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Equilibrium in FX Swap Markets: Funding Pressures and the Cross-Currency Basis

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Abstract

Departure from Covered Interest Parity (CIP), known as the cross currency basis, is not just a staple of crises: it can build up slowly and persist. Some bases exacerbated in 2008 have not gone away since then. To understand this normality requires turning the CIP logic on its head. We look at the Foreign Exchange (FX) swap market as the very market where scarce funding capacities are exchanged; the basis becomes an equilibrium outcome that compensates one of the parties for the temporary loss in the possession of one of the currencies. Ultimately, the counterparty's funding pressure in that currency determines the willingness to pay for such endogenous possession value.

In our model, banks compete for funding in two currencies. Unsecured, secured and FX positions are bounded by *leverage ratio constraints* tying banks' equity. Currency-specific funding pressures are apparent in banks' secured *funding constraints*, governing how securities denominated in different currencies can be pledged (and short-sold). The latter, not the former, is what drives the basis; this explains why bases also arise with no crisis in sight. A basis occurs when secured funding becomes more binding in one currency than in the other; leverage constraints can only have an accessory effect through this channel. Equivalently, the basis depends on how different across currencies are the spreads between actual (bank specific) unsecured borrowing rates and the secured rates. To illustrate, we look at central banks' actions targeting international funding pressures, in particular FX swaps lines and collateral policies.

Keywords: Cross-currency basis, FX swap market, repo, Covered Interest-rate Parity.

JEL Classification: D5, E5,G15, G18

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1 Introduction

When nationals of given countries hold large quantities of foreign assets, their choice is between risky large FX exposure, which is limited for most institutions, and large funding exposure in the foreign currency. In the case of the dollar, a large funding exposure by foreign nationals can only be ultimately mitigated with the cooperation of the Fed[1]. In recent years, such cooperation has been granted through a central banks' swap line that was set up in the aftermath of the recent global financial crisis and still exists. But even when such swap lines are present, funding pressures visible in FX swap markets mount in tandem with increase in foreign assets.

During the financial crisis, the dollar funding pressure almost brought the European banking sector down as some of their dollar assets lost collateral status. The Fed saved the day, cooperating with the ECB to lend the needed dollars to European banks. But funding pressures are not just present in crisis: as a by-product of the current accumulation of foreign assets by Japanese institutions, there is a similar dependence of Japanese banks on dollar funding. This funding pressure is apparent in the FX swap market. Current deviations from frictionless arbitrage pricing in this market are comparable to the ones observed at the peak of the sovereign European crisis. That is, the funding pressures that affect the FX market clearing prices can be found also in normal times. Our paper studies the equilibrium mechanism by which the FX swap markets clear funding pressures in different currencies. To successfully harness this mechanism is a policy option.

1.1 Deviations from CIP

For any pair of currencies and a pair of respective funding channels made available with no limit or friction, a simple arbitrage argument -the CIP- establishes how the FX forward rate should relate to the FX spot rate. However, such an arbitrage argument relies on scalability in the funding channels. In a world where funding is limited, and currencies are in different relative scarcities, a

[1]Reserves are a country's first line of defense, after which the Fed's help is required.

significant discrepancy may occur. Funding pressures influence the deviation from CIP which is usually represented as a spread, called *cross currency basis*, being added to one of the domestic interest rates.

The *theoretical* insight that binding collateral requirements or other forms of funding constraints generate a failure of arbitrage pricing of assets is well understood and modelled in the existing literature. See, for example, Garleanu and Pedersen (2011), who address the failure of arbitrage pricing of securities and derivatives in the presence of binding margin requirements. The impact of margins on exchange rates was actually emphasized in the paper by Gabaix and Maggiori (2015), which addressed the determination of the outright levels of exchange rates, but did not allow for deviations from arbitrage pricing (CIP holds in their model).

On the *empirical* side, it was also observed that funding constraints can explain the limits to arbitrage, in particular to CIP, as in the papers by Mancini-Griffoli and Ranaldo's (2010) and Hrung and Sarkar (2012). The former brought in the importance of secured borrowing (and why basis should be computed over repo rather than Libor rates, which are averages of banks' reported offer rates but may not be the effectively charged rates) and showed that funding constraints kept traders from arbitraging away excess profits. The latter pointed out how unanticipated changes in repo funding volumes (due to difficulties in finding dollar denominated collateral that could be adequately pledged) may have led banks to seek FX funding and cause an increase in the basis. Very recently, another empirical study, by Du, Tepper and Verdelhan (2016), pointed out that persistent and large deviations from CIP should not be considered the result of credit risk or transaction costs, but, being significantly correlated with fixed-income spreads, reflect instead important financial frictions. These are just some examples of contemporary work on the subject.

These papers and the above observations on the presence of large deviations of the basis from CIP levels suggest that one needs to turn the CIP logic on its head. In this paper we consider the FX swap market as the very market where funding scarcities are exchanged.

1.2 Currency exchanges in a model of competitive funding for banks

Our theoretical approach to the currency basis is related to the approach that Garleanu and Pedersen (2011) followed to look at deviations from arbitrage pricing in security and derivative markets. However, we focus on the CIP deviations and work out a detailed analysis of such deviations. There are important differences between our model (or results) and theirs, but some of their insights carry over into our analysis.

In the model by Garleanu and Pedersen (2011), all deviations from arbitrage pricing in security or derivative markets were driven by the shadow price of a *binding* margins constraint. This constraint required total wealth to cover an unsecured position and the margins paid, both by going long or short, in several securities, pledgeable in repo as collateral for secured loans. Under this constraint, all financial positions become bounded and the constraint's shadow price is equal to the unsecured/secured interests spread. According to Garleanu and Pedersen (2011), the deviations from CIP could also be explained by their binding margins approach, since one needs capital (funding) to trade in order to profit from deviations from parity. It was suggested that the cross currency basis should also be driven by the unsecured/secured interests spread, or the related TED spread (the difference between 3-month unsecured interest and 3-month T-bills interest). However, a precise extension of their model to an international setting was not carried out. With more than one type of unsecured borrowing (at least one per currency) or in presence of FX trades, the margins constraint does not manage to bound positions [2]. Furthermore, its shadow value is no longer the TED spread. In such a setting, it is not obvious what might drive the cross-currency basis.

We build up a general equilibrium model of banking competition in several funding markets, for two currencies, and the FX swap market, in order to re-examine what determines the market clearing prices for FX swaps, and therefore, the basis. We model explicitly the *box constraint* that

[2] In general, if more than one variable can take negative values, such constraint is not enough to bound financial and FX positions. In fact, we can not take the long in one instrument and use the constraint to bound that long position (and, then, by market clearing, bound also the short positions), as the long may now be short in another instrument.

each agent faces for the security (a government bond) denominated in each currency: a constraint requiring, on one hand, the collateral pledged in repo to be bounded by the security long position and, on the other hand, the short sale of the security to be bounded by what was borrowed of the security by having accepted it as collateral in repo. Hence, repo positions have to be explicitly introduced as choice variables for banks and margins will be charged on those repo positions. The crucial constraint in Garleanu and Pedersen (2011) specified margins directly in terms of security long or short positions, rather than on repo trades. It implicitly assumed that: (i) the security long position had to be entirely pledged as collateral (long positions were formed on a leveraged basis), while (ii) the short sale might be lower than the borrowed security but just the short sale should be charged a margin. Assuming the box constraint to be always binding is not an option since there is a slack in the economy as the bond is in positive net supply, implying that, in equilibrium, some agent must also have a slack in the respective box[3]. Once we introduce explicitly these funding constraints (called box constraints), we open up the possibility that the currency basis may be due to funding frictions (captured by the shadow prices of box constraints) rather than by the mere fact that margins have to be paid. A basis may occur in the absence of margins but under a differential in how binding funding is in the two currencies.

Next, we contemplate *no-overdraft constraints*, currency by currency and date by date. Currency exchanges together with financial trades allow banks to meet these constraints. We contemplate several funding avenues: the deposit base, pledged securities and the unsecured interbank market. But the no-overdraft constraints together with the box constraints are not enough to bound all financial positions, as the security serving as collateral can be reused (short sold or re-pledged) and the resulting leverage would be unbounded. We overcome this difficulty by taking into account *banks' leverage requirements*. Our stylized version of regulatory leverage constraint frameworks requires the bank's equity to be at least equal to a certain fraction e of its exposure in assets (total assets minus cash balances). In our *Proposition 1*, by addding up the equity requirements of all

[3]Also, in a deflationary environment, banks sometimes find themselves replacing loans with bonds on the asset side (and can be holders of large quantities of bonds on an unlevered basis, as in the case of Japan).

banks, we obtain upper bounds on secured or unsecured debt and on FX positions, which must hold at any market clearing allocation. This enables us to establish existence of equilibrium, under quite mild assumptions, in *Proposition 2*.

1.3 Currency possession value and the basis

Our analysis explores the analogy between FX swaps and repo. This is our contribution to providing a direct link between the cross-currency basis puzzle and the literature on the collateral value of securities (as developed in Duffie's (1996) paper on repo specialness, and in the article by Brunnermeier and Pedersen (2009) on collateral margins and market liquidity). In an FX swap, possession of two currencies gets swapped (exchanged now and given back to the original holders later), while in repo a security is swapped against cash. Repo specialness (low repo interest rates on a particular security) occurs as a result of a possession value for a scarce security (in order to reuse it, by short selling or repledging it), just like a cross currency basis occurs as a result of a *possession value for a currency* that is scarce, due to funding needs in that currency.

What we mean by the possession value of a currency is the value that somebody is willing to pay (beyond local carry differences)[4] to possess it during the two dates period of the FX swap. If repo is the relevant funding channel, the possession value of dollars is measured in terms of how the ratio of shadow values of the dollar no-overdraft constraints at two dates exceeds the repo return rate in that two-date period. One can imagine the two counterparties in the swap contemplating what they would have done with the cash balance if it had stayed in their possession. The basis is just putting the financial benefit to the holder of the scarcer currency to make him lend it in the swap[5]. Given two comparable funding markets (one in each currency), the basis is a premium on the interest rate in the currency whose relative possession value is higher, that is, whose funding

[4]Carry in CIP follows from assumed funding channels, rather than the FX swap here.

[5]In reality, however, only the FX swap happens and the basis is more about clarifying incentives than actual transactions.

needs are binding to the point of creating a friction visible in market prices. Depending on which funding market we choose, we get a basis over repo rates or interbank unsecured rates.

The FX spot or forward rates just reflect the relative shadow values of the no-overdraft constraints in each currency, at the first or at the second leg of the FX swap, respectively (see *Proposition 3*). The basis over, say, repo rates reflects instead the difference in possession values for the two currencies, taking repo as the relevant funding channel (see *Proposition 4*). Underneath are the funding frictions themselves, that is, the shadow prices of the binding funding constraints in a particular (or in many) credit market(s). We find out in *Proposition 5* that the basis is driven by box shadow prices, that is, opportunity costs of meeting funding constraints, rather than of paying for the margins. *Even if the margins were zero (no haircuts in repo, so that the cash loan given in repo coincides with the value of the bond serving as collateral) a basis would still exist.* What matters is how the need to find collateral affects a bank differently, depending on which is the currency that denominates the collateral. This is an important difference with respect to the approach by Garleanu and Pedersen (2011), where deviations from arbitrage pricing were due to an opportunity cost of paying margins. Actually, their margin constraint (under the aforementioned implicit assumptions on how repo and security positions are related and pay margins) already embeds the funding constraints, and it is not possible to distinguish these two opportunity costs[6].

Putting aside this difference, it is interesting to notice that both in Garleanu and Pedersen (2011) and in our model, the non-linearity of the constraint that bounds the financial positions seems to be crucial for a basis to occur in equilibrium. In their margins constraint, margins were being charged on the absolute value of the security position. Such non-linearity is inspired on what is the practice in exchanges. In our model, we address both the case of bilateral repo, where margins are charged by the repo long (the borrower of the bond) to the repo short (the one that pledges the bond), and the case of centralized repo, where both the repo long and the repo short pay margins to the exchange.

[6]In our model the opportunity cost of paying a margin is the shadow value of the margin coefficient itself (i.e., the derivative of the Lagrangean of the bank's optimization problem with respect to the repo haircut coefficient) and, therefore, differs from the box shadow value.

We have nevertheless also a non-linearity in the equity requirements leverage constraint: secured or unsecured borrowing enter on the non-equity liability side, while secured or unsecured lending enter on the assets side, therefore with a different coefficient[7]. To avoid non-convexities we go around this non-linearity in signed variables, by decomposing them into debts and credits. But the non-linearity is just as crucial in Garleanu-Pedersen (2011) as in our model: the fact that first order conditions on debts differ from those on credits is what allows us to find a basis.

Finally, the basis can be expressed in terms of observable market levels. More precisely, we show in *Proposition 6* that the basis over repo rates is driven by the difference between the unsecured-secured interest rate spreads in each currency. In fact, these spreads are related to shadow values of binding box constraints[8]. Unsecured interest rates are bank specific; these are the rates that each bank pays when borrowing from others in a given currency. However, the basis over repo rates is obviously common to all banks and does not reflect counterparty risk (the latter affects both spreads, if a bank is an unsecured borrower in both currencies, and turns out not to affect the former). Actually, the relationship of the basis as a market outcome and counterparty *solvency* is of special interest. The current USD-JPY basis episode shows how the basis is driven by relative funding needs in the two currencies, rather than by credit wariness or solvency issues, which are now largely absent. If solvency is understood in terms of meeting a regulatory requirement, which is here the regulatory leverage constraint, it can not be the cause of a basis, since the two currencies (assets or liabilities) are treated symmetrically. Therefore, even when the leverage constraint is binding, the effect of its shadow price on the funding in one currency cancels out the analogous effect on the funding in the other currency. The cross currency basis reflects instead the relative possession value of the two currencies as frictions come up from the imbalance in the two

[7]Our equity requirement implies that non-equity liabilities cannot exceed cash balances plus a fraction $1 - e$ of the bank's assets exposure. Hence, borrowed values enter with a coefficient 1 while lent values enter with a coefficient $-(1 - e)$.

[8]For agents that have a slack in their box constraints (and we know that there must always be such agents), the basis instead of being driven by box shadow values will now be driven by the shadow values of the non-negativity constraints on credit variables.

currencies that banks may be experiencing[9]. If for example banks facing severe funding pressures in one currency (say, dollars) see their domestic funding conditions (e.g., in euro or yen) improve, our results show that the basis *will widen, not narrow.*

We briefly comment also on bases over unsecured interest rates (in *Proposition 7*) and show that such bases are null whenever the basis over repo rates is zero (in *Proposition 8*), which supports our choice to focus on the secured basis.

1.4 Central bank interventions and large departures from CIP

In the aftermath of the Lehman Brothers bankruptcy, many European banks were left with large long positions in US dollar credit markets and with a difficulty in rolling over the funding of these positions. This funding friction was responsible for the large cross currency basis in EURO vs USD. We use our model to rationalize this episode and also examine how coordinated ECB-Fed actions helped to alleviate that funding pressure and reduce the basis.

Our equilibrium approach is useful to understand the effect on the basis of (1) the establishment of the Fed's dollar liquidity swap line with the ECB, (2) the ECB's policy of accepting dollar denominated assets as collateral in its repo operations with European banks, and (3) the ECB movement to a policy of full allotment. We rationalize how these policies led to the relaxation of European banks' funding constraints. We allow for the collateral pledged by European banks to be denominated in both euros and dollars, and for each of these possibilities (in *Propositions 9 and 10*), we provide equilibrium pricing formulas that relate the EUR-USD basis to the ECB's policy repo rate and haircut.

Even though the basis events in the 2008 financial crisis were our initial motivation, subsequent episodes show that significant departures from CIP tend to occur often in modern financially so-

[9]The solvency premise of the Bagehot dictum seems to apply. Policy is tricky here as coordinated central bank action is required: the need is typically outside the jurisdiction of the central bank that can create the currency in demand. The solution: swap lines among central banks which are nothing but...FX swaps.

phisticated economies. During the 2011-12 European sovereign debt crisis, the basis between the euro and other key currencies had another peak. Moreover, and in our view more interestingly as this is not associated with a crisis, the Yen-dollar basis has been also quite high since mid-2015, as the easing by the BOJ led to a large build-up of Yen funding of US assets.

1.5 Structure of the paper

The paper is organized as follows. Section 2 defines FX swaps and the basis, providing some examples as a background. Section 3 presents the model, the equilibrium concept and several results relating the basis to equilibrium shadow prices or observable prices (interest rates in different markets). Section 4 discusses central banks' actions targeting the basis and presents results on these policies. Section 5 concludes. Proofs can be found in the online supplementary material.

2 FX swaps and the Cross-Currency Basis

We consider FX swaps exchanging the following currency amounts at dates 1 and 2.

Date	Domestic currency	Foreign currency
1 :	$-X$	1
2 :	χ	-1

The FX swap market works as follows: for any fixed X amount, the market will set an amount χ so that agents can enter the FX swap at no cost. An agent engaging in the cash flow in the above diagram is Buy-Selling 1 unit of foreign currency. A canonical swap occurs when date 1 is the spot settlement date for the FX market and X is *the spot rate of the FX market*. Such a swap is naturally collateralized, as the exchanged initial amounts have the same value. This self-collateralization enables many counterparties with various credit qualities to smoothly trade with each other in the

FX swap market. When X is the FX spot rate, χ above is called the forward FX rate for date 2. The difference $\chi - X$, is referred to in terms of *FX points.* There is *a term structure of different potential dates (and FX points) for date 1.* We will focus on two fixed dates 1 and 2 (say, year 1 and year 2). This is without loss of generality for our purposes[10].

The FX swap is a collateralised transaction without reference to market interest rates (collateralized or not) in each currency. The model we build will endogenously determine χ through market clearing. That is, the Forward FX rate χ is not necessarily determined from covered interest parity levels: an additional degree of freedom may be needed to clear the FX swap market.

There are many analogies between FX swaps and repo markets. In repo, a cash balance is temporarily exchanged against a security. Once in possession of the security, one can for example short it. The value of security possession sometimes comes in the form of specialness [11]. However, the securities' entire class may be all more or less desirable[12]. In an FX swap, cash in one currency is exchanged against cash in another currency. The desirability of one currency against another will be visible in the pricing of a market FX swap. To capture it we need to strip out the impact of the different funding rates and get a sense of what influences the supply and the demand of different currencies. Once χ (and X) have been determined in such a general equilibrium setting, we can examine how χ/X deviates from what CIP would predict for particular funding rates.

2.1 Why is the cross-currency basis not close to zero?

We begin by illustrating the basis with a standard CIP argument. Assume that an agent has unlimited access to the funding market at interest rates R_d and R_f for domestic and foreign (e.g., USD and EUR), respectively, both for borrowing and investment, and also assume without loss of

[10]Short term funding is very sensitive to short term scarcity; one of the most important short end markets occurs when date 1 is tomorrow and date 2 is the next spot date - the following valid settlement (*"Tom Next"*). The 3m point is also an important liquid point for FX markets. One year will be mostly used here for notational simplicity.

[11]The premium over the least desirable security in the class (GC security) is the specialness value.

[12]There may be a common possession value within the class of securities contemplated in the definition of GC.

generality a one year horizon. Assume also that a canonical FX swap can be done for that period: one unit of foreign currency (e.g., Euro or Yen) is bought in the spot market for X of domestic currency (e.g., US dollars) against selling it forward at χ. When an agent does not have a balance X in the domestic market, he borrows this balance in the domestic funding market and pays back $X \times (1 + R_d)$ one year later. Simultaneously with this transaction, he can invest one unit of foreign currency at a one-year euro rate R_d and sell the proceeds forward at a one-year forward rate χ to get $(1 + R_f) \times \chi$ as a domestic balance at the end of the year. In terms of domestic balances, the net proceeds of all these transactions are $(1 + R_f) \times \chi - (1 + R_d)X$.

Under the above assumptions the net proceeds cannot be positive because a scalable free-lunch strategy would be possible. By a similar argument the proceeds cannot be negative either if we exchange the role of the currencies. Hence, the CIP holds so that the theoretical forward rate is related to the spot rate and to the interest rates in the two currencies by $\chi = X\frac{1+R_d}{1+R_f}$.

However, the market forward rate χ routinely differs from the one implied by CIP. We can express the basis in units of US dollar interest rates as follows[13]:

$$\chi = X\frac{1 + R_d + \beta}{1 + R_f} \tag{1}$$

The economic interpretation of the basis β is intuitive: if the domestic currency (think dollar) is the currency in shortage, then the convenience yield associated with the physical ownership of dollars is reflected by the fact that $\beta > 0$. The owners of domestic currencies at date 1 will only part with their physical holdings of dollars and agree to a forward transaction if they are compensated at date 2 with the effective interest rate $R_d + \beta > R_d$.

Why is the cross-currency basis not close to zero? A key impediment is scalability: arbitrage commits funding capability or cash balance in the scarcer currency. Such a capability is typically

[13] β stands for the extra spread paid to borrow the domestic currency (e.g., dollars). The approach is equivalent to writing the basis in units of foreign currency interest rate, as $\alpha = -\beta\frac{1+R_f}{1+R_d+\beta}$. In the theoretical developments we use the more intuitive measure β.

bounded. It is a resource shared with many bank activities and commits bank capital.

2.2 Some examples

Lack of scalability follows from the difficulties of some foreign banks to raise dollar funding for their dollar assets. Cash dollar balances are harder to generate for foreign banks. Caught in a credit carry trade, they may find themselves in possession of lower quality collateral with more fragile funding markets (compared to US Treasuries). Demand for funding in US dollars was very high in the wake of the Lehman crisis, but also during the European Sovereign crisis that followed it, as well as nowadays, when the large dollar funding need of Japanese institutions is being exacerbated by the QQE of the BOJ (and the associated increase in foreign assets holdings).

In 2008, right after the failure of Lehman Brothers, as well as in 2011-2012 during the sovereign debt crisis and since 2013 for JPY USD basis, many factors concurred to make this market price for exchanging funding abilities extremely costly (see Goldberg, Kennedy, and Miu (2009) for similar arguments based only on unsecured borrowing). Banks were extremely reluctant to lend the scarcer currency for the reasons presented earlier and, as a result, the basis widened each time imbalances between assets held and funding were at work. However, the recent example of Japan shows that high levels of the basis are not necessarily associated with the presence of a crisis.

Therefore, agents who possess dollars will be price sensitive when allocating their scarce resource. At the individual level, the basis translates into the shadow prices associated with limited access to funding. Fundamentally, the FX forward level χ is the driving variable, since all cross currency basis are just a function of χ given different funding rates.

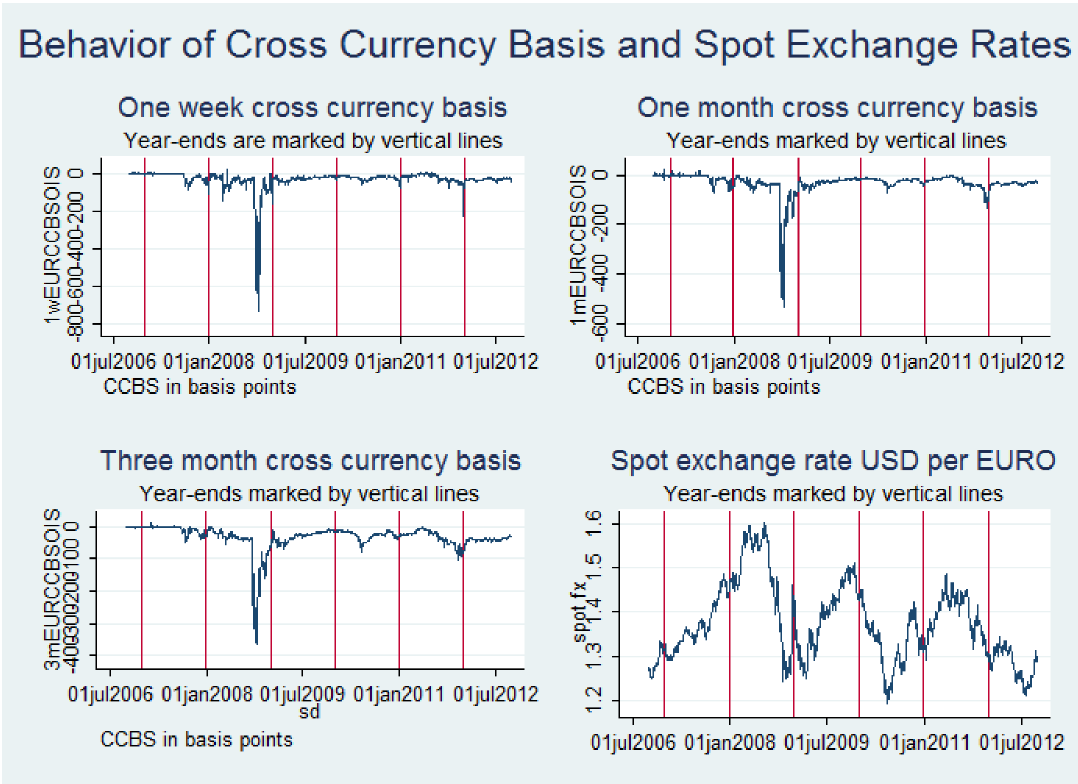


Figure 1: Cross Basis Currency History (Source: JP Morgan). The euro-dollar basis is computed as an annualized basis point spread on the euro interest rate received in basis points using the respective overnight interest swap rates.

2.2.1 Example 1: USD funding pressures in Europe

Figure 1 plots the euro-dollar cross-currency basis for 1-week, 1-month, and 3-months tenors, together with the spot exchange rates using OIS funding rates. It uses the market convention and measures the basis in annualized basis point spreads applied to the *euro* rate. Our main sample is from March 2008 to April 2012. In Figure 1, however, we display the basis for a slightly longer period, going back to October 2006, to emphasize that the basis prior to the onset of the credit crisis was broadly consistent with CIP. Several interesting facts emerge from this figure. First, the cross-currency basis is relatively small until mid-2007. After mid-2007, the basis becomes significantly negative and remains that way throughout the rest of the sample. Summary statistics

Table 1: Summary Statistics on Cross-currency Basis for March 2008 - April 2012

Tenor	Mean	Std.Error	95% Confidence interval
1 week ccbs	-38.498	1.916	[-42.258 to -34.737]
1 month ccbs	-40.595	1.573	[-43.682 to -37.509]
3 months ccbs	-39.718	1.306	[-42.281 to -37.156]

for the cross-currency basis are presented in Table 1 for the period March 2008 to April 2012.

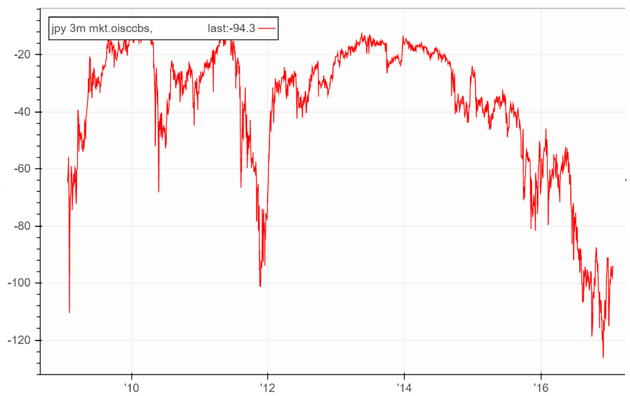
Figure 1 also shows that the basis tended to widen around some year-ends (especially in the 2011 year-end), a behavior well-documented in money market rates. The spot exchange rates plotted in Figure 1 show relative strengthening of the US dollar relative to the euro around year-ends, again notably at the 2011 year-end. Table 1 shows that the mean of the cross-currency basis for all tenors is significantly negative with the average around -40 basis points[14]. There is evidence that we can have a deviation from Covered Interest Parity (CIP) for sustained periods of time.

The basis widened dramatically at two stages in the sample period. The first widening of the basis occurred in 2008-2009 shortly after Lehman Brothers filed for bankruptcy on September 15, 2008. The second widening occurred later in the sample, around late 2011 when the European sovereign debt crisis escalated. In a nutshell we get a sense of the relative possession value of USD vs EUR as described by cross currency basis as a function of time. This suggests the existence of a possession value for currencies, with the USD attracting a larger possession value than the EUR over the contemplated period.

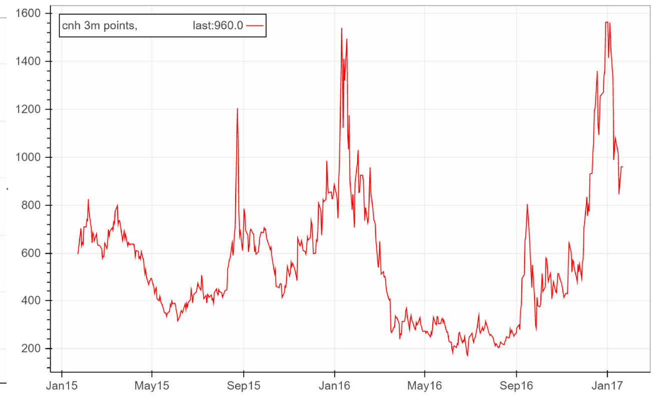
2.2.2 Example 2: USD funding pressures in Japan

In our next example there is also a widening around crisis, but then an intensification of the basis in non-crisis times. It is the example of Japan where the latest round of easing by the BOJ

[14]This finding was not heavily influenced by extreme outliers in the data, given the large number of daily observations (over 1,000). For the overall sample, we reject at conventional levels of significance that the mean of the basis during the sample period is zero (see Table 1).



(a) USDJPY Cross Basis Currency bp/ois annualized



(b) CNH 3M points

Figure 2: deviation from CIP: 2 examples (Source: JP Morgan)

saw expansion of the balance of USD assets funded in JPY. The widening of the USDJPY basis is associated with the gradual build-up of cross currency funding imbalance. Stricter regulation meant to mitigate the very funding pressures (as a multi-year buffer of reserve funding is promoted with the acquisition of foreign assets) has not managed, until now, to impede the imbalance. The persistence of a significantly negative basis (see Figure 2a) is clear.

2.2.3 CNH example.

There is another possible reason for the rise of a basis – simply the hedging demand for that currency, just like in the securities market. After a secular appreciation of the Chinese Yuan (CNY) things became less clear since the summer of 2015 and the hedging demand for CNY increased substantially for some time. The offshore CNH balances of Yuan suddenly played a key role. The hedging needs have been disproportionally fulfilled by the amount of currency freely tradable: the offshore CNH balances of Reminbi (Yuan) in Hong Kong. The possession value of these balances increased a great deal due to those hedging needs, as can be seen from the evolution of the points over time (see Figure 2b). The year-end effect, both in 2015 and 2016, should be noted[15]. This is

[15]Pressure mounts with the yearly allocation of USD 50000 each January for Chinese to acquire foreign currency

similar to a security getting special as the demand to short it increases.

2.3 Banks

There are many users of cross border finance, such as trading companies, cross border issuers and central banks. We chose banks for several reasons. One is their prevalence in the FX market, probably a consequence of their deposit function, and hence easy access to cash. Banks essentially serve a market-making function for FX products including FX swaps, and as such their incentives and constraints are likely to dominate the price discovery in those markets. Central banks have the power to create currency and, therefore, directly influence the relative scarcity of currencies. As such, central banks' constraints are not really suited to understand the market clearing mechanism of the FX swap market but more the associated policies (we will touch upon this later).

Most banks have a home currency, which is determined by its historical deposit base. A large deposit base of a bank automatically translates into ability to fund a large amount in this currency. In an international environment *the balance sheet* (with aggregates across different currencies expressed in the bank's home currency) is balanced: assets $\bar{A}$ equal equity $\bar{E}$ plus non-equity liabilities $\bar{L}$. This accounting identity - a direct consequence of double entry - uses book value (the transaction historical value). On the non-equity liabilities we find those denominated in the domestic currency, in the form of deposits (D), short-term collateralized or uncollateralized debt (D_s) and long-term debt (D_b), and also the analog components denominated in the foreign currency (D',D'_s and D'_b). On the assets side we have the assets denominated in the domestic currency, in the form of unemcumberd securities (B_u), encumbered securities (B_e), loans (l) and cash (C), together with the analog components denominated in the foreign currency (B'_u,B'_e,l' and C').

The amount of funding raised in the domestic currency ($D + D_s + D_b + E$) is usually greater than the foreign funding ($D' + D'_s + D'_b + E'$). In contrast with the liability side (which is very localized), the asset side can be much more international, and the foreign assets (A') can be commensurable with the domestic assets (A) especially in countries where excess saving is observed and domestic investment opportunities are not good due to a deflationary environment pushing down the domestic interest rate margin (the difference between interest received on a loan and the cost of deposits). The potential to raise funding in the foreign currency through foreign deposits (difficult due to a lack of a foreign branch network), long term debt and equity (name recognition is an issue abroad), and short term debt (relatively easier but volatile) is often outstripped by foreign investment, and an FX swap is used to make up for the mismatch in funding.

Typically, unemcumbered foreign securities (B'_u) tend to be small (if a foreign investment can be funded with repo, it is a desirable option), but unemcumbered domestic ones (B_u) can be quite large. Remark also that some securitization programs are available to banks, typically for their domestic loan portfolios, and this (by moving some of the assets from L to B_u) increases domestic funding options. Overall the international driver of the cross border funding role has been excess saving in some countries meeting borrowing first outside of the private sector in the form of government bonds and finally by some other countries (e.g., Europe and Japan versus US). In countries where borrowing is high, there will be lower incentives to look for foreign assets to invest in. So we can expect international funding demand for currencies of such countries.

One can determine the home currency of a bank by the currency where D_s, D, C, E and B_u can be large. Funding can be raised by increasing any of the liability items. The easiest avenues are: (1) pledging unpledged securities (typically bonds) from B_u, (2) using unused cash balances from C and (3) raising uncollateralised short term money (through CDs, interbank, money-markets). One can think of banks as juggling between secured (in the form of unpledged bond holding) and unsecured funding (in the form of unused cash and unsecured credit lines).

From the point of view of a bank trading FX swaps, it can source the currency it will lend either through its deposits and credit lines, or by pledging some of its securities in the repo market. From this perspective the money lent to the official sector preserves funding ability as government bond markets have a liquid repo market. Ultimately the deposit base of a bank will be the most fundamental anchor of its ability to fund. Wholesale avenues to fund (like bond issuance or repo outside of the core markets of government bonds) can prove quite volatile in times of crisis.

3 A Model of Currency Shortage

In this section, we develop a theory of funding constraints that will allow us to write the cross-currency basis as a function of the shadow prices of the underlying funding constraints.

3.1 FX, Repo, and Uncollateralized markets

The model is formulated having as the major players in FX markets both foreign (e.g. Japanese or European) and domestic (e.g. US) banks[16]. We model the transactions taking place at *three dates*, dates 1, 2 and 3 (issuance previously occurred at a date 0). To simplify, there is no uncertainty[17].

To keep the model simple, our endogenous variables are only interbank trades or trades between banks and bond issuers or central clearing houses. We do not model the decision problems of banks' customers, depositors, or non-financial borrowers and, for this reason, our model focuses on cash balances. We consider just two currencies, which we refer to as the domestic and the foreign currencies. Each bank maximizes a profit function which will be specified below, subject to several funding constraints. We denote by I the set of banks and, as explained below, we will consider some additional agents, namely the official sector (for each currency area) and possibly also clearing houses. We now present six relevant markets that banks have access to.

The markets for domestic and foreign cash: Each bank is assumed to be a member of one of the two currency systems and can only hold cash balances in that currency. For $t = 1, 2$, the cash balances y^i_{jt} in currency j left over at date t by a bank i member of system j, after all date t

[16]We have in mind the cross currency funding implications of the recent credit and sovereign crisis, when dollar-seeking European banks played the key role, as they were the most constrained and affected by the funding implication.

[17]This is just for simplicity (adding uncertainty is actually direct).

financial or FX trades have taken place, constitute a reserve at the respective central bank and earn interest on reserve (IOR) at a rate $i_{j,t+1}$. Denote by $\tilde{y}^i_{j,t+1}$ the non-negative cash balance that the same agent will hold at the next date prior to engaging in financial or FX trades. We denote the subset of banks that are members of system j by I_j.

Banks' initial cash holdings at date 1, $\tilde{y}^i_{j1}$, are predetermined, and stand for reserves that banks in I_j had at the respective central bank (possibly accrued of IOR). At a date $t > 1$, we have $\tilde{y}^i_{j,t} = (1 + i_{j,t})y^i_{j,t-1}$ for i member of system j.

At any date, for a bank not in the j' currency system, the j' cash in minus cash out, in trades in financial instruments denominated in currency j' is equal to the j' cash invested in FX spot or swap trades, as will be described below[18].

Cash markets clear at date t if $\sum_i y^i_{jt} = \sum_i \tilde{y}^i_{jt}$ for $j = d, f$. When we are not explicit about the domain of the summation index, we mean that the sum is over all agents, not just the banks.

The domestic and foreign bond markets: In our simple economy, there are just two bonds, one domestic government bond and one foreign. Before trade takes place at date 1, if foreign loan demand has been weak, foreign banks often start with relatively large initial government bond holdings (substitutes for lack of private borrowing), which we denote by $\tilde{b}^i_j$. Bonds have exogenous payments at the payment dates. We assume[19] that date 1 is not a payment date whereas date 2 is. In the case of treasury notes, the exogenous payments c_{jt} are either coupons (before maturity) or the principal (at maturity), whereas in the case of treasury bills there is just a principal to be paid at maturity. We can allow for the bond maturity being date 2 or a later date (date 3), depending on how we may want to combine the bond maturity with the maturity of the repo loans that use the bond as collateral. Bond positions of agent i, at dates 1 or 2, are denoted by b^i_{jt}. Bond markets clear at date t if $\sum_i (b^i_{jt} - \tilde{b}^i_j) = 0$. We denote the market clearing bond prices at date t by q_{jt} and

[18]We are aware that there are some facilities available, in particular through the BIS, to invest cash held in other currencies, but to keep the model simple we abstract from such opportunities.

[19]This is assumed without loss of generality as the bond trades ex-div in period 1.

the endogenous *return* that clears date 1 trades by $r_j = (q_{j2} + c_{j2})/q_{j1} - 1$[20].

The repo markets: Banks can secure their funding through repo rates. In a repo, collateral is lent to raise funding. This is probably the most relevant funding market, as the presence of government bonds on the balance sheet of banks translates into ability to fund. For repo at a date t, cash is exchanged against a bond serving as collateral and at $t+1$ the initial trade is reversed. Most repo trades tend to be done on bonds that mature after the repo maturity but the case of repo to maturity is also possible (the fact that a principal is paid at date 2 implies that the bond is still suitable as collateral and, at maturity, the cash lender surrenders the principal net of the loan settlement).

We denote by θ_d^i and by ψ_d^i repo long and repo short positions, respectively, using domestic bonds as collateral[21]. That is, ψ_d^i are the units of the collateral asset that bank i pledges in order to obtain a cash loan, whereas θ_d^i are the units of the collateral asset that bank i accepts in order to provide a cash loan. Repo positions on the foreign bond θ_f^i and ψ_f^i are defined analogously. The corresponding repo rates are denoted by ρ_d and ρ_f. Repo markets clear if $\sum_i(\theta_j^i - \psi_j^i) = 0$ for $j = d, f$. Following Bottazzi, Luque and Pascoa (2012), we write the collateral funding constraints (security box constraints) for bonds denominated in the two currencies, as follows:

$$box_d \equiv b_d^i + \theta_d^i - \psi_d^i \geq 0 \qquad \text{(multipliers)} : \tilde{\mu}_d^i \tag{2a}$$

$$box_f \equiv b_f^i + \theta_f^i - \psi_f^i \geq 0 \qquad : \tilde{\mu}_f^i \tag{2b}$$

The multipliers of the box constraints in the bank's profit maximization problem, once divided by the bond price (for convenience), are $\mu_j = \tilde{\mu}_j^i/q_{j1}$. If domestic cash is needed in period 1, an agent can sell domestic bonds (reduce b_d^i) or lend them ($\psi_d^i > 0$) against a loan denominated in the domestic currency. In both cases, actions in domestic bonds are constrained by (2a) or (2b).

[20]For a one-period treasury bill (that is, if the bond was not issued earlier and has principal c_j and zero coupons), r_j coincides with the yield.

[21]We need separate variables, rather than a single repo variable taking positive or negative signs, in order to avoid non-convexities that would occur in a leverage requirement constraint (14) that will be defined below.

To short sell the bond ($b_d^i < 0$), the agent needs to borrow ($\theta_d^i > 0$) enough of the bond (i.e., accept it as collateral when giving a dollar denominated secured loan) to do that. Symmetrically, lending the bond ($\psi_d^i > 0$), that is, pledging the bond as collateral in a repo loan, requires him to have a long position ($b_d^i > 0$) (and, analogously, for the box constraint (2b) for foreign bonds).

We accommodate both the case of bilateral repo and the case of repo trades done through a central counterparty clearing house (CCP), and we will show that our results are robust to whether repo is done over-the-counter (OTC) or cleared on exchange. The margin/haircut is reusable in the former but not in the latter.

In *bilateral repo*, the repo short pledges ψ_d^i units of the collateral (say, the domestic bond), worth $q_d\psi_d^i$, in order to obtain a cash loan $h_d q_d \psi_d^i$, for a given haircut $1 - h_d$ (which we specify exogenously). The counterparty, the repo long, accepts that collateral, provides a cash loan which is worth less, but is entitled to reuse the whole collateral. That is, in bilateral repo, repo longs have a haircut benefit[22].

In *centralized repo* (like in Garleanu and Pedersen (2011)), both parties pay a margin to the exchange: the cash loan provided by the repo long to the repo short is worth exactly the value of the pledged collateral (say, the domestic bond) but both must pay a margin $1-m_d$ over the collateral value. At the repo settlement date, collateral and loan repayments (accrued of repo interest) are given back to their providers, and the margins (accrued of repo interest) are also given back by the exchange to both parties[23].

The uncollateralized funding market: Different unsecured borrowers may be charged different interest rates. We denote by u_j^i the unsecured borrowing in currency j by bank i and denote by a_{ji}^k the credit line to bank i in currency j that counterparty k provides. The interest rate that bank i pays at date 2 for funding in currency j is π_{ji}. We denote $a_j^i \equiv \sum_{k \in I, k \neq i} a_{ji}^k$. Unsecured markets

[22]It is easy to see (using Lemma 1 in section 6.2 of the appendix) that in bilateral repo θ_d^i and ψ_d^i may be both positive only when there is a null shadow value for the leverage requirement constraint.

[23]It can be shown (using Lemma 1 in section 6.2 of the appendix) that in centrally cleared repo θ_d^i and ψ_d^i may be both positive only when the shadow values for the leverage requirement and the box are both null.

clear if $a^i_j = u^i_j$ for $j = d, f$ and every i.

The FX swap market: Action variable ϕ^i denotes the amount of foreign currency (say JPY or EUR) sold by bank i against $X\phi^i$ units of domestic currency (say USD) at date 1. Then, at date 2 the same amount of foreign currency ϕ^i is bought back against $\chi\phi^i$ of domestic currency, where χ is the rate that can be locked in at date 1 to trade foreign against domestic currency at date 2 (the forward FX rate). The FX swap market clears if $\sum_i \phi^i = 0$.

The FX spot market: Action variable σ^i denotes the amount of euros at date 1 that bank i exchanges against $X\sigma^i$ amount of dollars at date 1, where X is the date 1 spot exchange rate. FX spot market clears if $\sum_i \sigma^i = 0$.

We have a very simplified representation of *the retail sector* of banks. Customers are partitioned into two sets, S_d and S_f, each one consisting of agents trading goods denominated in just one currency $j = d, f$ and financing such trades through variations in deposits across every bank $i \in I_j$ or by borrowing from banks in I_j. Let l^i_{kjt} be the loan balance at date t for agent $k \in S_j$ at bank $i \in I_j$. The variation of loan balance occurs through repayment and new loans. For simplicity we neglect interest payments for both deposits and loans. Deposits held by customer k at bank $i \in I_j$ at date t are D^i_{kjt}.

Let G_{kjt} and $\tilde{G}_{kjt}$ be the demand and endowment vectors of consumer k at date t for the commodities (in number M, say) whose prices o_{jt} are denominated in currency j. Denote by G^{gj}_{jt} and $\tilde{G}^{gj}_{jt}$ the commodity demand and endowments by the official sector gj at this date. Consumer k's budget constraint requires $o_{jt}(G_{kjt} - \tilde{G}_{kjt}) + \sum\limits_{i \in I_j} (D^i_{kjt} - D^i_{kj,t-1} - l^i_{kjt} + l^i_{kj,t-1}) = 0$.

Let $l^i_{jt} \equiv \sum\limits_{k \in S_j} l^i_{kjt}$ and $D^i_{jt} \equiv \sum\limits_{k \in S_j} D^i_{kjt}$. We take these two aggregate variables to be exogenously given, for each $i \in I_j$ and denote $\Delta^i_{jt} \equiv D^i_{jt} - D^i_{j,t-1} - l^i_{jt} + l^i_{j,t-1}$. By commodity market clearing we get

$$\sum_{i \in I_j} \Delta^i_{jt} = o_{jt}(G^{gj}_{jt} - \tilde{G}^{gj}_{jt}) \tag{3}$$

Some consumers may be net borrowers, while others may be net lenders to banks but on aggregate, across banks and their customers, the private savings must match public expenditure[24].

We model banks as profit maximizing agents. Profits are measured as the own cash holdings in the "home currency" of each bank, that is, as cash holdings net of liabilities of the bank to its customers in the currency of the system that the bank belongs to. Such liabilities are deposits minus loans. We assume that profits are evaluated at the final date (which is date 2 or 3, depending on whether repo is done up to bond maturity or just up to an intermediate date), that is, bank i's funding profits[25] are of the form $\Pi(y) = y^i_{jT} - D^i_{jT} + l^i_{jT}$, where T is the final date, for some currency j which is the "home currency" of bank i. As D^i_{jT} and l^i_{jT} are predetermined, the objective function depends just on cash balances y^i_{jT} left over at the final date.

3.2 No-overdraft currency by currency

On top of the aforementioned funding constraints, banks face *no-overdraft constraints in each currency*[26]. Let Ω^i_{jt} be the net trade denominated in currency j, carried over in FX markets and in the financial instruments denominated in this currency. If bank i is a member of currency system j, cash trades in this currency may be non-null,

$$i \in I_j \Rightarrow y^i_{jt} - \tilde{y}^i_{jt} = \Omega^i_{jt} \tag{4}$$

[24]Excess savings by the private sector (as observed in Japan since the 90s) must have an offsetting official sector expenditure. In this model, the only way to accommodate deposits growing faster than loans, is by shrinking private consumption, for a fixed commodities supply.

[25]The approach of this paper is to assume normal domestic banking operations are given (hence we strip out its effect on cash balances) and focus on international funding and investment.

[26]The "no overdraft" constraint for a certain currency is somehow analogous to the box constraints (2a) and (2b). Securities can be shorted and loans in each currency can be arranged, but non-negative possession of such securities and currencies have to be monitored and enforced all along.

where $y^i_{jt} \geq 0$ and $\tilde{y}^i_{jt} = (1 + i_{jt})y^i_{j,t-1}$, for $t > 1$. Otherwise, the cash trade is null,

$$i \notin I_j \Rightarrow \Omega^i_{jt} = 0 \tag{5}$$

The initial cash balance $\tilde{y}^i_{d1}$ available to a bank member of the domestic system, before engaging in FX or financial trades, can be thought of as representing the reserves (accrued of IOR) that the bank had at the central bank of the domestic system. To keep the model as simple as possible, we focus on interbank trades and take as given the difference Δ^i_j between the variation in deposits and the variation in loans to customers outside the banking sector. We assume that $\Delta^i_j = 0$ for $i \notin I_j$.

Let us describe precisely what Ω^i_{jt} is. To capture both repo scenarios we introduce the variable δ taking values b or c, where $\delta = b$ stands for bilateral repo whereas $\delta = c$ stands for centralized repo. Date 1 net trade in the domestic currency is given by

$$\Omega^i_{d1} \equiv X(\sigma^i + \phi^i) + q_{d1}[\tilde{b^i}_d - b^i_{d1} - t^\delta_d \theta^i_d + s^\delta_d \psi^i_d] - a^i_d + u^i_d + \Delta^i_{d1} \qquad \text{multiplier} : \lambda_{d1} \tag{6}$$

The coefficients on the repo long and repo short positions are defined as follows. In bilateral repo ($\delta = b$), t^b_d and s^b_d are both equal to h_d (since the haircut, in a proportion $1 - h_d$, is paid by the repo short and collected by the repo long). In centralized repo ($\delta = c$), collateral values coincide with cash loans but both parties pay a margin to the exchange (in a proportion $1 - m_d$ of the collateral value), implying that $t^c_d = 2 - m_d$ (as the repo long must provide the cash loan and pay the margin), but $s^c_d = m_d$ (as the repo short receives the cash loan and pays the margin).

Equations (6) and (4) say that domestic cash balance increases when bank i sells or swaps foreign currency for domestic currency, sells domestic bonds, pledges domestic bond as collateral through repo or borrows at the uncollateralized interest rate π_{di}. Lending to its retail customers (captured by $-\Delta^i_{d1}$) reduces the domestic cash balance.

The net trade in the foreign currency at date 1 is captured by

$$\Omega^i_{f1} \equiv -(\sigma^i+\phi^i)+q_{f1}[\tilde{b^i}_f-b^i_{f1}-t^\delta_f\theta^i_f+s^\delta_f\psi^i_f]-a^i_f+u^i_f+\Delta^i_{f1} \qquad \text{multiplier}: \lambda_{f1} \quad (7)$$

where t^δ_f and s^δ_f are defined analogously to their domestic counterparts (t^δ_d and s^δ_d). To simplify, we ignore the unsecured funding market of date 2, and, therefore, we get

$$\Omega^i_{d2} \equiv -\chi\phi^i + \sum_{k\neq i}(1+\pi_{dk})a^i_{dk} - (1+\pi_{di})u^i_d + (1+\rho_d)q_{d1}(t^\delta_d\theta^i_d - s^\delta_d\psi^i_d) + c_{d2}b^i_{d1} + q_{d2}(b^i_{d1} - b^i_{d2}) + \Delta^i_{d2} : \lambda_{d2} \quad (8)$$

In the case of bilateral repo, the repo settlements appearing in (8) are just the repayment (accrued of repo interest) of the cash loan (the haircuted collateral value in this case) by the repo short to the repo long. In the case of centralized repo, on top of the settlement of the cash loan (the exact collateral value, in this case), there is also the repayment of the margins (accrued of repo interest) by the exchange to both sides of the repo market. Similarly, for the foreign currency,

$$\Omega^i_{f2} \equiv \phi^i + \sum_{k\neq i}(1+\pi_{fk})a^i_{fk} - (1+\pi_{fi})u^i_f + (1+\rho_f)q_{f1}(t^\delta_f\theta^i_f - s^\delta_f\psi^i_f) + c_{f2}b^i_{f1} + q_{f2}(b^i_{f1} - b^i_{f2}) + \Delta^i_{f2} : \lambda_{f2} \quad (9)$$

The monotonicity of bank i profits, in final date cash balances of the currency of the system that the bank belongs to (and, indirectly on previous cash balances, as $\tilde{y}^i_{jt} = (1+i_{jt})y^i_{j,t-1}$), allows us to rewrite (4) as $y^i_{jt} - \tilde{y}^i_{jt} \leq \Omega^i_{jt}$. Such inequality representation implies that the shadow value of (4) is non-negative. The non-negativity of the shadow values for the equality constraints (5) follows then from the first order conditions on FX spot and swap trades.

The bond issuance $\hat{b}_j$ is a choice variable of the official sector gj. We assume that, at some date t' before date 1, bond j was issued in the amount $\hat{b}_j$ and that what was issued becomes the aggregate initial holdings of the banks at date 1, $\hat{b}_j = \sum_{i\in I}\tilde{b}^i_j$. The way $\hat{b}_j$ gets distributed across banks would depend on how trade occurred between dates t' and 1.

In this section, we model *the official sector* in a very simple way, as raising debt cash flows from tax receipts. Date 1 initial cash holdings $\tilde{y}^{gj}_{j1}$ of the official sector stand for taxes collected at earlier dates and, therefore, are predetermined. We assume that subsequent dates ($t > 1$) are not tax collection dates either and, therefore, the initial cash holdings are equal to the cash held at the previous date minus the IOR on reserves. That is,

$$\tilde{y}^{gj}_{jt} = y^{gj}_{j,t-1} - i_{jt} \sum_{I_j} y^{i}_{j,t-1} \tag{10}$$

Notice that, for $t > 1$, cash markets clearing can be rewritten as requiring $\sum_i y^i_{jt} = \sum_i y^i_{j,t-1}$.

Let $\tilde{c}_{tj} = c_{tj}$ if $t > 1$ and $\tilde{c}_{tj} = 0$ at $t = 1$. To simplify we take the official sector commodity endowments and consumption as exogenous, satisfying (3). The official sector no-overdraft constraint at any date $t \geq 1$ is[27]

$$\tilde{y}^{gj}_{tj} - y^{gj}_{tj} = \tilde{c}_{tj}\hat{b}_j + o_{jt}(G^{gj}_{jt} - \tilde{G}^{gj}_{jt}) \tag{11}$$

Notice that, by market clearing in commodity markets, equation (3) implies

$$\sum_{i \in I_j} \Delta^i_{kjt} = \tilde{y}^{gj}_{jt} - y^{gj}_{jt} - c_{tj}\hat{b}_j \tag{12}$$

Using (12) and adding the no-overdraft constraints of all banks and the official sectors (for each date and each currency), we see that *Walras law holds*, in the sense that the sum of the values of aggregate (across banks and the official sector) excess demand in cash, FX, bond, secured and unsecured interbank credit markets must be zero.

Finally, to close the model, in the case of centralized cleared repo, we need to model two exchanges (CCP houses), ed and ef, handling the repo trades in each of the bonds. Exchanges are

[27] To simplify, we are not allowing for buy backs or tap issuance. For such minimalist representation, the official sector can be assumed to issue as much debt as possible, given the tax receipts pre-determined and presented through $\tilde{y}^{gj}$), that is, $\hat{b}_j = \min\{\tilde{y}^{gj}_{2j}/c_{2j}, \tilde{y}^{gj}_{3j}/c_{3j}\}$.

assumed to be passive agents that collect margins and invest them in repo in order to pay back later to the banks[28]. This implies that $\theta^{ej} = (1 - m_j)(\sum\limits_{i \in I} \psi_j^i + \sum\limits_{i \in I} \theta_j^i)$ and $\sum\limits_{i \in I} \psi_j^i = \sum\limits_{i \in I} \theta_j^i + \theta_j^{ej}$. Then, when the repo market for bond j is active, margin coefficients are given by $m_j = \frac{2 \sum_{i \in I} \theta_j^i}{\sum_{i \in I} \psi_j^i + \sum_{i \in I} \theta_j^i}$.

3.3 Leverage Constraints

Box and no-overdraft constraints are not enough to bound banks' choice variables[29] and guarantee a solution to the profit maximization problems. We introduce next *leverage requirements* in the spirit of the Basel framework that will allow us to bound debt and FX variables.

We assume that the equity E^i of each bank i must be at least a fraction $e < 1$ of its exposure in assets, which is the sum of assets A^i minus the cash balances C^i: $E^i \geq e(A^i - C^i)$, or equivalently, $(1 - e)(A^i - C^i) + C^i \geq L^i$, where L^i is the total non -equity liability of the banks. To be more specific, in the context of bilateral repo, these variables are defined by

$$A^i \equiv q_{d1} b_{d1}^i + h_d q_d \theta_d^i + a_d^i + y_{d1}^i + l_{d1}^i + X(q_{f1} b_{f1}^i + h_f q_f \theta_f^i + a_f^i + y_{f1}^i + l_{f1}^i) \tag{13a}$$

$$C^i \equiv y_{d1}^i + X y_{f1}^i \tag{13b}$$

$$L^i \equiv X h_f q_f \psi_f^i + h_d q_d \psi_d^i + X u_f^i + u_d^i + D_{d1}^i + X D_{f1}^i \tag{13c}$$

The leverage requirement implies that, for each bank i and, say in domestic currency terms, the sum of secured and unsecured debts incurred in both currencies plus deposits (all the non equity liabilities) must be bounded by the cash balances plus $(1 - e)$ times the exposure in assets. More

[28]Notice that investing the margin in the respective bond would not be a good modelling choice as the bond return r_j can't be greater than the repo rate ρ_j in the centralized repo case (whereas the opposite happens in bilateral repo) and, in case of equality, the margin becomes indeterminate.

[29]Recall that the reuse of the collateral prevents the collateral constraints from bounding secured loans. Moreover, there are other funding instruments (unsecured borrowing and FX) available at the same time, which makes it hard to bound the position of a long in one of the instruments as she may be short in another. In the appendix (section 6.1) we comment on some of the difficulties involved.

precisely, *in the case of bilateral repo*, we have

$$Lev \equiv Xh_f q_f \psi_f^i + h_d q_d \psi_d^i + Xu_f^i + u_d^i + D_{d1}^i + XD_{f1}^i - (1-e)(A^i - C^i) - (y_{d1}^i + Xy_{f1}^i) \leq 0 : \nu \tag{14}$$

In *the case of centralized repo*, each bank (either as a repo long or as a repo short) pays to the exchange a margin and, at the repo settlement date, that margin (accrued of repo interest) must be given back to the bank. Margins constitute claims of the bank on the exchange and, therefore, should enter on the assets' side of the leverage constraint. On the assets' side we also have the cash loans given in repo (as these are claims on the counterparties), and on the non-equity liability side we have the cash borrowed in repo (which is exactly equal to the collateral value in the case of centralized repo). Hence, assets denominated in currency j are $\tilde{A}_j^i \equiv q_j(b_j^i + (2-m_j)\theta_j^i + (1-m_j)\psi_j^i) + \sum_{k\in I, k\neq i} a_{jk}^i + l_{j1}^i + y_j^i$. That is, (14) should now be replaced by

$$Xq_f\psi_f^i + q_d\psi_d^i + Xu_f^i + u_d^i + D_{d1}^i + XD_{f1}^i \leq (1-e)(X\tilde{A}_f^i + \tilde{A}_d^i - C^i) + (y_{d1}^i + Xy_{f1}^i) \qquad : \nu \tag{15}$$

Let $\zeta = e/(1-e)$ and $\Lambda \equiv \frac{1}{\zeta}\sum_{i\in I}(Xq_f\tilde{b}_f^i + q_d\tilde{b}_d^i + l_{d1}^i + Xl_{f1}^i) + \frac{1}{e}(\sum_i \tilde{y}_{d1}^i + X\sum_i \tilde{y}_{f1}^i)$. Adding inequality (14) or (15) over banks and using the market clearing conditions we get the following

Proposition 1 *The equity requirement imposed on each bank limits the amount of interbank debt formation compatible with market clearing. In the case of bilateral repo,* $h_d\sum_{i\in I} q_d\psi_d^i \leq \Lambda$ *while* $h_f\sum_{i\in I} Xq_f\psi_f^i \leq \Lambda$. *For centralized repo* $q_{d1}\sum_{i\in I}\psi_d^i \leq \Lambda$ *and* $Xq_{f1}\sum_{i\in I}\psi_f^i \leq \Lambda$. *In both cases,* $u_d^i \leq \Lambda$ *and* $u_f^i X \leq \Lambda$.

This, together with the box constraints, limits the value of short sales. FX spot and swap trades become bounded as well. This is true irrespective of the amount of equity issued by banks.

The upper bounds in Proposition 1 are not being imposed as constraints of the optimization problem of an individual bank. On the contrary, the observance of such upper bounds is a property of any market clearing allocation[30].

3.4 Equilibrium

Each bank i chooses $\varphi^i \equiv ((y^i_j, (a^i_{jk})_{k\neq i}, u^i_j, b^i_j, \theta^i_j, \psi^i_j, \sigma^i, \phi^i)$ in order to maximize $\Pi(y^i)$ on a constraint set $K(P)$ given a parameters vector P describing prices (X, χ, π, q, ρ), cash and bonds initial holdings $\tilde{y}^i_1$ and $\tilde{b}^i$ and retail exogenous variables $(D_t, l_t)_{t=1,2,3}$. The constraint set is defined by (6),(7),(8),(9), together with (2a), (2b) and, either (14) or (15). Let the *indirect profit function* be $\tilde{\Pi(P)} \equiv \max\limits_{K(P)} \Pi(., P)$.

Issuing agents (the domestic official sector and its foreign counterpart) for the bonds choose a non-negative issuance $\hat{b}_j$ (equal to the initial net supply $\sum_{i\in I} \tilde{b}^i_j$) of each bond, together with cash balances y^{gj}_{jt} for each date t, so that the official sector's (thought of as the government together with the central bank) no-overdraft constraints (11) hold. Then, dates 2 and 3 commodity market clearing implies that coupons and the principal are paid back by the issuer, and vice-versa. The way that initial net supply ends up being allocated across banks is a process that took place first of all in the primary market of the issuance date t' and was then reshuffled in the secondary market between t' and date 1. We denote a plan for official sector j by $\varphi^j \equiv (\hat{b}_j, (y^{gj}_{jt})_{t=1,2,3})$.

An *equilibrium* consists of a price vector $(X, \chi, \pi, q, \rho, \pi)$ and an allocation of banks' choices $(\varphi^i)_{i\in I}$ and official sectors' choices φ^j ($j = d, f$), such that (i) each bank maximizes profits under the aforementioned constraints at these prices, given $\tilde{y}^i_{j1}$, $\tilde{b}^i$ and Δ^i_{jt}, (ii) official sector constraints hold, (iii) all markets clear and (iv) the issuances $\hat{b}_j$ are consistent with the initial holdings of the banks (that is, $\hat{b}_j = \sum_{i\in I} \tilde{b}^i_j$ for $j = d, f$).

To ensure existence of equilibrium, we assume that initial holdings of cash and bonds are sufficiently high to dominate retail proceeds in case these are negative. Under this assumption, banks'

[30] To be more precise, in the Arrow-Debreu tradition, an auxiliary truncated economy can be defined with upper bounds incorporated into the individual constraint set, but an equilibrium for the auxiliary economy is an equilibrium for the untruncated economy.

constraint sets have interior points and no-overdraft shadow values are bounded. The precise statement of the assumption is as follows:

Assumption A: For each i, $\tilde{b}_j^i > 0$, for $j = d, f$, and for $i \in I_j$ we have $\tilde{y}_{j1}^i + \Delta_{j1}^i > 0$ and $c_{jt}\tilde{b}_j^i + \Delta_{jt}^i > 0$, for $t > 1$.

In our simplified representation of retail, l_{jt}^i stands for bank *i*'s date t loan balance to its customers: new loans plus loans rollover minus loans repayments. Having assumed no interest on retail loans, there is no discrepancy between repayment and rollover and, therefore, l_{jt}^i captures what new loan balances are. When bank *i*'s customers are net savers, on the aggregate, assumption A will be trivially satisfied. In a deflationary environment, as for example in the 90's in Japan or in many countries during the 2008 global financial crisis, it is common for deposits to increase faster than loans and traditional banking activity generates a cash surplus increasing reserves: positive Δ_{jt}^i across banks is accommodated (in (3)) by government expenditure (or equivalently, as seen in (12), by large official cash holdings beyond the levels of public debt service). This large excess of cash in the banking system accommodates the acquisition of foreign assets.

Proposition 2 *Under assumption A, there exists an equilibrium.*

The quantity χ clears the market for FX swap. Bases are just an expression of this quantity for certain interest rates associated with funding scenarios. In fact, denoting the multiplier for the no-overdraft constraint in currency j at date t by λ_{jt}^i, one has

Proposition 3 *At equilibrium* $\chi = \frac{\lambda_{f2}^i}{\lambda_{d2}^i}$, *for any bank* i.

This result follows from the necessary first order conditions (FOC) with respect to σ^i (spot) and ϕ^i

(FX swap), which are, respectively[31],

$$X\lambda^i_{d1} = \lambda^i_{f1} \tag{16a}$$

$$\chi\lambda^i_{d2} + \lambda^i_{f1} = \lambda^i_{f2} + X\lambda^i_{d1} \tag{16b}$$

All basis formula follow by considering a certain pair of interest rates, corresponding to some funding assumption, as we shall see next.

3.5 Relationship Between Cross-Currency Basis and Funding Constraints

We start by relating the basis to the possession values of the two currencies for any bank. In order to define the possession value of each currency, we need to compare the currency marginal rate of substitution (MRS, the rate at which the bank is willing to substitute cash balances at one date for cash balances at another date, in that same currency) with a market funding rate in that currency (say the repo rate). The former is the ratio $\lambda^i_{j1}/\lambda^i_{j2}$ of the shadow values of the no-overdraft constraints at the two dates, in that currency. In fact, each shadow value λ^i_{jt}, for $t = 1, 2$, measures the impact on maximal profits, given by the indirect profit function $\tilde{\Pi}$, of a relaxation of the respective no-overdraft constraint.

When funding is done through repo markets, the *possession value of the domestic currency* to bank i is the premium of the indirect profit MRS in that currency over the repo return: $\frac{\lambda^i_{d1}/\lambda^i_{d2}}{1+\rho_d} - 1$. Loosely speaking, it measures how may dollars the bank wants to get tomorrow to compensate for a 1 dollar sacrifice today in initial holdings, by comparison with what dollar repo funding can provide tomorrow for that 1 dollar investment. As we tend to favor looking at the basis over secured rates, we will, from now on, use the notation β only for the basis defined by (1) when the funding rates are the repo rates ρ_d and ρ_f. Then, we have

[31] These are the FOCs of i's profit maximization problem, whose Lagrangean is $Lagr^i \equiv \Pi^i + \nu^i Lv^i + \sum_{j=d,f} \mu^i_j box^i_j + \sum_{j=d,f} \sum_{t=1,2} \lambda^i_{jt}\Omega^i_{jt}$, and all multipliers are non negative.

Proposition 4 *There is a positive basis β over repo rates if and only if, for any bank, the domestic currency possession value exceeds foreign currency possession value (relative to repo funding).*

In fact, $\frac{\lambda^i_{d1}/\lambda^i_{d2}}{1+\rho_d} > \frac{\lambda^i_{f1}/\lambda^i_{f2}}{1+\rho_f}$ if and only if $\chi/X > (1+\rho_d)/(1+\rho_f)$.

We can use the first order conditions on repo (reported in (17d) in section 6.2 of the appendix) to evaluate the currency possession value in terms of the shadow values of binding constraints. This will allow us to write the basis in terms of shadow interest rates. Let μ^i_d and μ^i_f be bank i's shadow values for the domestic and foreign bonds' box constraints (2a) and (2b), divided by the current bond prices q_d and q_f, respectively. These shadow prices measure the value that the bank attaches to the possession of these bonds at date 1. More precisely, these shadow values measure what the bank would gain at date 1 if the bank could short sell one unit of the bond without having to borrow that unit or if the bank could pledge one unit of the bond without having that unit as a long position.

Proposition 5 *If there are trades in repo markets for both the domestic and the foreign government bonds, then the cross currency basis, over repo rates, is driven by the difference in the shadow interest rates for repo funding using domestic and foreign bonds. More precisely, there is a bank i pledging the foreign bond and a bank k pledging the domestic bond such that*

$$\beta \in [\frac{\mu^k_d/s_d - \frac{\mu^k_f/s_f}{X}}{\lambda^k_{d2}}, \frac{\mu^i_d/s_d - \frac{\mu^i_f/s_f}{X}}{\lambda^i_{d2}}]$$

The agents i and k may be the same, in which case the above interval becomes degenerate. We can think of μ^i_d/λ^i_{d2} as a shadow interest rate for collateralized dollar funding, as it tells us - independently of how utility might be measured - how collateralized dollar funding is valued relative to the income valuation at date 2 (when such a loan is repaid).

Proposition 5 predicts that the basis should narrow when $\mu_f > 0$ increases. This prediction is also intuitive from an economic perspective. Say the dollar is the domestic currency and *jpy* the foreign one. If Yen also becomes scarce, the dollar funding needs relative to Yen funding needs are ameliorated, and therefore, the basis shrinks. In terms of our previous comparison between selling yen at date 1 versus locking in this sale at date 1 to be executed at date 2, the difference does not just reflect a difference in the prevailing interest rate on both currencies, but also the value attached to the ability to possess dollars during the interim period.

Even though the leverage constraint (14) is present, its shadow value ν does not play a role in explaining why a basis ocuurs. As the proof of Proposition 5 shows, the impact of ν on the domestic date1/date2 no-overdraft shadow values difference is exactly offset by its impact on the, spot converted, analogous difference for the foreign currency. Bases occur as a result of a difference in the possession values of two currencies and, more precisely, the possession values of securities denominated in these currencies, rather than solvency frictions.

Let us write the basis in terms of observable market variables. We find that the most interesting fact is the link between the basis and the difference in the unsecured-secured spreads for the two currencies[32]. Let $\tilde{sp}_{ji} = \frac{\pi_{ji} - \rho_j}{1+\rho_j}$ be the unsecured-secured spread (normalized over the repo return), for the unsecured interest rate π_{ji} paid by bank i on the currency j.

Proposition 6 *The basis, over repo rates, is* $\beta = \frac{\tilde{sp}_{di} - \tilde{sp}_{fk}}{1+\tilde{sp}_{fk}}(1+\rho_d)$ *for any bank* i *which is an unsecured borrower in the domestic currency and any bank* k *which is an unsecured borrower in the foreign currency.*

For β to be positive, it suffices to find a pair of such banks for which $\tilde{sp}_{di} > \tilde{sp}_{fk}$ holds[33]. Such inequality can be easily checked in the data (as will be done below), but we may wonder what is it that can make the domestic spread exceed the foreign spread, for some pair of banks?

Comments

[32]In the absence of other factors and simplying this shows how *relative* fra-ois in each currency can track basis changes.

[33]It may be happen that a bank is unsecured borrower in both currencies and, in this case, the formula in Proposition 6 would hold with i and k being the same, as will be illustrated below.

1) To illustrate a typical case generating a positive basis, suppose there is a *foreign* bank i holding a lot of domestic assets. This can result in its domestic box constraint being binding. Possibly the value of the domestic securities the bank holds, and can be pledged, is not enough to get the funding that needs to be rolled over. On the other hand, this bank has plenty of funding in the foreign currency and can provide it through repo to others. In this case, the unsecured-secured spread will be higher for its domestic offer rate than for the counterparties' foreign offer rates. The argument is as follows. Bank i has a higher possession value for the domestic bond than for the foreign bond. For given (unpersonalized) repo rates, bank i's MRSs in the two currencies adapts to such differences in the shadow values of the two box constraints and, as a result, the (personalized) unsecured rates will be such that the domestic unsecured-secured spread will be higher (and $\beta > 0$, if, in addition, bank i is an unsecured domestic borrower).

2) Leverage ratios may impact on the *magnitude* of the basis as the unsecured-secured spreads are affected. When e increases, as the lender i gets compensated for lost leverage capacity, the spread $\tilde{sp}_{jk}$ may increase (a rough estimate of such variation is given by $2\nu^i/\lambda^i_{j2}$ times the change in e, assuming constant repo and IOR rates and also a constant leverage shadow value ν^i for the lender, see section 6.3 of the appendix).

3) We can look at individual contribution rates into Libor to get a sense of how the basis computed according to the formula in Proposition 6 compares with observed basis[34]. For example in Figure 3 we take Deutsche Bank to be the bank on both sides (being at the same time bank i and bank k in the formula) and use the respective contribution to 3m Libor rate as its unsecured dollar rate and the respective euro contribution to the 3m Euribor rate as its unsecured euro rate. We see that the formula generates a pattern for the basis that follows closely the actual basis. In section 6.7 of the appendix we provide also satisfactory comparisons using data for other banks.

4) It is important to point out that Propositions 5 and 6 *still hold when margins are null* ($h_d =$

[34]Note that when most banks have such spreads in the same directions this gets aggregated up to the Libor level. But some banks will prefer using the FX route because it is cheaper than the rate they are ready to pay or being offered.

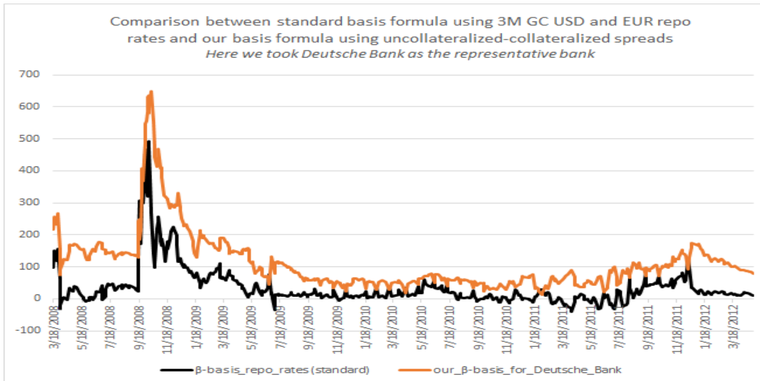


Figure 3: Secured vs. Unsecured (DB) basis (Source: JP Morgan).

$h_f = 1$ for bilateral repo and $m_d = m_f = 1$ for centralized repo). The basis is driven by relative funding pressures in the two currencies, rather than by relative opportunity costs of paying the funding margins. We are not claiming that margins are totally irrelevant. Margins may affect what the equilibrium interest rates and spreads are. But it is ultimately these spreads that matter, whether there are margins or not, and this is particularly important in terms of prediction and policy analysis, since haircuts may vary significantly in cross-section (depending on custodial agreements between the parties) and it is hard to collect data on haircuts.

Alternatively, we can find a basis over interbank unsecured rates. The unsecured version of the basis can often be found in the literature - see Baba and Packer (2009), Genberg, Hui, Wong, and Chung (2009), and Jones (2009). We opted to look at the basis in terms of secured rate for two reasons: secured rates are shared by all banks, and are transactional. Moreover, recently the provision of funding by central banks with secured rates has come to dominate the funding by banks and this is why we look at the currency basis expressed in terms of repo rates for government bonds. The latter sometimes closely follows the fed funds rate and OIS, and, when that happens, the basis over repo rates is close to the basis over OIS. Mancini-Grioffoli and Ranaldo (2010) were also against computing the basis in terms of Libor rates[35].

[35]Their work also contains an important result on excess returns from secured funding using GC rates.

To better understand why we should focus on basis over secured rather than unsecured rates, let us examine how the latter would look. We take a pair of banks, k and i, and pick for one the offer rate in the foreign currency and for the other the offer rate in the domestic currency, say π_{fk} and π_{di}. Then, $\chi = \frac{X}{1+\pi_{fk}}(1 + \pi_{di} + \hat{\beta}_{ik})$, where $\hat{\beta}_{ik}$ is the basis over the unsecured rates π_{di} and π_{fk}.

The analogue of Proposition 4 holds. Let $\frac{\lambda^i_{j1}/\lambda^i_{j2}}{1+\pi_{jk}}$ be the possession value of currency j when funding is done at the unsecured rate offered by bank k in currency j. Then, for funding at the unsecured rates π_{di} and π_{fk}, domestic currency possession value exceeds foreign currency possession value if and only if $\hat{\beta}_{ik} > 0$. It is immediate to see that when the basis over repo rates is zero, we have $\hat{\beta}_{ik} > 0$ if and only if $\tilde{sp}_{di} < \tilde{sp}_{fk}$. In general[36], we have the following results

Proposition 7 *(i) If the basis were defined over the offer rates of the same bank in the two currencies, a non-null basis would require that bank not to be an unsecured borrower in any of the two currencies. (ii) If all banks were offering the same unsecured rates when borrowing, then $\hat{\beta} \neq 0$ would require the unsecured credit markets in both currencies to be inactive. (iii) When there is no basis over repo rates, there will be a basis over unsecured rates π_{di} and π_{fk} if and only if the respective spreads ($\tilde{sp}_{di}$ and $\tilde{sp}_{fk}$) over secured rates are different.*

Proposition 8 *Proposition 6 and item (iii) of Proposition 7 imply that if there is no basis β over repo rates, then a basis $\hat{\beta}_{ik}$ over the unsecured rates π_{di} and π_{fk}, offered by banks i and k in the domestic and foreign currencies, respectively, will also be null whenever such banks are actually borrowing unsecured in these currencies in equilibrium.*

In fact, Proposition 6 says that $\beta = 0$ implies that there is no pair (i, k) of banks, with i as the unsecured borrower in the domestic currency and k as the unsecured borrower in the foreign currency, such that $\tilde{sp}_{di}$ and $\tilde{sp}_{fk}$ are different. Then, by item (iii) of Proposition 7, $\hat{\beta}_{ik} \neq 0$ implies that either bank i is not an unsecured borrower in the domestic currency or bank k is not an

[36]Notice that $\frac{1+\rho_d}{1+\rho_f} + \frac{\beta}{1+\rho_f} = \frac{1+\pi_{di}}{1+\pi_{fk}} + \frac{\hat{\beta}_{ik}}{1+\pi_{fk}}$.

unsecured borrower in the foreign currency. That is, when the secured basis is zero, we can only find a deviation from unsecured CIP by using unsecured rates of inactive markets.

Proposition 7 had already pointed out that it is absolutely misleading to compute basis over unsecured rates that would be common across banks. If such common rates would actually prevail in equilibrium, then the basis would be null. The only reason why basis over common rates have been found, is that such common rates are just hypothetical - an average of different announced offer rates, which may even differ from actual rates. Proposition 8 goes a step further and establishes that there is no advantage in looking at basis over bank specific unsecured rates. Whenever such basis occurs, there is always also a basis over secured rates.

4 Central Banks' Actions

We now extend our analysis to allow for policy actions, in particular for FX swaps done by central banks combined with funding to private banks in a currency which is not their home currency. The official sector is now seen in a more interesting way. Before it had a more passive role, only being able to choose the bond issue whose debt service would be affordable by the official commodity endowments. Now, it has more freedom on how to pay back the public debt and can also engage in FX swaps that will directly affect the cross currency basis. The formal reformulation of no-overdraft constraints of the official sector is left for sections 6.5 and 6.6 of the appendix. Here we just examine under the lens of our theory the effect on cross currency funding of different types of central banks' action[37].

We address a setting where European banks engage in repo with the ECB. We first discuss the

[37]We would like also to recall the work by Krishnamurthy, Nagel, and Orgov (2011) and Hrung and Sarkar (2012), which provided important empirical analyses of the relationship between central banks' interventions and the funding liquidity needs of major banks in the economy. The former examine funding of global banks in private markets and in central bank facilities, and find that the Fed's Primary Dealer Credit Facility is highly significant for easing funding constraints. The latter find evidence that the basis is lower the day after successful borrowing at the Fed's dollar liquidity facilities.

case of the ECB accepting euro denominated bonds as collateral (this period goes until October 15, 2008 and starts again on January 1, 2010), and then proceed to examine the case when the ECB accepted dollar denominated collateral in repo (from October 15, 2008 until the end of 2009).

4.1 ECB Accepts Collateral Denominated in Euros

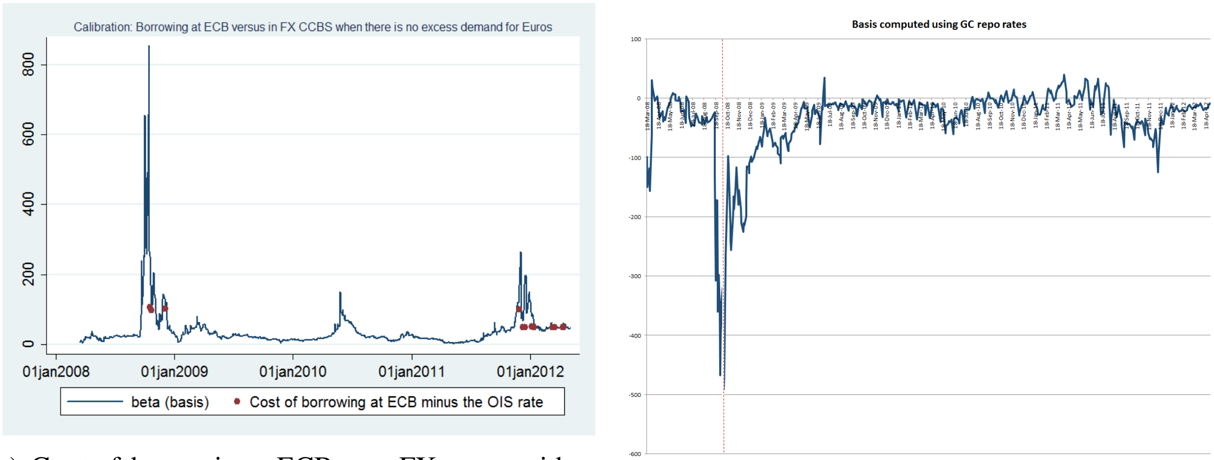
To simplify, we identify covered bonds with other regular eligible euro denominated bonds, and look at the introduction of *cross-currency repo* by the ECB, using all eligible bonds as an abstract representation of the overall funding capability of the European banks in euros.

In October 2008, all uncollateralized and collateralized markets were under significant stress and this situation was reflected in the market for the basis, which rose up to 400 basis points. The best option for European banks was to raise dollars using the ECB's repo facility, i.e., European banks turned to the ECB to borrow dollars through repo in exchange of euro covered bonds. This implies that the collateral has to be taken into account in the box constraint of the euro covered bond, whereas the cash loans will appear in the dollar no-overdraft box constraints of dates 1 and 2 multiplied in both cases by the spot rate X (see the Appendix for details). In this setting, denoting by P the repo rate chosen by the ECB, we have the following result.

Proposition 9 *For a bank i that is pledging euro denominated bonds at the ECB's dollar repo facility (to borrow dollars at rate P) and also in the free repo market (to borrow euros at rate ρ_f), with the same haircuts, the basis β relative to repo rates is equal to $P - \rho_d$.*

The basis β becomes the difference between the ECB repo rate and the US repo rate, $P - \rho_d$. For short repo maturities, the US T-bill (GC) repo rate is very close to the Fed Funds rate and, therefore, the basis becomes the spread over OIS at which the ECB is lending dollars. It is effective for the central bank to make the pool of eligible collateral as wide as possible; in the limit we will look at the case of a collateral that is abundant for users of the ECB's dollar operations.[38] In this case, the

[38]Easing the euro funding constraint did not significantly reduce the dollar shortage. Mancini-Grifolli and Ranaldo



(a) Cost of borrowing: ECB vs. FX swap with no excess demand

(b) Cross currency basis bp/GC

Figure 4: USDEUR basis and cost (Source: JP Morgan)

cross-currency basis is in fact equal to the spread between the policy repo rate and the US repo rate. The difference $P - \rho_d$ is the differential cost between raising dollars from the ECB or directly in the US market. If European banks were members of the Fed, or had plenty of dollar unencumbered collateral, they could raise dollars at (or close to) the rate ρ_d. But this is not feasible, and, therefore, these banks have to pay the differential $P - \rho_d$, which is then reflected in the basis. Interestingly, when the ECB provides dollars to key users of the basis, it can effectively cap the basis through this mechanism.

Figure 4a assesses the quantitative performance of the expression $P - OIS_d$. The figure is constructed as follows. The rate OIS_d is taken here to be the 3-month OIS rate. The repo rate P at which the ECB lent dollars was obtained from the ECB website.[39] Using the ECB's tender data, we examine auctions where (i) the excess demand for euros is zero ($\mu_f^i = 0$) and (ii) the best option for European banks was to raise dollars through the ECB's repo facility (i.e., 3-month OIS $+\beta >$ ECB's repo rate for lending USD). We plot β in basis points along with $P - OIS_{3m}$ in Figure 4a.

(2011) show how the holders of dollars in the euro-dollar spot exchange market demanded a very attractive exchange rate, reflecting risk and liquidity considerations, and in turn caused this channel to be very expensive.

[39]Notice that this rate is significantly higher than the TAF rate at which US banks could obtain dollars (see Goldberg, Kennedy, and Miu (2010) for a comparison between TAF stop-out rates to OIS and Libor for one-month term).

For all the ECB tenders with zero excess demand, when the best option is borrowing in the ECB tender, we do find evidence consistent with our prediction. Namely, the cost of borrowing at the ECB lines up exactly as predicted by our theory and common sense.

4.2 ECB Accepts Collateral Denominated in Dollars

We now proceed to examine the alternative setup, when the ECB accepts dollar denominated assets as collateral[40]. This period goes from the date when the ECB announced unorthodox measures to alleviate the demand for dollars, October 15, 2008, to the end of 2009, when dollar collateral was no longer accepted. During this period, the ECB provided dollar funding by accepting dollar collateral repo when the market ceased to accept it.

With this policy action, the central bank went back to the root of the problem. Originally, the dollar funding pressure had been created because European banks could not fund a dollar denominated asset in the market. A natural idea is for the ECB to provide such funds accepting the dollar denominated collateral on repo when the market ceases to accept it. Essentially, the ECB is doing a dollar repo better than the market could provide. This is according to Baba, McCaunley, and Ramaswamy (2009) and Coffey, Hrung, and Sarkar (2009), who point out that the cost of borrowing euros in unsecured markets (at the euro Libor) and swapping these euros for dollars was higher than borrowing dollars directly in the unsecured market (at the dollar Libor), which was in turn higher than borrowing dollars using the ECB repo facility. Also, Hrung and Sarkar (2012) show that anticipated reductions in repo funding compelled banks to go to the FX swap market and obtain dollars at a higher price.

Let us denote by $(1 - \tilde{h}_d)$ the haircut chosen by the ECB and by $\tilde{\rho}_d$ the repo rate for this closed repo operation at the ECB.

[40]Relevant to this policy is the work by Ashcraft, Garleanu and Pedersen (2010), who pointed out that enlarging the universe of collateral by the central bank, or lowering the haircut, relieves the funding pressure.

Proposition 10 *If the ECB accepts dollar collateral when lending dollars, then for a bank that uses this facility and is also pledging in the free repo market for European bonds, the basis* β *relative to repo rates is equal to* $\frac{\mu_d^i/\tilde{h}_d - X^{-1}\mu_f^i/h_f}{\lambda_{d2}^i} - (\rho_d - \tilde{\rho})$.

When compared with the free market result of Proposition 5, the result in Proposition 10 shows that a policy of collateral relaxation by the ECB can narrow the basis. The idea here is that the funding costs associated with dollar denominated collateral decrease (by $\rho_d - \tilde{\rho}$) and become low or even zero when the ECB starts accepting such collateral. This unorthodox measure permits European banks who issued vast amount of dollar denominated mortgages to post abundant eligible dollar denominated collateral in the form of mortgage-based securities. Funding costs decreased and the scarcity of dollars was alleviated, in turn narrowing the basis.

In Figure 4b we provide visual evidence that clearly shows the significant reduction of the basis following the ECB's policy change to full allotment policy and collateral relaxation after October 15, 2008. In this graph we plot the basis against the overnight GC repo rates, and mark with a vertical dashed line the date when the new policy was adopted.

5 Conclusion

Our equilibrium approach looked simultaneously at FX swaps and the markets for funding denominated in each currency. The FX swap market enables funding in different currencies to be exchanged against one another. The price that clears the FX swap market is not often easily derived by an arbitrage argument (CIP), due to the lack of scalability in the funding channel in some currency. The price has to be found by an equilibrium approach. Supply and demand in this market are often driven by the need to fund assets denominated in foreign currency. Such funding in a foreign currency may be hard to get unless FX swaps are done. When the FX swap price is looked at in terms of its deviation from CIP, for given funding channels, a basis is constructed for each pair of channels. In a world where agents have different access to various funding avenues, such bases will reflect the funding frictions that some agents face, for instance, the value attached to being in possession of a foreign government bond that can be pledged as collateral (that is, the shadow value of the box constraint for that bond). Finally, we show how central banks' intervention in the FX swap market can be effective in narrowing the basis by deploying funding across currencies (through a central banks FX swap followed by special repo channels) when private agents can't manage to use their assets to raise the foreign funding that they need.

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6 Online Supplementary Material: Appendix

In this Appendix we start by proving the proposition that establishes bounds on FX and financial trades. Next we present the first-order conditions for banks' profit maximization and use them to prove the propositions relating the basis to funding frictions in each currency. Then, we prove existence of equilibrium. To complement the existence proof, we reformulate the no-overdraft constraints of the official sector to allow for policy actions and show that financial and FX trades ares still bounded in that context. Finally, we present the graphs plotting the basis formula of Proposition 6 for several banks.

6.1 Banks' bounded positions

1) **Proof of Proposition 1**

Let $\zeta = e/(1-e)$. In the case of bilateral repo, adding inequality (14) over banks and noticing that $\sum_{i\in I} \psi^i_j = \sum_{i\in I} \theta^i_j$ implies that $h_d \sum_{i\in I} q_d \psi^i_d \leq \frac{1}{\zeta} \sum_{i\in I}(X q_f \tilde{b}^i_f + q_d \tilde{b}^i_d + l^i_{d1} + X l^i_{f1}) + \frac{1}{e}(\sum_{i\in I_d} y^i_{d1} + X \sum_{i\in I_f} y^i_{f1})$. By market clearing in cash markets $\sum_{i\in I_j} y^i_{j1} \leq \sum_{i\in I_j} \tilde{y}^i_{j1} + \tilde{y}^{gj}_{j1} \equiv \sum_i \tilde{y}^i_{j1}$ and, therefore, we get $h_d \sum_{i\in I} q_d \psi^i_d \leq \frac{1}{\zeta} \sum_{i\in I}(X q_f \tilde{b}^i_f + q_d \tilde{b}^i_d + l^i_{d1} + X l^i_{f1}) + \frac{1}{e}(\sum_i \tilde{y}^i_{d1} + X \sum_i \tilde{y}^i_{f1}) \equiv \Lambda$. Similarly, $\sum_{k\in I, k\neq i} a^k_{ji} = u^i_j$ implies that $u^i_d \leq \Lambda$. Analogously, $\sum_{i\in I} X q_f \psi^i_f \leq \frac{1}{h_f}\Lambda$ and $u^i_f X \leq \Lambda$. That is, market feasible unsecured debt positions and collateral pledged (and re-pledged) in repo are bounded in value as shown. By constraints (2a) and (2b) we have $(b^i_j)^- \leq \theta^i_j \leq \sum_k \psi^k_j$ and, therefore, $(b^i_j)^+ \leq \sum_k (\tilde{b}^k_j + (b^k_j)^-)$, which implies that there are upper bounds on the values of long positions (and also upper bounds on the values of short positions) compatible with market clearing. Hence, the leverage that can be done using the fixed initial holdings of the securities is bounded (captured by the ratio $\sum_k (b^k_j)^+ / \sum_k \tilde{b}^k_j = (l_j \# I)/(q_{1j} \sum_k \tilde{b}^k_j)$) and this is true no matter how much equity is issued by banks.

Notice that FX spot and swap trades become bounded as well. The bounds on secured and unsecured debt, together with the bounds on bond positions, imply, by (9), that $\sigma^i + \phi^i$ has an upper bound, which we denote by N^i.Also, by (9) there is a lower bound on ϕ^i, which we deonote by $-K^i$. By market clearing, $\phi^i \leq \sum_{\iota \neq i} K^\iota$ and, therefore, $\sigma^i \leq N^i - \phi^i \leq N^i + K^i$, while $\sigma^i \geq \sum_{\iota \neq i}(K^\iota + N^\iota)$.

The above argument can be easily adapted to the case of centralized repo. Now, $(2-m_j) \sum\limits_{i \in I} \theta^i_j = m_j \sum\limits_{i \in I} \psi^i_j$. Adapting the above proof, we see that $q_{d1} \sum_{i \in I} \psi^i_d \leq \Lambda$, while $X q_{f1} \sum_{i \in I} \psi^i_f \leq \Lambda$ and the bounds on unsecured borrowing are as before. FX trades are also bounded as before.

2) **Comment on why box and no-overdraft constraints are not enough to bound banks' choice sets.**

Even in the case of centrally cleared repo, the choice set would be bounded by these constraints only in an uninteresting setting with *just one more* funding instrument in each currency (either a *non-bank-specific* unsecured borrowing in each currency or FX trades), which would be unrealistic (and prevent us from relating the basis to the unsecured-secured spreads differential across currencies).

To see this, say that there were just one more funding instrument F^i_j in each currency j. Bounded positions would follow from (8) since $\tilde{y}^i_{j1} + q_{j1}\tilde{b}^i_j \geq q_{j1}(b^i_{j1} + \theta^i_j - \psi^i_j + (1 - m_j)(\theta^i_j + \psi^i_j)) + F^i_j \geq q_{j1}(1 - m_j)(\theta^i_j + \psi^i_j) + F^i_j$, implying that F^i_j has an upper bound $\bar{F}^i_j$. The lower bound on F^i_j would follow by market clearing, $F^i_j \geq -\sum_{k \neq i} \bar{F}^k_j$ and, therefore, θ^i_j and ψ^i_j would be bounded as well. This argument can not be extended to more than one funding instrument on top of centrally cleared repo. That is, it does not allow for unsecured borrowing to coexist with FX (and, even if FX were not considered, would not allow either for bank-specific unsecured interest rates). In the absence of unsecured borrowing, this argument enables us to bound $\sigma^i + \phi^i$ from above and from below (and bound θ^i and ψ^i) and then the bounds on ϕ^i and σ^i could be found as in the proof of Proposition 1.

Under bilateral repo, the haircut benefit for the repo long creates additional difficulties.

6.2 First order conditions for profit maximization

Denote the multiplier of (14) by ν. Only the first order condition on repo short positions becomes quite different in the two repo scenarios (the first order condition on repo long positions has the coefficient of the box shadow price always equal to $1/t_j$, where t_j has different values in the two cases). Let ϵ_j^δ, for $\delta = b, c$, be defined by $\epsilon_j^b = 1$ in the case of bilateral repo and equal to $\epsilon_j^c = [1-(1-e)(1-m_j)]/m_j > 1$ in the case of centralized repo. Let $\xi_d = 1$ and $\xi_f = X$.

We consider the Lagrangian function of each bank i: $Pi^i + \nu^i Lev^i + \sum_j \mu_j^i q_j box_j^i + \sum_{jt} \lambda_{jt}^i \Omega_{jt}^i$ where $j \in \{f, d\}, t \in \{1, 2\}$ and all multipliers are non-negative (dropping i superscript in multipliers for notational simplicity). Necessity of Kuhn-Tucker conditions follows from the weak reverse convex constraint qualification (given that all constraints are linear).

Lemma 1 *If the plan φ^i maximizes Π^i subject to no-overdraft constraints in each currency, leverage constraint (14), and collateral (box) constraints (2a) and (2b), then the following first order conditions (FOC) must hold, on top of the FOC with respect to spot and FX swap trades, ((16a) and (16b), respectively).*

$$\text{(1) wrt } y_{j1}^i, \textbf{\textit{if}}\ i \in I_j : S_{y_{j1}}^i \equiv \lambda_{j1} - \lambda_{j2}(1+i_j) - \nu e \xi_j \geq 0 \qquad S_{y_{j1}}^i y_{j1}^i = 0 \quad \text{(17a)}$$

$$\text{(2) wrt } y_{j2}^i, \textbf{\textit{if}}\ i \in I_j : S_{y_{j2}}^i \equiv \lambda_{j2} - 1 \geq 0 \qquad S_{y_{j2}}^i y_{j2}^i = 0 \quad \text{(17b)}$$

$$\text{(1) wrt } b_j^i : S_{b_j^i} \equiv \lambda_{j1} - \mu_j - \lambda_{j2}(1+r_j) - (1-e)\nu\xi_j = 0 \quad \text{(17c)}$$

$$\text{(2) wrt } \theta_j^i : S_{\theta j}^i \equiv \lambda_{j1} - \mu_j/t_j - \lambda_{j2}(1+\rho_j) - (1-e)\nu\xi_j \geq 0 \qquad S_{\theta j}^i \theta_j^i = 0 \quad \text{(17d)}$$

$$\text{(3) wrt } \psi_j^i : S_{\psi j}^i \equiv -\lambda_{j1} + \mu_j/s_j + \lambda_{j2}(1+\rho_j) + \nu\epsilon_j^\delta\xi_j \geq 0 \qquad S_{\psi j}^i \psi_j^i = 0 \quad \text{(17e)}$$

$$\text{(4) wrt } a_{jk}^i, k \neq i : S_{ajk}^i \equiv \lambda_{j1} - \lambda_{j2}(1+\pi_{jk}) - (1-e)\nu\xi_j \geq 0 \qquad S_{ajk}^i a_{jk}^i = 0 \quad \text{(17f)}$$

$$\text{(5) wrt } u_j^i : S_{uj}^i \equiv -\lambda_{j1} + \lambda_{j2}(1+\pi_{ji}) + \nu\xi_j \geq 0 \qquad S_{uj}^i u_j^i = 0 \quad \text{(17g)}$$

It follows from (17d) and (17e) that if $\theta_j > 0$ and $\psi_j > 0$, then $\nu = 0$ (and in such event we have *also $\mu = 0$ in the exchanges case with non-zero margin).* [41]

6.3 Proofs of propositions on the basis and funding variables

In order to prove Propositions 5 and 6 let us write the basis over repo rates in terms of the multipliers that enter in the first order conditions on repo long or repo short positions. Let us start with the former. From Proposition 3, we have $\chi\lambda_{d2} = \lambda_{f2}$ Recall that we defined the basis β such that $\chi = X\frac{1+\rho_d+\beta}{1+\rho_f}$, and therefore one can obtain the cross-currency basis from the forward FX rate formula.

$$\chi = \frac{\lambda_{f2}}{\lambda_{d2}} = \frac{\lambda_{f1} - \mu_f/t_f - X(1-e)\nu - S_{\theta f}}{(1+\rho_f)\lambda_{d2}} \tag{18a}$$

$$\chi = \frac{X}{(1+\rho_f)\lambda_{d2}}(\frac{\mu_d}{t_d} + \lambda_{d2}(1+\rho_d) + \nu(1-e) + S_{\theta d} - \frac{X^{-1}\mu_f}{t_f} - \nu(1-e) - X^{-1}S_{\theta f}) \tag{18b}$$

$$\chi = \frac{X}{1+\rho_f}(1+\rho_d + \frac{\mu_d/t_d - X^{-1}\mu_f/t_f}{\lambda_{d2}} + \frac{S_{\theta d} - X^{-1}S_{\theta f}}{\lambda_{d2}}) \tag{18c}$$

Notice that by using the first order conditions on repo short positions we get a formula similar to (18c), more precisely,

$$\beta = \frac{\mu_d/s_d - S_{\psi d}}{\lambda_{d2}} - X^{-1}\frac{\mu_f/s_f - S_{\psi f}}{\lambda_{d2}} \tag{19}$$

Take a bank i such that $\psi_f^i > 0$, then $\beta \leq \frac{\mu_d^i/s_d - X^{-1}\mu_f^i/s_f}{\lambda_{d2}^i}$. Next take a bank k such that $\psi_d^k > 0$, then $\beta \geq \frac{\mu_d^k/s_d - X^{-1}\mu_f^k/s_f}{\lambda_{d2}^k}$. This completes the proof of Proposition 5.

To prove Proposition 6 we combine the first order conditions of unsecured credit and secured credit (that is, (17f) and (17d)) to get $(\mu_j/t_j + S_{\theta j})/\lambda_{j2} = \pi_{jk} - \rho_j + S_{ajk}/\lambda_{j2}$. Now, we use (18c) and take a bank that is an unsecured creditor to bank i in the domestic currency, we get $\beta \leq (\pi_{di} - \rho_d) - X^{-1}\chi(\pi_{fk} - \rho_f)$ and take next a bank that is an unsecured creditor to bank k in

[41] $1/s - 1/t \leq 0$ both in the bilateral and central counterparty case as $1/(1+(1-m)) \leq 1/(1-(1-m))$

the foreign currency to get the converse inequality. Noticing that $\chi/X = \frac{1+\rho_d+\beta}{1+\rho_f}$ we conclude the proof.

To prove the assertion made in *Comment 1* on Proposition 6, notice that

$$\frac{1+\rho_d}{1+\rho_f} = (\frac{\lambda^i_{f2}}{\lambda^i_{d2}})\frac{S^i_{\psi d}+\lambda^i_{d1}-\mu^i_d/s^\delta_d-\nu^i\epsilon^\delta_d}{\lambda^i_{f1}-S^i_{\theta f}-\mu^i_f/t^\delta_f-(1-e)\nu^i X} \tag{20a}$$

while

$$\frac{1+\pi_{di}}{1+\pi_{fk}} = (\frac{\lambda^i_{f2}}{\lambda^i_{d2}})\frac{S^i_{ud}+\lambda^i_{d1}-\nu^i}{\lambda^i_{f1}-S^i_{afk}-(1-e)\nu^i X} \tag{20b}$$

where $\epsilon^\delta_d \geq 1$ (for $\delta = b, c$, that is, both when repo is bilateral or centralized). Recall also that t^δ_j and s^δ_j are both positive (irrespective of repo being bilateral or centralized). Then, the hypothesis made on bank k implies $\frac{1+\rho_d}{1+\rho_f} < \frac{1+\pi_{di}}{1+\pi_{fk}}$, or equivalently, $\tilde{sp}_{di} > \tilde{sp}_{fk}$ as claimed.

Comment 2 pointed out an impact of e on the bank-specific unsecured-secured spreads. This follows from the fact that for an unsecured lender i to bank k we have $\pi_{jk} - \rho_j = (2e-1)\nu^i/\lambda^i_{2j} + (i_j - \rho_j) + S^i_{y_j 1}/\lambda^i_{2j}$.

To prove Proposition 7 we use the first order conditions with respect to uncollateralized borrowing to get

$$\chi = \frac{X}{1+\pi_{fi}}[1+\pi_{di}+(X^{-1}S^i_{uf}-S^i_{ud})/\lambda^i_{d2}] \tag{21}$$

which implies that $\hat{\beta}_{ii} > 0 \Rightarrow u^i_f = 0$ and $\hat{\beta}_{ii} < 0 \Rightarrow u^i_d = 0$. Take now $k \neq i$ and use (17f) to get

$$\chi = \frac{X}{1+\pi_{fi}}[1+\pi_{di}+(S^k_{adi}-S^k_{afi}X^{-1})/\lambda^i_{d2}] \tag{22}$$

which implies that a positive basis over the two offer rates of a same bank i would imply that no other bank would lend to i in the domestic unsecured market, but we also knew from (21) that bank i would not borrow in the foreign unsecured market either. This implies item (i) and, therefore, item (ii) must hold also.

Let us show (iii). For a basis over π_{dk} and π_{fi}, for $k \neq i$, combining (17f) and (17g) we have

$$\chi = \frac{X}{1+\pi_{fi}}[1+\pi_{dk}+\frac{X^{-1}S^i_{uf}+S^i_{adk}-e\nu^i}{\lambda^i_{d2}}] = \frac{X}{1+\pi_{fi}}[1+\pi_{dk}-\frac{S^k_{ud}+S^k_{afi}X^{-1}-e\nu^i}{\lambda^k_{d2}}] \quad (23)$$

Proof of Proposition 9:

Any European bank i can now pledge the dollar bond (referred to as the domestic bond) to get a loan in dollars (referred to as the domestic currency). Let us denote by $z^i_f \leq 0$ this repo short position. The no-overdraft domestic currency constraint at dates 1 and 2 should now be written, respectively, as follows: $X(\sigma^i_1+\phi^i-hq_{1f}z^i_f)-b^i_d-a^i_d+u^i_d \geq 0$ and $-\chi\phi^i+\sum_k(1+\pi_{dk})a^i_{dk}-(1+\pi_{dk})u^i_d+(1+r_d)b^i_d+(1+P)Xhq_{1f}z^i_f \geq 0$. For simplicity, we omit the trading of domestic goods for the domestic no-overdraft constraint of the foreign bank at date 1. Also, we do not include free market repo operations on the foreign bond (and, therefore, short sales of such bonds) due to the high illiquidity in those repo markets by that time.

The first order conditions with respect to σ^i and ϕ^i are again (16a) and (16b), respectively. Also here, we can simplify the initial form of (16b) and get $\chi\lambda_{d2} = \lambda_{f2}$.

Notice that the box constraint for the European ('foreign') bond is now $b^i_f+\theta^i_f-\psi^i_f+z^i_f \geq 0$. Moreover, $hq_{1f}|z^i_f|$ enters into the computation of the bank's liabilities L^i. For a bank using this ECB facility, the FOC with respect to z^i_f is therefore

$$\lambda^i_{d1}Xh = \mu^i_f + \lambda^i_{d2}(1+P)Xh + \nu Xh \quad (24)$$

Now, using (17e) for the foreign bond and then (24) we get the following, when $h = h_f$,

$$\beta = P - \rho_d + S^i_{\psi f}X^{-1}/\lambda^i_{d2} \quad (25)$$

Proof of Proposition 10: The case under consideration is one where collateral accepted by the

ECB is denominated in USD. Now, the dollar denominated collateral pledged at the ECB is a repo short position denoted by $z_d^i < 0$. The box constraint for the dollar denominated bond is $b_d^i + z_d^i \geq 0$[42]. For a bank using this new ECB facility, the FOC with respect to z_d is the following[43]

$$\lambda_{d1}^i \widetilde{h} = \mu_d^i + \lambda_{d2}^i (1+\tilde{\rho})\tilde{h} + \nu\tilde{h} \tag{26}$$

Now, similar to the proof of the previous proposition, we can use the FOCs with respect to σ, ϕ, and ψ_f together with (26) to get

$$\beta = \tilde{\rho} - \rho_d + [\frac{\mu_d^i}{\tilde{h}} + X^{-1}(S_{\psi f}^i - \frac{\mu_f^i}{h_f}]/\lambda_{d2}^i \tag{27}$$

6.4 On the existence of equilibrium

We start by considering an auxiliary economy with two durable goods, one denominated in each currency, whose bundle values may stand for the cash balances of the original economy.

Assuming that these fictitious durable commodities are perfectly durable goods with no new endowments occurring at $t > 1$, we can relate endowments with previous consumption bundles, for $t > 1$, by $\omega_{jt}^i = x_{j,t-1}^i$ if i belongs to the j system (otherwise, $\omega_{jt}^i = 0$ and $x_{jt}^i = 0$). The link with the original economy is established as follows: for $t > 1$, we have $y_{j,t-1}^i = p_{j,t-1}x_{j,t-1}^i$ and $\tilde{y}_{jt}^i = p_{jt}\omega_{jt}^i$, implying for $i \in I_j$ that $\tilde{y}_{jt}^i = p_{jt}x_{j,t-1}^i = (p_{jt}/p_{j,t-1})y_{j,t-1}^i$. That is, $p_{jt}/p_{j,t-1} - 1 \equiv i_{jt}$ is the rate at which cash can be transferred into date t from the previous date, by a bank member of the j system; it is the IOR rate, the central bank reserves rate. For the initial date and $i \in I_j$, we have $p_{j1}\omega_{j1}^i = \tilde{y}_{j1}^i + \Delta_{j1}^i$. If i does not belong to the j system, we get $\tilde{y}_{jt}^i = 0$ and $y_{jt}^i = 0$.

To simplify let us assume repo to maturity or that the official sector buys back entirely the bond

[42] Again, we are ignoring the free repo market for this bond.

[43] Recall that now the collateral is denominated in dollars. Hence, in the dollar no-overdraft and capital adequacy constraints, X does not multiply the repo loan $\widetilde{h}_d q_{1d} z_d^i$.

supply at date 2. The former implies that $q_{2j} = 0$ and $1 + r_j = c_j/q_{1j}$. If the bond matures at date 3, the latter implies that the official sector must pay at date 2 a price $q_{j2} = c_j' \max_{i\in I} \lambda^i_{j3}/\lambda^i_{j2}$, given by the banks' highest willing to pay for bond j at date 2. Let us focus on the former and see next how the latter constitutes a slight variation. The general case, where the bond maturity exceeds the repo maturity and the buy-back decision is endogenous (so that banks may still end up holding bonds again at date 2) is a bit more elaborate but follows essentially what we present now.

Suppose $c_j \geq 1$ (without loss of generality, as bond positions can always be scaled down accordingly).

Each bank i is maximizing (in this two date case) profits in a "home currency" j, or equivalently (for predetermined and constant deposits), maximizing $y^i_{j2} = p_{j2}x^i_{j2}$. In the auxiliary economy we take the bank's payoff function to be $U^i(x) = x^i_{j2}$ and show that, in any equilibrium for the auxiliary economy, we have $p_{j2} > 0$, due to the monotonicity of the payoff of $i \in I_j$ with respect to x^i_{j2}. Then $p_{j1} > 0$ follows from the FOC on y^i_{j1} (or, equivalently, from the FOC on x^i_{j1}, in the auxiliary economy, $\lambda^i_{j1}p_{j1} \geq \lambda^i_{j2}p_{j2} + \nu e\xi_j p_{j1}$).

In this case, there is no reason for the issuer gj to keep any cash at date 2 ($y^{gj}_{j2} = 0$) and, therefore, he chooses the issuance $\hat{b}_j$ so that $c_j\hat{b}_j = \tilde{y}^{gj}_{j2}$. As he had no motive to use any cash at date 1, we have y^{gj}_{j1} equal to the predetermined $\tilde{y}^{gj}_1$. Recall that $\tilde{y}^{gj}_{j2} = y^{gj}_{j1} - i_j \sum_{i\in I_j} y^i_{j1}$.

Now, the issuer's date 1 fictitious commodity endowments in the auxiliary economy are such that $\tilde{y}^{gj}_{jt} = p_{jt}\omega^{gj}_{jt}$, implying that for the issuer of bond j the date 2 endowments of the j-th fictitious commodity must be such that $p_{j2}\omega^{gj}_{j2} = p_{j1}\omega^{gj}_{j1} - (p_{j2}/p_{j1} - 1)\sum_{i\in I_j} p_{j1}x^i_{j1} = p_{j1}\omega^{gj}_{j1} + (p_{j1} - p_{j2})\sum_{i\in I_j} x^i_{j1}$.

STEP 1

Let us set up a generalized game played by banks, bond issuers, and auctioneers choosing prices and some artificial players. This game is parametrized by natural number n which will bound some strategies and we will later, in step 4, let $n \to \infty$.

Given $\tilde{y}^i_{j1}$ for any bank $i \in I_j$, we introduce artificial players that choose $\omega^i_{j1} \in [0, n]$ in order to minimize $(p_{1j}\omega^i_{j1} - \tilde{y}^i_{j1} - \Delta^i_{j1})^2$. Moreover, ω^i_{j2} is set equal to x^i_{j1}, also for $i \in I_j$.

For the construction of the issuers' fictitious commodity endowments, we introduce artificial agents that choose $\omega^{jg}_{j1} \in [0, n]$ in order to minimize $(p_{j1}\omega^{gj}_{j1} - \tilde{y}^{gj}_{j1})^2$ and choose $\omega^{jg}_{j2} \in [0, n]$ in order to minimize $(p_{j2}\omega^{gj}_{j2} - [p_{j1}\omega^{gj}_{j1} + (p_{j1} - p_{j2}) \sum\limits_{i \in I_j} x^i_{j1}])^2$.

We start by reformulating the optimization problem of each bank, taking as choice variables the values of the bond positions and the repo trades. That is, we replace $q_{j1}b_j$ by B_j, $q_{j1}\theta_j$ by Θ_j and $q_{j1}\psi_j$ by Ψ_j. The parameter $q_{j1}\tilde{b}^i_j$ is replaced by $\tilde{B}^i_j$ and the coupon $c_j b_j$ is replaced by $(1 + r_j)B_j$. The box constraints (2a) and (2b) are written in value terms in the form $B_j + \Theta_j - \Psi_j \geq 0$.

It is easy to see that the interior of the constraints of the reformulated optimization problem of any bank has a non-empty intersection, for any vector $(\tilde{B}^i_j, p, (r_j, \rho_j, \pi_{jk})_{jk})$ provided $\tilde{B}^i_j > 0$ and $r_j \geq 0$ for $j = d, f$. These two conditions will hold, as will be seen in step 2. This ensures that the constraint correspondence of any bank is lower semi-continuous (while upper semi-continuity is trivially satisfied).

In fact, let $B_j = \xi\tilde{B}_j$, for $\xi \in (0, 1)$, while $\theta_j = \psi_j = 0$ and $u_j = a_j = 0$. Then, the box constraints (2a) and (2b) hold as strict inequalities and the leverage requirement holds also as a strict inequality. Now, by Assumption A, the no-overdraft constraints in currency j will hold as strict inequalities for ξ close enough to one, at both dates, for $x_{j1} = 0$, $\phi = 0$, $\sigma = 0$ and $x_{j2} = 0$.

STEP 2

We introduce artificial agents that convert the variables $(B^i_j, \Theta^i_j, \Psi^i_j)$ of the modified problem of bank i (considered in step 1) into the variables that auctionners will take as given. For the bond positions, there is one agent that chooses, for each bank i, the variable $b^i_j \in [0, (1+n)l_j/c_j]$ in order to minimize $((1 + r_j)B^i_j - c_j b^i_j)^2$, which is legitimate since we know that $b^i_j \in [(1 + r_j)s_j/c_j, (1 + r_j)l_j/c_j]$ where l_j is an upper bound on values of long positions $\tau_j B^i_j$ (for $\tau_d = 1$ and $\tau_f = X$), set above the ceiling established in the proof of Proposition 1, while s_j is the associated lower bound on $\tau_j B_j^i$ (that is, the upper bound on the value of short sales obtained from l_j by market clearing).

We want auctioneers to clear repo markets by picking prices that are the inverse of repo returns. To prepare this we must convert repo values (Θ_j^i, Ψ_j^i) into values multiplied by returns, which auctionners will then take as given and multiply by their choice variables (the inverse of repo returns). That is, one artifical agent chooses $\hat{\Theta}_j^i$ in order to minimize $((1+\rho_j)\Theta_j^i - \hat{\Theta}_j^i)^2$ and another chooses $\hat{\Psi}_j^i$ in order to minimize $((1+\rho_j)\Psi_j^i - \hat{\Psi}_j^i)^2$, the former constrained to choose within $[0,(1+n)\bar{\Theta}]$ and the latter within $[0,(1+n)\bar{\Psi}]$, where $\bar{\Psi}$ and $\bar{\Theta}$ are the bounds on the values of repo and reverse repo, respectively, established in Proposition 1.

For unsecured borrowing and lending, one agent chooses $\hat{u}_j^i$ in order to minimize $((1+\pi_{ji})u_j^i - \hat{u}_j^i)^2$ and another chooses $\hat{a}_{jk}^i$ in order to minimize $((1+\pi_{jk})a_{jk}^i - \hat{a}_{jk}^i)^2$, constrained to choose within $[0,(1+n)\bar{u}]$ and $[0,(1+n)\bar{a}]$, respectively, given the bounds established in Proposition 1 for unsecured borrowing and lending, $\bar{u}$ and $\bar{a}$, respectively.

This allows us to model auctioneers that choose prices. Let $\gamma^i \equiv -(\sigma^i + \phi^i)$. For date 1 and currency d, there is a date 1 auctioneer choosing $(p_{d1}, X, q_{d1}, H_d, P_{di})$ in the $3+I$ dimensional simplex in order to maximize $\sum_i [p_{d1}(x_{d1}^i - \omega_{d1}^i) + X\gamma^i + q_{d1}(b_d^i - \tilde{b}_d^i) + H_d h_d(\hat{\Theta}_d^i - \hat{\Psi}_d^i) + P_{di}(\sum_k \hat{a}_{di}^k - \hat{u}_d^i)]$.

For currency f, there is an auctioneer choosing $(p_{f1}, q_{f1}, H_f, P_{fi})$ in the $2+I$ dimensional simplex in order to maximize $\sum_i [p_{f1}(x_{f1}^i - \omega_{f1}^i) + q_{f1}(b_f^i - \tilde{b}^i{}_f) + H_f h_f(\hat{\theta}_f^i - \hat{\psi}_f^i) + P_{fi}(\sum_k \hat{a}_{fi}^k - \hat{u}_f^i)]$.

There is just one auctioneer at date 2, who chooses (p_{d2}, χ) in the 1 dimensional simplex in order to maximize the sum, over banks and issuers, of $p_{d2}(x_{d2}^i - \omega_{d2}^i) + \chi\phi^i$.

Finally, other artificial agents make the auctioneers' prices compatible with the parameters that banks take as given in the above reformulated problem. First, an initial holdings paramenter, in value terms, $\tilde{B}_j^i$ is chosen within $[1/n, \tilde{b}_j^i]$ in order to minimize $(q_{1j}\tilde{b}_j^i - \tilde{B}_j^i)^2$. Other artificial agents choose (r_j, ρ_j, π_{ji}). For instance, r_j in chosen in $[-1, n]$ in order to minimize $((1+r_j)q_{j1} - c_j)^2$

subject to the constraint $r_j + q_{j1}/c_j \geq 1$ (which implies $r_j \geq 0$). For $q_{j1} \neq 0$, if the upper bound n is not binding, we get $1 + r_j = c_j/q_{j1}$ (and the constraint is trivially satisfied, as it becomes equivalent to $r_j^2 \geq 0$). For $q_{j1} = 0$ the constraint implies $r_j \geq 1$. For the moment we take n as given and, therefore, when $q_{j1} \neq 0$, $1 + r_j$ may end up being lower than c_j/q_{j1} but later we will let $n \to \infty$. Similarly, ρ_j is chosen in $[-1, n]$ in order to minimize $((1 + \rho_j)H_j - 1)^2$ and π_{ji} is chosen in $[-1, n]$ in order to minimize $((1 + \pi_{ji})P_{ji} - 1)^2$.

Finally, to handle issuance, we assume that the issuer gj chooses $\hat{b}_j$ in order to minimize $(c_{2j}\hat{b}_j - p_{2j}\omega_{2j}^{gj})^2$ subject to $\hat{b}^{gj} \in [1/m, \omega^{gj}/c_{2j}]$. Notice that $\hat{b}^{gj}$ fails to be equal to $p_{2j}\omega^{gj}/c_{2j}$ only if $\hat{b}_j = 1/m$. For the moment we take m as given but will make $m \to \infty$ in a later step of the proof[44].

In the case of centralized repo, we can also avoid writing explicitly the CCP's constraints and model the CCP as choosing $m_j \in [0, 1]$ in order to mimimize $(m_j(\sum\limits_{i \in I} \psi_j^i + \sum\limits_{i \in I} \theta_j^i) - 2\sum\limits_{i \in I} \theta_j^i)^2$.

An equilibrium exists for the generalized game played by banks, auctioneers and the two types of artificial agents (the agents performing paramenter (prices and initial holdings) changes and the agents performing the change in banks' choice variables), under ceilings on the banks' choices of financial variables $(B_j^i, \Theta_j^i, \Psi_j^i, u_j^i, a_j^i)$ that can be taken to greater or equal to the bounds $(\bar{B}_j, \bar{\Theta}_j, \bar{\Psi}_j, \bar{u}_j, \bar{a}_j)$ defined in the proof of Proposition 1, a usual upper bound on consumption (greater or equal to $\sum_i \omega_{jt}^i$) and an upper bound on issuance above the official sector's endowment ω_{j2}^j.

STEP 3

So far, the value $\tilde{B}_j^i$ of the initial holdings of bonds that parametrize banks' reformulated problems may overestimate the true values $q_{j1}\tilde{b}_j^i$, when the lower bound $1/n$ on the choice of $\tilde{B}_j^i$ is binding. As $\tilde{b}_j^i > 0$ for all i, such an overestimate can only occur when $q_{1j}^n \to 0$ as $n \to \infty$.

[44] In the general case, where the bond matures at date 3, the function to be minimized would be $(\hat{b}_j - \min\{p_{2j}\omega_{2j}^{gj}/c_{2j}, p_{3j}\omega_{3j}^{gj}/c_{3j}\})^2$ subject to $\hat{b}^{gj} \in [1/m, \min\{\omega_{2j}^{gj}/c_{2j}, \omega_{3j}^{gj}/c_{3j}\}]$.

It follows from the problem of the date 1 dollar auctioneer that, at an equilibrium for the generalized game, there won't be excess demand in the following markets: for dollar denominated commodity, repo and unsecured credit, and the transfers γ^i. For the dollar denominated bond, banks' demands may be distorted by the overestimate of initial holdings, but, relative to such possibly distorted demands, there is also no excess demand.

Moreover, an excess supply in these markets would require the respective component of the auctioneers' strategies to be zero. In the case of consumption (of the fictitious goods), that would lead to a contradiction. In the case of the financial instruments, an excess supply can also be ruled out.

For instance, if q_{1j} were zero, then by (17c) we get $2 \leq 1 + r_j \leq \lambda^i_{j1}/\lambda^i_{j2}$ for any bank not hitting the ceiling on long positions, and we know that such ceilings cannot be attained in an equilibrium for the generalized game, as the ceilings were chosen above the upper bounds established in the proof of Proposition 1 (and for n large enough, the overestimation of initial holdings is still compatible with non-binding ceilings). However, the aggregation of the no-overdraft constraints in currency j tells us that if one of the markets were in excess supply (and all without excess demand), then some bank must have that constraint non-binding and, therefore, λ^i_{j1} would be zero for some i, which is a contradiction (ρ_j or π_{ji} equal to 1 would also imply $\lambda^i_{j1}/\lambda^i_{j2} \geq 2$, by (17d) or (17f)). Finally, if $\sum_i \gamma^i < 0$ then $X = 0$, which implies, by (6) and (7), that $\gamma^i > 0$ for all i (with σ^i adjusting to have $\phi^i \equiv -(\sigma^i + \gamma^i)$ as desired in (8) and (9)), also a contradiction.

Then, from the problem of the date 1 foreign auctioneer, we get that the foreign currency denominated markets clear. Notice next that, at the equilibrium for the generalized game, we have $\sum_i [p_{j2}(x^i_{j2} - \omega^i_{j,2}) + \chi\phi^i] = (1 + r_j) \sum_i \tilde{b}^i_j$. From the issuers' no-overdraft constraint, we see that at an optimal solution to the problem of the date 2 auctioneer, we will have market clearing for the date 2 dollar denominated commodity and $\sum_i \phi^i = 0$. Hence $\sum_i \sigma^i = 0$. It also follows from (9) that the sum, over banks and the official sector, of $x^i_{e2} - \omega^i_{e2}$ will be zero.

That is, for any generalized game parametrized by n, all markets clear.

STEP 4

However, if the upper bounds n are binding, there might be an inconsistency between the interest rates $(r_j, \rho_j \pi_{ji})$ that banks take into account and the auctioneer's choices (q_{j1}, H_j, P_{ji}). Let $n \to \infty$ and suppose the upper bound n is binding all along the sequence (that is, one of the components in (q_{j1}, H_j, P_{ji}) goes to zero, for some currency j). A contradiction would result.

To see this, take the price vector $(p_{j1}, (q_{j1}, (1+\rho_k)^{-1}, (1+\pi_{ki})^{-1})_k)/(p_{j1} + (1+r_j)^{-1})$ and write banks' constraints in terms of $(b, \hat{\Theta}, \hat{\Psi}, \hat{a}, \hat{u})$ and $e^i_j \equiv q_{1j}\tilde{b}^i_j$. The FOC on the long position whose price tends to zero implies $\lambda^{in}_{j1}/\lambda^{in}_{j2}$ would be unbounded along the sequence for every i (notice that the FOC is as reported in Lemma 1 since the ceiling of the long position is not attained when markets clear, as the ceiling was chosen above the upper bound in Proposition 1).

Now, $\lambda^{in}_{j1}/\lambda^{in}_{j2} = D_{j1}U^i/(\hat{p}_{j1}\lambda^i_{j2}) + p_{j2}/\hat{p}_{j1} + e\nu(1+\zeta) \leq (D_{j1}U^i/D_{j2}U^i + 1)p_{j2}/\hat{p}_{j1} + e\nu(1+\zeta)$. Here, $D_{j1}U^i/D_{j2}U^i$ is bounded and $\hat{p}^n_{j1}/p^n_{j2}$ is bounded away from zero (where $\hat{p}^n_{j1} = p^n_{j1}/(p^n_{j1} + (1+r^n_j)^{-1})$). In fact, adding $\min\{\omega^i_{j1}, e^i_j\}$ (positive for some i) units of good $(j,1)$ to the bundle $(1-\hat{p}^n_{j1})x^{in}$ would constitute, when n is large enough, an improving move for a bank $i \in I_j$, which is also feasible (by multiplying the whole financial and FX plan by $(1-\hat{p}^n_{j1})$).

Moreover, from (17d) we have $(1-e)\nu^i \leq \lambda^i_{j1}$. Now, $\lambda^{in}_{j1}/p^n_{j1}$ is bounded (due to the interiority of endowments for $i \in I_j$) and p^n_{j1} was shown to be bounded away from zero.

Notice that, from p^n_{j1} being bounded away from zero, we can infer that the lower and upper bounds on the construction of ω^i_{j1} and the upper bound of the construction of ω^i_{j2} become irrelevant, for n large enough.

Since the cluster point q_{1j} was shown to be non-zero, we find that the lower bound $1/n$ in the computation of $\tilde{B}^i_j$ is not binding, for n large enough (that is, initial holdings are not overestimated for n large enough). Moreover, we can find notional repo and reverse repo positions, $\theta^i_j = \Theta^i_j/q_{1j}$ and $\psi^i_j = \Psi^i_j/q_{1j}$, at a cluster point of the sequence of equilibria for the generalized games, as $n \to$

∞. It follows that such a cluster point, $(p, q_1, X, \chi, \pi, \rho)$ together with $(x^i, b^i, \theta^i, \psi^i, u^i, a^i, \sigma^i, \phi^i)_i$, is an equilibrium for the truncated economy (where banks and both Treasury agents are constrained by the above ceilings on all choice variables).

To conclude the existence proof, notice that an equilibrium for the truncated economy is also an equilibrium for the original economy by a standard argument, since payoffs are concave and choice sets are convex.

The case where the bond matures after date 2 but the official sector buys back the whole supply at this date, can be accommodated by adding, for each bond, another artificial agent that chooses q_{2j} in order to minimize $(q_{j2} - c_j' p_{j2} \max_{i \in I} DU^i_{j3}(x^i)/DU^i_{j2}(x^i))^2$, under the constraint $q_{j2} \in [0, c_j'\Delta]$, where Δ is an upper bound for $DU^i_{j3}(x)/Du^i_{j2}(x))^2$ on $[0, \sum_i \omega^i]$ (whose existence follows from Assumption A1).

6.5 Reformulation of the official sector no-overdraft constraints to allow for policy actions

$$\Omega^{gk}_{j1} = \iota_{j1}(\phi^{gk} + \sigma^{gk}) + q_{j1}[\tilde{b}^{gk}_j - b^{gk}_j - (t_j\theta_j - s_j\psi_j)] - o_{j1}(G^{gj}_{j1} - \tilde{G}^{gj}_{j1}) \tag{28a}$$

$$\Omega^{gk}_{j2} = -\eta_{kj}c_j\hat{b}_j + \iota_{j2}\phi^{gk} + (c_j + q_{2j})b^{gk}_j + q_{j1}(t_j\theta_j - s_j\psi_j)(1+\rho_j) - o_{j2}(G^{gj}_{j2} - \tilde{G}^{g2}_{jt}) \tag{28b}$$

The issuer gk faces a no-overdraft constraint in currency j, at date 1, specified by (28a) where $\iota_{f1} = -1$ and $\iota_{d1} = X$. For $\eta_{kj} = 1$ if $j = k$ and 0 otherwise, at date 2 we have (28b) where $\iota_{f2} = 1$ and $\iota_{d2} = -\chi$. We assume also that the public sector does not short sell: $b^{gk}_j \geq 0$ for any j, k, but also faces the box constraint $b^{gk}_j + \theta^{gk}_j - \psi^{gk}_j \geq 0$ governing what can be pledged out of the long position in the bond. Moreover, we assume $b^{gj}_j \leq \hat{b}_j$, that is, buybacks do not exceed the issuance. Otherwise, the private sector would be short selling on the aggregate, and if that would occur for both bonds, then banks' aggregate assets would be negative (and some banks would have negative assets).

The official sector can now repay the public debt using its initial cash holdings (which stand for taxes collected at earlier dates) or commodity endowments, by doing FX swaps (having sold its own currency before and getting it back at the debt repayment date, possibly combining this with an investment or a repo trade in the alien bond) or by trading in repo denominated in its own currency. Notice that equity requirements still manage to bound values of repo trades, in spite of the fact that the official sector may also trade in repo markets (as shown in the next subsection).

So far, bond cash flows are still tax funded, through $\tilde{y}^{gk}_{j1}$, which stands for taxes that were previously collected, possibly in both currencies (from nationals and foreigners). But we can take a step further and allow for the official sector to change its own money supply. This can be accommodated by adding an autonomous component z^{gj}_{jt} to $\tilde{y}^{gk}_{j2}$ and making it a choice variable, interpreted as an increase in money supply at date t.

An increase in money supply can have the purpose of servicing bond j's debt, selling currency j in an FX swap ($\phi^{gj} < 0$ at date 1), lending cash in repo ($\theta^{gj}_{j} > 0$, thereby increasing the cash balances in private hands) or buying back bonds ($b^{gj}_{jt} - \tilde{b}^{gj}_{jt} > 0$, with the same impact on private cash holdings). We assume that z^{gj}_{tj} has an upper bound M_{jt} set by public authorities.

Clearly, the official sector might also want to decrease the money available to the private sector, and this can be done by repoing the own bond (that is, taking a repo short position, $\psi^{gj}_{j} > 0$) or selling previous holdings of the own bond ($b^{gj}_{jt} - \tilde{b}^{gj}_{jt} < 0$). Actually, nowadays, open market operations tend to be done more in the form of repo trades than through actual purchases and sales of bonds. A decrease of the money supplied to private agents is accommodated by adjusting the official cash holdings (that is, by increasing y^{gj}_{jt}) with no need to change the money supply ($\tilde{y}^{gj}_{jt}$ stays the same).

The propositions on the cross-currency basis still hold in this policy framework.

6.6 On bounds on repo and bond positions when there are official trades

Take the case of centralized repo. We have $q_{d1} \sum_{i \in I} \psi_d - (1-e) q_{d1} \sum_{i \in I \cup \{ed\}} \theta_d \leq (1-e) \sum_{i \in I} (q_{d1} b^i_{d1} + X q_{f1} b^i_{f1})$. Now, $\sum_{i \in I} \psi^i_d = \sum_{i \in I \cup \{ed\}} \psi^i_d = \sum_{i \in I \cup \{ed\}} \theta^i_d + \theta^{gd}_d - \psi^{gd}_d$, implying $e q_{d1} \sum_{i \in I \cup \{ed\}} \theta^i_d \leq q_{d1} b^{gd}_{d1} + (1-e)F$, for $0 < F \equiv q_{d1} \sum_i \tilde{b}^i_{d1} + X q_{f1} \sum_i \tilde{b}^i_{f1}$. This bounds $q_{d1} \theta^i_d$ for $i \in I \cup \{ed\}$.

To see that for $i \in I$, ψ^i_d is also bounded, notice that $q_{d1}[\sum_{i \in I} \psi^i_d - (b^{gd}_d + \theta^{gd}_d - \psi^{gd}_d)] \leq F/e$. Now, $b^{gd}_d + \theta^{gd}_d - \psi^{gd}_d = \sum_i \tilde{b}^i_{d1} - \sum_{i \in I \cup \{ed\}} (b^i_d + \theta^i_d - \psi^i_d)$. Then, (2a) implies $q_{d1} \sum_{i \in I} \psi^i_d \leq q_{d1} \sum_i \tilde{b}^i_{d1} + F/e$.

Hence, $q_{d1} \theta^{gd}_d$ and $q_{d1} \psi^{gd}_d$ are also bounded (by repo market clearing). For $i \in I$, (2a) implies $b^i_{d1} \geq -\theta^i_d$, and, therefore, short sales are bounded in value. This implies by market clearing, that b^{gd}_{d1} and banks' long positions will be bounded as well. Finally, $\theta^{gd}_d \psi^{gd}_d = 0$ (as the official sector does not have the analog of a leverage constraint), which implies by (2a) that, when $\theta^{gd}_d > 0$, we get $\theta^{gd}_d = \sum_{i \in I \cup \{ed\}} (\psi^i_d - \theta^i_d) \leq \sum_{i \in I} b^i_d$ and, when $\psi^{gd}_d > 0$, we get $\psi^{gd}_d \leq \sum_{i \in I \cup \{ed\}} \theta^i_d$, so $q_{d1} \theta^{gd}_d$ and $q_{d1} \theta^{gd}_d$ are also bounded.

6.7 Basis formula of Proposition 6 plotted for several banks

Figure 5 compares the standard basis over 3M GC USD and EUR repo rates and the basis given by the formula in Proposition 6 using uncollateralized-collateralized spreads for the following banks: JPM, DB, Rabobank, Barclays, Citi, RBS, HSBC, Lloyds, RBC, MUFJ, CS, and UBS. There are not enough observations for BoA and Nochu to compute the basis (standard formula), so we don't calculate the basis for these two banks. For the repo rates we use the following. The Repo-Funds-Rate index reports rates on euro repo trades executed on both the BrokerTec and the MTS electronic platforms and uses sovereign government bonds as collateral. All eligible repo trades are centrally cleared. Data on USD repo GC rates is obtained from the Depository Trust & Clearing Corporation (DTCC). The DTCC GC Repo Index is comprised of the weighted average of the interest rates paid each day on transactions involving GC repos. To our knowledge, this is the only index to track GC USD repo transactions (no disaggregated repo data is publicly available at this point). It reflects actual, fully collateralized and centrally cleared repo transactions. Note how JPM stands out and does not satisfy the assumptions of Proposition 6 [45].

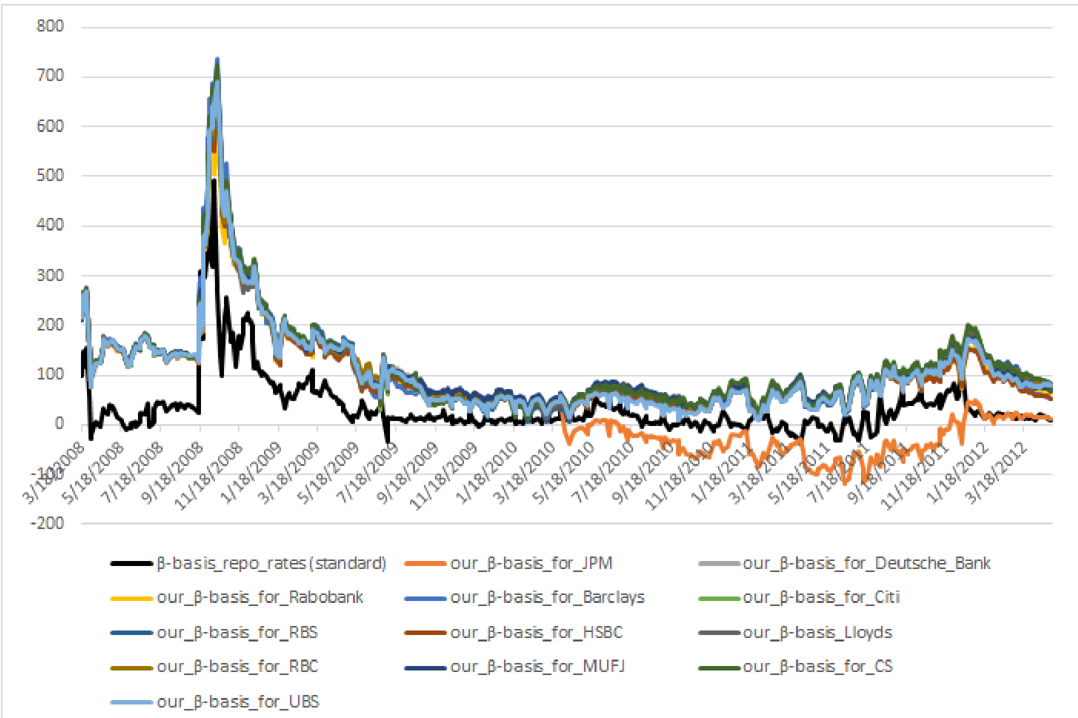

Figure 5: Comparison between standard basis formula using 3M GC USD and EUR repo rates and our basis formula in Proposition 6 using uncollateralized-collateralized spreads.

[45] Note also that while the interbank market stays deep in the high possession value currency as foreign banks need to borrow, in currency where there is excess cash in the banking system such market becomes smaller.