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Review of "Continuum modeling in the physical sciences" by van Groesen and Molenaar

van Groesen, E. and Molenaar, J. **Continuum Modeling in the Physical Sciences.** SIAM, Philadelphia, 2007. x+228 pp. US$65.00, ISBN 978-0-898716-25-2.

Successful mathematical modeling requires the ability to formulate a mathematical description of the phenomenon of interest as well as to apply appropriate tools to glean results from the model. An effective modeling text must give attention to developing both of these abilities. The intriguing, well-written book by van Groesen and Molenaar is successful in both respects.

Authors of modeling texts must decide whether to present mathematical concepts through a series of case studies, or whether to present mathematical principles in and of themselves and then apply them in examples. Van Groesen and Molenaar make the latter choice. Their philosophy (described in the preface) is that the case study approach "could disappoint the student, since having digested many particular models does not guarantee that one knows how to proceed when confronted with a new situation." Thus, they have written a text that highlights mathematical concepts relevant to the formulation of models. In contrast to modeling texts such as Fowler (1997) and Howison (2005), case studies do not play a central role. The exposition of general principles is peppered with examples; only at the conclusion of the book do the authors present full modeling case studies.

The book is based on material originally developed for advanced undergraduates at the University of Twente, and is published by SIAM. Excluding a few minor editing issues (for instance, I noticed that the notation in some figures and figure captions was not consistent with that used in the body of the text) the book lives up to the high production standard typical of SIAM texts. It is divided into six chapters, some of which end with a set of well-thought-out "challenging problems" that interweave the exposition of a particular problem with exercises for the reader to complete.

The first two chapters present concepts useful for the formulation of models. Chapter 1 covers dimensional analysis and scaling and applies those tools to problems from classical mechanics, including pendulums and springs. Also showcased is the well-known but still fascinating work of G.I. Taylor, who used dimensional analysis to estimate the strength of the first atomic bomb explosion by using film data showing the expansion of the resulting mushroom cloud. Chapter 2 focuses on two key model components, namely conservation laws and constitutive relations, and provides examples from heat flow, fluid flow, and traffic flow. One of my favorite features of this chapter is a pedagogically useful discussion of continuous versus discrete models, including a derivation of the former from the latter in the case of one spatial dimension.

The next two chapters present the core mathematical ideas that the authors believe to be key tools for dynamical modelers. The authors' unified mathematical approach is to treat ordinary and partial differential equations both as evolution equations describing movement through an appropriately defined state space, and to develop all mathematical tools in that context. This is a sensible approach, and one that I have not seen taken before in a modeling text. Chapter 3 defines the basic notions of state space, presents some fundamental tools including linearization and expansion in basis vectors, and introduces some properties of waves. Chapter 4 discusses stability and robustness. The emphasis on stability is standard. However, students first foraying into modeling often miss the importance of robustness, essentially "stability" with regards to perturbations of the form of the model. They will benefit from the clear treatment given to this topic.

Chapter 5 departs from the dynamical modeling emphasized in earlier chapters to take a look at variational modeling, including a presentation of the classic hanging chain (catenary) example as well as forays into Lagrangian and Hamiltonian mechanics. The authors also use the variational perspective to motivate the development of numerical methods (such as finite element) by an appropriate discretization of the state variables. For information on variational methods, modelers typically have to look beyond modeling texts, and so the inclusion of the variational chapter here is a nice addition to the literature.

Chapter 6 brings the previous chapters together in four cases studies. The applications examined are polymer dynamics, fiber spinning, surface water waves, and optics. Each case study begins with a convenient list of the specific concepts from earlier in the book (and their section number) called upon in the subsequent exposition. This makes for a tight integration with the rest of the text. Each case study also includes a useful set of exercises.

As mentioned above, the authors intend the book for an undergraduate audience. I think the material will be best appreciated by students with some background not only in multivariable calculus, ordinary and partial differential equations, and linear algebra, but also elementary physics, fluid dynamics and/or continuum mechanics, and some basic notions from functional analysis. I believe the text may be accessible to the most motivated undergraduates, and will definitely be a useful resource for beginning graduate students. The examples and applications in the book are not from the biological realm (as indicated by the title, the authors' interest is in physics-based models) but because of the generality of the modeling approach, a student or scientist with an interest in biological modeling should find the book to be useful. Supplemented with biological modeling texts such as Edelstein-Keshet (2005), Murray (2002), or de Vries et al. (2006), this book will be a valuable resource for students. It is an equally nice addition to the bookshelf of teachers and researchers – indeed, I plan to use it in both endeavors.

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