Benefit-Cost in a Benevolent Society

By THEODORE C. BERGSTROM*

Alice and Bob live together. Though they are fond of each other, they maintain separate budgets. They have been offered a chance to rent a larger apartment. It has two extra rooms; a study for Alice and a lounge for Bob. Alice would be willing to pay \$100 a month for the study and Bob would be willing to pay \$100 a month for the lounge. Alice would never use the lounge and Bob would never use the study, but each likes the other to be happy. For this reason, Alice is willing to pay \$50 a month for Bob to have the lounge and Bob is willing to pay \$50 a month for Alice to have the study. The additional rent for the larger apartment is \$250 per month. Should they accept the offer on the grounds that total benefits from the larger apartment are \$300, or reject it on the grounds that total benefits are only \$200?

To pose the question more generally, how should benefit-cost analysis account for the value that benevolent individuals place on other people's pleasure from public goods? When adding up benefits to be compared with costs, should we sum the private valuations, the altruistic valuations, or something else?

W. Kip Viscusi et al. (1998) proposed that the benefits from improvements in public health "consist of two components, the private valuation consumers attach to their own health, plus the altruistic valuation that other members of society place on their health." They conducted a survey, ingeniously designed to isolate these two components. They asked their subjects to state their willingness to pay for a hypothetical product that would reduce their own personal risks of contracting a carefully described illness. In a separate question, they asked subjects for their willingness to pay for an advertising campaign that would result in an equivalent reduction in risk for all members of a larger population. The authors point out that even a slight concern for the well-being of each member of a large population could amount to a substantial total willingness to pay for benefits to others. In a sample of citizens of Greensboro, North Carolina, Viscusi et al. found that, on average, subjects were willing to pay about five times as much to reduce a specific hazard for all North Carolinians as to reduce this hazard for themselves alone. For a similar benefit to all U.S. citizens, subjects would be willing to pay about six times as much as for themselves only. Even if these hypothetical claims of altruism are overstated, the magnitude of the altruistic component of public benefits appears to be significant. Thus, the question of how to treat altruistic valuations in benefit-cost analysis is a matter of the first order of importance.

Before attempting a general answer, let us try to resolve the dilemma of Alice and Bob. Suppose they decide to take the new apartment and split the rent equally. If they do this, then considering only her self-interest, Alice will be worse off. She is giving up \$125 in return for a study that she values at only \$100. Perhaps she will be compensated by an improvement in Bob's well-being? But Bob is now paying \$125 for a lounge that he values at \$100. It does not seem reasonable that two people who care about each other could both benefit from an outcome that makes each of them privately worse off.

The fable of Alice and Bob suggests a general principle. If we are to count the sympathetic *gains* each obtains from the other's enjoyment of the shared public good, then we should not forget also to count sympathetic *losses* each bears from the share of its cost paid by the other. To deepen our understanding of this principle, we need an explicit model of interpersonal benevolence.

We begin with a simple utility model that is consistent with the story of Alice and Bob. Alice and Bob have *private utility functions*

$$v_A(x_A, y) = x_A + 100y \tag{1}$$

$$v_B(x_B, y) = x_B + 100y \tag{2}$$

where x_A and x_B are the amounts of money that

* Department of Economics, University of California, Santa Barbara, 2127 North Hall, Santa Barbara, CA 93106 (e-mail: tedb@econ.ucsb.edu).

each spends on private consumption and where y is either 0 or 1 depending on whether they take the new apartment. Alice and Bob's affections take the form of *social utility functions*

$$(3)\qquad U_A(v_A, v_B) = v_A + v_B/2 = x_A + x_B/2 + 150y$$

$$(4)\qquad U_B(v_A, v_B) = v_B + v_A/2 = x_A/2 + x_B + 150y.$$

Equation (1) implies that Alice values her private benefits from the new apartment at \$100. Equation (3) implies that when she accounts for Bob's benefits from the apartment, she would be willing to pay \$150. Similarly, Equations (2) and (4) imply that Bob would be willing to pay just \$100 for his private benefits, but accounting for Alice's benefits, he would be willing to pay \$150.

Assume that Alice and Bob have incomes w_A and w_B, which they can spend on their own consumptions if they do not take the new apartment. In this case, their private utilities will be $v_A = w_A$ and $v_B = w_B$, and their social utilities will be $U_A = w_A + w_B/2$ and $U_B = w_B + w_A/2$. We can show the alternatives available to Alice and Bob with a *private utility possibility diagram,* in which private utilities, v_A and v_B, are registered on the axes.

In Figure 1, we draw indifference curves reflecting the social preferences of Alice and Bob. Since $U_A(v_A, v_B) = v_A + v_B/2$, the set of all points (v_A, v_B) that Alice likes as well as the allocation $(v_A, v_B) = (w_A, w_B)$ is the line AA' through this point with slope -2. Similar reasoning shows that Bob's indifference curve through this point is the line BB' with slope $-\frac{1}{2}$. Alice and Bob can both increase their social utilities by taking the apartment if and only if they can achieve a distribution of private utilities that lies above the two lines AA' and BB'.

Let additional rent for the new apartment be c and suppose that Alice and Bob divide the costs, with Alice paying c_A and Bob paying $c - c_A$. Their private utilities will then be $v_A = w_A - c_A + 100$ and $v_B = w_B - (c - c_A) + 100$. Adding these expressions, we see that if they take the apartment, possible distributions of private utilities lie on the line $v_A + v_B = w_A + w_B + 200 - c$. Where $c = \$250$, as in the story of Alice and Bob, these possible distributions are represented by the line segment PP' in Figure 1.[1] This line passes below the point (w_A, w_B) and does not intersect the area above AA' and BB'. Therefore, there is no way that Alice and Bob can both be made better off if they rent the new apartment at \$250.

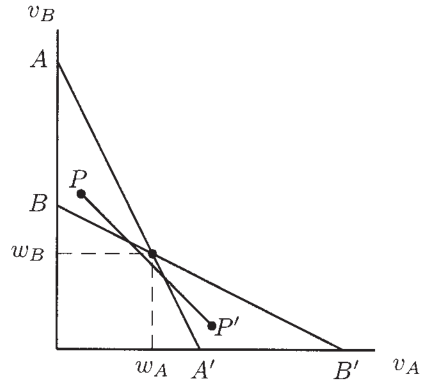

FIGURE 1. PRIVATE UTILITY POSSIBILITIES FOR ALICE AND BOB

Since all points (v_A, v_B) on PP' satisfy the equation $v_A + v_B = w_A + w_B + 200 - c$, it must be that PP' extends into the region above AA' and BB' if and only if $c < 200$. Therefore, a necessary and sufficient condition for the new apartment to be Pareto improving in terms of their social utilities is that the sum of their *private values* for this apartment exceeds the extra rental cost. The remainder of this paper explores the generality of this conclusion.

I. Benevolence and Benefit-Cost

A. *Nonmalevolent and Benevolent Preferences*

An informal description of benevolent preferences is that Alice agrees with Bob about what is good for Bob, and vice versa. Our formal definition applies to an economy that has n people, one private good and m public goods. Let x_i be the quantity of the private good con-

[1] The diagram is drawn to scale for $w_A = w_B = 400$. The endpoints P and P' correspond to the extreme allocations where all of the private goods remaining after rent is paid go to one or the other individual.

sumed by person i and let y be an m-vector of public goods. An allocation $(x, y) = (x_1, \dots, x_n, y)$ lists the private consumption of each i and the vector of public goods. Private preferences of consumer i are represented by a *private utility function* $v_i(x_i, y)$ defined on i's own consumption of private goods and the vector of public goods.

DEFINITION 1 (Nonmalevolent preferences): *Person i has nonmalevolent preferences if i's preferences over allocations can be represented by a "social utility function"*

$$(5) \quad U_i(x, y) = U_i(v_1(x_1, y), \dots, v_n(x_n, y))$$

where U_i is an increasing function of v_i and a nondecreasing function of v_j for all $j \neq i$. We say that i is selfish if U_i is constant with respect to all $j \neq i$, and we say that i is benevolent if i is nonmalevolent and not selfish.

Benevolent preferences were defined in this way by Serge-Christophe Kolm (1969), Sidney G. Winter (1969), and Bergstrom (1970). G. C. Archibald and David Donaldson (1976) refer to preferences representable in the form of equation (5) as nonpaternalistic. According to Kolm (1969, 2000), this model can be traced to Wilfredo Pareto (1913) who used the word "ophelimity" for the measurement of private utility and "utility" for that of social utility.[2]

We consider nonmalevolent preferences that satisfy the following technical assumptions.

ASSUMPTION 1 (Social and private utility functions): *Each consumer i has nonmalevolent preferences, with a social utility function $U_i(v_1, \dots, v_n)$ that is continuous and nondecreasing in all v_j and increasing in v_i. The private utility functions $v_i(x_i, y)$ are continuous and strictly increasing in x_i.*

B. *Technology, Utility Possibilities, and Private Values*

We assume that the technology allows redistribution of private goods in any way that does not change their total, that private goods are freely disposable, and that there is a well-defined cost function $C(y)$ for public goods such that:

ASSUMPTION 2 (Public goods technology): *The set of all feasible allocations is*

$$\{(x_1, \cdots x_n, y) | \textstyle\sum x_i + C(y) \leq W \text{ and } x_i \geq 0 \text{ for all } i\}$$

where $C(y)$ is continuous and $W > 0$ is the total resource endowment.

We define nonwasteful allocations in which no private goods are discarded.

DEFINITION 2 (Nonwasteful allocations): *A feasible allocation is nonwasteful if $\sum x_i + C(y) = W$.*

Each feasible allocation $(x_1, \dots, x_n, y)$ determines a feasible distribution of private utilities, $(v_1(x_1, y), \dots, v_n(x_n, y))$. Thus we can define private utility possibility sets and utility possibility frontiers, contingent on the quantity of public goods.

DEFINITION 3 (Contingent private utility possibility set): *The y-contingent private utility possibility set is the set of all feasible distributions of private utilities in which the vector of public goods is y. The "upper boundary" of this set, which is generated by nonwasteful allocations, is called the y-contingent private utility possibility frontier.*

Let us define a consumer's private value for a change in the amount of public goods to be the amount of private goods that she would have to give up in order to have exactly the same *private utility* after the change as before.[3] In order

[2] Kolm (1969) suggested that social utility functions of the form in equation (5) can be derived from a more fundamental theory in which the social utility of each i depends on i's private utility $v_i(x_i, y)$ and on the social utilities U_j of other consumers, $j \neq i$. Bergstrom (1989, 1999) establishes conditions under which such a derivation is possible.

[3] This definition is essentially the same as John R. Hicks's (1942) definition of *compensating variation,* which he defined as the amount one would have to deduct from a

for private values to be conveniently defined, we assume that every possible change in the amount of public goods can be compensated by *some* change in a consumer's private consumption.

ASSUMPTION 3 (Compensability): *Where the initial allocation is $(x_1, \ldots, x_n, y)$, for every possible vector y' of public goods and for all consumers i, there exists $x_i' \geq 0$ such that $v_i(x_i', y') = v_i(x_i, y)$.*

We define private values for a change in public goods.

DEFINITION 4 (Private value of changes): *If the initial allocation is $(x_1, \ldots, x_n, y)$, then i's private value for a change in the vector of public goods from y to y' is the unique quantity w_i such that $v_i(x_i - w_i, y') = v_i(x_i, y)$.*

C. *Potential Pareto Improvements*

Without explicit instructions about how to compare one person's benefits with the losses of another, we cannot expect benefit-cost analysis to determine whether a public project should or should not be adopted. The best we can hope for is to learn whether a project is *potentially* Pareto improving, in the sense that there is some way to divide its costs so that the resulting allocation is Pareto superior to the initial allocation. We define two distinct notions of potential Pareto improvement. One of these measures Pareto improvement in terms of the private utility functions v_i, and the other measures Pareto improvement in terms of the social utility functions U_i.

DEFINITION 5 (Potentially privately Pareto improving): *With initial allocation (x, y), a change in the vector of public goods from y to y' is potentially privately Pareto improving if there exists a feasible allocation (x', y') such that $v_i(x_i', y') \geq v_i(x_i, y)$ for all i, with at least one strict inequality.*

Similarly, a change in the vector of public goods is *potentially Pareto improving* if costs can be distributed so as to achieve an allocation that is Pareto improving in terms of the *social* utility functions U_i.

person's income to make him just as well off after a change in prices as in the initial situation (see Hicks, 1942; John S. Chipman and James C. Moore, 1980).

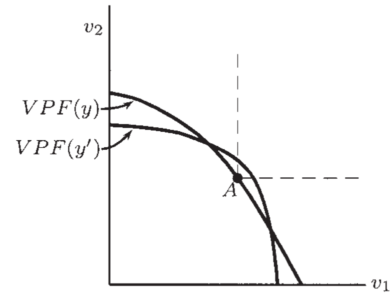

FIGURE 2. A POTENTIALLY PRIVATELY PARETO IMPROVING CHANGE

DEFINITION 6 (Potentially Pareto improving): *With initial allocation (x, y), a change in the vector of public goods from y to y' is potentially Pareto improving if there exists a feasible allocation (x', y') such that*

$$U_i(v_1(x_1', y_1'), \ldots, v_n(x_n', y_n')) \geq U_i(v_1(x_1, y_1), \ldots, v_n(x_n, y_n))$$

for all i, with at least one strict inequality.

Since with nonmalevolent preferences, the social utility U_i of each i is increasing in i's own private utility v_i and nondecreasing in the private utilities of all other consumers, a Pareto increase in private utilities implies a Pareto increase in the social utilities. Therefore, we can conclude that:

REMARK 1: *If preferences are nonmalevolent and if an allocation (x', y') is privately Pareto improving over allocation (x, y), then (x', y') is also Pareto improving in terms of the social utility functions, U_i.*

Figure 2 represents private utility possibilities for two consumers. The *y-contingent private utility possibility frontier* is the curve labelled $VPF(y)$, and the corresponding utility possibility set contains all points on or below $VPF(y)$. Point A shows the private utilities of the *initial distribution,* $(v_1(x_1, y), v_2(x_2, y))$. The curve $VPF(y')$ is the y'-contingent private utility possibility frontier, and points on or below it constitute the corresponding utility possibility set. In Figure 2, neither of the two utility possibility sets is contained in the other. However, $VPF(y')$ includes points that lie above and to the right of the initial distribution A. Thus, it is possible to increase the supply of public goods from y to y' and to pay for it in such a way that private utilities for both consumers are increased. This means that the change from y to y' is potentially privately Pareto improving.

Figure 3 demonstrates that the converse of Remark 1 is not true. The initial allocation is at A and the private utility possibility frontier corresponding to the initial quantity of public goods is $VPF(y)$. The diagram has indifference curves reflecting the two consumers' *social* utility functions. The indifference curve U^1U^1 passes through A and includes combinations of v_1 and v_2 that give consumer 1 the same social utility as A. Similarly, U^2U^2 is consumer 2's social indifference curve passing through A. The set of points that are Pareto superior to A includes all points that lie above and to the right of both lines, U^1U^1 and U^2U^2. The curve $VPF(y')$ is the utility possibility frontier corresponding to the vector y' of public goods. The curve $VPF(y')$ does not extend into the region V^+ above and to the right of A. Therefore the change from y to y' is not potentially privately Pareto improving. But this curve does extend into the Pareto improving region above the curve U^1AU^2, which implies that the change from y to y' is potentially Pareto improving.

When preferences are benevolent, it is also possible that a Pareto improvement can be achieved from some initial allocations, simply by redistributing private goods. We define an allocation to be *distributionally efficient* if there is no way to achieve a Pareto improvement by redistribution of private goods with no change in the amounts of public goods.

DEFINITION 7 (Distributionally efficient): *A feasible allocation* (x, y) *is distributionally efficient if there is no feasible allocation* (x', y) *that is Pareto superior to* (x, y).

The allocation (x_1, x_2, y) that produces the private utility distribution A in Figure 3 is seen to be distributionally efficient, since the curve $VPF(y)$ does not extend into the Pareto improving area above the broken curve U^1AU^2.

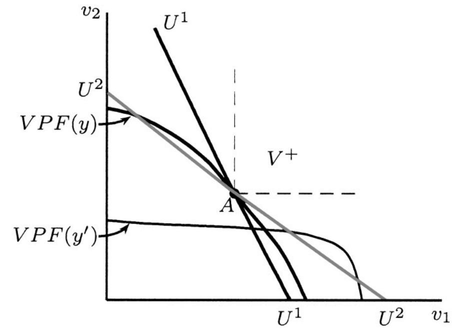

FIGURE 3. POTENTIALLY PARETO IMPROVING, BUT NOT PRIVATELY

If the initial allocation is not distributionally efficient, the criterion of potential Pareto improvement is not useful for deciding whether a change in the amount of public goods is justified. Where Pareto gains can be accomplished simply from distribution of private goods, some changes in public goods would satisfy the definition of potential Pareto improvement, even though these changes are wasteful in the sense that a better outcome for all consumers could be achieved by redistribution without the change in public goods.

D. *A Private Values Benefit-Cost Test*

A simple benefit-cost test determines whether a change is potentially privately Pareto improving.

DEFINITION 8 (Private values benefit-cost test): *A change in the amount of public goods from* y *to* y' *passes the private values benefit-cost test if the sum of all consumers' private values for this change exceeds the cost of this change,* $C(y') - C(y)$.

A proof of Lemma 1 is found in the Appendix.

LEMMA 1: *If the initial allocation is* (x, y), *then a change in the amount of public goods from* y *to* y' *is potentially privately Pareto improving if and only if this change passes the private values benefit-cost test.*

From Remark 1 and Lemma 1 it is immediate that the private value benefit-cost test is a sufficient condition for a change in the amount of public goods to be potentially Pareto improving.

THEOREM 1: *If preferences are nonmalevolent, then a change in the amount of public goods is potentially Pareto improving if it passes the private value benefit-cost test.*

E. *Sufficient but Not Necessary*

When preferences are nonmalevolent, passing the private value benefit-cost test is a sufficient condition but not a necessary condition for a change to be potentially Pareto improving. Figure 3 shows an example of a change that is potentially Pareto improving, although it fails the private values benefit-cost test. It is instructive to consider another example in algebraic form.

Example 1: There are two consumers, one private good, and one public good. The initial allocation is $(x_1, x_2, y) = (4, 1, 0)$. Private utility functions are $v_1(x_1, y) = \sqrt{x_1}$ and $v_2(x, y) = \sqrt{x_2(1+y)}$. Altruistic utility functions are $U_1(v_1, v_2) = v_1 + \frac{1}{2} v_2$ and $U_2 = v_2 + \frac{1}{2} v_1$. The initial private utilities are $v_1(4, 0) = 2$ and $v_2(1, 0) = 1$. The initial altruistic utilities are $U_1(4, 1, 0) = 2 + \frac{1}{2} = 2.5$ and $U_2(4, 1, 0) = \frac{2}{2} + 1 = 2$. The initial allocation is distributionally efficient since neither consumer would prefer to give private goods to the other.[4] Let the cost function for public goods be $C(y) = \frac{1}{3} y$. Consider a change in the amount of public goods from $y = 0$ to $y' = 3$. This change fails the private benefit-cost test, since the sum of private values for this change is $\frac{3}{4}$, while the cost of the change is $C(3) - C(0) = 1$.[5] But suppose that Consumer 1 pays the entire cost of supplying 3 units of y. In the resulting allocation, private utilities would be $v_1(3, 3) = \sqrt{3}$ and $v_2(1, 3) = 2$. The altruistic utilities are $U_1(3, 1, 3) = \sqrt{3} + \frac{1}{2} 2 = 2.73$ and $U_2 = 2 + \frac{1}{2}\sqrt{3} = 3.86$. Since both consumers achieve a higher social utility, a change from y to y' is potentially Pareto improving.

[4] Not only is this allocation distributionally efficient. Calculation shows that this allocation is preferred by Consumer 1 to *any* other way of allocating five units of private goods between 1 and 2.

[5] Consumer 2's private value for the change is the solution to $\sqrt{(1 - w_2)(1 + 3)} = \sqrt{1(1 + 0)}$, which is $w_2 = \frac{3}{4}$. Consumer 1's private value for the change is $w_1 = 0$.

II. Private Value Tests as Necessary Conditions

A. *A Marginal Private Values Test*

Although we have seen that some Pareto improving projects can fail the private values benefit-cost test, there is a useful necessary condition that compares the sum of *marginal* private value benefits to *marginal* cost. In Example 1, the change from $y = 0$ to $y = 3$ does not pass the private values benefit-cost test, but a smaller increase in the amount of public goods would do so. For example, a change from $y = 0$ to $y = 1$ passes the private value benefit-cost test, since consumer 2's private value for this change is $w_2 = \frac{1}{2}$, while the cost of the change is $\frac{1}{3}$. More generally, we will show that under suitable assumptions, the private value benefit-cost test is a necessary condition for potential Pareto improvement if the change is sufficiently small.

ASSUMPTION 4 (Differentiability and convexity): *The social utility functions $U_i(v_1(x_1, y), \ldots, v_n(x_n, y))$ are differentiable and quasiconcave in (x, y). The cost function C is differentiable and convex.*

Given that private utilities are differentiable, we can define private marginal rates of substitution and a private values Samuelson test.

DEFINITION 9 (Private marginal rate of substitution): *Consumer i's private marginal rate of substitution between public good j and private goods is the ratio $m_{ij}(x_i, y)$ between the partial derivative of $v_i(x_i, y)$ with respect to x_i and the partial derivative of $v_i(x_i, y)$ with respect to y.*

DEFINITION 10 (Private values Samuelson test): *At the allocation (x, y), public good j is said to pass the private values Samuelson test if*

$$\sum_i m_{ij}(x, y) > \frac{\partial C(y)}{\partial y_j}. \tag{6}$$

and to fail the private values Samuelson test if this inequality is reversed.

The following lemma, which is proved in the Appendix, is a key ingredient in proving our theorem on the private values Samuelson test.

LEMMA 2: *Given Assumptions A1–A4, if an increase in the amount of public good j fails the private values Samuelson test, then some decrease in the amount of public good j is potentially Pareto improving.*

THEOREM 2: *In an economy where Assumptions A1–A4 are satisfied and where the initial allocation* (x, y) *is distributionally efficient:*

(*a*) *If public good j fails the private values Samuelson test, then no increase in the amount of public good j can be potentially Pareto improving.*
(*b*) *If public good j passes the private values Samuelson test, then no decrease in the amount of public good j can be potentially Pareto improving.*

PROOF:

Suppose that the initial allocation is (x, y) and that public good j fails the private values Samuelson test. Then according to Lemma 2, a small decrease in the amount of good j is potentially Pareto improving. Therefore, there is a feasible allocation (x', y') that is Pareto superior to (x, y) where $y'_j < y_j$ and $y'_k = y_k$ for $k \neq j$. If an increase in the amount of public good j is potentially Pareto improving, there must be a feasible allocation (x'', y'') that is Pareto superior to (x, y) such that $y''_j > y_j$, and $y''_k = y_k$ for $k \neq j$. Since $y'_j < y_j$ and $y''_j > y_j$, there exists $\lambda \in (0, 1)$ such that $\lambda y' + (1 - \lambda)y'' = y$. From Assumptions 2 and 4 it follows that the set of feasible allocations is convex. Therefore, the allocation $\lambda(x', y') + (1 - \lambda)(x'', y'')$ is feasible, and since $y = \lambda y' + (1 - \lambda)y''$, this allocation can be achieved from (x, y) by a redistribution of private goods. Assumption 4 on convexity of preferences implies that the allocation $\lambda(x', y') + (1 - \lambda)(x'', y'')$ is Pareto superior to (x, y). But this contradicts the assumption that the allocation (x, y) is distributionally efficient. This proves Assertion 1 of the theorem. A straightforward parallel argument proves Assertion 2.

B. *Transferable Private Utility*

We have shown that for sufficiently small changes in the amount of public goods, passing the private goods benefit-cost test is both necessary and sufficient for a change in public goods to be potentially Pareto improving. For discrete changes, however, the private goods test is not a necessary condition. In Figure 3, a change from y to y' is potentially Pareto improving, even though it fails the private values benefit-cost test. In this example, a change from y to y' flattens the private utility possibility frontier, making it "cheaper" to increase in v_1 at the expense of v_2. Likewise, in Example 1, where the private test fails as a necessary condition, the slope of the private utility possibility frontier changes with the quantity of public goods.

There is an interesting special class of economies for which, over the relevant range, the y-contingent utility possibility frontiers are parallel to each other and lie in hyperplanes described by the equations $\sum v_i = H(y)$. Such economies are said to have *transferable utility.* Bergstrom and Richard Cornes (1983) prove that a public goods economy with technology satisfying Assumption 2 has transferable utility if and only if utility functions of all consumers can be represented in the form $v_i = A(y)x_i + B_i(y)$. A special case of this form is the quasi-linear family, $v_i(x_i, y) = x_i + f_i(y)$. Given the results of Bergstrom and Cornes, it is appropriate to define transferable private utility in a public goods economy as follows.

DEFINITION 11 (Transferable private utility): *In a public goods economy, there is transferable private utility if technology satisfies Assumption 2 and if the private utility functions of all i can be written in the form* $v_i(x_i, y) = A(y)x_i + B_i(y)$, *where* $A(y) > 0$.

If there is transferable private utility, then y-contingent private utilities can be redistributed additively, so long as all utilities remain above some threshold levels. This observation is formalized in Lemma 3, which is proved in the Appendix.

LEMMA 3: *If there is transferable private utility then the y-contingent private utility possibility set contains the set of distributions* $(v_1, \ldots, v_n)$ *such that* $v_i \geq B_i(y)$ *for all i, and* $\sum v_i \leq H(y)$, *where*

$$(7) \quad H(y) = A(y)(W - C(y)) + \sum_i B_i(y).$$

Figure 4 shows y and y'-contingent private utility possibility sets for two consumers with transferable utilities, where $H(y') > H(y)$. The y-contingent private utility possibility frontier is the line segment LL' consisting of all (v_1, v_2) such that $v_1 + v_2 = H(y)$ and $v_i \geq v_i(0, y)$ for $i = 1, 2$. The y'-contingent private utility possibility frontier is MM', described by $v_1 + v_2 = H(y')$ and $v_i \geq v_i(0, y')$ for $i = 1, 2$. The y and y' utility possibility sets $VP(y)$ and $VP(y')$ consist of all points on or below LL' and MM', respectively. Although the segments LL' and MM' are parallel to each other and MM' lies "above" LL', we see that neither of the private utility possibility sets $VP(y)$ and $VP(y')$ contains the other. Figure 4 represents a situation in which for fixed x_i, $v_i(x_i, y') > v_i(x_i, y)$, but $C(y') > C(y)$. Although the y'-contingent frontier, MM', lies strictly above the y-contingent frontier, LL', the set $VP(y)$ contains points like P that are not contained in $VP(y')$. This is possible because the less expensive vector y of public goods leaves more resources for private goods, which may be allocated very unequally so as to achieve highly unequal utility distributions that are not possible when more is spent on public goods.

It is easy to show that if the initial allocation is (x, y) and if a change from y to y' is potentially privately Pareto improving, then $H(y') > H(y)$. Since the y-contingent private utility possibility sets are not nested, the converse is not, in general, true. To obtain the converse, we need Assumption 3—that changes in the amount of public goods are compensable.

LEMMA 4: *If there is transferable utility and if the initial allocation* (x, y) *is nonwasteful and satisfies Assumption* 3, *then a change from y to* y' *is potentially privately Pareto improving if and only if* $H(y') > H(y)$.

For the private values test to be necessary, we also must rule out the possibility that some consumer is so benevolent as to prefer an allocation (x', y') to the initial allocation (x, y), even though her private utility would decline to a level lower than it would be if she gave up all private consumption. In realistic economies, no private consumption would entail starvation. Thus we call this the "no voluntary martyrs" assumption.

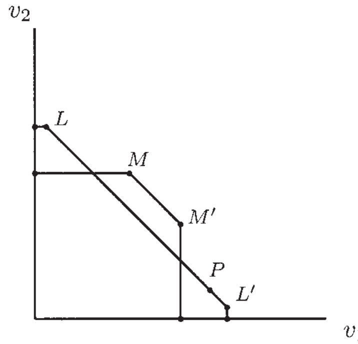

FIGURE 4. TRANSFERABLE PRIVATE UTILITY POSSIBILITY FRONTIERS

ASSUMPTION 5 (No voluntary martyrs): *For all i, if* $U_i(x', y') \geq U_i(x, y)$, *then* $v_i(x_i', y') \geq v_i(0, y)$.

THEOREM 3: *If Assumptions A*1–*A*3 *and A*5 *are satisfied, if the initial allocation is distributionally efficient, and if there is transferable utility, then a necessary condition for a change in the vector of public goods to be potentially Pareto improving is that this change satisfies the private values benefit-cost test.*

PROOF:

If Theorem 3 is false, then for some initial allocation (x, y), there is a change from y to y' that fails the private values benefit-cost test, but is potentially Pareto improving. Then there is a feasible allocation (x', y'), Pareto superior to (x, y). Assumption 5 requires that $v_i(x_i', y') \geq v_i(0, y) = B_i(y)$. Since the change fails the private values benefit-cost test, Lemma 4 implies that $H(y') \leq H(y)$. It follows from Lemma 3 that $(v_i', \ldots, v_n')$ belongs to the y-contingent utility possibility set. Therefore, there exists a feasible allocation (x^*, y) such that $U_i(v_1(x_1^*, y), \ldots, v_n(x_n^*, y)) = U_i(v_1(x_1', y'), \ldots, v_n(x_n', y'))$ for all i. Since (x', y') is Pareto improving over (x, y), so is (x^*, y). But this contradicts the assumption that (x, y) is distributionally efficient. This proves the theorem by contradiction.

III. Discussion and Applications

A. *Alice and Bob, Revisited*

Imagine that Alice and Bob hired a naive benefit-cost analyst to decide whether they should take the new apartment. If the analyst asked them, "How much would you be willing to pay to have the larger apartment?" each would reply \$150. The analyst would report total benefits of \$300 and recommend that they take the apartment so long as the extra rent did not exceed \$300. But if the rent is more than \$200 and is split equally between them, they will both be worse off if they take the apartment. What went wrong? The analyst evidently asked the wrong question. This question encouraged answers that include the sympathetic benefits that they place on each other's utility for the new apartment, but neglect the sympathetic cost that each would feel because the other has to pay more rent.

The analyst might instead have asked, "If the cost of moving to the new apartment is split equally between you, what is the most that you yourself would be willing to pay for the larger apartment?" Then, each would take account of the costs as well as the benefits to the other, and would answer \$100. The analyst would correctly recommend taking the apartment only if the extra rent did not exceed \$200. The symmetry of this example makes it natural for the analyst to propose splitting costs *equally*. But in less symmetric circumstances, it would not be obvious what division of costs to propose. In principle, the analyst could discover potential improvements, but this might require asking many different questions, each of which proposed a different division of costs.

Our results suggest that for many purposes, a simpler approach will suffice. The analyst could ask Alice and Bob a single question: "How much would you be willing to pay for the benefits that you yourself realize from the larger apartment, ignoring any benefits to the other person?" Moving to the larger apartment will be potentially Pareto improving if and only if the sum of these two measures of benefit exceeds the additional cost.

B. *Contingent Valuation Studies*

Government agencies routinely use benefit-cost studies based on "contingent evaluation surveys," where individuals are asked their willingness to pay for public amenities. As W. Michael Hanemann (1994) and Richard T. Carson (1999) observe, these studies vary widely in design and in quality. While there has been energetic debate over the validity of contingent evaluation studies, Hanemann reviews a body of evidence suggesting that carefully conducted contingent valuation studies exhibit reliability and consistency with other measures of willingness to pay.

Per-Olav Johansson (1994) recognized that if people have altruistic motives, contingent valuation studies are likely to overestimate the benefits of public projects. Subjects, when asked their willingness to pay for a public amenity, may include their altruistic valuations as well as their private valuations. To remedy this, Johansson proposed that subjects be asked to state their own willingness to pay for a public project, *conditional on the assumption that all others are taxed at rates equal to their private valuations.*

Some contingent valuation studies of public health and safety have framed their questions to distinguish between subjects' private values and the value that they place on benefits to others. Viscusi et al. (1988) asked their subjects what they would pay for a product that would increase their own safety, but not that of others, and also asked about their willingness to pay for extending the same benefits to a larger population. Michael Jones-Lee et al. (1985) asked a sample of British adults about their willingness to pay for a hypothetical safety device on their own cars that would increase only the driver's safety. Mark Dickie and Victoria Messman (2004) and Dickie and Shelby Gerking (2003) asked parents how much they would be willing to pay to spare their own children from illnesses with a specific list of unpleasant symptoms. They also asked their willingness to pay to avoid the same symptoms for themselves, and for others outside the family.

Viscusi and Joseph E. Aldy (2003) survey several "revealed preference" studies that attempt to estimate private valuations of public hazards by examining economic decisions in which individuals pay for reduced personal risk. Examples include studies of the relation of wages to occupational hazard; the relation of house prices to their distance from hazardous waste sites; and the relation of prices of automobiles to their optional safety features.

For many public goods, it is not easy to distinguish private from social values. The measurement of "existence value" of wilderness areas or of endangered species, which the subjects never expect to see, is an especially problematic case. John B. Loomis and Douglas S. White (1996) survey a large number of contingent valuation studies of the value of preserving various animal species. Don Coursey (1997) describes a typical question of such studies: "What is the maximum dollar amount your household would be willing to pay for a program that will lead to a 10-percent increase in the whooping crane population?"[6]

How are we to interpret the answers to Coursey's question? A subject might believe that this question means either: (a) How much would you be willing to pay for a 10-percent increase in the whooping crane population, if this increase depended solely on your own contribution? (b) What is the largest tax increase that you would be willing to accept for you and those like you in order to accomplish a 10-percent increase in the whooping crane population? People with strong altruistic motives are likely to respond with a much larger answer to Question A than to Question B. This paper has argued that Question B is conceptually appropriate for benefit-cost analysis. Contingent valuation studies may be improved by sharpening the line of inquiry to clarify that what is being asked is Question B and not Question A.

C. *Related Theoretical Work*

The idea that "altruistic values" should not be counted in benefit-cost analysis has been debated in the economic literature, but not satisfactorily resolved. An earlier paper by Bergstrom (1982) claimed that with benevolent preferences, the appropriate way to measure benefits is to sum private valuations, excluding altruistic considerations. Bergstrom's argument was based on the observation that, with benevolence, the marginal first-order necessary conditions for optimality are the same as those that apply if account is taken only of the private valuations. Jones-Lee (1991, 1992) and Johansson (1994) presented more thorough discussions and extensions of Bergstrom's result. Jones-Lee showed that when concern for others is "safety-focused" rather than purely altruistic, the appropriate benefit measures are intermediate between the private and social values. Kenneth E. McConnell (1997) concluded from study of first-order conditions that if altruism is nonpaternalistic, then "altruism has no impact on the benefit-cost outcome." McConnell suggested, however, that altruism is likely to take a paternalistic form in the case of natural resource allocation.

In a critical discussion of contingent valuation methods, Paul Milgrom (1993) maintained that the appropriate measure of benefits must be the sum of private valuations. Milgrom's paper, which appeared in a conference volume, provoked a lively exchange between Milgrom and several environmental economists who found it difficult to accept the assertion that altruistic values should be ignored in benefit-cost studies. Hanemann (1994) disputed Milgrom's conclusion on the grounds that altruistically motivated valuation is as legitimate as valuation for any other reason.

Nicholas E. Flores (2002) noticed that earlier discussions lacked a general proof that the private values benefit-cost test is a *necessary* condition for a change in the amount of public goods to be potentially Pareto improving. He produced an example in which this claim is false. He also conjectured that the private goods test is necessary for potential Pareto improvement if there is transferable utility.

IV. Conclusion

What have we learned that can guide policymakers and practitioners? If altruism is nonpaternalistic, can altruistic values be ignored in benefit-cost analysis? Our theorems do not justify such a sweeping conclusion. The assumptions under which the private values benefit-cost test is necessary for potential Pareto improvement need not always be satisfied. Our examples show that discrete changes in the amount of public goods may be Pareto improving even though they fail the private values test. If convexity is assumed, then the marginal "private values Samuelson condition" provides guidance about the direction in which Pareto improvements can and cannot be found, but not about the magnitude of change. If there is transferable utility, then the private values benefit-cost test is a necessary condition for discrete changes, but this conclusion requires the com-

[6] See, for example, J. Michael Bowker and John R. Stoll (1988) for whooping cranes, or Thomas Stevens et al. (1991) for bald eagles, Atlantic salmon, wild turkeys, and coyotes.

pensability assumption and the no-voluntary-martyrs assumption. With or without transferable utility, our results depend on the assumption that the initial allocation is distributionally efficient.

Despite these qualifications, our theorems indicate that for a broad class of economies, a comparison of the sum of private values to the cost of a project is the appropriate test for determining whether it can lead to a Pareto improvement. Understanding this should help policymakers avoid the mistake of Alice and Bob's "naive benefit-cost analyst." Such understanding may also improve decision-making for real couples like Alice and Bob, when they decide whether to purchase a new automobile or other household public good.

Appendix

PROOF OF LEMMA 1:

Suppose that the change from y to y' passes the private values benefit-cost test. For each i, let w_i be i's private valuation of the change, such that $v_i(x_i-w_i, y') = v_i(x_i, y)$. Let $\Delta = \sum_i w_i - (C(y') - C(y)) > 0$, and let $x_i' = x_i - w_i + \Delta/n$. Then $v_i(x_i', y') > v_i(x_i, y)$ for all i and $\sum x_i' = \sum x_i - \sum w_i + \Delta = \sum x_i + C(y) - C(y')$. Therefore, $\sum x_i' + C(y') = \sum x_i + C(y) \leq W$. It follows that the allocation (x', y') is feasible and privately Pareto preferred to (x, y), which means that the change from y to y' is potentially privately Pareto improving.

Conversely, suppose that there exists a feasible allocation (x', y') that is Pareto superior to (x, y). Since $v_i(x', y') \geq v_i(x, y)$ for all i with at least one strict inequality, it must be that $x_i' \geq x_i - w_i$ for all i, with strict inequality for some i, and therefore $\sum x_i' + \sum w_i < \sum x_i$. Feasibility implies that $\sum x_i' \leq W - C(y')$ and $\sum x_i \leq W - C(y)$. Substituting these two inequalities into the previous inequality and rearranging terms, we have $\sum w_i < C(y') - C(y)$. Therefore, the change satisfies the private values benefit-cost test.

PROOF OF LEMMA 2:

Suppose that an increase in the amount of public good j from the initial allocation, (x, y), fails the Samuelson test. Then, since $\sum_i m_{ij}(x_i, y) < C_j(y)$, there exists $\varepsilon > 0$ such that $\sum_i (m_{ij}(x_i, y) + \varepsilon) < C_j(y)$. For $t > 0$, define $x_i(t) = x_i + (m_{ij}(x_i, y) + \varepsilon)t$ and $y(t) = (y_1, \ldots, y_j - t, \ldots, y_m)$. Define $\tilde{v}_i(t) = v_i(x_i(t), y(t))$. Then, $(d/dt)\tilde{v}_i(0) > 0$ and, therefore, there exists $t^* > 0$ such that for all i, $\tilde{v}_i(t^*) = v_i(x_i(t^*), y(t^*)) > v_i(x_i, y)$.

The allocation $(x(t^*), y - t^*)$ is Pareto superior to (x, y), and $y(t^*)$ differs from y only by a reduction in y_j. We now show that this allocation is feasible, which will complete the proof. We have

$$\sum_i x_i(t^*) - \sum_i x_i = t^*(m_i(x_i, y) + \varepsilon) < t^*C_j(\bar{y}).$$

Since $C(\cdot)$ is a convex function, $t^*C_j(y) \leq C(y) - C(y - t^*)$. Therefore, $\sum_i x_i(t^*) - \sum_i x_i < C(y) - C(y - t^*)$ and, hence, $\sum_i x_i(t^*) + C(y - t^*) < C(y) + \sum x_i \leq W$, which implies that $(x(t^*), y - t^*)$ is feasible.

PROOF OF LEMMA 3:

Let $(v_1', \ldots, v_n')$ be a vector of private utilities such that $\sum v_i' \leq H(y)$ and $v_i' \geq B_i(y)$. Let $x_i' = (v_i' - B_i(y))/A(y)$. Assumption 5 implies that $x_i' \geq 0$. Also, $v_i(x_i', y) = A(y)x_i' + B_i(y) = v_i'$, and

$$\sum x_i' = \frac{\sum v_i' - \sum_i B_i(y)}{A(y)} \leq \frac{H(y) - \sum_i B_i(y)}{A(y)}$$

$$= W - C(y).$$

This implies that the allocation $(x_1', \ldots, x_n', y)$ is feasible and hence that $(v_1', \ldots, v_n')$ belongs to the y-contingent utility possibility set.

PROOF OF LEMMA 4:

Let (x, y) be the initial allocation and suppose that $H(y') > H(y)$. For each i, let $\hat{x}_i = [A(y)x_i + B_i(y) - B_i(y')]/A(y')$. Assumption 3 implies that $\hat{x}_i \geq 0$. Then $A(y')\hat{x}_i + B_i(y') = A(y)x_i + B_i(y)$. Summing both sides of this equation over i, and using the assumption that (x, y) is nonwasteful, we have $A(y') \sum \hat{x}_i + \sum B_i(y') = A(y) \sum x_i + \sum B_i(y) = A(y)(W - C(y)) + \sum B_i(y) = H(y)$. Since $H(y') > H(y)$, it follows that

$$H(y') = A(y')(W - C(y')) + \sum_i B_i(y')$$

$$> A(y') \sum \hat{x}_i + \sum_i B_i(y').$$

Since $A(y') > 0$, it must then be that $W - C(y') > \Sigma\, \hat{x}_i$. Let $\delta = W - C(y') - \Sigma\, \hat{x}_i > 0$, and for each i, let $x_i' = \hat{x}_i + \delta/n$. The allocation $(x_1', \dots, x_n', y')$ is feasible since $\Sigma\, x_i' = W - C(y')$ and privately Pareto improving over (x, y) since $v_i(x_i', y') > v_i(x, y)$ for all i. Therefore, a change from y to y' is potentially privately Pareto improving.

Conversely, suppose that the initial allocation (x, y) is nonwasteful and a change from y to y' is potentially privately Pareto improving. Then for some feasible (x', y'), $A(y')x_i' + B_i(y') > A(y)x_i + B_i(y)$ for all i with at least one strict inequality. Therefore, $A(y)\, \Sigma\, x_i' + \Sigma\, B_i(y') > A(y)\, \Sigma\, x_i + \Sigma\, B_i(y)$. Nonwastefulness of (x, y) implies that $\Sigma\, x_i = W - C(y)$, and feasibility of (x', y') implies that $\Sigma\, x_i' \leq W - C(y')$. Substituting these expressions in the previous inequality implies that $H(y') > H(y)$.

REFERENCES

Archibald, George C. and Donaldson, David. "Non-Paternalism and the Basic Theorems of Welfare Economics." *Canadian Journal of Economics*, 1976, *9*(3), pp. 492–507.

Bergstrom, Theodore C. "A 'Scandinavian Consensus' Solution for Efficient Income Distribution among Nonmalevolent Consumers." *Journal of Economic Theory*, 1970, *2*(4), pp. 383–98.

Bergstrom, Theodore C. "When Is a Man's Life Worth More Than His Human Capital?" in Michael Jones-Lee, ed., *The value of life and safety*. Amsterdam: North-Holland, 1982, pp. 3–26.

Bergstrom, Theodore C. "Love and Spaghetti, the Opportunity Cost of Virtue." *Journal of Economic Perspectives*, 1989, *3*(2), pp. 165–73.

Bergstrom, Theodore C. "Systems of Benevolent Utility Functions." *Journal of Public Economic Theory*, 1999, *1*(1), pp. 71–100.

Bergstrom, Theodore C. and Cornes, Richard C. "Independence of Allocative Efficiency from Distribution in the Theory of Public Goods." *Econometrica*, 1983, *51*(6), pp. 1753–65.

Bowker, J. Michael and Stoll, John R. "Use of Dichotomous Choice Nonmarket Methods to Value the Whooping Crane Resource." *American Journal of Agricultural Economics*, 1988, *70*(2), pp. 372–81.

Carson, Richard T. "Contingent Valuation: A User's Guide." University of California at San Diego, Economics Working Paper Series: No. 99-26, 1999.

Chipman, John S. and Moore, James C. "Compensating Variation, Consumer's Surplus, and Welfare." *American Economic Review*, 1980, *70*(5), pp. 933–49.

Coursey, Don L. "The Revealed Demand for a Public Good: Evidence from Endangered and Threatened Species." *New York University Environmental Law Journal*, 1997, *6*(2), pp. 411–50.

Dickie, Mark and Gerking, Shelby. "Valuing Children's Health: Parental Perspectives." University of Central Florida, Department of Economics Working Paper Series: No. 03-24, 2003.

Dickie, Mark and Messman, Victoria L. "Parental Altruism and the Value of Avoiding Acute Illness: Are Kids Worth More Than Parents?" *Journal of Environmental Economics and Management*, 2004, *48*(3), pp. 1146–74.

Flores, Nicholas E. "Non-Paternalistic Altruism and Welfare Economics." *Journal of Public Economics*, 2002, *83*(2), pp. 293–305.

Hanemann, W. Michael. "Valuing the Environment through Contingent Valuation." *Journal of Economic Perspectives*, 1994, *8*(4), pp. 19–43.

Hicks, John R. "Consumer's Surplus and Index Numbers." *Review of Economic Studies*, 1942, *9*(2), pp. 126–37.

Johansson, Per-Olov. "Altruism and the Value of Statistical Life: Empirical Implications." *Journal of Health Economics*, 1994, *13*(1), pp. 111–18.

Jones-Lee, Michael W. "Altruism and the Value of Other People's Safety." *Journal of Risk and Uncertainty*, 1991, *4*(2), pp. 213–19.

Jones-Lee, Michael W. "Paternalistic Altruism and the Value of Statistical Life." *Economic Journal*, 1992, *102*(410), pp. 80–90.

Jones-Lee, Michael W.; Hammerton, Max and Philips, Peter R. "The Value of Safety: Results of a National Sample Survey." *Economic Journal*, 1985, *95*(377), pp. 49–72.

Kolm, Serge-Christophe. "Optimum Production of Social Justice," in Henri Guitton and Julius Margolis, eds., *Public economics: Analysis of public production and consumption and their relations to the private sector*. New

York: St. Martin's Press, 1969, pp. 145–200.

Kolm, Serge-Christophe. "A Historical Introduction to Normative Economics." *Social Choice and Welfare*, 2000, *17*(4), pp. 707–38.

Loomis, John B. and White, Douglas S. "Economic Benefits of Rare and Endangered Species: Summary and Meta-Analysis." *Ecological Economics*, 1996, *18*(3), pp. 197–206.

McConnell, Kenneth E. "Does Altruism Undermine Existence Value?" *Journal of Environmental Economics and Management*, 1997, *32*(1), pp. 22–37.

Milgrom, Paul. "Is Sympathy an Economic Value? Philosophy, Economics, and the Contingent Valuation Method," in Jerry A. Hausman, ed., *Contingent valuation: A critical assessment*. Amsterdam: Elsevier Science, North-Holland, 1993, pp. 417–35.

Pareto, Vilfredo. "Il massimo di utilitá per una collectivá." *Giornale degli Economisti*, 1913, *3*, pp. 337–41.

Stevens, Thomas; Echeverria, Jaime; Glass, Ronald; Hager, Tim and More, Thomas. "Measuring the Existence Value of Wildlife: What Do CVM Estimates Really Show?" *Land Economics*, 1991, *67*(4), pp. 390–400.

Viscusi, W. Kip; Magat, Wesley A. and Forrest, Anne. "Altruistic and Private Valuations of Risk Reduction." *Journal of Policy Analysis and Management*, 1988, *7*(2), pp. 227–45.

Viscusi, W. Kip and Aldy, Joseph E. "The Value of a Statistical Life: A Critical Review of Market Estimates throughout the World." *Journal of Risk and Uncertainty*, 2003, *27*(1), pp. 5–76.

Winter, Sidney G., Jr. "A Simple Remark on the Second Optimality Theorem of Welfare Economics." *Journal of Economic Theory*, 1969, *1*(1), pp. 99–103.