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Generalized solitary solution and compacton-like solution of the Jaulent–Miodek equations using the Exp-function method

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Abstract

A new generalized solitary solution of the Jaulent–Miodek equations is obtained using the Exp-function method. By a transformation, the solitary solution can be easily converted into a generalized compacton-like solution. The free parameters in the obtained generalized solutions might imply some meaningful results in physical process.

Keywords: Exp-function method; Jaulent–Miodek equation; Solitary solution; Periodic solution

1. Introduction

In this Letter we consider the Jaulent–Miodek equations [1]

$$\begin{aligned} &u_t + u_{xxx} + \frac{3}{2}vv_{xxx} + \frac{9}{2}v_x v_{xx} - 6uu_x - 6uvv_x - \frac{3}{2}u_x v^2 = 0, \\ &v_t + v_{xxx} - 6u_x v - 6uv_x - \frac{15}{2}v_x v^2 = 0 \end{aligned} \tag{1}$$

which associate with energy-dependent Schrödinger potential [2–4]. This equation has many interesting characters, such as finite-band solution [5], solitary solution [6], compacton-like solution [7]. There are many methods to solve Eq. (1), such as F-function method [6], Adomian method [8], and tanh method [9]. A heuristic review on recently developed analytical methods is available in Refs. [10,11]. Those methods can only solve a special kind of solutions. Generally speaking the obtained solutions cannot satisfy actual initial/boundary conditions. Furthermore, the solution procedures become very complex as the degree of nonlinearity increases. For example, the main demerit of the decomposition method is the difficulty in calculating the so-called Adomian polynomial [8]. Ghorbani and Saberi-Nadjafi [12] suggested a very simple approach to calculating Adomian polynomial using the homotopy perturbation method [13,14]. Recently study showed that the variational iteration method [15–19] and the homotopy perturbation method [20–26] can completely overcome the difficulty arising in Adomian method.

Another approach to Eq. (1) is the numerical method [27]. Though it is very easy for us now to find the solutions of linear systems by means of computer, it is, however, still very difficult to solve nonlinear problems numerically. This is possibly due to the fact that the various discredited methods or numerical simulations apply iteration techniques to find their numerical solutions of nonlinear problems, and nearly all iterative methods are sensitive to initial solutions, so it is very difficult to obtain converged results in cases of strong nonlinearity [28–30]. In addition, most important information, for instance the dependence of the maximum wave

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height, wave celerity, and the length of a solitary wave of a nonlinear wave equation on the initial/boundary conditions, will be lost during the procedure of numerical simulation. Furthermore, the numerical solution cannot outline, for example, the dispersion relation between the wave speed and frequency, which plays a pivotal role in physics.

The Exp-function method [10,31–34] emerged not only as a new mathematical tool for searching for a generalized solitary solution, but also as compacton-like and periodic solutions. Furthermore the method is also valid for differential-difference equations [35,36].

2. Exp-function method

Introducing a complex variation ξ defined as $\xi = kx + \omega t + \varphi$, we can convert Eq. (1) into ordinary different equations, which read

$$\omega u' + k^3 u''' + \frac{3}{2}k^3 v v''' + \frac{9}{2}k^3 v' v'' - 6kuu' - 6kuvv' - \frac{3}{2}ku'v^2 = 0, \tag{2}$$

$$\omega v' + k^3 v''' - 6k(uv)' - \frac{15}{2}kv'v^2 = 0, \tag{3}$$

where the prime denotes the derivative with respect to ξ.

Solving u from Eq. (3), we obtain

$$u = \frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}, \tag{4}$$

where c_1 is an integration constant.

Substituting u into Eq. (2) yields:

$$\begin{aligned} &\omega\left(\frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}\right)' + k^3\left(\frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}\right)''' + \frac{3}{2}k^3 v v''' + \frac{9}{2}k^3 v' v'' \\ &\quad - 6k\left(\frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}\right)\left(\frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}\right)' \\ &\quad - 6k\left(\frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}\right)vv' - \frac{3}{2}k\left(\frac{\omega}{6k} + \frac{k^2 v''}{6v} - \frac{5}{12}v^2 + \frac{c_1}{v}\right)' v^2 = 0. \end{aligned} \tag{5}$$

According to the Exp-function method [31–34], we assume that the solution of Eq. (5) can be expressed in the following form:

$$v(\xi) = \frac{a_c \exp[c\xi] + \cdots + a_{-d}\exp[-d\xi]}{b_p \exp[p\xi] + \cdots + b_{-q}\exp[-q\xi]}. \tag{6}$$

To determine values of c and p, we balance the linear term of highest order of Eq. (5) with the highest order nonlinear term. By simple calculation, we have

$$\frac{v^{(5)}}{v} = \frac{c_1 \exp[(32p + c)\xi] + \cdots}{c_2 \exp[33p\xi] + \cdots} \tag{7}$$

and

$$v^3 v' = \frac{c_1 \exp[(p + 4c)\xi] + \cdots}{c_2 \exp[5p\xi] + \cdots} = \frac{c_1 \exp[(29p + 4c)\xi] + \cdots}{c_2 \exp[33p\xi] + \cdots}. \tag{8}$$

Balancing the highest order of Exp-function in Eqs. (7) and (8), we have $32p + c = 29p + 4c$, which leads to the result $p = c$.

Similarly we balance the lowest order in Eq. (5) to determine values of d and q, we obtain

$$\frac{v^{(5)}}{v} = \frac{c_1 \exp[-(32p + d)\xi] + \cdots}{c_2 \exp[-33p\xi] + \cdots}$$

and

$$v^3 v' = \frac{c_1 \exp[-(p + 4d)\xi] + \cdots}{c_2 \exp[-5p\xi] + \cdots} = \frac{c_1 \exp[-(29p + 4d)\xi] + \cdots}{c_2 \exp[-33p\xi] + \cdots}$$

which leads to the result $q = d$.

For simplicity, we set $p = c = 1$ and $q = d = 1$, then Eq. (6) reduces to

$$v(x,t) = \frac{a_1 \exp[\xi] + a_0 + a_{-1}\exp[-\xi]}{b_1 \exp[\xi] + b_0 + b_{-1}\exp[-\xi]}.$$

In case $b_1 \neq 0$ Eq. (6) can be simplified as

$$v(x,t)=\frac{a_1\exp[\xi]+a_0+a_{-1}\exp[-\xi]}{\exp[\xi]+b_0+b_{-1}\exp[-\xi]}. \tag{9}$$

Substituting Eq. (9) into Eq. (5), we have

$$\begin{aligned}&\frac{1}{B^4A^5}\big[C_8\exp(8\xi)+C_7\exp(7\xi)+C_6\exp(6\xi)+C_5\exp(5\xi)+C_4\exp(4\xi)+C_3\exp(3\xi)\\&\quad+C_2\exp(2\xi)+C_1\exp(\xi)+C_0+C_{-1}\exp(-\xi)+C_{-2}\exp(-2\xi)+C_{-3}\exp(-3\xi)\\&\quad+C_{-4}\exp(-4\xi)+C_{-5}\exp(-5\xi)+C_{-6}\exp(-6\xi)+C_{-7}\exp(-7\xi)+C_{-8}\exp(-8\xi)\big]=0,\end{aligned} \tag{10}$$

where $A=(\exp(\xi)+b_0+b_{-1}\exp(-\xi))^5$, $B=(a_1\exp(\xi)+a_0+a_{-1}\exp(-\xi))^4$, C_i $(i=-8,-7,\ldots,7,8)$ are constants obtained by Matlab.

Equating the coefficients of $\exp(n\xi)$ to be zero, we obtain

$$\begin{cases}C_8=0,\quad C_7=0,\quad C_6=0,\quad C_5=0,\quad C_4=0,\quad C_3=0,\quad C_2=0,\quad C_1=0,\\ C_0=0,\\ C_{-8}=0,\quad C_{-7}=0,\quad C_{-6}=0,\quad C_{-5}=0,\quad C_{-4}=0,\quad C_{-3}=0,\quad C_{-2}=0,\quad C_{-1}=0.\end{cases} \tag{11}$$

Solving the system, Eq. (11), simultaneously, we obtain the following solution

$$\begin{aligned}&a_0=3a_1b_0\frac{k^2+a_1^2}{3a_1^2+k^2},\qquad b_{-1}=\frac{1}{4}b_0^2\frac{9a_1^4+10k^2a_1^2+k^4}{(3a_1^2+k^2)^2},\qquad a_{-1}=\frac{1}{4}a_1b_0^2\frac{9a_1^4+10k^2a_1^2+k^4}{(3a_1^2+k^2)^2},\\&c_1=\frac{5}{12}k^2a_1+\frac{5}{12}a_1^3,\qquad \omega=\frac{k}{8a_1^2}\big(15a_1^4+3k^4+10k^2a_1^2\big).\end{aligned} \tag{12}$$

We, therefore, obtain the following generalized solitary solution

$$v(x,t)=\frac{a_1\exp[\xi]+3a_1b_0\frac{k^2+a_1^2}{3a_1^2+k^2}+\frac{1}{4}a_1b_0^2\frac{9a_1^4+10k^2a_1^2+k^4}{(3a_1^2+k^2)^2}\exp[-\xi]}{\exp[\xi]+b_0+\frac{1}{4}b_0^2\frac{9a_1^4+10k^2a_1^2+k^4}{(3a_1^2+k^2)^2}\exp[-\xi]}, \tag{13}$$

where $\xi=kx+\omega t+\varphi$, a_1 and b_0 are free parameters which can be determined by initial or boundary conditions

$$\omega=\frac{k}{8a_1^2}\big(15a_1^4+3k^4+10k^2a_1^2\big).$$

The Jaulent–Miodek equations also have compacton-like solutions, for this reason transferring the above generalized solitary solution to a generalized compacton-like solution is far of being a trivial problem. Exp-function method [29–32] is at this moment the most promising candidate theory for this purpose. Using the transformations

$$\begin{cases}\xi=i\zeta=Kx+\Omega t+\varphi,\\ \exp(\xi)=\cos(\zeta)+i\sin(\zeta),\\ \exp(-\xi)=\cos(\zeta)-i\sin(\zeta)\end{cases}$$

and $k=iK$, where K is a real number, Eq. (13) becomes

$$\begin{aligned}v(x,t)&=a_1+\frac{3a_1b_0\frac{-K^2}{3a_1^2-K^2}}{\cos(\varsigma)+i\sin(\varsigma)+b_0+C(\cos(\varsigma)-i\sin(\varsigma))}\\&=a_1-\frac{3a_1b_0\frac{K^2}{3a_1^2-K^2}}{(1+C)\cos(\varsigma)+b_0+i(1-C)\sin(\varsigma)},\end{aligned} \tag{14}$$

where $C=\frac{1}{4}b_0^2\frac{9a_1^4-10K^2a_1^2+K^4}{(3a_1^2-K^2)^2}$.

If we search for a periodic solution or compacton-like solution, the imaginary part in the denominator of Eq. (14) must be zero, that requires

$$1-C=1-\frac{1}{4}b_0^2\frac{9a_1^4-10K^2a_1^2+K^4}{(3a_1^2-K^2)^2}=0. \tag{15}$$

Solving b_0 from Eq. (15) we obtain

$$b_0 = \pm \frac{-3a_1^2 + K^2}{\sqrt{9a_1^4 - 10K^2a_1^2 + K^4}}. \tag{16}$$

Substituting Eq. (16) into Eq. (14) results in a compacton-like solution, which reads

$$v = a_1 \pm \frac{3a_1 \frac{K^2}{\sqrt{9a_1^4 - 10K^2a_1^2 + K^4}}}{2\cos(\varsigma) \pm \frac{-3a_1^2 + K^2}{\sqrt{9a_1^4 - 10K^2a_1^2 + K^4}}}, \tag{17}$$

where a_1 and K are free parameters.

3. Conclusions

In this Letter, the Exp-function method with a computerized symbolic computation has been proposed to obtain a generalized solitary solution and a generalized compacton-like solution. The Exp-function method is a straight, concise, reliable and promising mathematical tool to solve nonlinear evolution equations arising in mathematical physics.

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References

[1] M. Jaulent, J. Miodek, Lett. Math. Phys. 1 (1976) 243.
[2] H.T. Ozer, S. Salihoglu, Chaos Solitons Fractals 33 (4) (2007) 1417.
[3] J.L. Zhang, M.L. Wang, X.R. Li, Chaos Solitons Fractals 33 (5) (2007) 1450.
[4] J.M. Zhu, Z.Y. Ma, Chaos Solitons Fractals 33 (3) (2007) 958.
[5] R. Zhou, J. Math. Phys. 38 (1997) 2535.
[6] E. Fan, Chaos Solitons Fractals 16 (2003) 819.
[7] J.H. He, X.H. Wu, Chaos Solitons Fractals 29 (1) (2006) 108.
[8] J.B. Chen, X.G. Geng, Chaos Solitons Fractals 30 (4) (2006) 797.
[9] A.-M. Wazwaz, Phys. Lett. A 366 (1–2) (2007) 85.
[10] J.H. He, Non-Perturbative Method for Strongly Nonlinear Problems, Dissertation, De-Verlag im internet GmbH, Berlin, 2006.
[11] J.H. He, Int. J. Mod. Phys. B 20 (10) (2006) 1141;
J.H. He, Int. J. Mod. Phys. B 20 (18) (2006) 2561.
[12] A. Ghorbani, J. Saberi-Nadjafi, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 229.
[13] J.H. He, Int. J. Non-Linear Mech. 35 (1) (2000) 37.
[14] J.H. He, Int. J. Nonliner Sci. Numer. Simul. 6 (2) (2005) 207.
[15] D.D. Ganji, M. Jannatabadi, E. Mohseni, J. Comput. Appl. Math. 207 (1) (2007) 35.
[16] Z.M. Odibat, S. Momani, Int. J. Nonlinear Sci. Numer. Simul. 7 (1) (2006) 27.
[17] N. Bildik, A. Konuralp, Int. J. Nonlinear Sci. Numer. Simul. 7 (1) (2006) 65.
[18] E. Yusufoglu, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 153.
[19] H. Tari, D.D. Ganji, M. Rostamian, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 203.
[20] T. Ozis, A. Yildirim, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 243.
[21] A. Belendez, A. Hernandez, T. Belendez, et al., Int. J. Nonlinear Sci. Numer. Simul. 8 (1) (2007) 79.
[22] P.D. Ariel, T. Hayat, S. Asghar, Int. J. Nonlinear Sci. Numer. Simul. 7 (4) (2006) 399.
[23] T. Ozis, A. Yildirim, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 239.
[24] Q.K. Ghori, M. Ahmed, A.M. Siddiqui, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 179.
[25] M.A. Rana, A.M. Siddiqui, Q.K. Ghori, et al., Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 185.
[26] D.D. Ganji, A. Sadighi, Int. J. Nonlinear Sci. Numer. Simul. 7 (4) (2006) 411.
[27] D. Kaya, S. El-Sayed, Phys. Lett. A 318 (2003) 345.
[28] T.L. Bo, L. Xie, X.J. Zheng, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 223.
[29] E. Tlelo-Cuautle, J.M. Munoz-Pacheco, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 249.
[30] Y.C. Zeng, Y. Wu, Z.G. Pei, et al., Int. J. Nonlinear Sci. Numer. Simul. 7 (4) (2006) 385.
[31] J.H. He, X.H. Wu, Chaos Solitons Fractals 30 (3) (2006) 700.
[32] J.H. He, M.A. Abdou, Chaos Solitons Fractals 34 (5) (2007) 1421.
[33] X.H. Wu, J.H. He, Comput. Math. Appl. 54 (2007) 966.
[34] A. Bekir, A. Boz, Int. J. Nonlinear Sci. Numer. Simul. 8 (2007) 505.
[35] S.-D. Zhu, Int. J. Nonlinear Sci. Numer. Simul. 8 (3) (2007) 461.
[36] S.-D. Zhu, Int. J. Nonlinear Sci. Numer. Simul. 8 (3) (2007) 465.