,\theta) < 1$. In addition, $\bar{c}(N,N,\theta) < (1+\theta)/2$.

Proof. See Appendix. □

The lower bound provided in this proposition for the majority threshold that minimizes $\bar{c}$ is rather loose. In fact, it can be shown that $\bar{c}$ always assumes its minimum either at $\lceil(1+\theta)N/2\rceil$ or at $\lceil(1+\theta)N/2\rceil - 1$. However, since the proof of this result is considerably more cumbersome, we refrain from stating this result formally.

3.3 Ex ante payoffs under different majority rules

Proposition 3 shows how the majority rule affects individual voting behavior: When c is low, individual voters behave more conservatively under lower majority rules, and, when c is high, individual voters behave more conservatively under higher majority rules. We now consider what this implies for voters' ex-ante payoffs and the optimal majority rule.

Denote a player's ex-ante payoff, that is, his expected payoff given m, c and θ, but before the player's type is known, by $\pi(m,N,c,\theta)$. Since we focus on the effect of m on payoffs, we will from now on suppress (when no confusion can arise) the variables N and θ as arguments of all functions in order to save on notation. For $c \le \underline{c}(m)$, all voters vote for implementation in period 1, so that $\pi(m,c) = \tilde{\pi}(m,c) = 2(\theta - c)$. If $c > \bar{c}(m)$, then all voters reject the project in period 1 and so $\pi(m,c)$ coincides with the value of waiting, $U_W(c,m) = q(m)(\theta - c) + \theta p(m)(1-c)$. Finally, if $c \in (\underline{c}(m), \bar{c}(m)]$, then the project is approved in period 1 if and only if there are at least m high types. After a first period rejection, which occurs with probability $[1 - q(m) - \theta p(m)]$, the project may (in contrast to Section 3.1) still be implemented in period 2, so that $\pi(m,c) = \tilde{\pi}(m,c) + [1 - q(m) - \theta p(m)]U_W(c,m)$. Rearranging terms and dropping the arguments from the functions q and p, we thus have

$$\pi(m,c) = \begin{cases} 2(\theta - c) & \text{if } c \le \underline{c}(m) \\ 2q(\theta - c) + \theta p\,(1 + \theta - 2c) + (1 - q - \theta p)\,[q\,(\theta - c) + \theta p(1-c)] & \text{if } c \in (\underline{c}(m), \bar{c}(m)] \\ q\,(\theta - c) + \theta p(1-c) & \text{if } c > \bar{c}(m). \end{cases} \quad (5)$$

Clearly, like $\tilde{\pi}(\cdot)$ in the benchmark case, $\pi(m,\cdot)$ is a piecewise linear function of c that exhibits a downward jump at $\underline{c}(m)$ for any m, and, unless $m = N$, an upward jump at $\bar{c}(m)$. Figure 2 depicts the ex-ante payoff for $N = 15$, $\theta = 1/2$ and the cases $m = 8$ and $m = 9$.

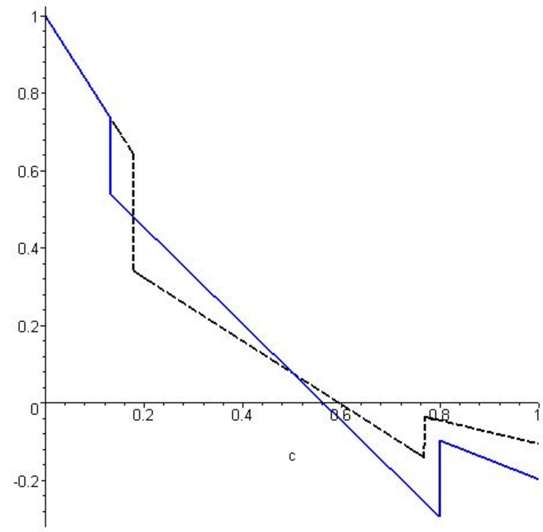


Figure 2: Ex-ante payoffs $\pi(8, 15, c, 1/2)$ (solid curve) and $\pi(9, 15, c, 1/2)$ (dashed curve)

We now turn to an analysis of the optimal majority rule for a given level of c, which may differ markedly from the benchmark case of Section 3.1. There are three different cases. First, if $c \leq \theta/2$, then each voter receives a positive ex-ante expected payoff from such a project, even if it is implemented independently of his own type. Thus, a rule that maximizes the probability of implementation is ex-ante optimal. If $c \leq \underline{c}(m)$, then the project is unanimously approved under rule m, and also under any larger majority rule $m' \geq m$ (as $\underline{c}(\cdot)$ is increasing in m). In particular, unanimity rule leads to implementation for the largest set of projects, $c \in [0, \underline{c}(N)]$. If, instead, $c \in (\underline{c}(N), \theta/2]$, then simple majority rule maximizes the implementation probability.

Second, if $c \in (\theta/2, \bar{c}(\lceil (1 + \theta)N/2 \rceil)]$,[19] then high types vote for and low types against first period implementation under any majority rule. For low values of c, the optimal rule is simple majority, because a high probability of first period implementation is socially optimal. In contrast, when c is relatively large, higher majority rules perform better.[20]

Third, if $c \geq (1 + \theta)/2$, then rules for which $\bar{c}(m) > (1 + \theta)/2$ cannot be optimal: At $c = \bar{c}(m)$, high types under majority rule m are indifferent between implementing immediately and waiting, and if $\bar{c}(m) > (1 + \theta)/2$, then their implementation payoff is negative. Consequently, the ex-ante expected payoff is negative, because all three components of this weighted average (the implementation payoffs of high and low types, and the payoff from waiting) are negative for all $c \geq (1 + \theta)/2$. In contrast, ex-ante expected payoffs are positive for all c under unanimity rule. Thus, for all $c \geq (1 + \theta)/2$, the optimal m satisfies $\bar{c}(m) \leq (1 + \theta)/2$. Hence, projects are unanimously rejected in the first period, so that the

[19]Remember that $\lceil (1 + \theta)N/2 \rceil$ is the majority rule at which $\bar{c}$ assumes its minimum.

[20]An exact analytical characterization of the optimal rule is more cumbersome here than in the benchmark case, since π is a nonlinear function of p and q. Since there are no deeper insights to be gained, we refrain from doing so.

ex-ante expected payoff coincides with the value of waiting $U_W(c, m)$. By Lemma 2, the value of waiting increases in m if and only if $m < cN$. Thus, the majority rule that maximizes the value of waiting is given by $m^*(c) = \lceil cN \rceil$. In particular, $m^*((1 + \theta)/2) = \lceil (1 + \theta)N/2 \rceil$. We show in the proof of Proposition 4 that this majority rule is also optimal for $c \in (\bar{c}(\lceil (1 + \theta)N/2 \rceil), (1 + \theta)/2]$.

Proposition 4. *1. For $c < \underline{c}(m)$ the ex ante expected payoff is maximized by any $m' \geq m$. In particular, unanimity rule is an optimal rule for all $c \leq \underline{c}(N)$ and strictly dominates simple majority for all $c \in (\underline{c}((N + 1)/2), \underline{c}(N))$.*

2. For $c > \bar{c}(\lceil (1 + \theta)N/2 \rceil)$ the (generically unique) optimal majority rule is given by $\max\{\lceil (1 + \theta)N/2 \rceil, \lceil Nc \rceil\}$.

Proof. See Appendix. □

It is also interesting that, for $c > (1 + \theta)/2$, voters may be willing to increase a low majority rule even *after* the first period types are realized. That is, such rules are not only suboptimal in an ex-ante sense, but also ex-post. Consider, for example, a project with $c \in \left[(1 + \theta)/2, \bar{c}\left(\frac{N+1}{2}\right)\right]$, and a society with a majority of high types. Under simple majority rule, the project is implemented in the first period by the support of high types. However, *all* voters (including high types) would be better off if society switched to unanimity rule, thereby killing the project in the first period. Thus, a change from simple majority rule to unanimity rule may be an ex-post Pareto improvement.[21]

3.4 Average ex-ante payoffs

The option to wait produces effects both in favor and against low majority rules, so it is again natural to ask which of these effects dominates when the voting rule cannot be conditioned on the project type, i.e. when c is drawn from some distribution F. To gain tractability for the proofs, we focus – in this and the following section – on the case that $\theta = 1/2$ (that is, voters have an equal chance of being either type). From the nature of our results (and continuity of solutions and value functions in the parameters), it is immediate that the results for $\theta = 1/2$ are robust in a neighborhood of $\theta = 1/2$. Moreover, in Section 3.6 we show that the results are also qualitatively robust for larger changes in θ if N is sufficiently large.

We denote the average ex-ante payoff of an individual voter under majority rule m by $\Pi(m, N)$. That is, $\Pi(m, N) = \int_0^1 \pi(m, N, c, 1/2) dF(c)$. Our first result obtains in the case of a large electorate and shows that the optimal majority rule with the option to wait is weakly larger than the optimal majority rule in the benchmark case. This result holds for a very large class of distributions of c.

[21] The fact that high types may choose to implement a project with a negative expected return even for themselves is an example of what Bai and Lagunoff (2007) call the *Faustian trade off* in politics, where today's policy is determined by a desire to influence either the identity or the set of feasible choices of a future policy maker. By implementing immediately, today's pivotal voters make sure that tomorrow's "leaders" have no power to make a decision.

Proposition 5. *Let $s \equiv m/N$ denote the proportional majority rule. Let S^* denote the set of ex-ante optimal proportional majority rules when voters have the option to wait, c is distributed according to F and N goes to infinity. That is,*

$$S^* = \{s^* | \lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N) \geq \lim_{N\to\infty} \Pi(\lceil sN \rceil, N) \text{ for all } s \in [1/2, 1]\}$$

Similarly, let

$$\tilde{S}^* = \{\tilde{s}^* | \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^* N \rceil, N) \geq \lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N) \text{ for all } s \in [1/2, 1]\}$$

be the set of optimal proportional majority rules in the limit when there is no option to wait. Then $\inf S^* \geq \sup \tilde{S}^*$ *for all distributions F with* $1 - F(3/4) + F(1/4) - F(1/6) > 0$ *(i.e., whenever* $\text{Prob}(c \in (1/6, 1/4] \cup (3/4, 1]) > 0$*).*

Proof. See Appendix. □

What are the intuitive benefits of increasing the majority rule? Consider a distribution that is symmetric around $c = 1/2$, so that (by Proposition 2) simple majority rule is optimal in the benchmark case. In a large society, all projects are implemented in period 2 with probability 1/2, and so, for given c, the value of waiting is $U_W = \frac{1}{2}\left(\frac{1}{2} - c\right)$. Thus, in period 1, low types agree to implement if and only if $\frac{1}{2} - 2c \geq \frac{1}{2}\left(\frac{1}{2} - c\right)$, hence whenever $c \leq 1/6$. Similarly, high types vote for immediate implementation whenever $c \leq 5/6$. Thus, in the first period, projects with $c \leq 1/6$ are always implemented those with $c > 5/6$ are never implemented, and those with $c \in (1/6, 5/6]$ are implemented with probability 1/2. Rejected projects are reconsidered in period 2, and each has a probability of 1/2 of gaining sufficient support for implementation.

Now consider the effect of a supermajority rule that requires approval of sN voters, where $s > 1/2$. Since the proportion of high types among voters is almost certainly close to 1/2, the project will not be implemented in period 2 under any supermajority rule. Thus, the value of waiting is zero. Furthermore, a project is implemented in period 1 if and only if low types agree, that is, if $\frac{1}{2} - 2c \geq 0$. Hence, under a supermajority rule, all projects with $c \leq 1/4$ are implemented in the first period, and no projects with $c > 1/4$ are ever implemented.

In period 1, a supermajority rule therefore makes low types more willing to implement low-c projects, and high types more reluctant to implement high-c projects, relative to simple majority rule. Obviously, both of these effects are socially desirable. Moreover, a supermajority rule prevents implementation of projects in the period 2. Note that projects that are considered in period 2 are an adverse selection from the set of all projects, because the projects with the lowest c have already been implemented in period 1. Since the initial distribution of c was symmetric around 1/2, the average ex-ante payoff from a project that is still available in period 2 is negative, so that a supermajority rule is strictly better in that period.

Since Proposition 5 holds for a large class of cost distributions, it cannot characterize the optimal majority rule with the option to wait precisely. Thus, while the optimal majority rule cannot decrease relative to the benchmark case, it remains unclear whether and by how much the optimal majority rule increases, and how this depends on N. In particular, as we show in the proof of Proposition 5, *any* supermajority rule yields the same ex-ante expected surplus in the limit, and thus the limit case cannot inform us on whether (in small or medium-sized electorates), the optimal majority rule is close to simple majority or unanimity rule. Proposition 6 below is therefore an important complement to Proposition 5: It shows that, if c is uniformly distributed on $[0, 1]$ (i.e., a case where simple majority rule is optimal without the option to wait), the option to wait leads to a substantial increase in the optimal majority rule for any N.

Proposition 6. *Suppose that c is ex-ante uniformly distributed on* $[0, 1]$*, so that* $\Pi(m) = \int_0^1 \pi(m, c)dc$.

i) $\Pi(m + 1) - \Pi(m) < 0$ *for all* $m \geq 2N/3$ *and*

ii) $\Pi(m + 1) - \Pi(m) > 0$ *for all* $(N + 1)/2 \leq m < 7N/11$.

Moreover, $\Pi((N + 1)/2) < \Pi(N)$.

Proof. See Appendix. □

While Proposition 6 does not determine the optimal majority rule exactly, it is clear from (i) that the optimal majority rule is at most $\lceil 2N/3 \rceil$, i.e., the lowest majority rule that is higher than a two-thirds majority. From (ii), it follows that the optimal majority rule is a supermajority rule with $m/N \geq 7/11 \approx 0.636$. In particular, if the number of voters N is large, then the optimal majority rule as a percentage of the electorate lies either within or arbitrarily close to the interval $[7/11, 2/3]$.

Interestingly, when the option to wait is introduced, simple majority not only loses its status as the optimal majority rule, but it actually becomes the worst majority rule. It is dominated even by unanimity (which is the worst of all supermajority rules that have $m \geq \lceil 2N/3 \rceil$). Thus, loosely speaking, choosing a "too high" supermajority rule has a lower welfare cost than choosing a majority rule that is "too low".

While Proposition 6 holds for the uniform cost distribution, it is intuitive that the result is robust. For different cost distributions that are 'close' to a uniform distribution, the optimal majority rule would be close to the one characterized in Proposition 6, and thus a supermajority rule. For example, one can show that, for any density of the distribution that satisfies $1/4 \leq f(c) \leq 2$ for all c, a supermajority rule is ex-ante better than a simple majority rule.[22]

3.5 Does the option to wait increase the welfare of voters?

In settings with a single decision maker, the option to postpone a decision always weakly increases the decision maker's expected profit: The decision maker can still choose to go ahead and invest immediately,

[22]The (rather tedious) proof of this claim is available from the authors upon request. Also note that, while there may be even weaker assumptions under which supermajority rules are optimal, there are some distributions for which the result does not hold. For example, if $c = 1/4$ with certainty, then simple majority rule is optimal for any N, with or without the option to wait.

but sometimes he may strictly prefer to wait. While our setup here is similar, the answer to the title question is not obvious, as the option to wait influences a *game* between different voters, rather than the decision problem of a single decision maker.

Indeed, for some values of c, the option to wait hurts citizens from an ex-ante perspective.[23] For example, projects with $c > 3/4$ are never implemented without the option to wait, as even the first period high types have a negative expected profit from their implementation. In contrast, with the option to wait and simple majority rule, each project that was rejected in period 1 has a 50 percent chance of being accepted in period 2, and for most of these projects, the percentage of winners is smaller than c, making the project socially undesirable. Moreover, $\bar{c}(m) > 3/4$ for many majority rules, so that some projects that would definitely be rejected without the option to wait are actually implemented in period 1 with positive probability.

However, there are also project types for which the option of waiting increases expected social welfare. For instance, if $c = 1/4 + \varepsilon$, then a project may be rejected in period 1. Without the option to wait, such a rejection is final, while there is a second period chance for implementation with the option to wait, which is socially beneficial (for small ε).

Thus, there exist some cost levels for which ex-ante welfare increases, and others where welfare decreases with the option to wait. Again, it is interesting to see which effect dominates from an ex-ante perspective. Proposition 7 shows that the option to wait often harms voters in expectation. Part 1 considers the limit case of $N \to \infty$; under some condition on the distribution (which is satisfied, for example, by any distribution that is symmetric around 1/2), the option to wait cannot strictly benefit voters. Part 2 again specializes to a uniform distribution of c and shows that, for a large range of low supermajority rules, the option to wait lowers ex-ante payoffs, and only under very high majority rules, the option to wait is guaranteed to have a positive social value in terms of average ex-ante payoffs. In particular, for $N > 5$, we show that the expected payoff without the option to wait and simple majority rule dominates the expected payoff with the option to wait and the optimal supermajority rule.

Proposition 7.

1. *Let s^* and $\tilde{s}^*$ be defined as in Proposition 5, and assume that F satisfies $\int_{1/4}^{3/4}(1-2c)dF(c) \leq 0$. Then, in a large electorate, the ex-ante expected utility is weakly lower with the option to wait:* $\lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N) \leq \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^* N \rceil, N)$.

2. *Suppose that $F(c) = c$. If $N > 3$ and $m \leq \lfloor 3N/4 \rfloor$ then $\tilde{\Pi}(m) > \Pi(m)$. If, instead, $m \geq 13N/16$ then $\tilde{\Pi}(m) < \Pi(m)$.*

3. *Suppose that $F(c) = c$. For $N > 5$, $\max_m \tilde{\Pi}(m) > \max_m \Pi(m)$: The maximal average ex-ante payoff is strictly lower if voters have the option to wait.*

Proof. See Appendix. □

[23]The earliest paper that has shown that the value of the option to wait may be negative is Gersbach (1993b), who constructs this result in an example with three players and correlated valuations in the second period. See also Gersbach (1993a).

Parts 2 and 3 of Proposition 7 provide a generic and robust example that contrasts starkly with the value of waiting in individual decision problems, where an individual decision maker would always strictly benefit from the option to wait. For an intuition, consider a setting where N is large. Under simple majority rule without the option to wait, all projects with $c \leq 1/4$ are unanimously implemented, just as under the optimal supermajority rule. In addition, however, projects with $c \in (1/4, 3/4]$ are implemented under simple majority rule if and only if a majority of voters has a high type, and this is, on average, better (from an ex-ante perspective) than not implementing any of these projects.[24] Again, nothing in this argument relies on c being drawn from a uniform distribution, and the result appears thus quite robust.

3.6 Average ex-ante payoffs for general θ

In this section, we generalize the results of Propositions 6 and 7 to the case of a generic θ. In doing so, we focus on the case of a uniformly distributed c. Proposition 8 describes the approximately optimal proportional majority rule for a large electorate.

Proposition 8. *Suppose that c is ex-ante uniformly distributed on* $[0, 1]$ *so that* $\Pi(m, N, \theta) = \int_0^1 \pi(m, N, c, \theta)$. *Let* $m^*(\theta, N) \in \arg\max_m \Pi(m, N, \theta)$.

i) Let $\underline{\theta} \in [0, 1]$ *be the solution of the equation* $6 - 12\theta + 5\theta^2 = 0$. *If* $\theta < \underline{\theta}(\approx 0.63)$, *then* $\lim_{N\to\infty} m^*(\theta, N)/N = \bar{s}(\theta) = 3(2 - \theta^2)/[4(3 - 2\theta)]$.

ii) Let $\bar{\theta}$ *be the solution of the equation* $3 - 6\theta + 2\theta^2 = 0$. *If* $\theta > \bar{\theta}(\approx 0.71)$, *then* $\lim_{N\to\infty} m^*(\theta, N)/N \approx \underline{s}(\theta) = (3 - 2\theta)/(4 - 2\theta)$.

iii) If $\theta \in [\underline{\theta}, \bar{\theta}]$ *then* $\lim_{N\to\infty} m^*(\theta, N)/N \in [\underline{s}(\theta), \bar{s}(\theta)]$.

$\bar{s}$ is increasing and satisfies $\bar{s}(0) = 1/2$ *and* $\bar{s}(\underline{\theta}) = (9 - 2\sqrt{3})/8 \approx 0.69$. $\underline{s}(\theta)$ *instead is decreasing and satisfies* $\underline{s}(\bar{\theta}) = (3 + 2\sqrt{6})/(8 + 2\sqrt{6}) \approx 0.61$ *and* $\underline{s}(1) = 1/2$.

Proof. See Appendix. □

The approximately optimal proportional majority rule (or its bounds) is depicted in Figure 3. For all θ the optimal majority rule is a supermajority rule. Moreover, the optimal rule is largest for some θ in the neighborhood of $2/3$, where $s^*(\theta)$ crosses θ.

In Proposition 8 we assume that there are many voters. It is very difficult to obtain any analytic results for general N. However, it can be verified numerically that for $\theta \notin [\underline{\theta}, \bar{\theta}]$ the optimal proportional rule, $s^*(\theta)$, is optimal also for small N in the sense that $m^*(\theta, N) = \lceil s^*(\theta)N \rceil$. That is, the difference in

[24] Clearly, this argument requires that N is large, but finite, because when we take N to infinity, then $\lim \Pi(m, N) = \lim \tilde{\Pi}(m, N)$.

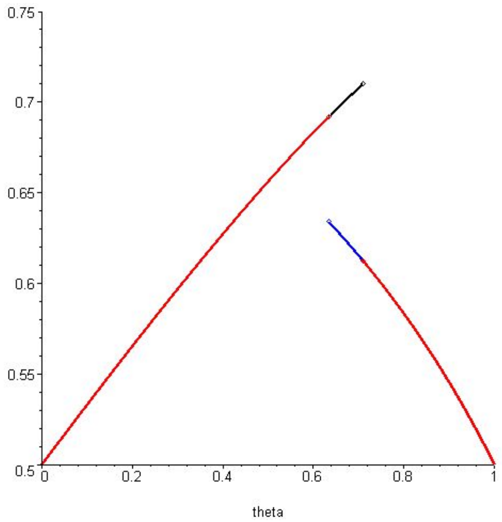


Figure 3: $s^*(\theta)$ (red) and upper and lower bound for $s^*(\theta)$ (black, blue)

the optimal proportional rules for small and large N is a consequence only of the fact that m must be an integer.

Proposition 9 below analyzes the same setup as Proposition 7 (large electorate, c is uniformly distributed), and compares the payoff under the optimal majority rules with and without the option to wait. Remember that $\tilde{\Pi}$ achieves its maximum always at simple majority. On the other hand, Proposition 8 shows that the majority rule that maximizes Π is always a supermajority rule. Thus, for $\theta < 1/2$, the optimal proportional rules under both regimes are larger than θ, and thus the implied difference in the implementation probabilities are small when N is large (the project is unlikely to be implemented both in the first and in the second period). Therefore, for all $\theta < 1/2$, $\lim_{N\to\infty}(\max_m \tilde{\Pi} - \max_m \Pi) = 0$.

If, instead, $\theta \in (1/2, \bar{\theta})$, we have $1/2 < \theta < s^*(\theta)$. That is, the respectively optimal majority rules lie on opposite sides of θ so that implementation probabilities differ substantially even for large N. Since $\theta > 1/2$ also means that the project yields on average a positive payoff it follows that lower majority rules are better on average, that is $\max_m \tilde{\Pi} > \max_m \Pi$.

For $\theta > \bar{\theta}$ we know that both optimal proportional rules are smaller than θ which again means that the first period implementation probabilities are similar (and large). But under the regime where the decision may be reconsidered after a rejection there is a high chance that the worst projects (those which are unanimously rejected in period 1) are implemented in period 2. This again implies that $\max_m \tilde{\Pi} > \max_m \Pi$. Finally, for $\theta \in [\underline{\theta}, \bar{\theta}]$ either one or both of the previously described effects are present. Consequently, the expected payoff is higher when the electorate cannot delay the decision on the project.

Proposition 9. *Suppose that c is ex-ante uniformly distributed on $[0, 1]$ so that $\Pi(m, N, \theta) = \int_0^1 \pi(m, N, c, \theta)dc$ and $\tilde{\Pi}(m, N, \theta) = \int_0^1 \tilde{\pi}(m, N, \theta)dc$.*

i) $\lim_{N\to\infty} \max_m \tilde{\Pi}(m, N, \theta) = \lim_{N\to\infty} \max_m \Pi(m, N, \theta)$ *for all* $\theta \leq 1/2$*;*

ii) $\lim_{N\to\infty} \max_m \tilde{\Pi}(m, N, \theta) > \lim_{N\to\infty} \max_m \Pi(m, N, \theta)$ *for all* $\theta > 1/2$.

Proof. See Appendix. □

Figure 4 shows the loss as percentage of $\max_m \tilde{\Pi}(m, N, \theta)$ due to the option to wait (assuming that the majority rule is optimally adjusted).

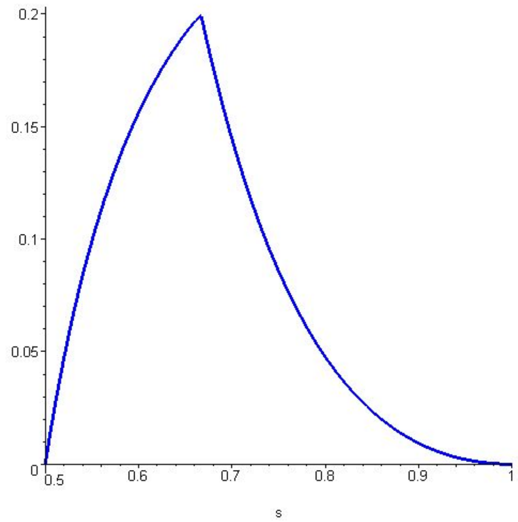


Figure 4: Percentage loss $\frac{\max_m \tilde{\Pi} - \max_m \Pi}{\max_m \tilde{\Pi}}$ due to the option to wait

4 Robustness and Extensions

In this section, we want to explore the robustness of the model when we loosen some of our assumptions. In Section 4.1, we briefly discuss some modeling choices. In Section 4.2, we analyze a setting where second period valuations of individuals are correlated with each other, so that voters are more likely to agree with the majority of other voters ex-post. In Section 4.3, the first- and second-period valuations of each individual are correlated, that is, first period high types are more likely to be second period high types than first period low types.

4.1 Discussion of modeling choices

Available choices. We restrict society to make a decision through voting and assume that project proposals cannot contain transfer payments between different voters. If, instead, types are observable and transfer payments are feasible, then, by the Coase theorem, any majority rule leads to implementation if and only if the project creates more benefits than costs. The assumption that transfer payments are not feasible is standard in most of the political economy literature and also appears to be quite realistic in many applications, for example because of informational constraints or legal provisions against vote buying.[25] However, we do not model explicitly why this is the case.

The decision in the first period is restricted to the first-period implementation decision. For example, the first period electorate cannot choose to wait *and* commit the second period electorate to implement in the second period, or cannot choose to wait *and* forbid the second period electorate to consider implementation. Similarly, they cannot change the majority rule for the second period. We also do not allow society to choose different majority rules to apply for "first-period projects" and "second-period projects". Again, the reason is that it might be very difficult to describe ex-ante whether an investment proposal falls in the first or the second category.

While there are cases in which a majority of the first period electorate would like to take such measures, the assumption that today's electorate cannot commit a future electorate is both standard in the literature, and quite realistic for most democratic institutions, as such attempts would be very controversial (at least ex-post).

Dynamic framework. The purpose of the model is to provide a simple framework for the analysis of the effects of intertemporal information arrival under different majority rules on implementation decisions. To keep the setup as simple as possible, our model has only two time periods. It is, in principle, not too difficult to extend this model to a setup with payoffs in infinitely many periods; however, a key assumption is that voters detect their preference for or against the project after some finite time, so that uncertainty is concentrated in early periods. In many applications, this appears realistic.

For example, we could generalize our model as follows. Once a project is implemented, it generates an infinite stream of payoffs for each voter (depending on the voter's type, as in our model, and discounted using a discount factor of δ). In the first period, voters know only their first period type. In the second period, they detect whether they are a high or low type for the remaining periods (or, more generally, the frequency with which they will be high types in the future). Thus, voting behavior from the second period on will be type dependent and thus, implementation either occurs in one of the first two periods, or not at all. As in our model, backwards induction can then be used to determine first period voting behavior.

Distribution of valuations. In our basic model, there are only two different payoff types of voters, high types with payoff $1 - c$ and low types with payoff $-c$. This formulation has the advantage that voting behavior in the second period depends only on the voter's type and not on the type of the project c. Consequently, the probability of implementation in the second period (conditional on no first period implementation) is independent of c, and the value of waiting is a linear function of c.

If, instead, the distribution of voter payoff types is continuous, then voter i votes for project implementation in the second period if $V_2^i \geq c$. Thus, the probability of second period implementation, as well as the analogues of $p(m)$ and $q(m)$, also depend on c. Consequently, the value of waiting is a rather complicated polynomial of c and $F_V(c)$, where F_V is the cumulative distribution function of V. Even

[25] For similar reasons, it is not our objective to solve the general mechanism problem for our setting.

if one were to assume specific distribution functions for V and for c, the continuous model is relatively intractable compared to our basic model. A more useful and tractable way of introducing an influence of the type of project on second period voting behavior is discussed in Section 4.2 below, where the second period type realizations of voters are correlated.

4.2 Systematic second-period risk

We now consider the effect of correlation between voter types in period 2. Specifically, in period 1, let $\theta = 1/2$; at the beginning of period 2, nature draws a parameter μ from a uniform distribution on $[0, 1]$; then, each voter is assigned a high type with probability μ (and, correspondingly, a low type with probability $1 - \mu$). Note that this variation of the model would not at all affect the expected utility or first period actions of a single decision maker (relative to a reference case where $\theta = 1/2$ in both periods), as, from the perspective of period 1, the expected probability of being a high type in period 2 remains at $1/2$. Consequently, both the implementation payoff and the value of waiting remain unchanged.

However, the model variation introduces correlation between the types of different voters in the second period: The probability of voter i being a high type in the second period, conditional on voter $j \neq i$ being a high type in that period, is $\text{Prob}(V_2^i = 1|V_2^j = 1) = \frac{\text{Prob}(V_2^i=1 \cap V_2^j=1)}{\text{Prob}(V_2^j=1)} = \frac{\int_0^1 \mu^2 d\mu}{\int_0^1 \mu d\mu} = 2/3$, while this probability is equal to $1/2$ when $\theta = 1/2$ in both periods. Effectively, while voters still do not know in the first period whether they will like the project in the second period, they are now more likely than in the basic model to agree with the majority of the other voters about the desirability of the project ex-post: Say, if μ turns out to be high, then it is likely that a particular voter i is a high type, and also likely that the majority of other voters agrees.

For example, consider the job market example from Section 2.2. Suppose that, whether a particular faculty member "likes" next year's candidate (i.e., receives a positive net payoff from the candidate being hired) depends stochastically on that candidate's "quality" (i.e., μ). A high quality candidate is more likely to be liked by each existing faculty member than a low quality candidate, so there is correlation between the opinions of different voters. However, if the department rejects this year's candidate, its voters do not know the quality of the (feasible) candidates next year, so next year's μ is a random variable from today's perspective.

What is the value of waiting in this setup? Just as in the basic model, we can condition on whether voter i is or is not pivotal for second period implementation. Writing $p_2(m, N, \mu)$ and $q_2(m, N, \mu)$ for the obvious generalizations of the functions $p(m, N)$ and $q(m, N)$ from the basic model, we have

$$U_W(c, m) = E_\mu \left\{ E[\max\{V_2^i - c, 0\}|\mu] p_2(m, \mu) + (E(V_2^i|\mu) - c) q_2(m, \mu) \right\} =$$
$$\int_0^1 \left[\binom{N-1}{m-1} \mu^{m-1} (1-\mu)^{N-m} \cdot \mu(1-c) + \sum_{k=m}^{N-1} \binom{N-1}{k} \mu^k (1-\mu)^{N-1-k} \cdot (\mu - c) \right] d\mu. \tag{6}$$

The first term in (6) refers to the case that individual i is pivotal, which happens with probability $p_2(m, N, \mu) = \binom{N-1}{m-1} \mu^{m-1} (1-\mu)^{N-m}$; then, with probability μ, individual i is a high type and votes for implementation, in which case his payoff is $1 - c$. The second term refers to the case that there are at least m high types among the other voters; in this case, voter i's expected type is μ, so that his expected implementation payoff is $(\mu - c)$.

As in the basic model, intersecting $U_W(c, m)$ with $U_I(0, c)$ and with $U_I(1, c)$ yields the values for $\underline{c}(m)$ and $\bar{c}(m)$, respectively. All projects with $c \leq \underline{c}(m)$ are unanimously approved in the first period; those with $c > \bar{c}(m)$ are unanimously rejected in the first period; and for those with $c \in (\underline{c}(m), \bar{c}(m)]$, voting is type dependent and the voting outcome in period 1 depends on the realization of voter types.

Observe that the function $U_I(V_1, c)$ is linear in V_1, so $E_{V_1} U_I(V_1, c) = U_I(EV_1, c)$. Therefore, if we assume in addition that c is uniformly distributed on $[0, 1]$, we can write the ex-ante expected utility, given majority rule m, as

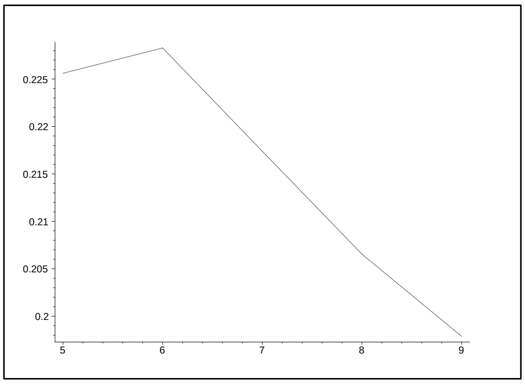
$$\begin{aligned}\Pi(m) = &\int_0^{\underline{c}(m)} U_I(0.5, c)dc + \int_{\bar{c}(m)}^1 U_W(c, m)dc + \\ &\int_{\underline{c}(m)}^{\bar{c}(m)} \left[\sum_{k=m}^{N} \binom{N}{k} \left(\frac{1}{2}\right)^N U_I\left(\frac{k}{N}, c\right) + \left(1 - \sum_{k=m}^{N} \binom{N}{k} \left(\frac{1}{2}\right)^N\right) U_W(c, m)\right] dc\end{aligned} \tag{7}$$

The first of these terms corresponds to those projects that have $c \leq \underline{c}(m)$ and are all implemented in period 1. All projects with $c > \bar{c}(m)$ are rejected in period 1, and each voter obtains the value of waiting. The third integral corresponds to projects with a cost between $\underline{c}(m)$ and $\bar{c}(m)$; if there are $k \geq m$ high types in period 1, these projects are implemented immediately and generate a per-capita utility of $U_I(k/N, c)$; otherwise, if $k < m$, the project is delayed and each voter obtains the value of waiting.

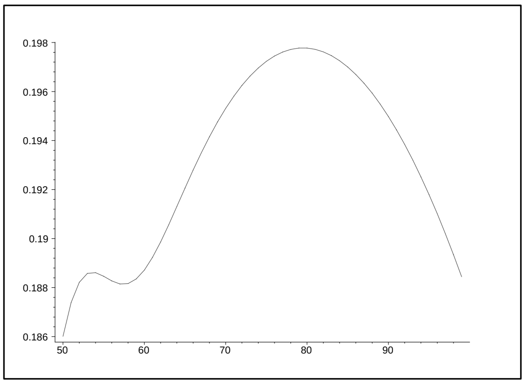
Figure 5 shows the function $\Pi(m, N)$ for $N = 9$, $N = 99$ and $N = 199$. For $N = 9$ in part (a), the optimal majority rule is $m = 6$, that is, a two-thirds majority rule, just as in the comparison case of $\theta = 1/2$ in both periods. For $N = 99$ in part (b) and $N = 199$ in (c), the optimal m is approximately equal to $0.8N$, respectively. Thus, the optimal majority rule in these cases increases relative to the comparison case without correlation.

Intuitively, why does the optimal supermajority rule increase? Remember that, in the comparison case without correlation, the optimal supermajority rule of approximately $2/3$ prevents almost all second period implementations when N is large. Almost all projects that are implemented at all have $c \leq \underline{c} \approx 1/4$, and are implemented unanimously in period 1.

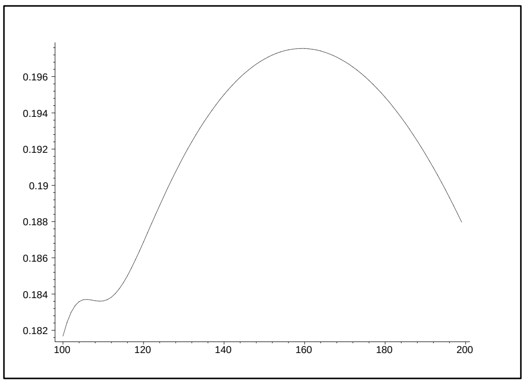
Now consider what happens with correlation under the same majority rule. If the draw of μ is low, then the project will still be rejected, but if μ is high, then it is quite likely that sufficiently many voters agree in the second period to implement the project. Thus, the value of waiting increases, and low types become less inclined to implement a project already in period 1 (so $\underline{c}$ decreases). From an ex-ante perspective, projects with types between the old and the new level of $\underline{c}$ are socially very valuable; since their c is less than $1/4$, from a per-capita perspective, they should be implemented based on their first period payoffs for high types alone (the average per-capita surplus of these projects is positive even if all voters turn out to be low types in the second period). Thus, the decrease of $\underline{c}$ is very inefficient. Increasing the majority rule reduces the value of waiting, and thus, more projects are accepted (unanimously) in the



(a) $N = 9$



(b) $N = 99$



(c) $N = 199$

Figure 5: Ex-ante expected utility for different majority rules

first period. This effect outweighs the cost that, with a higher majority rule, slightly more, on average beneficial, projects are rejected in the second period.

To gain more insight, consider the case of a very large society with proportional majority rule s. In period 2, the project is implemented if and only if $\mu > s$. Thus, the value of waiting is

$$\mathcal{U}_W(c,s) \equiv \lim_{N\to\infty} U_W(c, \lceil sN \rceil, N) = \text{Prob}(\mu \geq s)\left[E(\mu|\mu \geq s) - c\right] = (1-s)\left(\frac{1+s}{2} - c\right). \tag{8}$$

It is easy to check that $\mathcal{U}_W(c,s)$ is maximal for $s = c$. Intuitively, in the second period a utilitarian social planner would accept a project if and only if the realized percentage of winners μ is greater than c.

When are first period low types just indifferent between implementing the project and waiting? Solving $U_I(0,c) = \mathcal{U}_W(c,s)$ yields

$$\underline{c} = \frac{s^2}{2(1+s)} \tag{9}$$

Similarly, high types are indifferent between implementing in period 1 and waiting if

$$\bar{c} = \frac{1 + \frac{s^2}{2}}{1+s} \tag{10}$$

For simple majority rule, (9) and (10) imply $\underline{c} = 1/12$ and $\bar{c} = 3/4$, and projects with types between $\underline{c}$ and $\bar{c}$ are implemented with probability $1/2$. Thus, expected utility under simple majority rule is

$$\begin{aligned}\lim_{N\to\infty} \Pi\left(\frac{N}{2}, N\right) = \int_0^{1/12} (1-2c)dc + \int_{1/12}^{3/4} \left[\frac{1}{2}(1-2c) + \frac{1}{4}\left(\frac{3}{4} - c\right)\right] dc \\ + \int_{3/4}^{1} \frac{1}{2}\left(\frac{3}{4} - c\right) dc = \frac{11}{64} = 0.171875.\end{aligned} \tag{11}$$

Consider now a supermajority rule $s \geq 0.5 + \varepsilon$ (for some $\varepsilon > 0$). Since the percentage of high types in period 1 is almost certainly within ε of 0.5, if $c > \underline{c}(m,N)$, there are almost never enough high types to implement the project in period 1. Thus, expected ex-ante utility under supermajority rule s is

$$\begin{aligned}\lim_{N\to\infty} \Pi(\lceil sN \rceil, N) &= \int_0^{\underline{c}} (1-2c)dc + \int_{\underline{c}}^{1} \left(\frac{1+s}{2} - c\right)(1-s)dc \\ &= (1-\underline{c})\left[\underline{c} + \frac{1-s^2}{2} - \frac{1-s}{2}(1+\underline{c})\right] = \frac{s(2-s)(2+2s-s^2)}{8(1+s)}\end{aligned} \tag{12}$$

where the last line follows from substituting (9). Differentiating (12) with respect to s yields

$$\frac{3s^4 - 4s^3 - 10s^2 + 4s + 4}{8(1+s)^2}$$

Setting this equal to zero and solving yields that the optimal supermajority rule is approximately $s^* = 0.7985$.[26] Substituting the optimal value into Π yields an expected utility of about 0.1973, which is larger than the ex-ante utility under simple majority rule. Therefore, a supermajority rule of approximately 80% is optimal in the limit, which corresponds very well to the maximum in the graphs of Figure 5.

[26]The second order condition is satisfied at s^*.

4.3 Intertemporal correlation

We now consider the case that each voter's first and second period type are positively correlated. Specifically, we assume that each voter has an equal chance of being a high or a low type in period 1, and that his period 2 type coincides with his period 1 type with probability $r \in [0.5, 1]$, i.e. $Prob(V_2^i = 1|V_1^i = 1) = Prob(V_2^i = 0|V_1^i = 0) = r$.

Benchmark: No option to wait. We start by considering the case of a one-off vote in period 1. A high type gets an immediate payoff of 1, and is a second period high type with probability r. A low type gets zero in period 1, and is a second period high type with probability $1 - r$. The net present value for voter i, given c and r, is thus

$$U_I(V_1^i, c, r) = V_1^i + E[V_2^i|V_1^i] - 2c = \begin{cases} 1 + r - 2c & \text{if } V_1^i = 1 \\ 1 - r - 2c & \text{if } V_1^i = 0. \end{cases} \tag{13}$$

If the first period decision is final, high types vote in favor of projects with $c \leq (1 + r)/2$, while low types only vote in favor of projects with $c \leq (1 - r)/2$. The stronger is the correlation across periods (i.e., the higher is r), the more extreme are these cost thresholds, because low types have only a very slight hope that they will profit from the project in period 2, while high types are very confident that they will remain high types. Essentially, the higher is r, the more the social decision problem resembles a situation with known benefits.

The ex-ante expected payoff of a project with cost c, given r and m, is

$$\tilde{\pi}(m, c, r) = \begin{cases} 1 - 2c & \text{if } c \leq \frac{1-r}{2} \\ q(m)(1 - 2c) + \frac{p(m)}{2}(1 + r - 2c) & \text{if } \frac{1-r}{2} < c \leq \frac{1+r}{2} \\ 0 & \text{if } c > \frac{1+r}{2} \end{cases} \tag{14}$$

Again, the ex ante equilibrium payoff is a piecewise linear function of c that exhibits a downward jump at $(1 - r)/2$ and an upward jump at $(1 + r)/2$. Following the same arguments as in Proposition 2, we can show that the optimal majority rule for a given c is

$$m^* = \lceil N(2c - (1 - r))/2r \rceil,$$

which is decreasing in r. Thus, correlation strengthens the case for low majority rules, and it is intuitive that the result of Proposition 2 also holds here: Without the option to wait, if project types are drawn from some distribution that is symmetric around 1/2, then simple majority rule is optimal.

The option to wait and intertemporal correlation. We now turn to the case that society can implement the project in period 2, if it was turned down in period 1. In order to characterize first period voting behavior, we need to find the value of waiting. In contrast to the basic model, it matters here whether individuals can observe the payoff types of other voters. The number of first period high types influences the distribution of the number of second period high types, and thus the probability of implementation in period 2. Thus, if voters can observe the first period types of other voters, they will condition their behavior on it. If, instead, types are only privately observed, then each voter has to take into account the first period type distribution conditional on the event that his vote is decisive in the first period election.

In what follows, we assume that payoff types are publicly observed. We make this assumption for two reasons. First, this assumption is probably reasonable for applications with small electorates. Second, a model with publicly observed types is more tractable than a model with privately observed payoff types. In particular, with publicly observable payoff types, iterated elimination of weakly dominated strategies still delivers a unique strategy profile (up to tie breaking in situations where individuals are indifferent between their two first period actions, independently of other voters' behavior). The same is not true for a model with privately observed types, where the voting game may exhibit multiple (sequential) equilibria in iteratively weakly undominated strategies. A sufficiently detailed exposition of such a game would considerably increase the length of the paper. Moreover, in qualitative terms, the interesting results do not change substantially from the case considered here.[27]

In our setting, voter i's value of waiting depends not only on the project type c and the majority rule m, but also on the intertemporal correlation parameter r, voter i's first period type and the number of high types among other voters, h, which determines the distribution of the number of second period high types and hence the probability of second period implementation. Formally, we have

$$\begin{aligned} U_W(V_1^i, c, m, h, r) &= E[\max\{V_2^i - c, 0\}|V_1^i]p_2(m,h,r) + (E[V_2^i|V_1^i] - c)q_2(m,h,r) \\ &= \begin{cases} (1-c)(1-r)p_2(m,h,r) + (1-r-c)q_2(m,h,r) & \text{if } V_1^i = 0 \\ (1-c)rp_2(m,h,r) + (r-c)q_2(m,h,r) & \text{if } V_1^i = 1. \end{cases} \end{aligned} \tag{15}$$

The functions p_2 and q_2 are generalizations of the functions $p(\cdot)$ and $q(\cdot)$ and represent the probability of being pivotal in period 2, and the probability that the project will pass in period 2 independently of voter i's will, respectively. In order to formally define p_2 and q_2, consider the transition function

$$t(\ell, k, r) = \sum_{j=0}^{k} \binom{\ell}{j} \binom{N-1-\ell}{k-j} (1-r)^{k+l-2j} r^{N-\ell-1-k+2j},$$

which describes the probability of moving from a first period type profile in which ℓ of the $N-1$ other players have high types to a second period profile in which k of them have high types. The functions p_2 and q_2 can now be written as

$$p_2(m,h,r) = t(h, m-1, r) \quad \text{and} \quad q_2(m,h,r) = \sum_{j=m}^{N-1} t(h, j, r).$$

Just as in the basic model, both (13) and (15) are linearly decreasing functions of c. Moreover, (13) decreases faster than (15). Clearly, for c sufficiently close to 0, implementing the project immediately is

[27] A formal analysis of the voting game with privately observed types is available from the authors upon request.

strictly better than waiting, irrespective of the values of V_1^i, h, m and r. Similarly, for c sufficiently close to 1, delaying the project dominates investing immediately for all parameter values. Thus, for each triple (h, m, r), there are thresholds $\underline{c}(h, m, r)$ and $\bar{c}(h, m, r)$ at which low and high types switch from approval to rejection, respectively. Since

$$U_W\left(0, \frac{1-r}{2}, m, h, r\right) = \frac{(1-r)}{2}((1+r)p_2(m, h, r) + q_2(m, h, r)) > 0 = U_I\left(0, \frac{1-r}{2}, r\right),$$

it follows that $\underline{c}(m, h, r) < (1-r)/2$ for all (m, h, r). Similarly, for high types we have

$$U_W(1, m, r, h, r) = (1-r)rp_2(m, l, r) < 1 - r = U_I(1, r, r),$$

which implies that $\bar{c}(m, h, r) > r$ for all (m, h, r). Results that parallel Proposition 3 for the behavior of $\underline{c}$ and $\bar{c}$ can be obtained, but, in order to save some space, we refrain from presenting them explicitly.

Turning to the definition of the ex-ante expected payoff, it is convenient to calculate this in two steps: First, we integrate over a player's payoff type, given that h of the other players are high types. Let $C^0_{m,h,r} = \{c | c \leq \underline{c}(m, h, r)\}$, $C^1_{m,h,r} = \{c | c \in (\underline{c}(m, h, r), \bar{c}(m, h, r)]\}$, and $C^2_{m,h,r} = \{c | c > \bar{c}(m, h, r)\}$. For any triple (c, m, r), voter i's expected payoff, conditional on h of the other voters being high types, is

$$\pi_h(m, N, c, r) = \begin{cases} \dfrac{U_I(1, c, r) + U_I(0, c, r)}{2} & \text{if } c \in C^0_{m,h,r} \vee (c \in C^1_{m,h,r} \wedge h \geq m) \\ \dfrac{U_I(1, c, r) + U_W(0, m, c, h, r)}{2} & \text{if } c \in C^1_{m,h,r} \wedge h = m - 1 \\ \dfrac{U_W(1, m, c, h, r) + U_W(0, m, c, h, r)}{2} & \text{if } c \in C^2_{m,h,r} \vee (c \in C^2_{m,h,r} \wedge h < m - 1) \end{cases}$$

Second, we now take the expectation with respect to h, which gives

$$\pi(m, N, c, r) = \frac{1}{2^{N-1}} \sum_{h=0}^{N-1} \binom{N-1}{h} \pi_h(m, N, c, r).$$

Again, this is a piecewise linear function of c. Since both high and low type voters have multiple thresholds (one for each h) at which their behavior switches, $\pi(m, c, r)$ exhibits multiple discontinuities. Figure 6 shows the expected payoffs for $m = 8$ and $m = 9$ for the case that $r = 2/3$ and $N = 15$.

In comparison to the basic model, intertemporal correlation strengthens the case for simple majority rule. This is clear for $r = 1$, because the dynamic structure of our model then becomes irrelevant: If individuals' benefits are constant over time, then the set of voters who approve remains constant, and hence, a project is either implemented at once or never. This is exactly the same behavior as in the benchmark model without the option to wait, where we know that simple majority maximizes the ex-ante payoff. Intuitively, as r increases from $1/2$ (i.e., the basic model) to $r = 1$ (i.e., perfect correlation), the optimal supermajority rule decreases. It is an interesting quantitative question to consider for which levels of r the optimal supermajority rule switches.

Figure 7 shows, for $N = 15$, the average ex-ante payoff $\Pi(m, N, r) = \int_0^1 \pi(m, N, c, r)dc$ as a function of r for different majority rules. Consider first the three lower curves that intersect each other. The blue

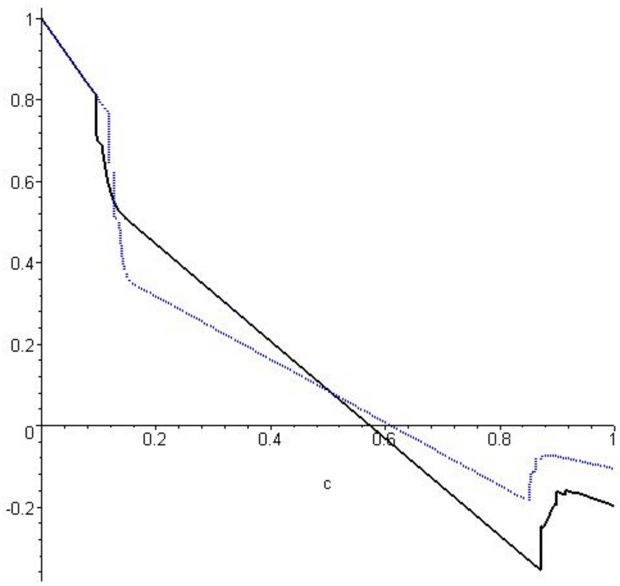


Figure 6: Ex-ante payoff $\pi(c)$ for $N = 15$, $r = 2/3$, $m = 8$ (solid) and $m = 9$ (dashed)

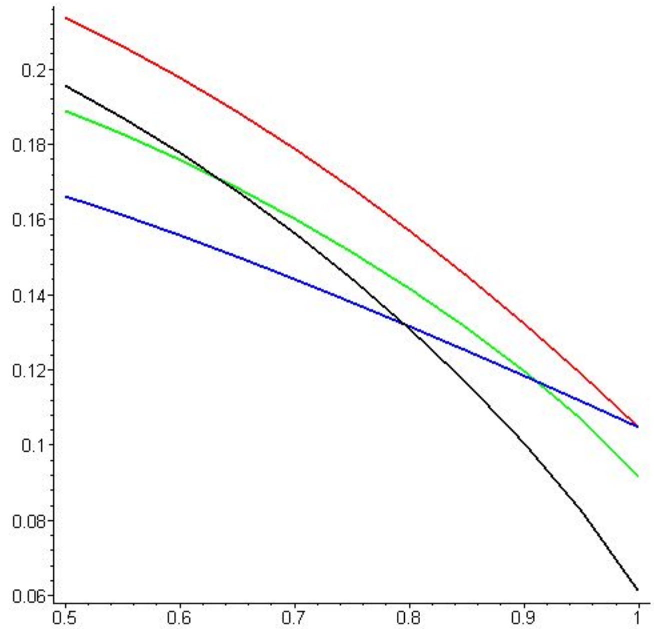


Figure 7: Π as function of r for $N = 15$, $m = 8$ (blue),$m = 9$ (green),$m = 10$ (black); and $\tilde{\Pi}$ for $m = 8$ (red)

curve in Figure 7, the flattest of the three, represents the payoffs under simple majority, i.e. $m = 8$, while the green and the black one (the steepest one) show the payoffs under the supermajority rules $m = 9$ and $m = 10$, respectively. A two-thirds majority rule ($m = 10$) is the optimal majority rule in the basic model, and remains optimal up to approximately $r = 0.64$. For r between about 0.64 and 0.92, $m = 9$ (i.e. a 60% rule) is the optimal majority rule, and only for higher levels of correlation, simple majority rule is optimal. Thus, while intertemporal correlation decreases the optimal majority rule, our qualitative result concerning the optimality of supermajority rules is very robust even at high levels of correlation.[28]

The highest curve (red) shows the ex-ante expected payoff without the option of waiting. From the basic model, we know already that, for $r = 0.5$ this payoff is higher than the payoff with the option to wait, even under the optimal supermajority rule. Figure 6 shows that this result continues to hold for $r > 1/2$, except at $r = 1$ where the difference between the payoffs with and without the option to wait becomes zero. This is intuitive, as with perfect correlation, a project is either implemented immediately or not at all, so payoffs are the same, whether or not second period implementation is, in principle, possible.

5 Discussion

5.1 Supermajority rules

Supermajority rules are used in many international organizations like the European Union, and in most countries for a change of the constitution, and, often implicitly, for "normal" legislation. For example, in parliamentary systems with a strong committee organization, a legislative proposal usually needs the support of *both* the respective committee and the house. In parliamentary systems with two chambers, certain legislative proposals need the support of both chambers.

Our model provides a fundamentally new rationale for societies choosing supermajority rules, which relies on voters' uncertainty over the consequences of project implementation, and the option value of waiting until new information is available. Thus, our model is most relevant for societies that frequently face decision problems with such characteristics.

For example, one can argue that the European Union fits this description quite well. The most important decisions in the EU concern the admission of new members, transnational investment projects like the introduction of the Euro and the harmonization of industry regulations. Many of these projects are less "standard" (relative to the most important policy issues in the member states) and have uncertain payoff consequences for the member states. Consistent with this view, the European Council (the council of member state governments that makes the most significant decisions) uses a relatively high supermajority rule.

Also, most countries require a supermajority to change their constitution. Again, this area appears closer to the setting of this model than ordinary legislation issues: When the initial constitution is written, future needs are difficult to foresee and potential winners and losers are unclear, and even once a proposal

[28]This result is even more surprising as N is relatively small in our example, so that even a simple majority rule requires the approval of $8/15 = 53.3\%$ of the population, and the smallest possible supermajority rule is already a 60% rule.

arises, the consequences of changes for the distribution of gains and losses are not necessarily clear.

In contrast, most ordinary legislation in national legislatures concerns social or economic issues where preferences are more stable. As we have seen in Section 4.3, the higher the correlation of voter types over time (and therefore, the less new information is expected to arise over time), the closer is the ex-ante optimal majority rule to simple majority rule. In this context, it is interesting to note that the European Union has recently adopted a new, lower supermajority rule for their decisions. With the expansion of their fields of responsibility, the EU appears to become more like a normal state in which standard decisions become more important, and thus the optimal supermajority rule decreases.

5.2 Previous literature on supermajority rules

In this section, we review several previous papers that have analyzed different arguments for supermajority rules from an economic point of view. Buchanan and Tullock (1962) argue for unanimity rule as the suitable rule governing social choices. Under a simple majority rule, a majority of people could be tempted to implement certain projects that are not socially desirable because they can "externalize" part of the cost associated with this project to the losing minority. Under unanimity rule, only Pareto improving projects are implemented. However, Guttman (1998) has argued that the unanimity rule leads to a rejection of many projects that are not Pareto improvements, but nevertheless worthwhile from a reasonable social point of view. Assuming that the social goal is to minimize the sum of both types of mistakes, he shows that simple majority rule is optimal. Our model is constructed in a way that simple majority rule would also be optimal if voters have to make a once-and-for-all decision about the project in the first period. However, with the option to postpone a decision to the second period, we show that (in the same symmetric setting), a supermajority rule is optimal.

The implications of different majority rules have also been analyzed in settings where voters have congruent interests, but are only imperfectly informed about the consequences of the different alternatives. Inspired by Condorcet's famous Jury Theorem, several authors have analyzed which majority rule is most efficient in aggregating the information that is dispersed in the electorate. Nitzan and Paroush (1985) find that the probability of a correct choice is maximized under simple majority rule. Feddersen and Pesendorfer (1998) analyze information aggregation with strategic voters and show that simple majority rule is optimal for information aggregation purposes, while unanimity rule is dominated by all other majority rules, if there are sufficiently many voters.

As is well known, simple majority rule may lead to cycles in electoral preferences. A higher required majority reduces the possibility of cycles. Indeed, Caplin and Nalebuff (1988) show that a $(1-(n/(n+1))^n)$ supermajority rules out cycles, if voters have single peaked Euclidean preferences in an n–dimensional space. In our model, the decision is binary in each period, so cycles never arise in our model.

Another rationale for a supermajority rule is that it counteracts the problem of time inconsistency of optimal policies (see, e.g., Gradstein (1999) or Dal Bo (2006)). For example, a constitution that protects investment by inhibiting nationalization is valuable only if the constitution cannot be easily changed after investment has taken place. In our model, time inconsistency is not an issue.

Aghion and Bolton (2003) analyze the optimal choice of the majority rules in a model where a polity has to decide simultaneously about public good provision and costly redistribution, and redistribution creates a deadweight loss. They assume that the constitutional rules are written before voters learn the costs and benefits of the public good. The optimal majority rule trades off the higher ex-post flexibility of low majority rules (which lead to more efficient public good provision) against the protection against excessive redistribution afforded by supermajority rules.

Eraslan and Merlo (2002) analyze an advantage of unanimity rule over all other majority rules in a model of bargaining with stochastic surplus. Under any majority rule requiring less than unanimity, the proposer and voters he selected into the minimum winning coalition have to fear that they might not be a part of a winning coalition in the future if no agreement is reached today. Therefore, agreement may be reached too early from a social point of view (i.e., for a too small surplus).

Messner and Polborn (2004) analyze an overlapping generations model in which voters know that their preferences over reform projects will become more conservative over the remainder of their lives. The initial population decides on the majority rule to be used for later decisions. The median voter in the constitutional election prefers to implement a supermajority rule, which allows him to transfer power to his (more conservative) "average future self". In contrast to Messner and Polborn (2004), the electorate remains constant over time in our model, thus removing the incentive for the initial generation to use supermajority rules in order to transfer power from future voters to themselves. Also, the implementation decision on any reform project in Messner and Polborn (2004) is a simple one-time, up-or-down vote, while our focus here is on the timing of the implementation of reforms.

6 Conclusion

We analyze a model in which voters have to choose whether to implement a project immediately, or wait till the second period and reconsider the decision then. Our main focus is the effect of the majority rule on individual voting behavior and social decisions in this framework.

We show that the option to wait makes voters in the first period more conservative towards projects that have a positive expected second period value, and more inclined to implement projects that have a negative expected second period value. To counteract these socially undesirable tendencies, the optimal majority rule increases when society has the option to wait relative to the case where postponing is not possible. For example, for a uniform project type distribution, the optimal majority rule changes from simple majority without the option to wait to a supermajority rule that is between $7/11 \approx 63.6$ percent, and about $2/3$. The benefits of supermajority rules are further strengthened if individual voters' preferences are correlated with each other. In contrast, correlation between first and second period valuations reduces the size of the optimal majority rule. Perfect correlation re-establishes simple majority rule as the optimal rule; however, even if correlation is high, but not perfect, the optimal majority rule remains a supermajority rule.

Another important result was that the option of waiting, which is always positive for individual decision problems, can be negative for our social decision problem. Indeed, we show that this is the case when the project cost is uniformly distributed from an ex-ante perspective, even if society adopts the optimal majority rule in the case that they have the option to wait.

One direction in which future research can expand on our model framework is as follows. In our model, individuals only choose how to vote. In some instances, individuals may also be able to adapt to the policy enacted and thereby influence the distribution of their payoff in the second period. This may be important, for example, in issues where the project is some sort of environmental regulation, say, increasing the private cost of some polluting activity. Adaptation (say, buying a smaller car, isolating one's home) may make compliance less costly over time, but the enacted policy (as well as the expectation of which regulation will be in force in the next period) will affect the optimal extent to which individuals adapt.

7 Appendix

In this appendix, we will often suppress the argument m of functions (e.g., we write p for $p(m)$), and generally denote functions evaluated at $m+1$ by primes (e.g, $p' \equiv p(m+1)$).

Lemma A1. $\frac{p'}{p} = \frac{\theta}{1-\theta}\frac{(N-m)}{m}$.

Proof. Using the definition of p, $\frac{p'}{p} = \frac{\binom{N-1}{m}\theta^m(1-\theta)^{N-1-m}}{\binom{N-1}{m-1}\theta^{m-1}(1-\theta)^{N-m}} = \frac{\theta}{1-\theta}\frac{N-m}{m}$. □

Proof of Proposition 1, part 2. For $c \in [\theta/2, (1+\theta)/2)$ we have

$$\begin{aligned}
\tilde{\pi}' - \tilde{\pi} &= 2(q'-q)(\theta - c) + (p'-p)\theta(1+\theta-2c) \\
&= p'[\theta(1+\theta-2c) - 2(\theta-c) - p\theta(1+\theta-2c) \\
&= p[(p'/p)(1-\theta)(2c-\theta) - \theta(1+\theta-2c)] \\
&= p\theta\left[\frac{N-m}{m}(2c-\theta) - (1+\theta-2c)\right] = \frac{p\theta}{m}[N(2c-\theta)-m], \qquad (16)
\end{aligned}$$

where the second line uses the identity $q = q' + p'$, and the third line uses Lemma A1.

Equation (16) is positive if and only if $m \leq (2c-\theta)N$. Thus, if $c > (1+2\theta)/4$, the optimal majority rule is given by $m^* = \lceil N(2c-\theta)\rceil$. If $c \leq (1+2\theta)/4$, then $\tilde{\pi}' - \tilde{\pi}$ is never positive, which implies that the optimal majority rule in this case is simple majority, $m^* = (N+1)/2$. □

Proof of Proposition 2. Using (16), we can express the difference $\tilde{\Pi} - \tilde{\Pi}'$ as

$$\begin{aligned}
&\frac{p\theta N}{m}\left\{\int_{\theta/2}^{(1+\theta)/2}[2c-(1+2\theta)/2]dF(c) - \left[\frac{m}{N}-\frac{1}{2}\right]\int_{\theta/2}^{(1+\theta)/2}dF(c)\right\} \\
&= \frac{p\theta N}{m}\left[-x + \left(\frac{1}{2}-\frac{m}{N}\right)y\right]. \qquad (17)
\end{aligned}$$

The sign of this expression is given by the sign of the term in the square brackets. That term is strictly decreasing in m, which implies that the difference in the unconditional ex-ante payoffs changes sign at most once. Finally, observe that (17) is positive (so that $\tilde{\Pi}(\cdot, N, \theta)$ is nondecreasing at m) if and only if

$$m \leq \frac{y-2x}{2y}N. \quad \square$$

Lemma A2. *For all N and all $m \geq \theta N$,* $\frac{q(m,N,\theta)}{p(m,N,\theta)} \leq \frac{\theta(N-m)}{m-\theta N}$.

Proof. Let $s = m/N$. For $s \geq \theta$, we have

$$\begin{aligned}
\frac{q(m,N,\theta)}{p(m,N,\theta)} &= \sum_{l=m+1}^{N}\frac{p(l,N,\theta)}{p(m,N,\theta)} = \sum_{l=m}^{N-1}\frac{\binom{N-1}{l}\theta^l(1-\theta)^{N-1-l}}{\binom{N-1}{m-1}\theta^{m-1}(1-\theta)^{N-m}} = \sum_{l=m}^{N-1}\left[\left(\frac{\theta}{1-\theta}\right)^{l-m+1}\prod_{k=m}^{l}\frac{N-k}{k}\right] \\
&\leq \sum_{l=m}^{N-1}\left(\frac{(1-s)\theta}{s(1-\theta)}\right)^{l-m+1} \leq \frac{(1-s)\theta}{s(1-\theta)}\sum_{l=0}^{\infty}\left(\frac{(1-s)\theta}{s(1-\theta)}\right)^{l} = \frac{(1-s)\theta}{s-\theta} = \frac{\theta(N-m)}{m-\theta N}. \quad \square
\end{aligned}$$

Proof of Lemma 1. Observe that $\frac{p'}{q'} \geq \frac{p}{q}$ if and only if

$$\frac{p'}{p} \geq \frac{q'}{q} = \frac{q'}{q'+p'} = \frac{1}{1+\frac{p'}{q'}}. \tag{18}$$

Using Lemma A1, and $\dfrac{p'}{q'} \geq \dfrac{(m+1)-\theta N}{\theta(N-(m+1)}$ by Lemma A2, a sufficient condition for (18) to hold is

$$\frac{\theta}{1-\theta}\frac{(N-m)}{m} \geq \frac{1}{\frac{\theta(N-(m+1))}{\theta(N-(m+1))}+\frac{(m+1)-\theta N}{\theta(N-(m+1))}} = \frac{\theta(N-(m+1))}{(1-\theta)(m+1)}$$

which is always true. □

Proof of Lemma 2. The difference between the value of waiting at m' and at m,

$$U_W(c,m',N,\theta) - U_W(c,m,N,\theta) = \theta(1-c)(p'-p) + (c-\theta)p', \tag{19}$$

is positive if and only if

$$\frac{p}{p'} \geq \frac{c}{1-c}\frac{1-\theta}{\theta} \tag{20}$$

Using Lemma A1, (20) holds if and only if

$$\frac{1-\theta}{\theta}\frac{m}{N-m} \leq \frac{c}{1-c}\frac{1-\theta}{\theta} \Leftrightarrow \frac{m}{N} \leq c. \quad \square$$

Proof of Proposition 3. The cutoff structure follows immediately from the fact that $\frac{\partial U_I}{\partial c} = -2 < \frac{\partial U_W}{\partial c} = -\theta p - q$. Moreover, $U_I(1,c,\theta) > U_I(0,c,\theta)$ implies that $\underline{c} < \bar{c}$.

We now show that $\underline{c} < \theta/2$. Since $\theta/2$ is the value of $\underline{c}$ in the benchmark case (without the option to wait), $\underline{c} < \theta/2$ if and only if the value of waiting at this threshold is strictly positive. We have

$$U_W(\theta/2,m,N,\theta) = \theta p(m,N,\theta)(1-\theta/2) + q(m,N,\theta)(\theta-\theta/2) > 0, \tag{21}$$

for all θ. Thus, low types become more conservative with the option to wait.

We next show that $\underline{c}$ is an increasing function in m. Since $m \geq (N+1)/2$, this claim follows immediately from the Lemma 2: For all $m \geq (N+1)/2$ the value of waiting is decreasing in m for all $c < 1/2$. Thus, $\underline{c} < \theta/2 < 1/2$ means that $\underline{c}$ must be increasing in m if $m \geq (N+1)/2$.

We now turn our attention to the threshold of high types, $\bar{c}(\cdot,N,\theta)$. Solving $U_I(c,1,\theta) = U_W(c,m,N,\theta)$ for c yields

$$\bar{c}(m,N,\theta) = \frac{1+\theta(1-p(m,N,\theta)-q(m,N,\theta))}{2-\theta p(m,N,\theta)-q(m,N,\theta)} \tag{22}$$

The claim that $\bar{c}(N,N,\theta) < (1+\theta)/2$ follows immediately from the fact that at $c = (1+\theta)/2$ the value of implementing the project immediately is just zero, while the value of waiting is $p(N,N,\theta)\theta(1-\theta)/2 > 0$.

Since

$$U_I(1, 1/(2-\theta), \theta) - U_W(1/(2-\theta), m, N, \theta) =$$
$$1 + \theta - \frac{2}{2-\theta} - \theta p\left[1 - \frac{1}{2-\theta}\right] - q\left[\theta - \frac{1}{2-\theta}\right] = \frac{1-\theta}{2-\theta}[\theta(1-p) + (1-\theta)q] > 0, \tag{23}$$

it follows that $\bar{c}(\cdot, N, \theta) > 1/(2-\theta)$.

By Lemma 2, $\bar{c}(m) \geq \bar{c}(m')$ if and only if $\bar{c}(m) \geq m/N$. Using (22), we obtain

$$\frac{1 + \theta(1-p-q)}{2 - \theta p - q} \geq \frac{m}{N} \quad \Leftrightarrow \quad (1+\theta)N - 2m - [p\theta(N-m) - q(m - \theta N)] \geq 0. \tag{24}$$

Lemma A2 implies that the term in square brackets is positive for all $m \geq \theta N$. This means that condition (24) can never hold for $m \geq (1+\theta)N/2$. Hence, $\bar{c}$ is increasing at least from $m = \lceil (1+\theta)N/2 \rceil$ onwards.

Finally, since $2 > \theta p + q$, the left hand side of (24) is decreasing in m, which implies that (24) changes sign at most once. □

Lemma A3. $Np(\lceil 3N/4 \rceil, N, 1/2) \leq 4$ *for all* N.

Proof. Given that N is an odd number we have that either $3N+1$ is divisible by 4 (for $N = 5, 9, 13, \ldots$) or $3(N+1)/4$ is an integer (for $N = 3, 7, 11, \ldots$). In the first case we have that $\lceil 3N/4 \rceil = (3N+1)/4$, while in the latter case we have $\lceil 3N/4 \rceil = 3(N+1)/4$. Notice also that in either case we have $\lceil 3(N+4)/4 \rceil - \lceil 3N/4 \rceil = 3$.

Let $f(N) = Np(\lceil 3N/4 \rceil, N, 1/2)$. In what follows we will show that $f(N+4) - f(N) < 0$. This is sufficient for proving our statement since it implies that $\max_N f(N) = \max\{f(3), f(5)\} = \max\{3\binom{2}{2}/2^2, 5\binom{4}{3}/2^4\} = 5/4 < 4$.

Letting $m = \lceil 3N/4 \rceil$ the increment $f(N+4) - f(N)$ is given by

$$\frac{N+4}{2^{N+3}}\binom{N+3}{m+2} - \frac{N}{2^{N-1}}\binom{N-1}{m-1} = \frac{N}{2^{N-1}}\binom{N-1}{m-1}\left[\frac{(N+4)(N+3)(N+2)(N+1)}{16(N+1-m)(m+2)(m+1)m} - 1\right].$$

Observe that $(N+j+2) < 4(m+j)$ for $j = 0, 1, 2$ and that $N+1 \leq 4(N+1-m)$. Thus the first term in the square brackets is a product of four numbers which are smaller than one, and thus the term in square brackets is negative. □

Proof of Proposition 4. The main arguments are in the text. It remains to be shown that $m^*(c) = \lceil (1+\theta)N/2 \rceil$ for all $c \in (\bar{c}(\lceil (1+\theta)N/2 \rceil), (1+\theta)/2]$. The ex-ante payoff under majority rule m is a weighted average of the values of immediate implementation for low and high types and the value of waiting associated with rule m (where the weights depend on the behavior of players). Thus, it is sufficient to show that for all c in the specified range we have $U_W(c, \lceil (1+\theta)N/2 \rceil) > \max(U_W(c, m), U_I(V_1, c))$ for all m and all V_1. The definition of $\bar{c}(\lceil (1+\theta)N/2 \rceil)$ implies immediately that $U_W(c, \lceil (1+\theta)N/2 \rceil) > U_I(V_1, c)$ for all $c > \bar{c}(\lceil (1+\theta)N/2 \rceil)$ and V_1. For $m > \lceil (1+\theta)N/2 \rceil$, $U_W(c, \lceil (1+\theta)N/2 \rceil) > U_W(c, m)$ follows immediately by Lemma 2. Furthermore, if for any majority rule $m < \lceil (1+\theta)N/2 \rceil$ we had

$U_W(c, \lceil(1+\theta)N/2\rceil) < U_W(c, m)$ for some $c \in (\bar{c}(\lceil(1+\theta)N/2\rceil), (1+\theta)/2]$, then this inequality would also hold for any lower c, and in particular for $\bar{c}(\lceil(1+\theta)N/2\rceil)$. But this implies $\bar{c}(m) < \bar{c}(\lceil 3N/4\rceil)$, a contradiction to the fact that $\bar{c}(\cdot)$ is minimized at $m = \lceil(1+\theta)N/2\rceil$. □

Proof of Proposition 5. In the limit of $N \to \infty$, a voter's ex-ante expected utility for simple majority rule and without the option to wait is

$$\lim_{N\to\infty} \tilde{\Pi}(\lceil N/2\rceil, N) = \int_0^{1/4} (1-2c)dF(c) + \frac{1}{2}\int_{1/4}^{3/4} (1-2c)dF(c), \tag{25}$$

because all projects with $c \leq 1/4$ are implemented unanimously, and those with $c \in (1/4, 3/4]$ are implemented with probability $1/2$.

A voter's ex-ante expected utility for a proportional supermajority rule s, with $s > 1/2$, and without the option to wait, is

$$\lim_{N\to\infty} \tilde{\Pi}(\lceil sN\rceil, N) = \int_0^{1/4} (1-2c)dF(c) \tag{26}$$

because all projects with $c \leq 1/4$ are implemented unanimously, and those with $c > 1/4$ are (almost) never implemented, because they are only supported by high types, and the proportion of high types is almost surely less than s by the law of large numbers.

Similarly, when waiting is possible, a voter's ex-ante expected utility for a proportional supermajority rule s, with $s > 1/2$, is $\lim_{N\to\infty} \Pi(\lceil sN\rceil, N) = \lim_{N\to\infty} \tilde{\Pi}(\lceil sN\rceil, N)$. The reason is that, even when society can reconsider the decision in the second period, the proportion of high types in the second period is (almost) never sufficient for implementation.

Last, consider the ex-ante expected utility for simple majority rule and with the option to wait. In the second period, all projects are implemented with probability $1/2$, and so, for given c, the value of waiting is $U_{W,1/2} = \frac{1}{2}\left(\frac{1}{2} - c\right)$. Thus, in the first period, low types agree to implement if and only if $\frac{1}{2} - 2c \geq \frac{1}{2}\left(\frac{1}{2} - c\right)$, hence whenever $c \leq 1/6$; similarly, first period high types vote for immediate implementation whenever $c \leq 5/6$. Thus, if $c \leq 1/6$, the project is implemented in the first period. Projects with $c \in (1/6, 5/6]$ are implemented with probability $1/2$ in the first period, and projects with $c > 5/6$ are not implemented in the first period. In the second period, all projects that were not implemented in the first period are implemented with probability $1/2$. A voter's ex-ante expected utility under simple majority rule with the option to wait is thus

$$\begin{aligned}\lim_{N\to\infty} \Pi(\lceil N/2\rceil, N) = &\int_0^{1/6} (1-2c)dF(c) + \frac{1}{2}\int_{1/6}^{5/6} (1-2c)dF \\ &+\frac{1}{4}\int_{1/6}^{5/6} \left(\frac{1}{2} - c\right) dF(c) + \frac{1}{2}\int_{5/6}^{1} \left(\frac{1}{2} - c\right) dF(c)\end{aligned} \tag{27}$$

Suppose the claim in the proposition is false for some distribution F. Since expected utility under any supermajority rule, and with or without the option to wait, is equal to $\int_0^{1/4}(1-2c)dF(c)$, a contradiction to the claim can only arise if there exists $\tilde{s}^*(F) \in \tilde{S}^*$, with $\tilde{s}^*(F) > 1/2$, and $s^*(F) = 1/2 \in S^*(F)$. In this case,

$$\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) \geq \lim_{N\to\infty} \Pi(\lceil \tilde{s}^*(F)N \rceil, N) = \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^*(F)N \rceil, N) \geq \lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) \tag{28}$$

where the inequality signs follow from the optimality of $s^*(F)$ and $\tilde{s}^*(F)$, respectively, and the equality sign follows from $\lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N) = \lim_{N\to\infty} \Pi(\lceil sN \rceil, N)$ for all $s > 1/2$. The last inequality in (28) implies that

$$\int_{1/4}^{3/4} (1 - 2c) dF(c) \leq 0. \tag{29}$$

Furthermore, note that $\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) - \lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) =$

$$-\frac{1}{4}\int_{1/6}^{1/4}(1-2c)dF(c) + \frac{1}{4}\int_{1/4}^{3/4}(1-2c)dF(c) + \frac{3}{4}\int_{3/4}^{5/6}(1-2c)dF(c) + \frac{1}{2}\int_{5/6}^{1}(1-2c)dF(c) \tag{30}$$

must be greater or equal to zero by (28). However, this inequality cannot hold, as all terms are nonpositive (the second term is nonpositive by (29)), and either the first, the third or the fourth term are strictly negative, by the assumption on F. This provides the desired contradiction.

Note that, while our assumption on F is not necessary for the Proposition to hold, some assumption is required. To see this, suppose that c has a one-point distribution with all mass on $c = 1/2$. In this case, without the option to wait, all majority rules yield the same ex-ante expected surplus of 0. Also, with the option to wait, all majority rules yield an ex-ante surplus of 0. Thus, for this example, $\sup \tilde{S}^* = 1 > \inf S^* = 1/2$. □

Proof of Proposition 6. We have to show that the difference $\Pi(m') - \Pi(m)$ is negative whenever $m \geq 2N/3$. It is a matter of tedious but straightforward algebraic manipulations that this difference is equal to the ratio[29]

$$\frac{2pqp' + 8qp - 11p - 8pp' - q^2p - p(p')^2 - 16qp' + 21p' - 2q(p')^2 + 4(p')^2 + 3q^2p'}{4(4 - 2q + p')(4 - 2q - p)}. \tag{31}$$

The denominator of this expression is clearly positive. Thus the sign of the difference in average ex-ante payoffs coincides with the sign of the numerator of this expression. Denote this numerator by $d(p, q, p')$.

We first show that for any p and q, d is monotonically increasing in p'. We have

$$\begin{aligned} \frac{\partial d(p,q,p')}{\partial p'} &= 2pq - 8p - 2pp' - 16q + 21 - 4qp' + 8p' + 3q^2 \\ &\geq -8p - 2pp' - 16q + 21 - 4qp' \geq 13 - 2pp' - 4qp' > 0, \end{aligned}$$

where the second inequality sign in this expression follows from the fact that $q + p/2 \leq 1/2$.

Since $p' = (N - m)p/m$, we have that $p' \leq p/2$ for all $m \geq 2N/3$, and thus

$$d(p,q,p') \leq \max_{p' \leq p/2} d(p,q,p') = d(p,q,p/2) = p\left[\frac{pq}{2} - \frac{1}{2} - 3p + \frac{q^2}{2} - \frac{p^2}{4}\right] < \frac{p}{2}\left[pq - 1 + q^2\right].$$

[29] The following expression is obtained by using $q' = q - p'$.

Given that $p, q \leq 1/2$ we thus have that $d(p, q, p') < 0$ whenever $m \geq 2N/3$. This proves the first part.

Since $p' = (N - m)p/m = \frac{1-s}{s}p$ for $s = m/N$, we have $p' \geq ap$ for $m \leq sN$, where $a = (1 - s)/s$. Monotonicity of d in p' therefore implies that

$$\begin{aligned} d(p, q, p') &\geq \min_{p' \geq ap} d(p, q, p') = d(p, q, ap) \\ &= \{[a(4 - p) - 8]ap + [2pa(1 - a) + (3a - 1)q - 8(2a - 1)]q + 21a - 11\}p. \end{aligned}$$

Now observe that the sign of $\partial d(p, q, ap)/\partial q$ coincides with the sign of $2pa(1-a)+(6a-2)q-8(2a-1)$. Since

$$2pa(1 - a) + (6a - 2)q - 8(2a - 1) < 8 - a(16 - 2(2q + p)) + 2q(1 - a) \leq 8 - 14a$$

it follows that whenever $a \geq 8/14 = 4/7$ then $\min_{p' \geq ap} d(p, q, p')$ is decreasing in q. Using the fact that $q \leq (1 - p)/2$ we thus have that for all $1 \geq a \geq 4/7$

$$\begin{aligned} d(p, q, p') &\geq \min_{q \leq (1-p)/2} \left\{ \min_{p' \geq ap} d(p, q, p') \right\} = d(p, (1 - p)/2, ap) \\ &= \frac{p}{4}(55a - 29 - 2ap - ap^2 - 14p + 12a^2 p - p^2) =: D(p, a). \end{aligned}$$

$D(p, a)$ is clearly increasing in a. In the case of simple majority we have $a = (N - 1)/(N + 1)$, which is increasing in N. Thus under simple majority we have that $a \geq 2/3$ if $N \geq 5$ (for $N = 3$ all majority rules satisfy $m \geq 2N/3$). Since $D(p, 2/3) = p(23 - 30p - 5p^2)/12 > 0$ for all $p \in (0, 1/2)$ we can therefore conclude that at simple majority the increment of Π is positive.

The preceding observations allow us to restrict our attention in the remainder of the proof to supermajority rules $m > (N + 1)/2$. Since for all such rules we have $1/2 \geq p(m - 1)/2 + q(m - 1)$ and $p(m) \geq p(m - 1)/2$, the identity $q(m - 1) = p(m) + q(m)$ implies $3p(m)/2 + q(m) \leq 1/2$ or equivalently $q \leq (1 - 3p)/2$. Exploiting this fact we can thus claim that if $m > (N + 1)/2$ then we have for all $a \in (4/7, 1)$ that

$$\begin{aligned} d(p, q, p') &\geq \min_{q \leq (1-3p)/2} \left\{ \min_{p' \geq ap} d(p, q, p') \right\} = d(p, (1 - 3p)/2, ap) \\ &= \frac{p}{4}(50ap + 15p^2 a - 29 - 42p - 9p^2 + 8a^2 p^2 + 55a + 12a^2 p) =: \hat{D}(p, a). \end{aligned}$$

Since $\hat{D}(p, a)$ is strictly increasing in a it follows that for all $a \geq 4/7$ we have

$$d(p, q, p') \geq \hat{D}(p, 4/7) = \frac{p(119 - 466p + 107p^2)}{196}.$$

It is straightforward to see that this expression is strictly positve for all $p \in (0, 1/4]$. Since $p(m)$ is decreasing in m for every N and also $p((N + 3)/2)$ decreases with N, it follows that for $2N/3 > m > (N + 1)/2$ we must have $p(m) \leq p(7, 11) = 105/520 < 1/4$ (notice that only for $N \geq 11$ there are majority rules in the specified range). Hence, for all p which may arise for $2N/3 > m > (N + 1)/2$ we know that $\hat{D}(p, a) > 0$, whenever $a \geq 4/7$. The condition $a = (1 - s)/s \geq 4/7$ in turn is equivalent to

$s = m/N \leq 7/11 \approx 0.636$. Thus we can conclude that $d(p, q, p') > 0$ for all $(N+1)/2 < m < 7N/11$. This proves statement ii).

Finally, using $p((N+1)/2)/2 + q((N+1)/2) = 1/2$ and $q(N) = 0$ it is straightforward to show that

$$\Pi(N) - \Pi((N+1)/2) = \frac{16 - 27p + 10p^2 + p^3}{48(4-p)},$$

which is strictly positive for all $p < 1$. □

Proof of Proposition 7. Suppose the first claim is false, i.e.

$$\lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N) > \lim_{N\to\infty} \tilde{\Pi}(\lceil \tilde{s}^* N \rceil, N). \tag{32}$$

In the proof of Proposition 5, we have shown that $\lim_{N\to\infty} \Pi(\lceil sN \rceil, N) = \lim_{N\to\infty} \tilde{\Pi}(\lceil sN \rceil, N)$ for any $s > 1/2$, so that (32) cannot hold when s^* and $\tilde{s}^*$ are both greater than $1/2$. Furthermore, by Proposition 5, we cannot have that $\tilde{s}^* > 1/2$ and $s^* = 1/2$. If $\tilde{s}^* = 1/2$ and $s^* > 1/2$, then

$$\lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N) \geq \lim_{N\to\infty} \tilde{\Pi}(\lceil s^* N \rceil, N) = \lim_{N\to\infty} \Pi(\lceil s^* N \rceil, N), \tag{33}$$

where the inequality follows from the optimality of $\tilde{s}^* = 1/2$. Last, if $\tilde{s}^* = 1/2$ and $s^* = 1/2$, then equation (30) shows that $\lim_{N\to\infty} \Pi(\lceil N/2 \rceil, N) < \lim_{N\to\infty} \tilde{\Pi}(\lceil N/2 \rceil, N)$. Thus, (32) cannot hold, the desired contradiction.

For the proof of the second statement, we drop the arguments from the functions p and q (like in earlier proofs). Calculating the difference between $\Pi(m)$ and $\tilde{\Pi}(m)$ gives

$$\Pi(m) - \tilde{\Pi}(m) = \frac{2p + 3 + q^2 - 4q}{4(4 - 2q - p)} - \frac{3 + 2p}{16} = \frac{3p + 4q^2 - 10q + 4qp + 2p^2}{16(4 - 2q - p)}. \tag{34}$$

The denominator of this expression is clearly positive and thus the sign of the difference is determined by the numerator. Denote this numerator by $d(p, q)$.

We first show that $d(p, q)$ is negative for $m = (N+1)/2$. Remember that, in this case, we have $p = 1 - 2q$ and thus

$$d = 3p + 4q^2 - 10q + 4qp + 2p^2 = 5 - 20q + 4q^2.$$

This expression is negative iff $q((N+1)/2, N) \leq 5/2 - \sqrt{5} \approx 0.26$, which is satisfied for all $N > 5$.

Next consider any supermajority m such that $(N+1)/2 < m \leq \lfloor 3N/4 \rfloor$. For any such majority rule we have that $m \leq N - 2$. Therefore, it follows that

$$q(m) \geq p(m+1) + p(m+2) = p(m+1)\left(1 + \frac{N-m-1}{m+1}\right) = p(m)\frac{N-m}{m}\frac{N}{m+1}.$$

Notice that the last term in this expression is decreasing in m. Thus we may write $p \leq (q\lfloor 3N/4 \rfloor \lceil 3N/4 \rceil)/((N - \lfloor 3N/4 \rfloor)N)$. Using Lemma A3, it can be shown that the right-hand side of this last inequality is smaller than $(12/5)q$.[30]

[30]If $N = 7, 11, 15, \ldots$, then $12/5 - (\lfloor 3N/4 \rfloor \lceil 3N/4 \rceil)/((N - \lfloor 3N/4 \rfloor)N) = (N + (N-1)/2 + 8)/10N > 0$.

Next observe that the fact that m is a supermajority rule implies that $1/2 \geq p(m-1)/2 + p(m) + q(m) \geq 3p(m)/2 + q(m)$. Combining this observation with the preceding one, we obtain $p \leq \min\{(1-2q)/3, 12q/5\}$, or equivalently,

$$p \leq \begin{cases} 12q/5 & \text{if } q \leq 5/46 \\ (1-2q)/3 & \text{if } q > 5/46. \end{cases}$$

Notice that $d(12q/5, q) = -(14/5 - 628q/25)q < 0$ for all $q \leq 5/46$ and $d((1-2q)/3, q) = (11 - 104q + 20q^2)/9 < 0$ for all $5/46 < q \leq 1/2$. Since $d(p,q) \leq d(\min\{(1-2q)/3, 12q/5\}, q)$, this proves the first claim.

As for the second part of number 2, observe that a sufficient condition for the numerator of (34) to be positive is $3p > 10q$. From Lemma A2, we have $\frac{q}{p} \leq \frac{N-m}{2m-N}$ so that for all $m \geq \frac{13}{16}N$, the numerator of (34) is positive.

For number 3, note that Proposition 6 implies that the optimal majority rule with the option of waiting is lower or equal to $\lceil 2N/3 \rceil$, which is lower or equal to $\lfloor 3N/4 \rfloor$ for all $N > 5$. By the second statement of Proposition 7, for all such rules, the expected ex-ante payoff is higher without the option to wait. □

Proof of Proposition 8. It is a matter of straightforward - though tedious - algebra to show that the difference $\Pi(m', N, \theta) - \Pi(m, N, \theta)$ is proportional to the following expression

$$\begin{aligned} d(s,p,q,\theta) = &-(1-s)^2\theta^2 p^2 - \theta p(1-s)[2s(1-\theta)(2-q) + (1-s)q + 2(2s-1)] \\ &-s(1-\theta)q[\theta - q(1+\theta-2s)] - sq\theta - s(1-q)[8s(1-\theta) + 2s + 3\theta^2 - 4] + 2(1-s)s \end{aligned}$$

where $s = m/N$. Notice, that the pivot probability p converges to zero for all (m,θ) as N grows. Thus, for N sufficiently large the sign of $d(s,p,q,\theta)$ must coincide with the sign of

$$d(s,0,q,\theta) = s\left[(1-q)(8\theta s - 10s + 4 - 3\theta^2 + \theta^2 q) - 2q\theta + 2(1-s) - 2sq^2(1-\theta) + q^2\right]. \quad (35)$$

We first argue that $d(s,0,q,\theta)$ is decreasing in s. Writing $d_s(s,0,q,\theta)$ for $\partial d(s,0,q,\theta)/\partial s$, we have

$$d_s(s,0,q,\theta) = -(1-\theta)(2-q)^2[4s - (1+\theta)] - 2(2s-1) - 4s(1-q) - \theta(2q-\theta).$$

The first three components of this sum are negative for all $s \geq 1/2$ and all (q,θ). If $2q - \theta \geq 0$ then the last term is also negative. If, instead $2q - \theta < 0$, then the sum of the last two terms is

$$-4s(1-q) - \theta(2q-\theta) \leq -2(1/2) + \theta < 0$$

since $s > 1/2$, $q < 1/2$ and $(2q - \theta) \geq -1$. Next, observe that

$$\begin{aligned} d(1/2,0,q,\theta) &= [(1-q)\theta(1-\theta)(3-q) + \theta(1-q) + q(1-\theta)]/2 > 0 \quad \forall\, (q,\theta) \quad \text{and} \\ d(3/4,0,q,\theta) &= 3\left[-(6-2q)(1-\theta)^2(1-q) - q(1-q+\theta q)\right]/8 < 0 \quad \forall\, (q,\theta). \end{aligned}$$

Monotonicity of d in s therefore implies that $s^*(\theta) = m^*(N,\theta)/N \in (1/2, 3/4)$.

Moreover, for each pair (q, θ) there is some $\tilde{s}(q, \theta) \in (1/2, 3/4)$ such that $d(\theta, s, 0, q) \geq 0$ if and only if $s \leq \tilde{s}(q, \theta)$. Solving (35) for $\tilde{s}(q, \theta)$ yields

$$\tilde{s}(q, \theta) = \frac{(1-q)[4 - 3\theta^2 + \theta^2 q] + 2(1 - \theta q) + q^2}{2[(1-q)(5 - 4\theta) + q^2(1 - \theta) + 1]}.$$

Remember that $d > 0$ implies that an increase in the majority rule leads to a higher ex ante expected payoff. Thus, $\min_q \tilde{s}(q, \theta) \leq s^*(\theta) \leq \max_q \tilde{s}(q, \theta)$. Therefore, whenever $\theta < \min_q \tilde{s}(q, \theta)$ then $s^*(\theta) > \theta$. But in this case, $\lim_{N\to\infty} q = 0$, which means that $s^*(\theta) = \bar{s}(\theta) := \tilde{s}(0, \theta) = 3(2-\theta^2)/[4(3-2\theta)]$. Similarly, if $\theta > \max_q \tilde{s}(q, \theta) \geq s^*(\theta)$, then $\lim_{N\to\infty} q = 1$ and thus $s^*(\theta) = \underline{s}(\theta) := \tilde{s}(1, \theta) = (3 - 2\theta)/(4 - 2\theta)$.

To conclude the proof of parts (i) and (ii) of the proposition it remains to show that $\min_q \tilde{s}(q, \theta) > \theta$ for all $\theta < \underline{\theta}$ and $\max_q \tilde{s}(q, \theta) < \theta$ for all $\theta > \bar{\theta}$. As for part (iii), we have to argue that for all $\theta \in [\underline{\theta}, \bar{\theta}]$, $\tilde{s}(q, \theta)$ is monotonically decreasing in q so that $\min_q \tilde{s}(q, \theta) = \tilde{s}(1, \theta) = \underline{s}(\theta)$ and $\max_q \tilde{s}(q, \theta) = \tilde{s}(0, \theta) = \bar{s}(\theta)$.

For any q, $\tilde{s}(q, \cdot)$, is a function which maps the unit interval into $(1/2, 3/4)$. Hence, for any q, there must exist some $\theta \in (1/2, 3/4)$ such that $\tilde{s}(q, \theta) = \theta$. It is easy to show that this equation has a unique solution in $(1/2, 3/4)$ for each q. Denoting this solution by $\tilde{\theta}(q)$ and calculating it explicitly, delivers

$$\tilde{\theta}(q) = \frac{q^2 + 6 - 4q - \sqrt{6 - 4q + q^2}}{q^2 - 4q + 5}.$$

One can verify that $\tilde{\theta}$ is a strictly decreasing function that satisfies $\tilde{\theta}(0) = \bar{\theta} = (6 - \sqrt{6})/5 \approx 0.71$ and $\tilde{\theta}(1) = \underline{\theta} = (3 - \sqrt{3})/2 \approx 0.63$.

We finally have to show that, for $\theta \in [\underline{\theta}, \bar{\theta}]$, the function $\tilde{s}(\theta, q)$ is decreasing in q. Taking the derivative of $\tilde{s}(q, \theta)$ with respect to q and factoring out and canceling all strictly positive terms we obtain

$$Z(q, \theta) = 6(1-\theta)^3 - \theta^2 + q(1-\theta)^2(2\theta - q) + 2(1-\theta)\theta(q - 1 - \theta).$$

The last term of this sum is strictly increasing in q, while the second term is maximized for $q = \theta$. Thus we have that

$$Z(q, \theta) \quad \leq \quad 6(1-\theta)^3 - \theta^2 + \theta^2(1-\theta)^2 - (1-\theta)\theta^2 = 6(1-\theta)^3 - 2\theta^2 + \theta^4.$$

Clearly, this upper bound on Z is a strictly decreasing function of θ. Evaluating it at $\underline{\theta}$ delivers a value of -0.348. Thus, $\tilde{s}$ is a decreasing function of q. □

Proof of Proposition 9. If c is uniformly distributed we have $\tilde{\Pi}(m, N, \theta) = \int_0^1 \tilde{\pi}(c, m, N, \theta)dc$ and $\Pi(m, N, \theta) = \int_0^1 \pi(c, m, N, \theta)dc$. Using the definitions of $\tilde{\pi}$ (see (2)) and π (see (5)) and the explicit formulae for the thresholds $\bar{c}$ (according to (24) we have $\bar{c}(m, N, \theta) = (1 + \theta(1 - p - q))/(2 - \theta p - q))$) and $\underline{c}$ (it is straightforward to show that $\underline{c}(m, N, \theta) = \theta(1 - p - q)/(2 - \theta p - q))$, we obtain

$$\tilde{\Pi}(m, N, \theta) = \int_0^{\theta/2} 2(\theta - c)dc + \int_{\theta/2}^{(1+\theta)/2} [2(\theta - c)q + p\theta(1 + \theta - 2c)]dc = \frac{3\theta^2 - q + 2q\theta + p\theta}{4} \quad (36)$$

$$\begin{aligned} \Pi(m, N, \theta) &= \int_0^{\underline{c}} 2(\theta - c)dc + \int_{\underline{c}}^{\bar{c}} [2(\theta - c) + p\theta(1 + \theta - 2c) + (1 - \theta p - q)((\theta - c)q + p\theta(1 - c))]dc \\ &+ \int_{\bar{c}}^{1} [(\theta - c)q + p\theta(1 - c)]dc = \frac{p\theta(3 - 4\theta) + (3 - q)[\theta^2 - (1 - \theta)^2 q]}{2(2 - p\theta - q)}. \end{aligned} \quad (37)$$

If N is large then $p \approx 0$ for all (m, θ). Remember that the optimal majority rule without the option to wait is simple majority. Thus, for N large, we must have that $q \approx 0$ for all $\theta < 1/2$ and $q \approx 1$ for all $\theta > 1/2$. Combining these observations, it follows that

$$\lim_{N\to\infty} \max_m \tilde{\Pi}(m, N, \theta) = \begin{cases} 3\theta^2/4 & \text{if } \theta \leq 1/2 \\ [3\theta^2 + 2\theta - 1]/4 & \text{if } \theta > 1/2. \end{cases} \quad (38)$$

Next, we determine $\lim_{N\to\infty} \max_m \Pi(m, N, \theta)$. Remember that, when waiting is feasible, then the optimal (proportional) rule exceeds θ for all $\theta \leq \underline{\theta}$ (for the definition of $\underline{\theta}$ see the proof of Proposition 8). Thus, we have $\lim_{N\to\infty} q = 0$ in this case. Moreover, for $\theta > \bar{\theta}$ the optimal proportional rule is smaller than θ, which implies $\lim_{N\to\infty} q = 1$.

Finally, when $\theta \in [\underline{\theta}, \bar{\theta}]$, Proposition 8 does not determine exactly the optimal proportional majority rule with the option to wait. However, substituting $p = 0$ in (37), and taking the second derivative with respect to q yields $[\theta^2 - 2(1 - \theta)^2]/(2 - q)^3 > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$); this implies that Π is convex in q and hence maximized at either $q = 0$ or $q = 1$; thus, Π is less or equal to $\max\{3\theta^2/4, 2\theta - 1\}$. Summarizing, we have

$$\lim_{N\to\infty} \max_m \Pi(m, N, \theta) \begin{cases} = 3\theta^2/4 & \text{if } \theta < \underline{\theta} \\ \leq \max\{3\theta^2/4, 2\theta - 1\} & \text{if } \theta \in [\underline{\theta}, \bar{\theta}] \\ = 2\theta - 1 & \text{if } \theta > \bar{\theta}. \end{cases} \quad (39)$$

Comparing (39) and (38) immediately delivers part i). As for part ii), notice first that for $1/2 < \theta < \underline{\theta}$ we have $\tilde{\Pi}(m, N, \theta) - \Pi(m, N, \theta) = (2\theta - 1)/4 > 0$. Similarly, for $\theta > \bar{\theta}$ we have $\tilde{\Pi}(m, N, \theta) - \Pi(m, N, \theta) = 3(1 - \theta)^2/4 > 0$. This also proves part (iii) for $\theta \in [\underline{\theta}, \bar{\theta}]$. □

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