Economic Growth and the Environment:

Matching the Stylized Facts

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Abstract: The relationship between economic growth and the environment is not well understood: we have only a limited understanding of the basic science involved and very little data. Because of these difficulties it is especially important to develop a series of relatively simple theoretical models that generate stark predictions. This paper develops a simple endogenous growth model that matches the three most salient features of the growth and environment data. In addition to matching the facts it provides two new hypotheses: a set of predictions linking the path of environmental quality to pollutant characteristics (stocks vs. flows; toxics vs. irritants) and a novel cross-country prediction we dub the Environmental Catch-up Hypothesis.

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1. Introduction

The relationship between economic growth and the environment is, and will likely remain, controversial. Some see the emergence of new pollution problems, the lack of success in dealing with global warming and the still rising population in the developing world as proof positive that humans are a short-sighted and rapacious species. Others however see the glass as half full. They note the tremendous progress made in providing urban sanitation, improvements in air quality in major cities and marvel at continuing improvements in the human condition made possible by technological advance. The first group focuses on the remaining and often serious environmental problems of the day; the second on the long, but sometimes erratic, history of improvements in living standards.

These views are not necessarily inconsistent and growth theory offers us the tools needed to explore the link between environmental problems of today and the likelihood of their improvement tomorrow. A first step in predicting future outcomes is however an understanding of the past. The purpose of this paper is to develop a theory of economic growth that replicates the broad features of existing evidence on economic growth, pollution levels and abatement costs. Our thesis is that ongoing technological progress in both goods production and pollution abatement are together responsible for the three most salient features of the growth and pollution data: declining emissions to output ratios; pollution abatement costs that are roughly constant as a percentage of national output or manufacturing value-added; and a general tendency for the environment to at first worsen but then improve as income levels rise.

In addition to matching these broad features of the data, we present two new testable hypotheses. The first is a cross-country prediction relating environmental quality to current income levels and country specific initial conditions. We dub this prediction the Environmental Catch-up Hypothesis: initially poor countries experience greater environmental degradation than rich countries and worse environmental quality at all levels of income; despite this, differences in environmental quality between rich and poor diminish over time.

The second is a set of within country but across pollutant predictions. These predictions link the onset and stringency of regulation to primitives such as natural regeneration rates, toxicity of pollutants, and the degree to which income gains affect the demand for a cleaner environment. By providing these new hypotheses we widen the scope for empirical work in this area considerably.

In developing our model we make two key simplifying assumptions. First, we adopt assumptions that generate growth via a very simple mechanism that links productivity in goods production with an intensive measure of capital per worker. This formulation leads to the familiar AK production technology, where ongoing knowledge spillovers offset diminishing returns to capital accumulation leaving the social marginal product of capital constant. We then extend this formulation to a standard abatement production function, showing how knowledge spillovers created in abatement can play a similar role and produce an aggregate abatement function where diminishing returns to lowering emissions per unit output are likewise undone.

Our second assumption is that a benevolent social planner chooses policy. This is a standard and perhaps reasonable assumption for an analysis that investigates the broad features of the growth and environment link.

The cost of adopting these assumptions is that the source of technological progress in either production or abatement is left unexplained so that we can focus instead on its implications. We cannot examine the role intergenerational conflict or interest group politics may play in determining pollution policy; and we likewise give up any ability to gauge whether market forces or government policies will ensure that technological progress will be sufficiently rapid to offset diminishing returns in the future. In return, we get analytical tractability and a model with a manageable number of state variables. We use this breathing space to go further than earlier work in providing a simple explanation for our three stylized facts, a reasonably complete characterization of transition paths for various pollutants, and a novel Environmental Catch-up Hypothesis.

Our formal model is related to many others in the literature. It is similar to the one sector models discussed in Smulders (1993), Smulders and Gradus (1996), Stokey (1998) and Aghion and Howitt (2000) but differs from each of these significantly because of our treatment of abatement.[1] It is also related to work seeking to explain the EKC (Andreoni and Levinson (2001), Jones and Manuelli (2000), Lopez (1994), and John and Pecchenino (1994)) but differs from these both in purpose and in many specifics. Its closest connection may be to the much earlier work of Keeler, Spence, and Zeckhauser (1972) that sought to understand the growth and environment link quite generally and adopted several alternative formulations for abatement and pollution.

The rest of the paper proceeds as follows. In section 2 we set out stylized facts on emissions to output ratios, abatement costs and overall emissions. To limit the discussion we rely on plots of raw data rather than econometrics, but also direct the reader to formal

[1] See Brock and Taylor (2003) section 5 for a detailed account of the differences. This paper effectively replaces Brock and Taylor (2003).

econometric work. Following this introduction to the facts, in Section 3 we detail the model assumptions. Section 4 investigates the balanced growth path of the model. Section 5 constructs transition paths and presents our Environmental Catch-up Hypothesis. A short conclusion sums up. All proofs are relegated to an appendix.

2. Stylized Facts

We present three stylized facts drawn from the historical record of the US. Since we are primarily concerned with pollutants that are presently under active regulation but were not in the past, we discuss six criteria air pollutants. These are: sulfur dioxide, nitrogen oxides, carbon monoxide, lead, large particulates and volatile organic compounds. [2] With the exception of lead, these air pollutants all typically classified as irritants and so we also briefly discuss the US history of regulation of long-lived and potentially harmful chemical products. For the most part we present data on emissions rather than concentrations because data on emissions covers a much longer time period and is unaffected by industry location and zoning regulation. On the other hand, the longest time spans of data (from 1940 onwards) reflect some changes in collection and estimation methods.[3] Nevertheless, this data is the best we have available and where possible we direct the reader to concentration data and related empirical work. In

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[2] The long series of historical data presented in the figures is taken from the EPA's 1998 report National Pollution Emission Trends, available at http://www.epa.gov/ttn/chief/trends/trends98.

[3] As methods of estimation improve new categories of emissions are included and some revision occurs as well. For example, prior to 1985 the PM10 data excluded fugitive dust sources and other miscellaneous emissions, so these are eliminated from the time series graphed in Figure 7. As well revision occurs. A close look at the 2001 Trends report shows that emissions reported for our pollutants during the 1970s and 1980s does not exactly match the figures given in the 1998 report. We use the 1998 figures rather than those from 2001 since the 2001 report only contains estimates to 1970, and we import the EPA's graphics directly into our figures because we cannot match them precisely from the raw data.

addition we present data on industry pollution abatement costs from Vogan (1996), although these are only available for the 1972-1994 period.

We start by presenting in Figure 1 emissions per dollar of GDP for all pollutants except lead. Lead is excluded since data is only available over a much shorter period. As shown, emissions per unit of output for sulfur, nitrogen oxides, particulates, volatile organic compounds, and carbon monoxide all fall over the 1940-1998 period. For ease of comparison emission intensities were normalized to 100 in 1940. Although there is a tendency to see good news in falling emission intensities, real economic activity increased by a factor of 8.6 over this period and this masks the fact that emissions of many of these pollutants rose during this period.

Our second stylized fact is presented in Figure 2. In it we plot pollution abatement costs per dollar of GDP over the period 1972-1994. These twenty-two years are the only significant time period where data is available.[4] As shown, pollution abatement costs as a fraction of GDP rise quite rapidly until 1980 and then remain relatively constant. As a fraction of overall output, these costs are relatively small. Generating a similar plot for costs as a fraction of manufacturing value-added leads to similar results. Alternatively, if we consider pollution abatement costs specifically directed to our criteria air pollutants and scale this by real US output, the ratio is then incredibly small – approximately one half of one percent of GDP - and has remained so for over twenty years (See Vogan (1996)).

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[4] In 1999 the PACE survey was run again this time as a pilot project. Using the 1999 survey we find the ratio of PACE to GDP of approximately 1.9% which is very much in line with figure 2. This 1999 survey is different in some respects from earlier ones. For details see the Survey of Pollution Abatement Costs and Expenditures, U.S. Census Bureau 1999 available at www.census.gov/econ/overview/mu1100.html

Our third and final fact is presented in Figures 3-8. These figures show a general tendency for emissions to at first rise and then fall with development. Note that the falling emissions/intensities reported in Figure 1 are necessary but not sufficient for this result. This pattern in the data is visible for all pollutants except nitrogen oxides that may now be approaching a peak in emissions. Conversely, particulate pollution peaked much earlier than the other pollutants, while lead has a dramatic drop in the mid-1970s. Since the US exhibited trend growth in real per capita income of approximately 2%/year over this period, the time scale in the figures could just as well be replaced by income per capita. Therefore, these raw US data offer a dramatic confirmation of the cross-country empirical findings of Grossman and Krueger (1993,1995) and others.

Combining our three stylized facts presents us with an important question. How did aggregate emissions and emissions per unit output fall so dramatically without raising pollution abatement costs precipitously?

There are several possible explanations. One possibility is that ongoing changes in the composition of US output have led to a cleaner mix of production that has lowered both aggregate measures of costs and emissions.[5] Similarly, the downward trend in emissions per unit output shown in Figure 1 prior to the advent of the Clean Air Act suggests some role for composition effects. While changes in the composition of US output are surely part of the story, there are reasons to believe that they cannot be the most important part. Over the 1971-2001 period of active regulation by the EPA, total emissions of the 6 criteria air pollutants (Nitrogen Dioxide, Ozone, Sulfur Dioxide,

[5] For example, Levinson and Taylor (2003) argue that aggregate ratios of pollution abatement costs to manufacturing value-added give a distorted view of true abatement costs since the composition of US manufacturing has been moving towards cleaner industries. See for example their Figure 2 where corrections are made to hold the composition of output constant.

Particulate matter, Carbon Monoxide, and Lead) decreased on average by 25%. Over this same period, gross domestic product rose 161% and pollution abatement costs have risen only slightly.[6] The magnitude of these emission reductions is too large for it to reflect composition changes alone.[7] It is also apparent from the figures that emissions for most pollutants have been falling since the early 1970s and there are limits to how far aggregate emissions can fall via composition effects alone.[8] Finally, there is little evidence that international trade is playing a major role in shifting dirty goods industries to other countries but stronger composition effects after the advent of federal policy in early 1970s would be necessary to explain the fall in emissions seen in the figures.[9]

Another possible explanation is that income gains arising from ongoing capital accumulation or technological advance generate a strong demand for environmental improvement. In these accounts, ongoing income gains produce the change in policy in the early 1970s and herald the start of emission reductions. While this explanation fits with the decline in emission to output ratios and the lowered emissions since the early 1970s, it relies on agents being willing to make larger and larger sacrifices in

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[6] These figures are from the EPA's Latest Findings on National Air Quality, 2001 Status and Trends, available at http://www.epa.gov/air/aqtrnd01/ published September 2002.

[7] To get a handle on the magnitudes involved think of aggregate emissions, E as the sum across n industries of (emissions per dollar of output in industry i) X (value share of industry i in national output) X (national output). In obvious notation, $E = \sum_{i=1}^{n} a_i s_i Y \ \ where \sum_{i=1}^{n} s_i = 1$. Differentiating with respect to time, we find: $\hat{E} = \sum_{i=1}^{n} p_i [\hat{a}_i + \hat{s}_i] + \hat{Y} \ where\, p_i = E_i / E$. If changes in the composition of output over the 1971-2001 period are to carry all the burden, then the changes in a_i are zero. Using the EPA's estimate of an average 25% reduction for E and the 161% increase for Y, we find that the weighted average of industry level changes must add up to - 186% change – a very large realignment in US productiont.

[8] Using the framework given in the previous footnote it is possible to prove that ongoing growth in Y and falling E are only compatible if emissions per unit of output fall (proof available on request). Composition effects cannot lower emissions indefinitely.

[9] See for example Antweiler et al. (2001) and Grossman and Krueger (1993).

consumption for improving environmental quality. For example, Stokey (1998), Aghion and Howitt (1998), Smulders (2001), Lopez (1994), etc. present formulations with these features, but all require a rapidly declining marginal utility of consumption to generate rising environmental quality and ongoing growth. As Aghion and Howitt note:

> "Thus it appears that unlimited growth can indeed be sustained, but it is ot guaranteed by the usual sorts of assumptions that are made in endogenous growth theory. The assumption that the elasticity of marginal utility of consumption be greater than unity seems particularly strong, in as much as it is known to imply odd behavior in the context of various macroeconomics models…" (p. 162.)

A rapidly declining marginal utility of consumption is required in earlier work because increasingly large investments in abatement are required to hold pollution in check.[10] This implies the share of pollution abatement costs in the value of output approaches one in the limit, which is inconsistent with available evidence.[11]

In this paper we put forward an alternative and related explanation of these data. Ongoing technological progress in production *and* abatement have simultaneously driven long run growth, while holding pollution abatement costs in check. Technological progress in production has driven growth, while technological progress in abatement has allowed emissions per unit of output to fall precipitously without raising costs skyward. Income gains from ongoing growth are responsible for the onset of serious regulation in the 1970s, but the advent of regulation then brought forth improvements in abatement

[10] This restriction also implies a large income elasticity of marginal damage and many question whether the demand for a clean environment can be so income elastic. For example, McConnell (1997) argues that current empirical estimates from contingent valuation and hedonic studies do not support the very strong income effects needed.
[11] See the discussion in Aghion and Howitt (1998, page 160-161) and our discussion of abatement in Section 5 of Brock and Taylor (2003).

methods and as a consequence agents have not been required to make increasingly large sacrifices in consumption for improving environmental quality.

Before we proceed we should note that the stylized facts given thus far exclude a discussion of many other pollutants. By selecting only pollutants for which data is available we may have erred on the side of optimism since the measurement of pollutants is almost always a precursor to their active regulation. One important omission from the above is any discussion of air toxics such as benzene (in gasoline), perchloroethylene (used by dry cleaners), and methyl chloride (a common solvent). These are chemicals are believed to cause severe health effects such as cancer, damage to the immune system, etc. At present the EPA does not maintain an extensive national monitoring system for air toxics, and only limited information is available.[12]

Another omission is any discussion of the set of long-lived chemicals and chemical by-products that have found their way into waterways, soils and the air. These products have very long half-lives and produce serious health and environmental effects. Prominent among these in US history are DDT, PCBs, Lead, and most recently CFCs. Official estimates on emissions of these pollutants is difficult to find, but historical accounts and partial data indicate their emissions follow a pattern roughly similar to that of lead shown in Figure 8. As shown by the figure the history of lead is one of strong initial growth in emissions, followed by a rapid phase-out and virtual elimination. In fact, lead continues to be emitted in small amounts, whereas PCB emissions rose from very low production levels in the 1930s to millions of pounds per year of production in

[12] See U.S. EPA, Toxic Air Pollutants, at www.epa.gov/airtrends/toxic.html.

the 1970s, to end with a complete ban in 1979. Similarly, DDT was used extensively after WWII but banned in 1972. CFC production in the US rose quickly with the advent of refrigeration and air conditioning, but this set of chemicals now faces a detailed phase-out with CFC-11 and CFC-12 already facing a complete production ban. The salient feature of these accounts is strong early growth followed by quite rapid elimination.

2. The Model

We adopt the conventional infinitely-lived-representative-agent model. There is one aggregate good, labeled Y, which is either consumed or used for investment or abatement purposes. There are two factors of production: labor and capital. We assume zero population growth and hence L(t) = L. The capital stock accumulates via investment and depreciates at the constant rate δ.

2.1 Tastes

Our representative consumer maximizes lifetime utility given by:

$$W = \int_0^{\infty} U(C,X)e^{-rt}dt \tag{1.1}$$

where C indicates consumption, and X is the pollution stock. Utility is increasing and quasi-concave in C and –X . When we treat pollution as a stock, X= 0 corresponds to a pristine state of the environment. When we treat pollution as a flow, X = 0 corresponds to a zero flow of pollution. A useful special case of U(C,X) is the constant elasticity formulation where:

$$U(C,X)=\frac{C^{1-e}}{1-e}-\frac{BX^{g}}{g}\ \ for\ e\neq 1$$
$$U(C,X)=\ln C-\frac{BX^{g}}{g}\ \ for\ e=1 \qquad (1.2)$$

where $g\geq 1$ and B measure the impact of local pollution on a representative individual. The elasticity of marginal utility from consumption is given by ε > 0.

2.2 Technologies

Our assumptions on production are standard. Each firm has access to a strictly concave and CRS production function linking labor and capital to output Y. The productivity of labor in goods production is augmented by a technology parameter T taken as given by individual agents. Following Romer (1986) and Lucas (1998) we assume the state of technology is proportional to an economy wide measure of activity. In Romer (1986) this aggregate measure is R&D, in Lucas (1988) it is average human capital levels; in AK specifications it is linked to either the aggregate capital stock or (to eliminate scale effects) average capital per worker. We assume T is proportional to the average capital to labor ratio in the economy, K/L, and by choice of units take the proportionality constant to be one.[13]

2.3 From Individual to Aggregate Production

Although we adopt a social planning perspective, it is instructive to review how firm level magnitudes aggregate to economy wide measures since this makes clear the

[13] As is well known, one-sector models of endogenous growth blur the important distinction between physical capital and knowledge capital and force us to think of “capital” in very broad terms. Extensions of our framework along the lines of Grossman and Helpman (1991) or Aghion and Howitt (1998) seem both possible and worthwhile. These extensions would however add additional state variables making our examination of transition paths difficult.

assumptions made regarding the role of knowledge spillovers. We aggregate across firms to obtain the AK aggregate production function as follows: [14]

$$\begin{aligned} Y_i &= F(K_i, TL_i) \\ Y &= \sum_i F(K_i, TL_i) \\ Y &= TLF(K/TL, 1) \\ Y &= KF(1,1) = AK \end{aligned} \tag{1.3}$$

where the first line gives firm level production; the second line sums across firms; the third uses linear homogeneity and exploits the fact that efficiency requires all firms adopt the same capital intensity. The last line follows from T = K/L and A=F(1,1).

Summarizing: diminishing returns to capital, given L_i, at the firm level are undone by technological progress linked to aggregate capital intensity, leaving the social marginal product of capital constant.

We now employ similar methods to generate the aggregate abatement technology. To start we assume that pollution is jointly produced with output and take this relationship to be proportional.[15] By choice of units we take the factor of proportionality to be one. Pollution emitted is equal to pollution created minus pollution abated. Denoting pollution emitted by P, we have:

$$P = Y^G - abatement \tag{1.4}$$

Abatement of pollution takes as inputs the flow of pollution, which is proportional to the gross flow of output Y^G, and abatement inputs denoted by Y_A. The abatement production

[14] Implementing our planning solution by way of pollution taxes and subsidies to investment and abatement should be straightforward. We leave this to future work.
[15] Nothing is lost if we assume pollution is produced in proportion to the services of capital inputs. The service flow of capital is proportional to the stock of capital, and the stock of capital is proportional to output.

function is standard: it is strictly concave and CRS. Therefore we can write pollution emitted by the ith firm as:

$$P_i = Y_i^G - a(Y_i^G, Y_i^A) \tag{1.5}$$

note that in this formulation, the abatement activity pollutes since it is counted in Y^G.

Now consider a Romeresque approach where individual abatement efforts provide knowledge spillovers useful to others in the economy. This knowledge arrives as a consequence of abatement efforts and spills over to other firms raising the productivity of their abatement efforts. To make this idea operational we introduce the technology shift parameter Γ that raises the marginal product of abatement. To be consistent with our earlier treatment of technological progress we assume *G* is proportional to the average abatement intensity in the economy, Y^A/Y^G. Then much as before we have the individual to aggregate abatement technology transformation as:

$$\begin{aligned}
&P_i = Y_i^G - a(Y_i^G\Gamma, Y_i^A) \\
&P_i = Y_i^G[1 - \Gamma a(1, Y_i^A / Y_i^G\Gamma)] \\
&\sum_i P_i = \sum_i Y_i^G[1 - \Gamma a(1, Y_i^A / Y_i^G\Gamma)] \\
&P = Y^G[1 - \Gamma a(1,1)], a(1,1) > 1 \\
&P = Y^G[1 - q\, a(1,1)], q \equiv Y^A / Y^G
\end{aligned} \tag{1.6}$$

where the first line introduces the technology parameter Γ; the second exploits linear homogeneity of the abatement production function; the third aggregates across firms; the fourth recognizes that efficiency requires all firms choose identical abatement intensities, uses the definition of Γ and notes that for abatement to be productive it must be able to clean up after itself. The fifth line defines the intensity of abatement, $q \equiv Y^A / Y^G$.

Summarizing: diminishing returns to lowering emissions per unit output at the firm level, that lead to rising marginal abatement costs, are undone by technological progress linked to aggregate abatement intensity leaving the social marginal cost of abatement constant.

Putting these pieces together our planner faces the aggregate production relations for output and abatement given by the last lines of (1.3) and (1.6) together with the atemporal resource constraint linking gross output, abatement and net production:

$$Y = Y^G - Y^A \tag{1.7}$$

2.4 Endowments

We treat pollution as a flow that either dissipates instantaneously – such as noise pollution – or a stock that is only eliminated over time by natural regeneration – such as sulfur or lead emissions. When X is a stock we have:

$$\dot{X} = AK[1-\theta a] - \eta X \tag{1.8}$$

where AK[1-θa] is the instantaneous inflow of pollution and ηX the amount cleansed by natural regeneration. $\eta > 0$ is the speed of natural regeneration, and where for economy of notation we have denoted a(1,1) by a. When we consider flow pollutants, the state of the environment, X, is just given by the instantaneous inflow of pollution:

$$X = AK[1-\theta a]$$

Since the flow of pollution can never be negative we require $\theta \leq 1/a$, and denote by θ^M the maximum intensity of abatement.

3. Balanced Growth

In order to replicate the broad features of the pollution data the regulation regime must change over time. This is apparent from Figures 3 through 8 and is consonant with the history of environmental regulation in the US and elsewhere. We refer to the period prior to regulation as Stage I and post regulation as Stage II. To generate a complete environment and growth profile we start by examining balanced growth paths in Stage II. The entire history is found by pasting Stage I solutions together with those from Stage II.[16]

Two features of our model deserve mention before proceeding. As is well known the AK formulation typically eliminates transitional dynamics. This allows us to concentrate on how changes in abatement intensity introduce transitional dynamics on their own, without having complications introduced by the well-known convergence features of the neoclassical model. The second feature is the constant marginal product of abatement at the aggregate level. This feature allows us to escape the implications of Inada conditions so that relative price changes can switch the economy from zero to active abatement and then through to an effective ban on emissions. Since many pollution problems are ignored, others actively regulated, and some eliminated by fiat, any model hoping to match the data must contain a similar assumption.

For the most part we present the analysis when pollution is a stock since it contains the flow problem as a limiting special case. Consider the following problem:

$$\begin{aligned}
&Maximize \int_0^\infty U(C,X)e^{-rt}\,dt \\
&s.t.\; K(0)=K_0,\;\; X(0)=X_0,\;\; and\;\; q \le 1/a \\
&\dot{K} = AK[1-q]-dK-C \\
&\dot{X} = AK[1-qa]-hX
\end{aligned} \qquad (1.9)$$

[16] A similar two-stage model is presented in Weitzman (2003), but his focus is quite different.

where we adopt U(C,X) from (1.2). Recall the fraction of gross output allocated to abatement is θ. Since there are no intermediate goods nor profits, θ represents the share of value added allocated to abatement; i.e. the same variable graphed in Figure 2. We can write the Hamilton-Jacobi-Bellman equation for our problem as:

$$\rho W(K,X) = Max\left\{ H = \frac{C^{1-\varepsilon}}{1-\varepsilon} - \frac{BX^{\gamma}}{\gamma} + \lambda_1[AK[1-\theta] - C - \delta K] + \lambda_2[AK[1-\theta a] - \eta X] \right\} \quad (1.10)$$

where ρW(K,X) is the maximized value of the program for the given initial conditions $\{K_0, X_0\}$, and H is the current value Hamiltonian for our problem. The controls for this problem are consumption, C, and abatement intensity, θ.

An important feature of (1.10) is its linearity in θ. To see why observe the term involving our control variable, θ,

$$Max\left\{ AK\theta[-\lambda_1 - a\lambda_2] \right\} \ s.t.\ 0 \le \theta \le 1/a$$

where λ_1 is the positive shadow value of capital and λ_2 is the negative shadow cost of pollution. The opportunity cost of more abatement is less capital accumulation and hence the shadow cost of abatement inputs is just λ_1. Since a marginal unit of abatement lowers pollution by a > 1 units, the marginal cost of abating a unit of pollution is λ_1/a. The marginal benefit is given by $-\lambda_2$. Therefore, marginal benefits to abatement exceed costs when the variable $S = [-\lambda_1 - a\lambda_2]$ is positive.

Accordingly, when abatement is cheap and the cost of ignoring the environment large, S > 0, and maximal abatement is undertaken with $\theta = \theta^M = 1/a$. There is an effective ban on emissions and the environment improves via natural regeneration.

When the shadow value of capital is high relative to that of pollution, S < 0, and zero abatement is undertaken and the environment worsens. Finally, when S = 0 pollution is only partially abated, and the evolution of the environment reflects both the intensity of abatement and natural regeneration. Putting these possibilities together, it is apparent that the economy can switch from zero-to-active-to-maximal abatement as the relative cost of abatement inputs varies over time.

3.1 Growth with Active Abatement

Consider growth paths with active abatement. These paths have S ≥ 0, and the capital co-state equation from the problem in (1.10) yields:[17]

$$-g \equiv \frac{\dot{l}_1}{l_1} = -\left[A[1-q^{M}] - r - d \right] < 0, \tag{1.11}$$

The shadow value of capital falls over time at a constant exponential rate provided g > 0. Note that while θ is endogenous, θ^M is a parameter and hence so too is g.

The assumption g > 0 is required for ongoing growth. It represents an assumption that the marginal product of capital, adjusted for the ongoing costs of abatement, be sufficiently high. A necessary condition is that $A[1-\theta^M]$ be positive, but this is guaranteed as long as abatement is a productive in lowering pollution. Recall that abatement, like all other economic activities, pollutes. One unit of abatement creates one unit of pollution, but cleans up a > 1 units of pollution. Therefore abatement is only productive as a means to lowering pollution when $1\text{-}1/a = 1\text{-}\ \theta^M > 0$. Given abatement is productive, we still require the adjusted marginal product of capital, $A[1-\theta^K]$, to offset

[17] The relevant first order conditions are given in the appendix.

both impatience and depreciation. If abatement is not very productive, then 1/a will be close to one and growth cannot occur. If capital is not very productive or if the level of impatience and depreciation are high then ongoing economic growth cannot occur. These are however very standard requirements for growth under any circumstances.[18]

Regardless of the choice for abatement, optimal consumption will always satisfy

$$\frac{\partial H}{\partial C} = C^{-e} - l_1 = 0 \tag{1.12}$$

Using (1.11) it is immediate that consumption rises at the constant rate $g_C = g/\varepsilon > 0$.

From the capital accumulation equation in (1.9) we can now infer that capital and output must grow at the same rate as consumption if the intensity of abatement θ is constant over time. To determine whether the intensity of abatement *is* constant over time, consider the accumulation equation for pollution:

$$\dot{X} = AK[1 - q\,a] - h\,X \tag{1.13}$$

There are two ways (1.13) can be consistent with balanced growth. The first possibility is that we have a maximal abatement regime where pollution is eliminated everywhere along the balanced growth path. In this situation, K grows exponentially over time and θ is set to the maximal level θ^M. This balanced growth path would have:

$$\dot{X} = -h\,X \quad and \quad q = q^{M} \tag{1.14}$$

The environment improves at the rate η over time and abatement is a constant fraction of output $1 > \theta^M > 0$. As time goes to infinity the lingering effects of any pollutant are

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[18] In Keeler, Spence and Zeckhauser (1972) a similar condition describes their Golden Age capital stock. In their model with no endogenous growth the Golden Age capital stock is defined by (in our notation) the equality $f'(K)[1-1/a]-\delta = \rho$; simulations of the model assume a to be 12 (see p.22). Chimeli and Braden (2002) assume a similar condition.

eliminated and the economy approaches a pristine level of environmental quality. Therefore one possible balanced growth path exhibits constant growth in consumption, output, capital and environmental quality. Consumption is a constant fraction of output and we have:

$$g_C = g_K = g_Y = g / e > 0, \, g_X = -h < 0 \tag{1.15}$$

It follows that technological progress in goods production is ongoing with T growing at the rate g_T equal to g_K. In contrast, since the pollutant is effectively banned by policy there are no ongoing spillovers from a rising abatement intensity and hence no ongoing technological progress in abatement, $g_\Gamma = 0$. [19]

A second possible growth path occurs when abatement is active but not maximal. Define the deviation of abatement from its maximal level as $D(\theta) = (\theta^M - \theta)/\theta^M$. Using this definition rewrite (1.13) finding:

$$\frac{\dot{X}}{X} = \frac{AK[D(q)]}{X} - h \tag{1.16}$$

It is now apparent that if the intensity of abatement rose towards its maximal level, then it may be possible for the environment to improve (X falls exponentially) while growth proceeds (K rises exponentially). Policy becomes increasingly tight, emissions fall and the share of pollution abatement costs in output rises. In obvious notation, a possible balanced growth path would have:

$$g_K + g_D = g_X \tag{1.17}$$

[19] All the efficiency gains from knowledge spillovers come in one lump as abatement jumps from zero to its balanced growth path level.

In this situation abatement is at an interior solution and approaches its maximal level asymptotically. There is ongoing technological progress in abatement as a result of its rising intensity, and ongoing technological progress in goods production as well. The inflow of pollution from production into the environment is always positive but environmental quality improves nevertheless.

3.2 The Intensity of Abatement

It is clear that the two growth paths primarily differ in their predictions for the intensity of abatement. To determine which of the two possible growth paths arise in a given situation, we assume abatement is interior and explore what this requires of the parameters. When abatement is interior, the shadow values for capital and pollution are related by:

$$l_1 = -a\, l_2 \qquad (1.18)$$

which implies, since *a* is constant, that the rate of change of the shadow cost of pollution must fall over time at the same rate the shadow value of capital falls over time. From (1.11) we know the shadow value of capital falls at rate g. And using the fact that the time rates of change and levels of the shadow prices are pinned down by (1.18), we find:

$$\frac{1}{a} = \frac{BX^{g-1}C^{e}}{h + g + r} \qquad (1.19)$$

Differentiating yields:

$$\frac{\dot{X}}{X} = \frac{e}{1-g}\frac{\dot{C}}{C} = \frac{g}{1-g} < 0 \qquad (1.20)$$

Since consumption rises along the balanced growth path at rate g/ε the environment improves at rate g/(γ–1). The only question remaining is whether the growth rate given by (1.20) for the pollution stock is in fact consistent with remaining active in abatement. If the environment improves too quickly the shadow cost of pollution will fall, and we will enter a zero abatement situation; conversely if the environment improves too slowly, maximal abatement is called for.

To investigate, manipulate the state equation for X using (1.20) and exploit our definition of D(θ) to find:

$$D(q) = \frac{X}{AK}\left[\frac{h(g-1)-g}{(g-1)}\right] \tag{1.21}$$

Recall that D(θ) is the deviation of current abatement from its maximum and cannot be negative. X and K are always positive. Therefore, balanced growth with an interior solution to abatement requires the parameter restriction:

$$h(g\text{ -}1)\text{>g} \tag{1.22}$$

This condition reflects two different requirements.

The first requirement is simply that γ cannot equal one. If it did then the (instantaneous) marginal disutility of pollution would be constant and the shadow cost λ_2 constant as well. To see this manipulate (1.19) and (1.18) to find when γ is one:

$$l_2(t) = -\frac{B}{h+g+r} < 0 \tag{1.23}$$

When γ = 1, the marginal benefit of *any* environmental improvement is the same. But marginal abatement costs are falling with ongoing growth in capital, and hence the two cannot equal but for a moment in time. Since our growth path requires a slowly adjusting interior solution for abatement, a constant marginal disutility rules this out.

The second requirement is less obvious. Natural regeneration, η, must be large relative to the growth rate g. If the rate of natural regeneration is high and the growth rate quite low, then the optimal plan is to use nature's regenerative abilities to partially offset the costs of abating. Exploiting nature's regenerative abilities makes sense in this situation because the shadow value of foregone output is high in slow growth situations.[20] Therefore in situations where pollutants are readily dissipated from the environment (most air pollutants) or cause no long lasting human effects, a program of slowly tightening policy is optimal. Emissions per unit output fall slowly along this path.

Conversely, if the rate of natural regeneration is low and the growth rate g relatively high, then no amount of abatement short of the maximal level will hold pollution to acceptable levels. In this situation, the optimal plan is to clamp down on emissions hard because their life in the environment is so long and the economy's unfettered production of them so rapid, that only a very aggressive abatement regime makes sense. Emissions per unit output are fixed at zero along this path.

To complete our analysis we need to rule out the logical possibility of a balanced growth path with no abatement. We leave this to the appendix and summarize:

Proposition 1. Assume $g > 0$, then positive economic growth with an ever improving environment is possible and optimal.

i) If $\eta(\gamma-1) > g$, the intensity of abatement approaches the maximal level θ^M, asymptotically.

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[20]This follows because when g is low, output will be scarce in the future since growth is slow. If it is going to be scarce in the future, it is scarce now.

ii) If $\eta(\gamma-1) < g$, $\theta = \theta^M$ everywhere along the balanced growth path.

iii) Balanced growth with zero abatement is not optimal.

Proof: See Appendix.

Proposition 1 suggests a natural corollary for the case of flow pollutants. If pollution has only a flow cost it is "as if" the environment is regenerating itself infinitely fast. This intuition suggests that as we let η get large, our results in the stock pollutant case should replicate those for a flow. This intuition is, in fact, correct.

Proposition 2. Assume $g > 0$ and X is a flow pollutant, then positive economic growth with an ever improving environment is possible and optimal.

iv) If $(\gamma-1)>0$, the intensity of abatement approaches the maximal level θ^M, asymptotically.

v) If $(\gamma-1)=0$, $\theta = \theta^M$ everywhere along the balanced growth path.

Proof: See Appendix.

Propositions 1 and 2 are important in several respects. Perhaps most importantly they provide partial evidence for our technology-based explanation of the stylized facts. We find that during the active regulation phase, emissions and emissions to output fall while growth is ongoing. At the same time, the share of pollution abatement costs in the value of output is either constant or growing only to asymptote to a positive constant less than one. Since we have not assumed anything in particular regarding tastes, it should be clear that increasingly large sacrifices in consumption are not required for our results.

The propositions also contain an interesting prediction concerning the intensity of abatement and the optimality of a complete emissions ban. The simple condition that determines whether abatement should be gradual or aggressive makes good economic sense, but is far from obvious. It also produces results consistent with the historical record because in many cases long-lived chemical discharges and gas emissions were eliminated very quickly by legislation, whereas short-lived criteria pollutants have seen active regulation, but not elimination, over the last 30 years.

4. The Transition to Active Abatement

To complete our analysis we present several transition paths for economies moving from inactive to active abatement in Figure 9. One of these paths is that of a Poor country having a small initial capital K^P but a pristine environment. The other is the path of a Rich country starting again with a pristine environment but with a much larger initial capital K^R. Each economy starts with a pristine environment in stage I and grows. During this stage the environment deteriorates, X rises, and the capital stock grows until the trajectory hits the Switching Locus labeled SL. Once the economy hits the Switching Locus it begins stage II. The exact position and shape of the locus depends on whether parameters satisfy the fast growth or slow growth scenario. For the most part we will proceed under the assumption that economic growth is fast relative to environmental regeneration; that is (1.22) fails and we have $\eta(\gamma-1) < g$. This implies $\theta(t) = \theta^M$ along the balanced growth path (Figure 9 implicitly assumes this result). For illustrative purposes we will sometimes discuss the parallel flow case (where we can think of η approaching infinity but (1.22) failing because $\gamma = 1$).

We first demonstrate that all growing economies follow the stage I – stage II life cycleries, and then turn to our stylized facts.

4.1 The Switching Locus

Stage II starts at the moment the economy's shadow prices for capital and pollution satisfy (1.18) and abatement begins. Denote this transition time t=t* and the associated capital and pollution stocks at the moment of transition K* = K(t*) and X* = X(t*). Recall the shadow values in (1.18) are partial derivatives of the value function W(K,X) evaluated at K*, X*. Where W(K*,X*) is the maximized value of the program starting from capital and pollution stocks {K*, X*} over the period t = t* until $t = \infty$. This implies the set of {K*, X*} at which a transition to active abatement occurs must satisfy:

$$-\frac{\partial W(K^*, X^*)/\partial K}{\partial W(K^*, X^*)/\partial X} = \frac{l_1(K^*, X^*)}{-l_2(K^*, X^*)} = a \tag{1.24}$$

We refer to the set of {K*,X*} satisfying (1.24) as the Switching Locus, since any trajectory of the economy switches from inactive to active abatement when it crosses it.[21]

To characterize the switching locus we need to solve for the shadow values at t* as functions of K* and X*. Plugging these into (1.24) gives us the algebraic relationship we have plotted in Figure 9. Since the shadow values at t* reflect the entire future path for the economy we need to solve the system completely to do so. To start note the switching time is t*, and hence at t* the S > 0 set of dynamics come into play. Using the condition S > 0 solve the differential equation for the shadow value of capital. Eliminate

[21] Trajectories in the fast growth case cross the Switching Locus; trajectories in the slow growth locus enter a similar Switching Locus and then follow it thereafter. We discuss the slow growth case subsequently.

consumption and then solve for the capital stock. This yields solutions for the shadow value of capital and the capital stock for any t after t*. These solutions are:

$$\begin{aligned} &\lambda_1(t) = \lambda_1(t^*)\exp[-gt] \\ &K(t) = \exp[[g+\rho]t]\left[K^* - \lambda_1(t^*)^{-1/\varepsilon}\int_{t^*}^{t}\exp[-[g(1-1/\varepsilon)+\rho]s]ds\right] \end{aligned} \qquad (1.25)$$

K* is fixed at t* since it is given by the economy's history. The initial shadow value of capital $\lambda_1(t^*)$ however must be chosen to satisfy the transversality condition, TVC, $\lim_{t\to\infty}[\exp[-\rho t]\lambda_1(t)K(t)] = 0$. Subbing (1.25) into the TVC we find it requires:

$$\lim_{t\to\infty}\left\{\left[K^* - \lambda_1(t^*)^{-1/\varepsilon}\int_{t^*}^{t}\exp[-[g(1-1/\varepsilon)+\rho]s]ds\right]\right\} = 0, \qquad (1.26)$$

Integrating and solving (1.26) gives us the solution for $\lambda_1(t^*)$ as:

$$\lambda_1(t^*) = [hK^*]^{-\varepsilon},\ h \equiv g(1-1/\varepsilon)+\rho > 0 \qquad (1.27)$$

The shadow value is a simple declining function of K* since we assume h > 0. Using (1.27) in (1.25) now allows us to solve for $\lambda_1(t)$ as a function of K*, which we can then substitute into (1.24). The shadow value of capital is only a function of K* because, in the fast-growth-slow-regeneration case, the economy's shadow value of capital falls so rapidly that abatement efforts are always maximal. Since abatement is maximal, the $\{K,\lambda_1\}$ part of the problem separates and solves a simple AK model with power utility. It is now possible to show that h > 0 is equivalent to the standard sufficient condition for existence of an optimum path in an AK model with power utility.[22]

[22]Denote the growth rate in an AK model with power utility by g*, then in terms of our parameters we have g* =g/ε and the standard condition is ρ+(ε–1)g*>0. This is equivalent to h>0. See Aghion and Howitt (1998, Equation (5.3)).

We now turn to solve for the shadow cost of pollution. Assuming we remain in the S > 0 regime forever, we can solve the state and co-state equations to yield:

$$
\begin{aligned}
&X(t) = X^* \exp[-\eta(t-t^*)], \\
&\lambda_2(t) = \exp[[\rho+\eta](t-t^*)][[\lambda_2(t^*) + BX^{*\gamma-1}[1-\exp[-c_1(t-t^*)]/c_1] \qquad (1.28) \\
&c_1 \equiv \rho + \gamma\eta
\end{aligned}
$$

where X* is given by history and we can again use a transversality condition to pin down $\lambda_2(t^*)$. Using $\lim_{t\to\infty}[\exp[-\rho t]\lambda_2(t)X(t)] = 0$ and manipulating yields:

$$\lambda_2(t) = \lambda_2(t^*)\exp[[1-\gamma]\eta(t-t^*)], \quad \lambda_2(t^*) = -BX^{*\gamma-1}/[\rho+\gamma\eta] \qquad (1.29)$$

From (1.29) it is apparent that the shadow cost of pollution falls at rate $\eta(\gamma-1)$, which in the fast growth case, is less than g, the rate the shadow value of capital falls. This implies we have abatement at its maximum throughout – pollution emissions are effectively banned – and MAC < MD(K(t), X(t)) for t > t* as assumed.

We can now employ (1.27) and (1.29) in (1.24) and set t = t* to find a closed form expression for the Switching Locus. Manipulating to obtain familiar terms we find:

$$MAC = \frac{1}{\alpha} = \frac{B}{[\rho+\gamma\eta]} X^{*\gamma-1}[\eta K^*]^{\varepsilon} = MD(K^*, X^*) \qquad (1.30)$$

which is a downward sloping and convex relationship between the pollution and capital stocks. The left hand side of (1.30) represents marginal abatement costs. The right hand side is marginal damage evaluated at {K*,X*}.

It is worth pausing for a moment to examine the determinants of marginal damage. Marginal damage is increasing in the pollution stock provided γ exceeds one, and is independent of the pollution stock otherwise. Marginal damage is also a function of contemporaneous income since the flow of national (and per-capita) income Y is always proportional to K. Given this interpretation it is apparent that the income elasticity of marginal damage with respect to flow income is given by $\varepsilon > 0$.

The discussion thus far is incomplete because it has implicitly assumed two key points yet to be proven. The first is simply that a country starting in a situation below the Switching Locus – such as the natural initial conditions $K(t=0) > 0$ and $X(t=0) = 0$ – will indeed reach the Switching Locus and make the transition to Stage II. Proposition 1 indicates a balanced growth path with zero abatement is not possible, but this does not imply every country must cross the Switching Locus in finite time. The second is that the locus does indeed represent an irrevocable change so that once we cross over to Stage II we remain there forever. These points are possible to establish and hence we record:

Proposition 3. Assume $g > 0$ and $g > \eta(\gamma-1)$, then:

i) every economy starting in Stage I must enter Stage II in finite time

ii) every economy in Stage II remains in Stage II.

iii) The locus of {K, X} at which Stage I ends is given by (1.30)

Proof: See appendix

It is possible to extend this result to flow pollutants. To remain in the fast growth case with $\eta(\gamma-1) < g$ we now assume $\gamma=1$ and find:

Proposition 4 Assume $g > 0$ and $\gamma = 1$, then:

iv) every economy starting in Stage I must enter Stage II in finite time

v) every economy in Stage II remains in Stage II.

vi) The Switching Locus is vertical at a finite $K^* > 0$.

Proof: See appendix

Proposition 3 and 4 prove that all economies follow the stage I – stage II life cycle.[23] We have assumed g > 0 and the standard sufficient condition required in models of endogenous growth given here as h > 0 in (1.27). Since we have now proven that all economies make the transition from inactive to active abatement the model's predicted time profile for emissions, emissions to output and pollution abatement costs roughly fit US historical experience. The model fits US experience because technological progress in goods and abatement technologies bound the marginal product of capital from below allowing for continual growth. Progress in abatement makes lowering emission to output ratios possible without increasingly large costs, and hence every economy capable of growth will also generate the time profile for pollutants given in Figures 3 to 8. Moreover, if our explanation is correct it represents a significant example of how induced technological progress can lower control costs and thereby not choke off growth.[24]

It is clear however that even within US experience there is variation. For example, our figures reveal that particulate pollution hit its maximum much earlier than others and nitrous oxides appear to be still rising. What are the potential explanations for

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[23] The previous analysis was undertaken under the fast-growth-slow-regeneration assumptions because this case illustrates the various forces at work very clearly. Since many of the same conclusions hold when growth is relatively slow we only provide a sketch here of some differences, leaving a complete analysis to the interested reader. We can again solve for a Switching Locus which divides Stage I from Stage II. If we let W(K,X) represent the state valuation function the new Switching Locus is again defined by (1.24). The interpretation given to the Switching Locus remains the same. The most important difference is that once a trajectory of the system hits this new Switching Locus it remains within it forever. This implies the economy's choice of abatement remains interior; i.e. the trajectory follows along the Switching Locus maintaining MAC = MD(K,X) throughout. As time progresses the intensity of abatement approaches the maximal level in the limit. Solving for the Switching Locus involves similar steps and is left to the appendix. The Switching Locus again defines a unique X* that is declining in K*. If the economy is initially below the locus then abatement is inactive and K rises at a rapid rate. X increases rapidly until the Switching Locus is reached in finite time. If the initial (K,X) is above the locus, maximal abatement is undertaken but this drives down the shadow cost of pollution very quickly and we again hit the locus, this time, from above. Once on the locus, countries remain trapped within it thereafter. See Figure 10.

[24] See for example, the work of Goulder and Mathai (2000) for an examination of how induced technological progress affects optimal CO2 policy, and the review of Jaffe et al. (2000).

these facts? In addition, there is good data on cross-country aggregate pollution concentrations and emissions that could be fruitfully explored, what are the model's cross-country predictions?

4.2 The Environmental Catch-up Hypothesis

We have now established that the transitions paths for initially Poor and Rich are as shown in Figure 9. As shown the Poor country experiences the greatest environmental degradation at its peak, and at *any* given capital stock, (i.e. income level) the initially Poor country has worse environmental quality than the Rich. Moreover, since both Rich and Poor economies start with pristine environments, the qualities of their environments at first diverge and then converge. This is our Environmental Catch-up Hypothesis.

It is relatively easy to see why divergence occurs. Divergence occurs because the opportunity cost of abatement (and consumption) is much higher in capital poor countries. A high shadow price of capital leads to less consumption, more investment and rapid industrialization in the Poor country. Nature's ability to regenerate is overwhelmed. The quality of the environment falls precipitously. In capital rich countries the opportunity cost of capital is lower: consumption is greater and investment less. Industrialization is less rapid and natural regeneration has time to work. The peak level of environmental degradation in the Rich country is therefore much smaller.

Convergence is driven by the fact that countries have a common rate of decline in marginal abatement costs along their balanced growth paths. This in turn reflects a common rate of decline in the opportunity cost of abatement inputs (the shadow value of capital) and a common marginal productivity in abatement. In our formulation, the marginal products of both capital and abatement are fixed by technology and this lends itself to the fulfillment of these conditions. But marginal products are constant along balanced growth paths in far more general circumstances, and hence the convergence tendency we have identified is more general than our framework suggests.[25]

4.3 The ECH and the EKC

A further implication of our analysis is that countries make the transition to active abatement at different income *and* peak pollution levels. This of course throws into question empirical methods seeking to estimate a unique income-pollution path. More constructively it suggests that an important feature of the data may well be a large variance in environmental quality at middle-incomes with little variance at either low or high incomes.[26]

In some cases however, (initially) Poor and Rich will make the transition at the same income level but still exhibit our Catch-up Hypothesis. Let γ approach one. Then the slope of the Switching Locus approaches infinity and all countries attain their peak pollution levels at the same K*. From (1.30), at γ equal to one, we have:

$$K^* = \frac{1}{[g(1-1/e)+r]}\left[\frac{[r+h]}{aB}\right]^{1/e} \qquad (1.31)$$

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[25] For example, the marginal product of capital is constant in our AK set up but it is also constant along the balanced growth path in a typical Solow model with ongoing technological progress. The marginal product of abatement is constant in our set up, but this could be constant along a balanced growth path in more general models if technological progress in abatement offsets diminishing returns to lowering emissions per unit of output.

[26] Empirical work by Carson et al. (1997) relating air toxics to U.S. state income levels is supportive of this conjecture. They state "Without exception, the high-income states have low per-capita emissions while emissions in the lower-income states are highly variable. We believe that this may be the most interesting feature of the data to explore in future work. It suggests that it may be difficult to predict emission levels for countries just starting to enter the phase, where per capita emissions are decreasing with income", p. 447-8.

Even with a common turning point differences in environmental quality remain. Moreover, these are not simple level differences because countries initially diverge and then converge after crossing K*.

To eliminate our Catch-up Hypothesis entirely we must assume regeneration is infinitely fast: X is a flow. The Switching Locus is now vertical at K* and given by:[27]

$$K^* = \frac{1}{[g(1-1/e)+r]}\left[\frac{1}{aB}\right]^{1/e} \tag{1.32}$$

Since pollution is proportional to production before K*, and policies are identical after K*: initial conditions no longer matter.

These results tell us that when pollution is strictly a flow, all countries share the same income pollution path regardless of initial conditions. We have generated an EKC, and empirical methods used to estimate a unique income-pollution path are appropriate. But when pollution does not dissipate instantaneously, initial conditions matter. We have the Environmental Catch-up Hypothesis, and empirical methods must now account for the persistent role of initial conditions.[28]

4.4 Across Pollutant Predictions

The ECH focuses on cross-country comparisons in pollution levels, but says little about how predictions vary with pollutant characteristics. There is however good data in

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[27] The turning point in the flow case is related to that of the stock in a transparent manner. Recall from (1.29) that the shadow cost of pollution is equal to the perpetuity value of the constant marginal disutility of – B (when γ=1). The perpetuity value arises since any unit of pollution has a constant instantaneous cost of –B but this cost is discounted by time preference, ρ, and eliminated by natural regeneration at rate η from the present time until infinity. Therefore, the perpetuity value of any unit of pollution is simply $-B/[\rho+\eta]$. Once pollution is a flow, it is eliminated immediately from the environment and has only a current flow cost of –B. Not surprisingly then (1.32) only differs from (1.31) by the absence of the perpetuity term.

[28] This result may explain why empirical research investigating the EKC has been far more successful with air pollutants like SO2, than with water pollutants or other long lasting stocks (see the review by Barbier (2000)).

the U.S. and elsewhere that could be fruitfully employed to test within country but across pollutant predictions. We have already examined how the rate of regeneration determines the intensity of abatement and the optimality of an emission ban, we now ask how characteristics of technology and pollution affect the timing of regulation.

Consider the impact of a level change in the productivity of goods production by raising A in (1.3). From (1.11) we know the rate at which the shadow price of capital falls during the active abatement stage is now higher when A is higher. But from (1.27) a change in goods productivity A also affects the level of the shadow price of capital at the start of Stage II when $t = t^*$. If $\varepsilon > 1$, then the shadow price of capital, for any K^*, is lowered with an increase in A; it is raised otherwise. Therefore, the income elasticity of marginal damage plays an important role in determining when a shift to active abatement begins.

To illustrate its role consider the fast growth regime and let the gross marginal product of capital, A, rise. This necessarily raises g and if $\varepsilon > 1$, the Switching Locus shifts in. Abatement is hastened. The same result holds with constant marginal damage, (1.31), and with a pure flow pollutant (1.32). When $\varepsilon < 1$, abatement is delayed and peak pollution levels shift right.[29]

A similar set of results holds for increases in the productivity of abatement although there is an additional conflicting force. From (1.31) it is clear that an increase in abatement productivity, holding g constant, necessarily lowers K^* and hastens abatement. This is direct impact of higher productivity on marginal abatement costs. But to this we

[29] Our use of the terms delayed or hastened does not refer to calendar time, but rather to whether actions occur at a higher or lower income level.

must add the income response created by the resulting change in g. When the income elasticity of marginal damage is greater than one both forces work to hasten abatement. When it is less than one, the result appears ambiguous.

Therefore we have found the income elasticity of marginal damage has an important role to play in determining *the income level* at which abatement occurs and therefore *pollution levels*, it plays virtually no role in determining if abatement will ever occur nor how fast the environment improves.

Next, consider the role of regeneration. Start from a position where $\eta = 0$ (radioactive waste). Then it is immediate from (1.30) that the Switching Locus shifts outwards as we raise η. The response is to delay action and allow the environment to deteriorate further. Delay is obvious when the marginal disutility of pollution is constant, because (1.31) shows K* necessarily rises with η. Once we raise η sufficiently the economy enters the fast regeneration regime and here we find abatement delayed in another manner – it is introduced slowly by the now gradual intensification of abatement efforts. Faster regeneration then implies that countries either begin abatement at higher income levels or allow their environments to deteriorate more before taking action. And hence surprisingly, fast regeneration will be associated with lower and not higher environmental quality - at least over some periods of time or ranges of income.[30]

[30] An especially colorful example of delay in abatement caused by rapid natural regeneration is that of the City of Victoria in British Columbia. Every day, the Victoria Capital Regional District (CRD) dumps approximately 100,000 cubic meters of sewage into the Juan de Fuca Strait. Scientific studies have long argued that since the sewage is pumped through long outfalls into cold, deep, fast moving water there is no need for treatment. The CRD has always used these studies to delay building a treatment facility. Current plans are for secondary treatment to begin in 2020, but until then over 40 square kilometers of shoreline remains closed to shell fishing. Background information can be found at the Sierra Legal Defense fund site http://www.sierralegal.org/m_archive/1998-9/bk99_02_04.htm

Pollutants also differ in their toxicity. The marginal disutility of toxics could exceed those classified as irritants, and damages from toxics may rise more steeply with exposure. The first feature of toxics implies their abatement should come early. This is clear from (1.30)-(1.32) where increases in B hasten abatement. Surprisingly very convex marginal damages call for the gradual and not aggressive elimination of pollution. The logic is that any reduction in the concentration of toxics has a large impact on marginal damage. Therefore, only by lowering emissions slowly can we match a steeply declining value of marginal reductions with a falling opportunity cost of abatement. Therefore, although toxics may have large absolute negative impacts on welfare, this argues for their early, but not necessarily aggressive, abatement.

Finally putting these results together reconsider the time profiles for nitrogen, particulates, and lead. Particulates are relatively easy to abate and dissipate quite quickly. This is consistent with our predictions when $\varepsilon > 1$ and η is quite large. In contrast, nitrogen oxides are difficult to abate but are also not long lived. The first feature delays abatement, the second argues for their gradual elimination. Lead is also relatively easy to abate, but has long lasting environmental and health effects. The first feature calls for rapid action, the second for a complete ban on emissions.

5. Conclusion

The relationship between economic growth and the environment is not well understood: we have only a limited understanding of the basic science involved – be it physical or economic – and we have very little data. Because of these difficulties it is important to develop a series of relatively simple theoretical models to generate stark predictions testable with currently available data.

At present the intersection of *theory and data* is limited. Grossman and Krueger's finding of an Environmental Kuznets Curve has played a useful role in stimulating a search for theoretical explanations, but the productive give-and-take characteristic of growth theory more generally has been largely absent. Kaldor's stylized facts, the Solow residual, and recent empirical debates over convergence have all played a major role in the evolution of thought regarding growth. In the hope of fostering a similar interaction between theory and evidence in the growth and environment literature, we presented three stylized facts drawn from US historical experience and developed a simple growth model that roughly matched these facts. While matching stylized facts is a good start, we will need to explore other sources of variation in the pollution data to go much further. In recognition of this, we provided two new sets of predictions. One set of predictions link pollutant characteristics such as toxicity, permanence in the environment, and immediate disutility to the income-pollution path. These within-country but across-pollutant predictions could be tested using the relatively short panels of pollution data already available in the U.S. and some OECD countries. Another set introduced a novel Environmental Catch-up Hypothesis predicting initial divergence but eventual convergence in environmental quality across both Rich and Poor countries. This cross-country prediction could be examined using the relatively long spans of cross-country data available for some important pollutants such as sulfur dioxide. Together these predictions widen the scope for empirical research in this area considerably.

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7. Appendix

Dynamics Under Different Abatement regimes

When S > 0, maximal abatement occurs and the dynamics are given by:

$$\begin{aligned}
&S > 0\\
&q = q^M, q^M \equiv 1/a\\
&\dot{l}_1 = -g\,l_1,\ \ g \equiv A[1-q^M] - d - r\\
&\dot{K} = [g + r]K - C(l_1),\ K(0) = K_0,\ \ C(l_1) \equiv l_1^{-1/e}\\
&\dot{l}_2 = l_2[r + h] + BX^{g-1}\\
&\dot{X} = -hX,\ X(0) = X_0
\end{aligned} \qquad \text{(A.1)}$$

When S= 0 we have an interior solution for abatement, with the following dynamics:

$$\begin{aligned}
&S = 0\\
&q \in [0, q^M]\\
&\dot{l}_1 = -g\,l_1\\
&\dot{K} = [A[1-q^M] - d]K - C(l_1),\ K(0) = K_0,\ \ C(l_1) \equiv l_1^{-1/e}\\
&\dot{l}_2 = l_2[r + h] + BX^{g-1}\\
&\dot{X} = AK[1 - aq] - hX,\ X(0) = X_0
\end{aligned} \qquad \text{(A.2)}$$

With no abatement at all we must have S < 0 yielding:

$$\begin{aligned}
&S < 0\\
&q = 0\\
&\dot{l}_1 = -l_1[A - d - r] - l_2 A,\\
&\dot{K} = [A - d]K - C(l_1),\ K(0) = K_0,\ \ C(l_1) \equiv l_1^{-1/e}\\
&\dot{l}_2 = l_2[r + h] + BX^{g-1}\\
&\dot{X} = AK - hX,\ X(0) = X_0
\end{aligned} \qquad \text{(A.3)}$$

Proof of Proposition 1

Start with part ii) and assume η(γ-1)< g then given our analysis in the text all that remains to prove is that the first order condition dynamics of S > 0 and θ = θ^M remain in the S > 0 set forever. To start write out the state/co-state equations for S > 0 given in (A.1). In Section 4 of the text we solve these to obtain the switching locus and the shadow values. In doing so we implicitly assumed:

$$-a\, l_2(t) > l_1(t),\ for\ \forall t > t^S \qquad \text{(A.4)}$$

where t^s is the switching time. Solving (A.1) for the shadow value of capital we have:

$$l_1(t) = l_1(0)\exp\{-gt\},\ l_1(0) = [hK(0)]^{-e} \qquad \text{(A.5)}$$

where the initial value can be found by using the transversality condition (see section 4 in the text). Similarly solve for the shadow cost of pollution to find:

$$l_2(t) = \frac{-BX(0)^{g-1}\exp\{(1-g)ht\}}{r+gh} \qquad \text{(A.6)}$$

where again we have employed a transversality condition to pin down the initial value of the shadow price in terms of X(0). Hence using (A.5) and (A.6) we find that (A.4) requires:

$$\frac{BX(0)^{(g-1)}}{[r+gh][hK(0)]^{-e}} > \exp\{[-g-[1-g]h]t\} \qquad \text{(A.7)}$$

for all t > t^S. By definition (A.7) is an equality when t = t^s. Using the definition of t^s and multiplying both sides of (A.7), we obtain:

$$1 > \exp\{-[g-(g-1)h][t-t^s]\} \qquad \text{(A.8)}$$

which is true for t > t^s if and only if

$$h(g-1) < g \qquad \text{(A.9)}$$

This is of course what we have assumed and this parameter restriction defines the fast growth-slow regeneration scenario. Note that when (A.9) fails – so we are in the fast regeneration-slow growth situation – and initial conditions put us above the switching locus so that S > 0, then (A.8) also implies the S > 0 dynamics drive the system to the switching locus and S=0 in finite time. This ends the proof to part ii).

Consider part i). The text does not show the economy approaches a balanced growth path with capital, output and consumption growing. To do so solve (1.16) for θ and substitute into the capital accumulation equation to obtain:

$$\dot{K} = \left[g + r\right]K + [X(0)/a]\exp\{((g/(1-g))t\} - C(0)\exp\{(g/e)t\} \qquad \text{(A.10)}$$

where use has been made of the solution for $\lambda_1(t)$ and the solutions for C(t) and X(t) given by:

$$\begin{aligned} C(t) &= C(0)\exp\{(g/e)t\} \\ X(t) &= X(0)\exp\{(-g/(g-1)t\} \end{aligned} \qquad \text{(A.11)}$$

Now integrate (A.10) to find:

$$\begin{aligned} &K(t) = -\Pi_1 X(0)\exp\{((g/(1-g))t\} + \Pi_2 C(0)\exp\{(g/e)t\} + \Pi_3\exp\{(g+r)t\} \\ &\Pi_1 \equiv \left[\frac{(g-1)}{g\,g - r(1-g)}\right] > 0, \Pi_2 \equiv \left[\frac{e}{re - g(1-e)}\right] > 0 \end{aligned} \qquad \text{(A.12)}$$

Multiply (A.12) by exp{-ρt}λ_1(t) and invoke the transversality condition, that:

$$\lim_{t\to\infty} l_1(t)K(t)\exp\{-r\,t\} = 0 \qquad \text{(A.13)}$$

This in turn implies

$$0 = \lim_{t\to\infty}\begin{bmatrix} -\Pi_1 X(0)\,l_1(0)\exp\{(gg/(1-g) - r)t\} \\ +\Pi_2 C(0)\,l_1(0)\exp\{(g(1-e)/e - r)t\} + \Pi_3\exp\{gt\} \end{bmatrix} \qquad \text{(A.14)}$$

The limit of the term multiplying X(0) goes to zero as its exponent is negative. The limit of the term multiplying C(0) goes to zero as long as: $g(1-\varepsilon)/\varepsilon - \rho < 0$ which is a standard condition for the transversality condition to be met in AK models. Together they imply Π_3 must be zero in (A.12). We can rewrite this condition in a useful way to find:

$$K(t) = \exp\{(g/e)t\}\left[\Pi_1 X(0)\exp\{-e/(g-1)t\} + \Pi_2 C(0)\right] \tag{A.15}$$

First note the term in square brackets approaches $\Pi_2 C(0)$ as time goes to infinity. This implies the growth rate of capital approaches that of consumption, g_C, in the limit. As the growth rate of capital and consumption converge, the abatement intensity must be approaching its maximal level. Finally we must ensure the TVC for X(t) holds as well along this balanced growth path. Solving for $\lambda_2(t)$ we find:

$$l_2(t) = \exp\{(r+h)t\}\left[l_2(0) + \int_0^t BX(s)^{g-1}\exp\{-(r+h)s\}ds \right] \tag{A.16}$$

The TVC requires the limit of $X(t)\lambda_2(t)\exp\{-\rho t\}$ go to zero as t goes to infinity. The solution for X(s) is given in (A.11). Substituting this into (A.16) and integrating shows the TVC requires:

$$\lim_{t\to\infty} X(0)\exp\{(h - g/(g-1))t\}\left[l_2(0) + \frac{BX(0)^{g-1}}{g+r+h}\left[1-\exp\{-(g+h+r)t\right] \right] = 0 \tag{A.17}$$

The first term multiplying the brackets goes to plus infinity since the exponent is positive in the fast regeneration-slow growth scenario we are investigating. Therefore, we are going to need the initial value of pollution to be given by:

$$l_2(0) = -\frac{BX(0)^{g-1}}{g+r+h} \tag{A.18}$$

Finally we need to prove that a balanced growth path with no abatement is not possible. To do so, recall from the text that this requires a growth rate of pollution equal to that of capital. By solving for the shadow value of pollution we obtain:

$$l_2(t) = \exp\{(r + h)t\}\left[l_2(0) + BX(0)^{(g-1)} \int_0^t \exp\{[g_K(g-1) - (r+h)]s\} ds \right] \quad \text{(A.19)}$$

and invoking the transversality condition yields:

$$\lim_{t\to\infty}\begin{bmatrix} X(0)\exp\{(h+g_K)t\}\left[l_2(0) - \dfrac{BX(0)^{g-1}}{g_K(g-1) - r - h}\right] \\ +X(0)\exp\{(g_K g - r)t\}\left[\dfrac{BX(0)^{g-1}}{g_K(g-1) - r - h}\right] \end{bmatrix} = 0 \quad \text{(A.20)}$$

Hence a necessary condition for the TVC to be met is $g_K\gamma<\rho$ as this sends the second term in brackets to zero. An additional condition is that $\lambda_2(0)$ take on a specific value to make the first term in the brackets zero. This in turn, implies, since this shadow value must be negative, that $g_K(\gamma\text{-}1)<\rho+\eta$. Using this information we can now write out the solution for the shadow value completely as:

$$l_2(t) = \exp\{g_K(g-1)t\}\left[\frac{BX(0)^{g-1}}{g_K(g-1) - (r+h)}\right] < 0 \quad \text{(A.21)}$$

and now note that it rises with t. Therefore, if S < 0 at some t, it will be contradicted in finite time since the shadow values move in opposite directions – exponentially.

This completes the proof of Proposition 1.

Proof of Proposition 2

The problem in the flow case is given by:

$$
\begin{aligned}
&Maximize \int_0^{\infty}\left[\frac{C^{1-e}}{1-e}-\frac{BX^{g}}{g}\right]e^{-rt}\,dt \\
&s.t.\ K(0)=K_0,\ \ and\ \ q \le 1/a \\
&\dot{K}=AK(1-q)-dK-C \\
&X=AK[1-qa]
\end{aligned}
\qquad (A.22)
$$

Forming the Hamiltonian we obtain:

$$
rW(K)=\underset{\{C,q\}}{Max}\left\{H=\frac{C^{1-e}}{1-e}-\frac{B[AK[1-qa]]^{g}}{g}+l\,[AK[1-q]-C-dK]\right\} \qquad (A.23)
$$

with first order conditions,

$$
\frac{\partial H}{\partial C}=C^{-e}-l=0 \qquad (A.24)
$$

$$
\underset{q}{Max}\left[-\frac{B[AK[1-qa]]^{g}}{g}+l\,AK[1-q],\ s.t.q \le 1/a\right] \qquad (A.25)
$$

$$
\dot{l}-r\,l=-\frac{\partial H}{\partial K}=-\left[\,l\,[A[1-q]-d\,]-BX^{g-1}A[1-qa]\right] \qquad (A.26)
$$

Define the sets, S_-, S_0, S_+ as follows:

$$
\begin{aligned}
S_- &= \{(K,l)\,|\,G'(q=0)=aB(AK)^{g-1}-l<0 \\
S_0 &= \{(K,l)\,|\,G'(q=0)=aB(AK)^{g-1}-l=0 \\
S_+ &= \{(K,l)\,|\,G'(q=0)=aB(AK)^{g-1}-l>0
\end{aligned}
\qquad (A.27)
$$

Then note that when S_- is relevant, no abatement would occur at all. When S_0 is relevant abatement is just worthwhile. And when S_+ is relevant some abatement is necessary.

We start with the simple $\gamma = 1$ case. Assume $\{K,\lambda\}$ are in S_+. Then, under this parameter restriction, we obtain from the above that:

$$
if\ \ G'(q=0)=aB-l>0,\ then\ q=q^{M} \qquad (A.28)
$$

since marginal damage from pollution is a constant, abatement is either nothing, or it is set to the maximum. When $\theta = \theta^M$ we have:

$$\begin{aligned} \lambda(t) &= \lambda(0)\exp\{-\gamma t\} \\ K(t) &= \exp[(\gamma+\rho)t][K(0) - \lambda(0)^{-1/\varepsilon}\int_0^t \exp\{-[\gamma(1-1/\varepsilon)+\rho]s\}ds] \end{aligned} \qquad (A.29)$$

Using the TVC we find $\lambda(0)=[hK(0)]^{-\varepsilon}$ where $h=g(1-1/\varepsilon)+\rho > 0$ is again the standard sufficient condition for the TVC to hold. To show that ongoing growth is possible in this situation, note that $\lambda(t)$ is a strictly decreasing function of K(0). Therefore choosing K(0) large enough we obtain must be in S_+. Once in S_+, the dynamics in (A.29) indicate that λ falls. Hence the dynamics are forward invariant. Balanced growth in consumption, capital and output occur at rates:

$$g_Y = g_K = g_C = \gamma/\varepsilon = [A[1-\theta^M] - (\delta+\rho)]/\varepsilon > 0 \qquad (A.30)$$

Suppose instead that K(0) puts us in S_-. Then $\theta = 0$ from (A.27), and (A.29) holds if we replace $[1-\theta^M]$ with 1. Since the solution for λ shows it must fall exponentially over time approaching zero, there exists a finite time t at which S_+ is entered at which the dynamics shift to the case previously discussed. This ends the γ=1 proof.

Let γ not equal one, and assume we start in S_+. Then from (A.27) we have

$$\alpha B(AK[1-\theta\alpha])^{\gamma-1} - \lambda = 0 \qquad (A.31)$$

and $\theta=\theta^M$ is never optimal if λ is positive. Using $G'(\theta) = 0$, we can now write the state-co-state equations as:

$$\begin{aligned} \dot{\lambda} &= -\gamma\lambda, \\ \dot{K} &= (\gamma+\rho)K + D\lambda^{1/(\gamma-1)} - C(\lambda), D \equiv (1/\alpha)[1/\alpha B]^{1/(\gamma-1)} \end{aligned} \qquad (A.32)$$

Using the TVC we can now obtain a relationship between the initial shadow value of capital and the initial capital stock. Manipulation of the TVC yields an implicit relationship between these two variables:

$$\begin{aligned} &K(0) + D\,l\,(0)^{1/(g-1)} / a_1 = l\,(0)^{-1/e} / a_2, \\ &-a_1 \equiv -(g + r) - g/(g-1), a_2 \equiv h \end{aligned} \tag{A.33}$$

Straightforward differentiation shows that λ(0) is a decreasing function of K(0). Therefore, for K(0) large we must be in S_{+}. Once in S_{+}, the dynamics given above shows λ falls exponentially over time leaving the solution in S_{+}. Note from (A.32) that consumption grows at rate $g_C = g/\varepsilon > 0$; and from the solution for λ(t) that as t goes to infinity, θ goes to θ^M as required. The growth rates of capital and output approach those of the γ equal to one case. Finally, suppose our initial conditions put us in S_{-}. Then it is again straightforward to show λ falls exponentially over time until some finite time t, such that S_{+} is entered. This completes the proof for Proposition 2.

Proof of Proposition 3

Part ii) has already been proven in Proposition 1. Part iii) is proven in the text. We prove part i) by contradiction. Assume it is never optimal to leave Phase I. This requires we stay in S < 0 for all t, or that:

$$0 \le -a\,l_2(t) < l_1(t) \tag{A.34}$$

The difficulty in proving this is false arises from the fact that when S < 0 we cannot solve for either shadow value independently. Accordingly we adopt an indirect approach by bounding $\lambda_1(t)$ from above and bounding $\lambda_2(t)$ from below and then showing these bounds violate (A.34) generating the contradiction. Note since S < 0 is assumed throughout we must have an upper bound on $\lambda_1(t)$ given by:

$$l_1(t) < l_1(0)\exp\{-gt\} \tag{A.35}$$

To bound $\lambda_2(t)$ we start with its solution which is given by:

$$l_2(t) = \exp\{(r+h)t\}\left[l_2(0) + \int_0^t BX(s)^{g-1}\exp\{-(r+h)s\}ds \right] \tag{A.36}$$

Use the TVC to solve for the constant and find:

$$l_2(0) = -\int_0^\infty BX(s)^{g-1}\exp\{-(r+h)s\}ds \tag{A.37}$$

And hence the solution becomes:

$$l_2(t) = \exp\{(r+h)t\}\left[-\int_t^\infty BX(s)^{g-1}\exp\{-(r+h)s\}ds \right] \tag{A.38}$$

The solution for X(t) involves $\lambda_1(t)$ so we bound $\lambda_2(t)$ without solving for X(t) by noting that when S < 0:

$$\begin{aligned} &\dot{X} = AK - hX \geq -hX, \\ &\Rightarrow X(t) \geq X(0)\exp\{-ht\} \end{aligned} \tag{A.39}$$

Now substitute this solution into (A.38) since this bounds $\lambda_2(t)$ from below. That is, we now have constructed the fictitious $\lambda_2(t)^*$ where:

$$\begin{aligned} &-l_2(t) \geq -l_2(t)^* = \exp\{(r+h)t\}\left[\int_t^\infty BX(0)^{g-1}\exp\{-(r+hg)s\}ds \right] \\ &-l_2(t)^* = \frac{BX(0)^{g-1}}{hg+r}\exp\{-h(g-1)t\} \end{aligned} \tag{A.40}$$

Now employing the inequality in (A.34), substituting using (A.35) and (A.40), and manipulating slightly we see that staying in S < 0 requires that for all t

$$\frac{aBX(0)^{g-1}}{hg+r}\exp\{g-h(g-1)t\} < l_1(0) \tag{A.41}$$

which is false for t sufficiently large since g > $\eta(\gamma\text{-}1)$.

Proof of Proposition 4

The proof of Proposition 2 demonstrates that $\lambda(t)$ is monotonically falling during Stage II. This is shown by (A.29). Moreover when γ=1 we can find the unique capital stock at which Stage II begins. Note when Stage II begins we must have from (A.27) and γ=1, that λ = aB. By definition then, K* must satisfy λ=aB = $[hK^*]^{-\varepsilon}$ Therefore, once K* is reached the Stage II dynamics will keep it in stage II since $\lambda(t)$ falls monotonically. Since K* is uniquely determined, the Switching Locus is vertical. To show that every economy must enter Stage II in finite time locate the dK/dt = 0 locus during Stage I. This locus must have $\lambda_1(t) = [(A-\delta)K]^{-\varepsilon}$ where we have denoted the shadow value in Stage I by the subscript 1. In stage II the dK/dt = 0 locus is given by $\lambda_2(t) = [(A[1-\theta^M]-\delta)K]^{-\varepsilon}$ where 2 stands for Stage II. Construct a phase diagram in $\{\lambda,K\}$space and note while both loci are necessarily downward sloping the Stage II dK/dt = 0 locus lies everywhere above that of the Stage I dK/dt = 0 locus. Also draw in the horizontal line λ=aB. K* is defined by the intersection of this horizontal line and the Stage II dK/dt = 0 locus. Notice for any K(0) < K*, a $\lambda(0)$ can be found such that Stage I dynamics carries the point (K(0), $\lambda(0)$) in finite time T to the point (K*, λ^*). This is true because the steady state value of λ assuming we stay in Stage I, is less than aB when g > 0 which we have assumed. For t > T the Stage II dynamics apply as discussed above. To prove that this constructed candidate solution in (K,λ) space, that joins the two candidate Stage I and Stage II dynamics, is optimal, apply the sufficiency theory of Arrow and Kurz (1970, pp. 43-49). First the candidate solution satisfies the state/co-state equations by construction. The TVC is met in Stage II by construction of the Stage II solution of the state/costate equations. It is easy to see that the projection of the graph of Stage I in (K,λ) space covers the set $(0,K^*)$ on the K-axis as can be seen by the relative location of the switching locus $aB = \lambda$ and $d\lambda/dt = 0$ in Stage I and by the construction of Stage I by running the Stage I state/costate dynamics backwards in time from the point $(K^*, \lambda^*=aB)$. Thus we have a well-defined candidate solution for all positive initial K(0). Using the location of the locus $aB = \lambda$ it is easy to see that $\theta = 0$ in Stage I and $\theta = \theta^M$ in Stage II. Thus the control θ actually optimizes H. The control C optimizes H. Hence we are done by sufficiency theory.

Switching Locus in the Slow Regeneration Case

We have from (A.18) one relationship between the shadow value of pollution and the pollution level when we move to Stage II. Along the Switching locus we also have from the text that:

$$dX/dt + hX = AK[1-qa] = (g_X + h)X^*\exp\{g_X t\} \tag{A.42}$$

And the accumulation equation for capital gives us:

$$\begin{aligned} dK/dt &= AK[1-q] - C(l_1) \\ &= [g+r]K + \left[\frac{(g_X+h)X^*}{a}\right]\exp\{g_X t\} - l_1{}^{*-1/e}\exp\{(g/e)t\} \end{aligned} \tag{A.43}$$

Integrate (A.43) and use the transversality condition to obtain:

$$\left[\frac{l_1{}^{*-1/e}}{g(1-1/e)+r}\right] = K^* + \frac{(g_X+h)}{(g+r-g_X)(X^*/a)} \tag{A.44}$$

Along the Switching Locus we have λ_1=-aλ_2 and using this and (A.18) we obtain:

$$\left[\frac{aBX^{*g-1}}{g+r+h}\right]^{-1/e} / \left[g(1-1/e)+r\right] = K^* + \left[\frac{g_X + h}{(g+r-g_X)(X^*/a)}\right] \qquad \text{(A.45)}$$

Now recall g_X=g/(1-γ) < 0. Since η(γ-1) > g together they imply g_X + η > 0 and the right hand side of (A.45) is an increasing function of X*. The left hand side is a declining function of X* since h > 0. Therefore we obtain a unique X* > 0 for each K*>0. Rearranging (A.45) yields the more friendly form:

$$\frac{1}{a} = \left[\frac{BX^{*g-1}}{g+r+h}\right]\left[hK^* + \frac{h(g_X + h)}{(g+r-g_X)(X^*/a)}\right]^{e} \qquad \text{(A.46)}$$

The optimal solution may now be sketched. If (K,X) lies below the SL we put θ = 0 and the FOC is increasing in K at rapid rate. X increases rapidly until SL is reached in finite time. If (K,X) lies above SL we put θ = 1/a. The FOC values of K and C grow slowly. Since η(γ–1)>g, the shadow value aλ_2 is falling faster than the shadow value of capital on the FOC solution above SL. Therefore SL will be hit in finite time and which θ is adjusted to keep the FOC dynamics in SL.

Figure 1

Emission Intensities, 1940-1998
Tons of Emissions/Real GDP

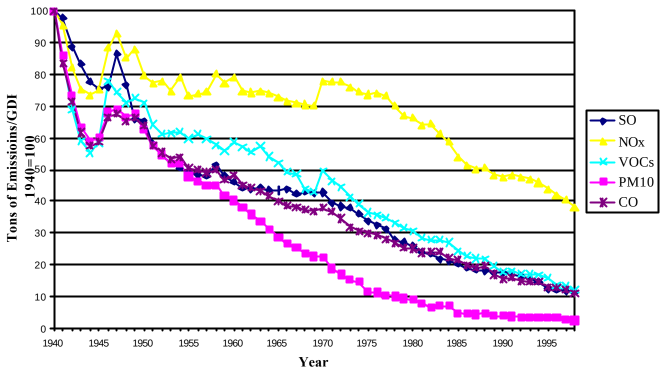


Figure 2

Pollution Abatement Costs, 1972-1994
PACE/GDP

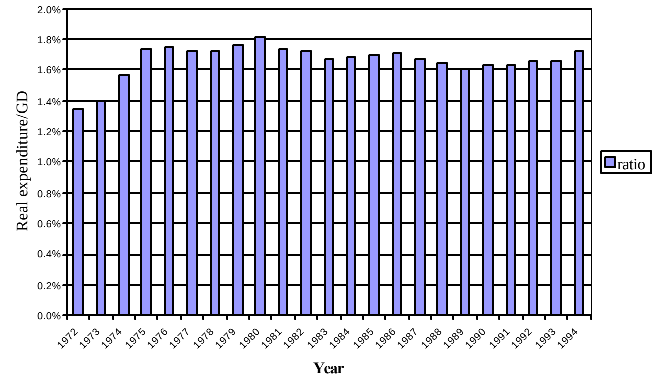


Figure 3

Sulfur Dioxide Emissions, 1940-1998

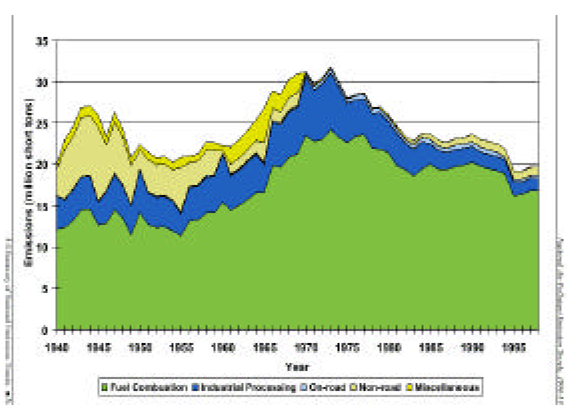


Figure 4

Nitrogen Oxide Emissions, 1940-1998

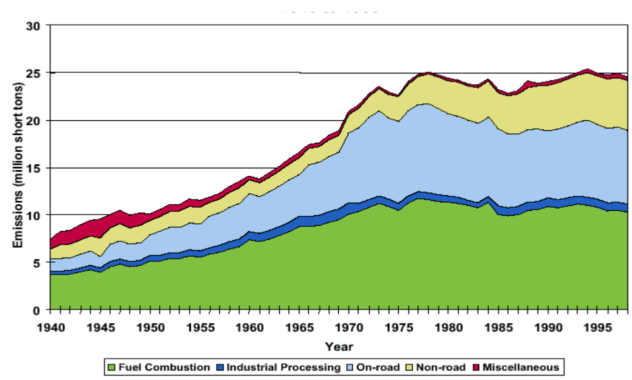


Figure 5

Carbon Monoxide Emissions, 1940-1998

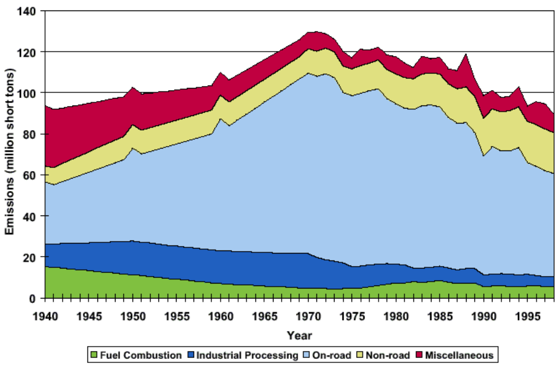


Figure 6

Volatile Organic Compounds 1940-1998

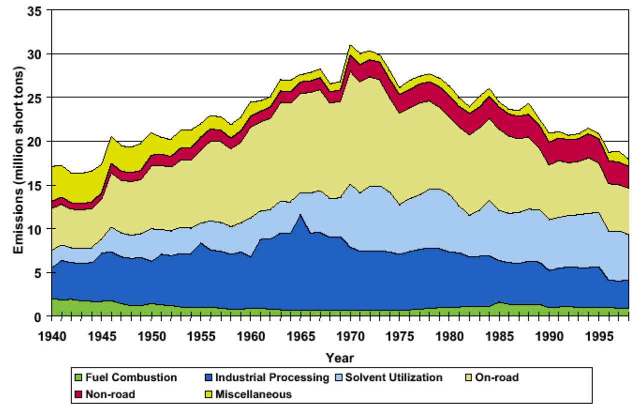


Figure 7

Particulate Matter PM10, 1940-1998

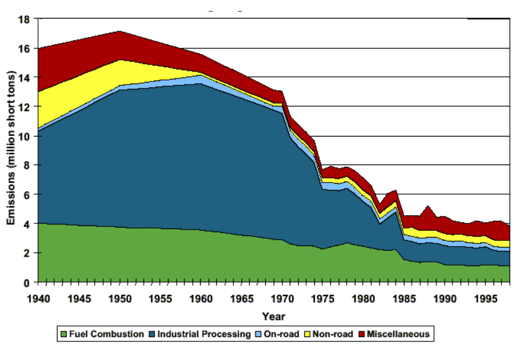
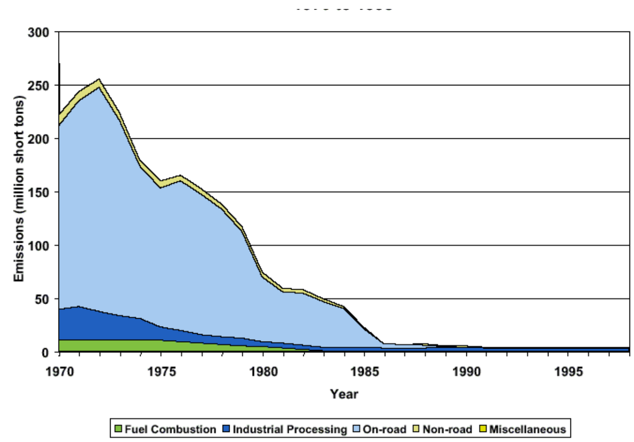


Figure 8

Lead Emissions 1970-1998



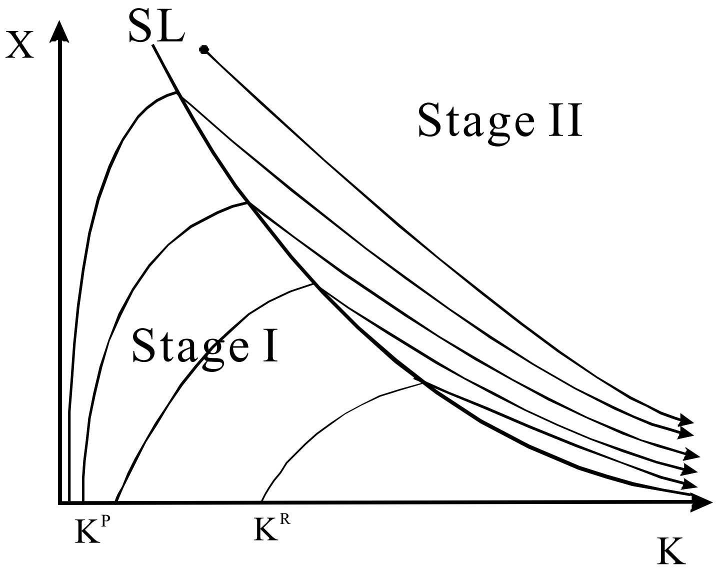


Figure 9
Slow Regeneration

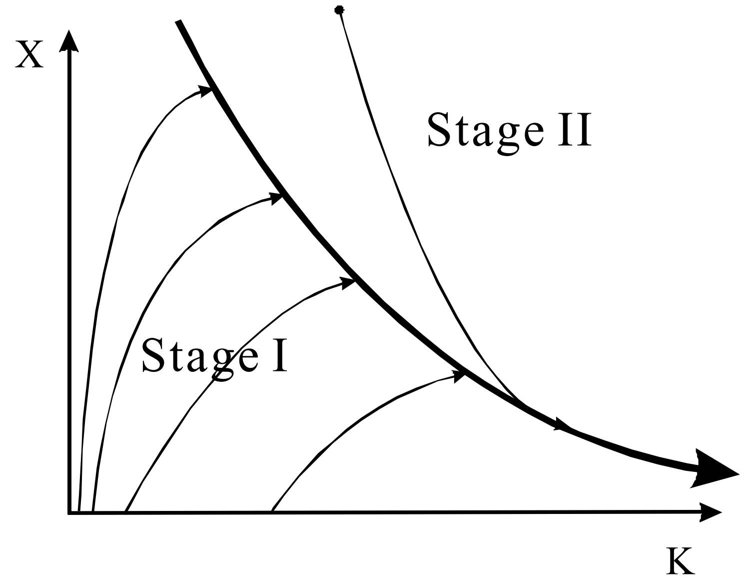


Figure 10
Fast Regeneration