LEARNING AND MAINTENANCE OUTSOURCING

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Abstract

In this paper, we discuss learning in maintenance outsourcing. We consider a manufacturer that outsources its maintenance activities to a contractor. We find that a payment scheme consisting of a fixed amount along with cost subsidization (CS) can maximize the manufacturer's profit under no learning. If the contractor achieves learning through experience, CS still maximizes the manufacturer's profit. If learning happens with costly effort, CS is still effective under special conditions.

Keywords: Maintenance Outsourcing, Learning, Cost Subsidization

INTRODUCTION

It is well-known in practice that a successful maintenance policy is a critical component of a company's financial well-being. For instance, [1] reports that maintenance costs can make up 5 to 40 % of total production costs. Maintenance has been in focus academically as well. There is a vast literature of academic papers on designing sound maintenance policies. [2] [3] [4] and most recently, [5], provide extensive surveys and reviews of the maintenance literature.

Even though most of the academic papers on maintenance assume the maintenance activities are carried out in-house, there is a clear tendency in practice to outsource these (usually non-core) activities to maintenance contractors who can provide high quality service at a lower cost [6]. This should not be surprising since companies are outsourcing their non-core processes and focusing on their core processes, or competitive advantages, in all aspects of business. Maintenance outsourcing is becoming more and more widespread in many industries, including office equipment, medical (hospital) equipment, airplane engines and brakes, computer hardware and networks, and manufacturing processes at significant monetary values. For example, GE Engine Services recently signed with Skywest Airlines a 16-year maintenance contract valued at $1 billion to perform engine maintenance on the carrier's fleet of Canadair Regional Jets [7]. In addition, it was estimated that outsourced equipment maintenance represents a $26 billion industry in healthcare [8].

Given the significance of maintenance outsourcing in practice, it can be somewhat surprising that there are not many academic papers analyzing this phenomenon [9][10][11][12][13]. [9] analyzed a maintenance contract between a manufacturer and contractor with the objective of maximizing the contractor's profit. They later extended their work to consider one contractor and multiple homogeneous manufacturers, still focused on maximizing the contractor's profit [10]. [11] applied classical principal-agent methods for assessing the value of outsourcing maintenance using a Markovian decision process. [12] examined channel coordination over an extended enterprise, where the service chain consisted of a manufacturer with a single production process and a maintenance contractor. They proposed incentive mechanisms that a manufacturer may offer to a contractor to achieve channel coordination, resulting in improved profits to the system, manufacturer, and contractor. [13] analyzed a single manufacturer running a multi-stage production system and outsourcing the maintenance activities to multiple contractors. Once again, they identified contracts which achieved channel coordination.

In this paper, we assume a model similar to that of [12] with three significant changes: 1) Instead of an infinite horizon period, we are looking at a short-term outsourcing contract; 2) We are maximizing the manufacturer's profit instead of coordinating the service chain and 3) We are analyzing the effects of learning on preventive maintenance activities. The importance of learning has been analyzed in the operations management area before, mainly in the production area. Learning in production results in a decrease in unit production-time as the plant produces additional units; this pattern is known as experience-based learning / learning-by-doing. [14] was the first one to recognize and model the experience-based learning, based on his observations in the aircraft industry. Later, several additional evidences for the learning phenomenon have been accumulated [15][16]. [17] provided a thorough review of the learning curve. [18] modeled the competitive interaction in the presence of a learning curve. [19] studied the effect of learning and quality improvement on production systems. [20][21] investigated stochastic learning curves using a Bayesian approach. [22] conducted an empirical study on the persistence and transfer of organizational learning. [23] studied the creation of knowledge (or learning) by investment in organizational resources such as science and technology, software development and worker training. Learning in their study is not experience-based. In maintenance literature, [24] estimate the learning curve for maintenance activities. In our paper, we assume both experience-based (natural) learning similar to the [24] and traditional learning models, and costly learning through investment similar to [23].

FINITE HORIZON WITH NO LEARNING

In this section, we describe our basic model and define the terminology. We consider a service chain consisting of a manufacturer and a maintenance contractor. We assume the manufacturer has a continuous process with a failure rate function $\lambda(t)$ in time t such that $\lambda(0) = 0$ and $\lambda'(t) > 0$. This implies that $\lambda(t)$ is an *increasing* failure rate (IFR) function. Many common probability distributions, including Weibull, Normal and Lognormal distributions, are IFR functions.

The manufacturer has a finite production horizon of length Y units. There are two types of maintenance activities used to improve the reliability of the manufacturing process: **preventive maintenance (PM)** and **minimal repair (MR)**. Preventive maintenance is performed periodically to avoid costly repairs and disruptions of production caused by process breakdowns. Minimal repair is performed immediately after the process breaks down. The manufacturer plans to outsource these maintenance functions because the contractor can perform the functions more efficiently in both time and cost. We use T_p and C_p to denote the average time and cost required for the contractor to perform a preventive maintenance operation, respectively. It is assumed that preventive maintenance overhauls the process and restores the failure rate to the original (best) state; i.e., $\lambda(0) = 0$ [25]. This assumption is commonly used in maintenance/reliability studies, and the results of the paper remain unchanged as long as the failure rate is constant after every PM. Furthermore, we use T_r and C_r to denote the average time and cost required for the contractor to perform a minimal repair operation, respectively.

We assume that the manufacturer delegates the control of maintenance decisions to the contractor. Therefore, the contractor selects the number and frequency of PM activities. We assume that the contractor starts by performing a PM at the beginning of the horizon. Let N be the number of PM activities performed by the contractor during the horizon Y. We also define T_i as the i^{th} PM interval; i.e., the time between the end of the i^{th} PM and the beginning of the $(i+1)^{th}$ one. T_N is defined to be the PM interval after the end of the N^{th} PM and the end of the horizon. The manufacturer's payment to the contractor has two components: i) A fixed payment of $P \geq 0$ over the time horizon, and, ii) A cost subsidization scheme for every preventive maintenance and minimal repair activity that the contractor performs. Let $0 \leq s_p \leq 1$ and $0 \leq s_r \leq 1$ represent the ratio of C_p and C_r that the manufacturer subsidizes, respectively. Therefore, the contractor's effective PM and MR costs become $(1- s_p)\, Cp$ and $(1- s_r)\, Cr$, respectively. This type of incentive mechanism, consisting of a fixed component and a cost subsidization scheme, has been shown to be more effective to coordinate a service chain and to improve the efficiency of maintenance activities than a forcing contract, which specifies the exact PM frequency for the contractor [12]. This cost subsidization mechanism can also be used by the manufacturer to induce the contractor to select a maintenance policy that maximizes the manufacturer's expected profit.

The maintenance contractor is responsible for performing preventive maintenance periodically according to the schedule (contractor's choice) and necessary minimal repairs (random events) whenever the process breaks down. We assume that the contractor has a pre-determined reservation (minimum) profit over the production horizon Y, denoted by P_0, for participating in the contractual relationship. Let the net revenue of the manufacturer, after taking into account the production related costs, be R per unit of time that the process is in operation.

We assume that minimal repair merely restores the process back to operation, and the deterioration of the process remains during minimal repair. Under this assumption, the process is restored to the deterioration state that it would have been in without breakdown and minimal repair. This assumption is used to simplify our analysis and is valid when T_r is relatively small or a minimal repair involves, for example, replacing minor components, while the rest of the process is not stopped [25][26][27][12]. Furthermore, we also assume that preventive maintenance will take place if there is a minimal repair operation in process near the end of a PM interval and the two types of maintenance coincide.

The contractor's expected profit over the time horizon depends on the payment/cost subsidization it receives from the manufacturer, on the number of PM activities it decides to perform and also the expected number of failures in each PM interval. We define $M(T_i)$ as the expected number of failures in time interval $[0, T_i]$. Then, $M(T_i) = \int_0^{T_i} \lambda(x)dx$ [28]. Then, the contractor's expected profit can be given by the following optimization problem: Π_{c1} =

$$\max_{N_c\, T_{c1}\, T_{c2} \ldots T_{cN}} \quad P - N_c\,[(1- s_p)\, C_p] - [(1- s_r)\, C_r] \sum_{i=1}^{N_c} M(T_{ci}) \quad \text{such that } N_c\, T_p + \sum_{i=1}^{N_c} T_{ci} = Y$$

The following Lemma gives an important result to facilitate the solution of Π_{c1}:

Lemma 1. For any value of N, the optimal PM intervals for the contractor that maximize Π_{c1} are equivalent; i.e. $T_{c1}^{*} = T_{c2}^{*} \ldots = T_{cN}^{*} = T_c^{*}$.

Then, we can rewrite the contractor's expected profit as follows: $\Pi_{c2} = \max_{N_c} \; P - N_c\,[(1- s_p)\, C_p] - N_c\,[(1- s_r)\, C_r]\, M\left(\frac{Y}{N_c} - T_p\right)$

Now that we are able to write the contractor's expected profit as a function of a single variable, N_c, the following Lemma gives the solution to Π_{c2}. In part ii, we note that the number of PM activities should be an integer value.

Lemma 2. i) Define $g(N) = \dfrac{Y\lambda\left(\frac{Y}{N} - T_p\right)}{N} - M\left(\frac{Y}{N} - T_p\right)$. Then, there is a unique N_c, denoted by N_c^{**} and not necessarily an integer, that maximizes Π_{c2}. N_c^{**} satisfies the following relationship: $g(N_c^{**}) = \dfrac{(1- s_p)\, C_p}{(1- s_r)\, C_r}$.

ii) The integral N_c value, denoted by N_c^{*}, that maximizes Π_{c2} can be found by rounding N_c^{**} up or down. In other words,

$N_c^{*} = \operatorname{argmax}_{N \in \lfloor N_c^{**} \rfloor, \lceil N_c^{**} \rceil} \; \Pi_{c2}$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x and $\lceil x \rceil$ represents the smallest integer greater than or equal to x.

We can now analyze the manufacturer's expected profit. The manufacturer needs to make sure that the contractor's expected profit, given by Π_{c2}, is satisfied at the reservation profit, P_0. The manufacturer's expected profit is governed by the contractor's selection for the number of PM activities, N, as well as the manufacturer's decisions for P, s_p, s_r. The manufacturer's expected profit is given by the following problem: $\Pi_{m1} = \max_{P\, s_p\, s_r} R[Y - N\,T_p] - N\,s_p\,C_p - N\,[\,s_r\,C_r + R\,T_r]\,M\left(\frac{Y}{N} - T_p\right) - P$

such that $P - N\,[(1-s_p)\,C_p] - N\,[(1-s_r)\,C_r]\,M\left(\frac{Y}{N} - T_p\right) = P_0.$

Substituting P from the constraint into the objective function, we get $\Pi_{m2} = R[Y - N\,T_p] - N\,C_p - N\,[C_r + R\,T_r]\,M\left(\frac{Y}{N} - T_p\right) - P_0$

Therefore, the manufacturer's expected profit can also be written as a single variable function of N, the number of PM activities scheduled and performed. Of course, it is the contractor who selects N. However, the manufacturer can induce the contractor to choose the optimal N value that would maximize Π_{m2} by selecting appropriate P, s_p, s_r values. In other words, although the manufacturer cannot select N directly, it can give the necessary incentive to the contractor to implement the PM schedule that maximizes the manufacturer's profit. Lemma 3 finds the optimal N value that maximizes the manufacturer's expected profit function Π_{m2} :

Lemma 3. i) Using the definition of g(N) from Lemma 2, there is a unique N_m, denoted by N_m^{**} and not necessarily an integer, that maximizes Π_{m2}. N_m^{**} satisfies the following relationship: $g(N_m^{**}) = \frac{R\,T_p + C_p}{R\,T_r + C_r}$.

ii) The integral N value, denoted by N_m^{*} , that maximizes Π_{m2} can be found by rounding N_m^{**} up or down. In other words,

$$N_m^{*} = \operatorname*{argmax}_{N \in \lfloor N_m^{**} \rfloor, \lceil N_m^{**} \rceil} \Pi_{m2}.$$

The manufacturer, equipped with the knowledge regarding its own and the contractor's expected profit, can apply the following three steps to maximize its profit: 1) Find N_m^{*} from Lemma 3. 2) Set s_p, s_r values such that $g(N_m^{*}) = \frac{(1-s_p)\,C_p}{(1-s_r)\,C_r}$. 3) Set $P = [(1-s_p)\,C_p] + N_m^{*}\,[(1-s_r)\,C_r]\,M\left(\frac{Y}{N_m^{*}} - T_p\right) + P_0$.

Step 1 finds the PM frequency that maximizes the manufacturer's expected profit as given in Π_{m2} . Step 2 enables the manufacturer to give the contractor the incentive to choose this PM frequency that benefits the manufacturer. Please note that there is an infinite number of s_p, s_r pairs that achieve this goal. Finally, Step 3 ensures that the contractor receives its reservation profit and thus is willing to participate in this transaction. Please note that even though we assume the contractor and the manufacturer are both risk neutral, different values of P, s_p, and s_r can be selected to accommodate for different risk levels. For instance, if the contractor were risk averse, the manufacturer could select a high fixed or guaranteed payment (P) and low subsidization ratios (s_p, s_r). This would keep the risk-neutral manufacturer's profit the same but increases the risk-adjusted profit of the contractor.

LEARNING IN MAINTENANCE OUTSOURCING

Natural Learning

In this section, we assume that the contractor performs the first PM at a cost of C_p and time of T_p. The second PM costs less money and takes less time due to learning and the third one even less and so on. Specifically, we assume that for the ith PM, the cost will be $C_p\,i^{-a}$ and the duration will be $T_p\,i^{-a}$, where $a \geq 0$ represents the steepness, or degree, of learning. A large a value corresponds to a high learning level. This type of learning model is commonly used in the operations and production management literature [29][30][31]. Please note that we assumed the same level of learning for time and cost to make the analysis more clear. Different levels of learning for time and cost can be incorporated into the model without any difficulty. Under this scenario, the contractor's expected profit is given by the following:

$$\Pi_{c1}(L) = \max_{N_c\, T_{c1}\, T_{c2} \ldots T_{cN}} P - [(1-s_p)\,C_p] \sum_{i=1}^{N_c} i^{-a} - [(1-s_r)\,C_r] \sum_{i=1}^{N_c} M(T_{ci}) \ \textbf{such that} \ \sum_{i=1}^{N_c} i^{-a}\ T_p + \sum_{i=1}^{N_c} T_{ci} = Y$$

Please note that in the function above, s_p still represents the subsidy ratio that the contractor receives for every PM it conducts. In other words, for the first PM, the contractor receives a subsidy amount of $s_p\,C_p$; for the second PM, this amount decreases to $s_p\,C_p\,2^{-a}$ and for the ith PM, it becomes $s_p\,C_p\,2^{-a}$, due to learning.

Lemma 4. For any value of N, the optimal PM intervals for the contractor that maximize $\Pi_{c1}(L)$ are equivalent; i.e. $T_{c1}^{*} = T_{c2}^{*} \ldots = T_{cN}^{*} = T_c^{*}$.

Therefore, as in the basic model with no learning, the contractor would select PM intervals such that their lengths are equalized. This result is analogous to what researches have found about the effects of learning in production: for EOQ models, the lot sizes are still the same if there is learning in setup costs [32]. We can rewrite the contractor's expected profit as follows:

$$\Pi_{c2}(L) = \max_{N_c} P - [(1-s_p)\,C_p] \sum_{i=1}^{N_c} i^{-a} - N_c\,[(1-s_r)\,C_r]\,M\left(\frac{Y}{N_c} - T_p \sum_{i=1}^{N_c} i^{-a}\,\frac{1}{N_c}\right)$$

To solve $\Pi_{c2}(L)$, we use the approximation, $\sum_{i=1}^{N_c} i^{-a} \approx \int_0^{N_c} i^{-a} di$. We note that this approximation works very well for values of a less than or equal to 0.6, which covers virtually all practical cases [33]. We note that this approximation would fail for values of a greater or equal to 1, which are very rare in practice.

Lemma 5. i) Define $h(N) = \left(\frac{Y}{N^{1-a}} - \frac{aT_p}{1-a}\right) \lambda \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) - N^a M \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right)$. Then, there is a unique N_c, denoted by $N_c^{**}(L)$ and not necessarily an integer, that maximizes $\Pi_{c2}(L)$. $N_c^{**}(L)$ satisfies the following relationship: $h(N_c^{**}(L)) = \frac{(1- s_p)\, C_p}{(1- s_r)\, C_r}$

ii) The integral N_c value, denoted by $N_c^{*}(L)$, that maximizes $\Pi_{c2}(L)$ can be found by rounding $N_c^{**}(L)$ up or down. In other words,

$$N_c^{*}(L) = \underset{N \in \lfloor N_c^{**}(L) \rfloor, \lceil N_c^{**}(L) \rceil}{\operatorname{argmax}} \Pi_{c2}(L)$$

Lemma 5 tells us that the contractor's selection of the optimal number of PM activities still depends on the simple ratio of PM cost to MR cost. Now that we know the solution to the contractor's problem, we divert our focus to the manufacturer's profit function. Acknowledging the result of Lemma 4 that the contractor will select PM intervals of equal length, we can write:

$$\Pi_{m1}(L) = \underset{P\, s_p\, s_r}{\text{Max}} \quad R\, [Y - \frac{T_p N^{1-a}}{1-a})\,] - s_p C_p \frac{N^{1-a}}{1-a} - N\, [s_r\, C_r + R\, T_r]\, M \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) - P$$

$$\text{s.t. } P - [(1- s_p)C_p]\frac{N^{1-a}}{1-a} - N\,[(1- s_r)\, C_r]\, M \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) = P_0$$

Substituting for P in the objective function, the manufacturer's expected profit becomes a function of N as follows:

$$\Pi_{m2}(L) = \quad R\, [Y - \frac{T_p N^{1-a}}{1-a})\,] - C_p \frac{N^{1-a}}{1-a} - N\, [C_r + R\, T_r]\, M \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) - P_0$$

Lemma 6. i) Using the definition of $h(N)$ from Lemma 5, there is a unique N_m, denoted by $N_m^{**}(L)$ and not necessarily an integer, that maximizes $\Pi_{m2}(L)$. $N_m^{**}(L)$ satisfies the following relationship: $h(N_m^{**}(L)) = \frac{R\, T_p + C_p}{R\, T_r + C_r}$.

ii) The integral N_m value, denoted by $N_m^{*}(L)$, that maximizes $\Pi_{m2}(L)$ can be found by rounding $N_m^{**}(L)$ up or down. In other words,

$$N_m^{*}(L) = \underset{N \in \lfloor N_m^{**}(L) \rfloor, \lceil N_m^{**}(L) \rceil}{\operatorname{argmax}} \Pi_{m2}(L).$$

iii) $N_m^{**}(L) \geq N_m^{**}$.

Lemma 6 has some important implications. In the first part, it indicates that the critical measure for the manufacturer, as in Lemma 3, is still the relative total cost (revenue loss and actual maintenance cost) of PM to MR activities. The third part states that if there is an improvement in the PM activities due to learning, the manufacturer's optimal (not necessarily integral) number of PM activities does not decrease. The PM cost obviously affects the contractor's profit; however, it influences the manufacturer's profit as well through the cost subsidization scheme. Therefore, the decrease in C_p makes PM more attractive cost-wise to the manufacturer. In addition, as T_p decreases, the manufacturer loses relatively less revenue due to downtime, which makes PM even more attractive to the manufacturer. Hence, the tendency to increase the frequency of PM activities arises.

Similar to Section 2, the following three steps would induce the contractor to select $N_m^{*}(L)$ thereby maximizing the manufacturer's profit, given by $\Pi_{m2}(L)$: 1) Find $N_m^{*}(L)$ from Lemma 6. 2) Set s_p, s_r values such that $h(N_m^{*}(L)) = \frac{(1- s_p)\, C_p}{(1- s_r)\, C_r}$.
3) Set $P = [(1- s_p)C_p]\frac{N_m^{*}(L)^{1-a}}{1-a} + N_m^{*}(L)\,[(1- s_r)\, C_r]\, M\left(\frac{Y}{N_m^{*}(L)} - \frac{T_p N_m^{*}(L)^{-a}}{1-a}\right) + P_0$.

Please note that, as in Section 2, there is an infinite number of s_p, s_r solutions. This again implies that different s_p, s_r values can be picked depending on risk levels of the parties involved.

At this point, we would like to discuss the "robustness" of the cost subsidization scheme. Suppose that s_p, s_r values are chosen such that $\frac{(1- s_p)\, C_p}{(1- s_r)\, C_r} = \frac{R\, T_p + C_p}{R\, T_r + C_r}$, and the P value is selected so that the contractor's expected profit at the beginning of the horizon is equal to P_0 under an expected learning level of a, which means the contractor will accept the contract. Then, it can be shown that the contractor's and the manufacturer's optimal number of PM activities will always be the same, regardless of the *actual* learning level, which may turn out to be different than initially estimated. Since the initial P is a sunk cost, this dynamic "robustness" feature of the cost subsidization method ensures that the manufacturer's profit is maximized if the learning level changes from its initial value and/or the learning level is not constant from one PM to the next. On the other hand, a forcing contract which dictates the PM schedule to the contractor is a static contract and if the learning level differs from expectations, the manufacturer's profit may not be maximized.

Learning through Effort

In this section, we assume that instead of being experience based, learning takes place through costly effort. Specifically, we assume the contractor incurs a cost of $K(a)$ in an effort to achieve the learning level of a. This cost is an initial investment and is incurred only once through the contractual horizon. We assume that $K(0) = 0$ and $K'(a)$, $K''(a) > 0$; i.e. it costs nothing to learn nothing and the higher the learning level, the more relatively costly it becomes to achieve this level. Thus, $K(a)$ is an increasing convex function.

Under this scenario, the contractor's profit function will be similar to the one in Section 3.1 but now it will have the learning effort cost as well. The contractor's profit can be given by:

$$\Pi_{c1}(L,e) = \max_{N_c\, T_{c1}\, T_{c2} \ldots T_{cN}\, a_c} P - [(1-s_p)\, C_p] \sum_{i=1}^{N_c} i^{-a_c} - [(1-s_r)\, C_r] \sum_{i=1}^{N_c} M(T_{ci}) - K(a_c) \textbf{ such that } \sum_{i=1}^{N_c} i^{-a_c}\ T_p + \sum_{i=1}^{N_c} T_{ci} = Y$$

Lemma 7. For any value of N_c and a_c, the optimal PM intervals for the contractor that maximize $\Pi_{c1}(L,e)$ are equivalent. That is, $T_{c1}^* = T_{c2}^* \ldots = T_{cN}^* = T_c^*$.

As in natural learning and no learning, learning with costly effort also results in equal time intervals. Using this fact, the contractor's profit function can be rewritten as:

$$\Pi_{c2}(L,e) = \max_{N_c\, a_c} P - [(1-s_p)\, C_p] \sum_{i=1}^{N_c} i^{-a_c} - N_c\, [(1-s_r)\, C_r]\, M\left(\frac{Y}{N_c} - T_p \sum_{i=1}^{N_c} i^{-a_c} \frac{1}{N_c}\right) - K(a_c)$$

Again, using the approximation $\sum_{i=1}^{N_c} i^{-a} \approx \int_0^{N_c} i^{-a} di$, we solve for the contractor's optimal choice of PM frequency and effort level.

Lemma 8. i) Define $h(N(a)) = \left(\frac{Y}{N^{1-a}} - \frac{aT_p}{1-a}\right) \lambda \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) - N^{a} M \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right)$. Then, there is a unique $N_c(a_c)$, denoted by $N_c^{**}(a_c)$ and not necessarily an integer, that maximizes $\Pi_{c2}(L,e)$ for any given value of a_c, the effort level. $N_c^{**}(a_c)$ satisfies the following relationship:

$$h(N_c^{**}(a_c)) = \frac{(1-s_p)\, C_p}{(1-s_r)\, C_r}.$$

ii) Let $m_c(a(N)) = N^{1-a}[(1-a)\ln(N) - 1][(1-s_p)C_p - (1-s_r)C_r T_p \lambda \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right)] - (1-a)^2 K''(a)$.

Then, there is a unique a_c, denoted by $a_c^{**}(N_c)$ that maximizes $\Pi_{c2}(L,e)$ for any given value of N_c, the number of PM activities. $a_c^{**}(N_c)$ satisfies the following relationship: $m_c(a_c^{**}(N_c)) = 0$.

Lemma 8 states the properties of the "best response function" of N_c given any value of a_c and vice versa. To find the optimal N_c, a_c pair that maximizes $\Pi_{c2}(L,e)$, the following iterative algorithm can be utilized: 1) Set $a_c = 0$. 2)Find $N_c^{**}(a_c)$ from the first part of Lemma 8.
3) Find $a_c^{**}(N_c^{**}(a_c))$ from the second part of Lemma 8. 4) Go to Step 2 and find the new N_c^{**} value. If the difference between the previous N_c^{**} value and the new one is within a small range, say ε, then go to Step 5. Otherwise go to Step 3. 5) Let N_c^{***} denote the optimal but not necessarily integral N_c value. 6) The optimal N_c, a_c pair that maximizes $\Pi_{c2}(L,e)$, denoted by $N_c^*(L,e)$ and $a_c^*(L,e)$, will be either $\lfloor N_c^{***} \rfloor$ and $a_c^{**}(\lfloor N_c^{***} \rfloor)$ or $\lceil N_c^{***} \rceil$ and $a_c^{**}(\lceil N_c^{***} \rceil)$.

Similar to Section 3.1, we can rewrite the manufacturer's profit function as follows:

$$\Pi_{m2}(L,e) = R\, [Y - \frac{T_p N^{1-a}}{1-a})\,] - C_p \frac{N^{1-a}}{1-a} - N\, [C_r + R\, T_r]\, M\left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) - K(a) - P_0$$

Lemma 9. i) Using the definition of $h(N(a))$ from Lemma 8, there is a unique $N_m(a_m)$, denoted by $N_m^{**}(a_m)$ and not necessarily an integer, that maximizes $\Pi_{m2}(L,e)$ for any given value of a_m, the effort level that the manufacturer would like the contractor to choose. Then, $N_m^{**}(a_m)$ satisfies the following relationship: $h(N_m^{**}(a_m)) = \frac{R\, T_p + C_p}{R\, T_r + C_r}$.

ii) Let $m_m(a(N)) = N^{1-a}[(1-a)\ln(N)-1][RT_p+C_p - (RT_r+C_r)T_p \lambda \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right)] - (1-a)^2 K''(a)$.

Then, there is a unique a_m, denoted by $a_m^{**}(N_m)$ that maximizes $\Pi_{m2}(L,e)$ for any given value of N_m, the number of PM activities. $a_m^{**}(N_m)$ satisfies the following relationship: $m_m(a_m^{**}(N_m)) = 0$.

Lemma 9 describes the properties of the optimal effort level and optimal number of PM activities for the manufacturer. To find this unique optimal N_m, a_m pair that maximizes $\Pi_{m2}(L,e)$, denoted by $N_m^*(L,e)$ and $a_m^*(L,e)$, we can utilize an iterative algorithm very similar to the one used to find $N_c^*(L,e)$ and $a_c^*(L,e)$ above. The details, therefore, are omitted. Since the ($N_m^*(L,e)$, $a_m^*(L,e)$) and ($N_c^*(L,e)$, $a_c^*(L,e)$) pairs are unique, it follows that the manufacturer can maximize its profit only if $N_m^*(L,e) = N_c^*(L,e)$ and also $a_m^*(L,e) = a_c^*(L,e)$. The following Lemma gives the properties of s_p and s_r which achieve this goal:

Lemma 10. The contractor selects the PM frequency and the effort level pair that maximizes the manufacturer's profit only if $s_p = -\frac{R\, T_p}{C_p}$ and $s_r = -\frac{R\, T_r}{C_r}$.

This Lemma has four important and interesting implications. First, if we limit ourselves to s_p and s_r both being non-negative, then the manufacturer cannot achieve its first-best solution via the traditional cost subsidization method. Instead, it needs to use the "reverse" cost subsidization method, where the contractor *pays* the manufacturer every time it carries out any type of maintenance activity. This method may seem impractical but its effects can be negated by setting a very high P value and by scheduling this payment to coincide with PM activities. Therefore, the contractor could receive a positive cash flow during each PM and in the end, achieve its reservation profit of P_0. For this reason, we assume that the manufacturer would be able to install this "reverse" cost subsidization scheme.

The second implication of Lemma 10 is that unlike the results in Sections 2 and 3.1 with an infinite number of solutions, there is only one solution for the s_p, s_r pair. Therefore, when the learning requires a costly effort on the contractor's behalf, this method can achieve the first-best solution for the manufacturer only if both are risk neutral.

Third, if we look at the implied cost of each PM for the contractor, we see that the 1st PM costs $C_p + R\,T_p$, the second PM costs $C_p\,2^{-a} + R\,T_p\,2^{-a}$, the 3rd one costs $C_p\,3^{-a} + R\,T_p\,3^{-a}$, etc. The first term for each cost value represents the actual cost of carrying out PM, which is usually incurred by the contractor. The second term represents the cost of revenue loss due to downtime, usually a cost associated with the manufacturer. However, this "reverse" cost subsidization actually enables the manufacturer to pass this cost along to the contractor. This allows the manufacturer to align its goals with the contractor and thus, the manufacturer can achieve its first-best solution.

Finally, the optimal s_p, s_r values are independent of the effort level or its cost function. This means that there is no need to incur any administrative costs to actually observe and enforce a certain effort level. The effort need not be even observable; the manufacturer only needs to know the general cost function of the effort.

Similar to Sections 2 and 3.1, the following steps enable the manufacturer to achieve its first-best solution – that is, $N_m^*(L,e)$, $a_m^*(L,e)$ – assuming the "reverse" cost subsidization can be implemented: 1) Find $N_m^*(L,e)$, $a_m^*(L,e)$ via Lemma 9. 2) If $a_m^*(L,e) = 0$, then the problem reduces to the one in Section 2. In that case, follow the steps at the end of Section 2. 3) If $a_m^*(L,e) > 0$, then set $s_p = -\frac{R\,T_p}{C_p}$ and $s_r = -\frac{R\,T_r}{C_r}$. 4) Set $P = [RT_p+C_p]\frac{N^{1-a}}{1-a} + N\,[RT_r+C_r]\,M \left(\frac{Y}{N} - \frac{T_p N^{-a}}{1-a}\right) + P_0 + K(a)$, where N stands for $N_m^*(L,e)$ and a stands for $a_m^*(L,e)$.

Assuming these negative values of s_p and s_r are acceptable, it can be shown that, similar to the discussion at the end of Section 3.1, the cost subsidization scheme is robust. In other words, if the four-step system above is implemented, then the manufacturer's profit will still be maximized even if the learning level turns out to be different than initially estimated.

CONCLUSION

In this paper, we focused on learning in maintenance outsourcing. First, we analyzed a short-term service contract between a manufacturer and a maintenance contractor under no learning. We adopted the fixed payment plus cost subsidization scheme from [12]. Similar to finite term EOQ models in production, we found that both parties' profits are maximized with equal preventive maintenance intervals. Next, we considered learning in preventive maintenance (PM) activities. We assumed two types of learning: experience based and investment based.

Under the experience based learning – the most popular learning model in academic literature – the contractor, through repetition, learns to carry out the PM activities more efficiently; i.e. faster and at a lower cost. We found that the manufacturer can utilize the cost subsidization scheme to maximize its profit. More interestingly, by setting the parameters of the contract carefully, the manufacturer can be assured that the contractor will still choose the PM schedule that maximizes the manufacturer's profit even if the learning level differs from initially expected values. Therefore, the robustness of the cost subsidization scheme can be very useful and makes it a very attractive tool for the manufacturer. A strictly buying contract such as a forcing contract that dictates the PM schedule to the contractor does not have this dynamic and adaptive quality.

With investment based learning, we assumed the contractor exerts costly effort to improve the PM efficiency. Therefore, there is a trade of between the cost of effort and the savings in cost and time for the PM activities. We found that with investment based learning, the cost subsidization scheme guarantees to maximize the manufacturer's profit with only one set of parameters. These parameters actually imply a *negative* cost subsidization; i.e. the contractor, rather than the manufacturer, makes a payment when a maintenance activity takes place! This may not seem very plausible; however, we are confident it can be taken care of by making the fixed payment from the manufacturer to the contractor coincide with or contingent on the maintenance activity. We also found that the cost subsidization method is robust under investment based learning as well; i.e. if the learning parameters change, the manufacturer can still maximize its profit with the existing contract. The cost subsidization scheme has an additional benefit under investment based learning: The manufacturer does not need to observe or verify the effort level of the contractor; the contract guarantees that the contractor's own optimal effort level will also maximize the manufacturer's profit. This feature can be especially useful if the effort level is unobservable or unenforceable due to legal issues.

[12] show that cost subsidization scheme is effective in coordinating the service chain. In this paper, we found that it also works very well in terms of maximizing the manufacturer's profit. In addition, it has a very attractive feature that guarantees the manufacturer's profit will still be maximized even with a changing learning level. The fact that it is easy to implement and does not require too much administration costs or duties is also useful.

We would like to close with further research possibilities: One immediate follow up is integrating forgetting into the model similar to what has been done in production literature [34]. It would also be interesting to analyze different payment schemes and how they compare with the cost subsidization scheme.

For a list of references and proofs, please contact Hakan Tarakci at h.tarakci@mbs.edu.