

Control Under Disagreement

—Preliminary and Incomplete—

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Abstract

In this paper, I study the effects of overconfidence in an investment-decision agency setting. Principal and agent agree on some investment rule that is contingent on a public signal. In a standard common-priors setting, the optimal contract provides full insurance to the agent: the principal pays a fixed wage to the agent and implements the efficient investment rule. When the agent overestimates his ability (the expected revenue of the project following a decision to invest), however, he is willing to “wager” on success against the relatively pessimistic principal and hence bears some project risk in equilibrium. In addition, because what the principal considers to be the optimal investment rule is too conservative according to the agent’s beliefs and the agent holds some stake in the project, he will accept a lower fixed payment in exchange for a more liberal investment rule. A straightforward interpretation of this result is that the principal is transferring some control to the agent. It is somewhat counterintuitive that the principal will surrender more control to an agent with whom she disagrees more sharply.

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1 Introduction

An important function of a firm’s manager is to guide investment decisions; a “good” manager is likely to be better at implementing potential investment projects than others. Consider the following setting, in which a risk-averse manager is hired by a risk-neutral owner. Imagine that owner and manager can define an investment rule that will depend on the information about investment projects available. In this principal-agent setting, under the assumption that manager and owner agree about the distribution of profits conditional on the information, the manager’s risk aversion does not pose an incentive problem. The owner can fully insure to the manager (i.e. offer him a fixed payment), and simply implement the investment rule that she deems optimal.

The common-priors assumption implicit in this logic is a strong one, however—in particular when the manager has some stake in the outcome of the investment decision. Studies in psychology suggest that individuals tend to be overly optimistic when evaluating their own skills and ability, and the likelihood of favorable outcomes. This bias seems to be stronger when evaluating outcomes that affect their own well being (Taylor and Brown (1988) is a good survey of the findings in psychology research regarding cognitive biases and well being). If the manager in our example overestimates his ability to implement a given investment project relative to the owner’s beliefs and if his payment can be made contingent on the project’s outcome following investment, then he will not receive full insurance in equilibrium. In this case, the manager will in fact “wager” to some extent with the owner, so that he receives outcome-contingent pay in equilibrium. For this reason, the manager’s well being is directly affected by the project’s outcome following investment, and therefore by the owner’s decision of whether or not to invest. Disagreement about the expected returns to investment most likely implies disagreement about the optimal course of action. As a result, the manager may be willing to “pay” for some degree of control over the investment decision. In contrast to the common-priors setting, in the presence of heterogeneous beliefs the investment rule implemented in equilibrium will almost surely not be the efficient investment rule according to the owner’s beliefs. We will characterize two sources of deviation from that rule.

Section 2 introduces a simple model that will allow us to explore the effects of agent overconfidence in an investment-decision setting. The structure of the model is similar to the one introduced by Holmström and Ricart i Costa (1986). Principals compete to hire the agent by making simultaneous contract offers. Agent and principals will observe some (verifiable) signal that is correlated with the expected profits of a potential investment project. A contract offer, thus, specifies both the payment scheme and the investment rule that will be followed (i.e. whether or not investment is undertaken), possibly contingent on the realization of such a signal. The expected revenue of the project depends stochastically on both the public signal and on the agent’s ability (which is un-

known to both parties, à la Holmström (1982)). Following the realization of the signal, investment is undertaken according to the contracted investment rule. Once final investment revenue is realized, payments are distributed according to the terms of the contract, and the agency relationship ends.

Section 3 isolates the issue of the equilibrium payment scheme from the issue of the equilibrium investment rule (which we incorporate in Section 4). In a common-priors setting, the equilibrium payment scheme is very simple: the principal offers full insurance to the agent, so that the agent is paid his expected productivity. We will allow principal and agent to hold heterogeneous beliefs, in particular about the agent's ability (i.e. about the distribution of revenue conditional on each signal realization). In equilibrium, the agent receives a higher payment following those realizations of revenue that he deems more likely than the principal does, and in return accepts a lower payment following those realizations that he believes are relatively unlikely. Even though the agent is risk averse, some degree of "wagering" of this kind will always be observed in equilibrium. A corollary of this result is that a more overconfident agent will tend to receive a higher payment in the event that no investment is made because of insurance motives. An overconfident agent overestimates his expected payment and his expected utility following the decision to invest; because he disagrees with the principal about his (unknown) ability but not about the ex-ante distribution of the signal, he receives as much insurance as possible in terms of signal variability. Wishing to smooth his consumption across "invest" and "do not invest" events, a more overconfident agent will receive a higher payment conditional on no investment, consistent with his expectation of higher payment if investment is undertaken.

Section 4 includes the investment rule as part of the optimization problem implicit in the equilibrium contract offer. Given that the agent receives full insurance in a common-priors setting, he is indifferent as to the implemented investment rule in that case. The principal thus implements the rule that she considers optimal, which, given her risk neutrality, is in fact the efficient rule. When allowing for heterogeneous beliefs, the agent's payment is contingent on the outcome of investment whenever it is undertaken because of Pareto-optimal wagering. Two sources of distortion (relative to the efficient rule according to the principal's beliefs) affect the equilibrium investment rule. First, note that wagering affects the principal's evaluation of the investment rule. If the expected payment to the agent conditional on investment is higher than the no-investment payment according to the principal's beliefs, then the principal is relinquishing some of the marginal benefits of investment to the agent; if it is lower, the principal's benefits from investment include savings in expected remuneration to the agent. As a consequence, the principal's objective function will in general not be aligned with efficiency. Second, an overconfident agent believes that his *expected utility*

will be higher conditional on investment than when no investment is made, so he will be willing to forego some of his remuneration in exchange for a more liberal investment rule. The agent is “willing to pay for control” in that case, and the implemented investment rule will in fact be skewed away from what the principal would optimally choose if her decision was unilateral and non-contractible—towards what the agent believes to be the optimal investment rule.

Section 5 provides an example of the equilibrium contract when the agent has an exponential utility function, and the productivity shocks are normally distributed. This purposefully simple example further illustrates the intuition behind the results of the model.

Section 6 concludes and discusses a testable implication of the presence of overconfidence in an expert-advice setting, given that firms tend to rely on both outsiders’ and insiders’ advice to guide their investment decisions (e.g. consultants and managers).

2 The Model

Consider a simple model, in which a principal and an agent wish to decide whether to invest in a potential project. We will follow the basic model structure used by Holmström and Ricart i Costa (1986), applied to a one-period setting with public information and allowing for the agent’s payment to be contingent on outcome.¹

Assume that several principals have access to investment projects, and compete to contract with one agent.² The timing of the model is as follows: first, principals make simultaneous contract offers to the agent. A contract offer consists of an investment rule and a payment schedule to the agent. The agent then chooses which offer (if any) to accept. If the agent chooses to accept a contract offer, signal s is realized. This signal is public and verifiable, and is correlated with the project’s profitability. Investment then is undertaken (or not) according to the agreed-upon investment rule.

¹In Holmström and Ricart i Costa (1986), the signal is private information to the agent, who can then choose to report it to the principal. There is no possibility of misrepresenting the information, so the implied “veto power” assigned to the agent by their assumption affects our results only in terms of a “bonus for investment” transfer to ensure reporting. The results are more transparent (and the main message of the model clearer) when we simply make the signal verifiable. The possibility of information misrepresentation, on the other hand, is an interesting and relevant alternative to our specification. Stocken (2000), for example, studies the credibility of information disclosure in a repeated cheap-talk game. Given that our focus right now is on the effects of disagreement, however, the simplification of making information verifiable allows us to separate them from other channels.

²The results in a setting where one principal makes a take-it-or-leave-it offer to an agent are nearly identical. These are dual problems, with the main difference being who retains the expected gains from trade, as was pointed out in (de la Rosa, 2006, Chapter 1). Different strands of the agency literature have traditionally converged to either of these settings; we follow Holmström and Ricart i Costa (1986) and use the setting studied in most career-concerns problems.

The profitability of the project also depends on the agent’s ability. In particular, we will assume that the project’s revenue y , conditional on a decision to undertake investment, can be written as

$$y = s + \theta,$$

where s has cumulative distribution function G , and the distribution of θ depends on the agent’s ability—which is unknown to both parties. If investment is undertaken, θ is then realized and correctly inferred by both parties (given that project profitability is publicly observed). Payoffs are then distributed according to the agreed-upon payment schedule, and the agency relationship ends. If the agent chooses not to accept any contract, no project will be undertaken, and the players receive payoffs according to their outside option. I assume that the agent’s outside option is low enough so that he always accepts some contract offer in equilibrium.

Under the assumption of common priors (which we will relax below), principal and agent share the belief that θ has cumulative distribution function F with mean μ .³ The agent can decide whether or not to report the true value of s (which can be interpreted as whether or not to “recommend” investment), but cannot misrepresent information in the report. The principal can always guarantee zero revenue by not investing. The principal is risk neutral, and his objective is thus to maximize expected profits:

$$\max \mathbb{E}[y - w]$$

where w stands for the agent’s “wage”—a money transfer to the agent. The agent is risk averse, and his objective is to maximize his expected utility of consumption: $\max \mathbb{E}[u(c)]$. Assume that the agent’s only income is his wage, so that the agent’s objective can be expressed as

$$\max \mathbb{E}[u(w)].$$

The equilibrium contract solves

$$\begin{aligned} & \max \mathbb{E}[u(w)] \\ & \text{s.t. } \mathbb{E}[y - w] \geq 0. \end{aligned}$$

Note that the non-negative expected profits restriction must bind in equilibrium: if a principal made positive expected profits in equilibrium, another principal could outbid the “equilibrium”

³Holmström and Ricart i Costa (1986) assume that the agent can be either talented or not, and the prior probability of the agent being talented is p . In their model, $\theta \sim G$ with mean μ_G if the agent is talented (G for “good”), and $\theta \sim B$ with mean μ_B if the agent is not talented, where $\mu_G > \mu_B$. This specific form is one way to parametrize agent ability, particularly useful in a career-concerns setting; the generalization that $\theta \sim F$ with mean μ allows for a more intuitive presentation of the results of this paper.

contract by offering δ more to the agent in every state of the world, attract the agent, and earn positive expected profits.

Let $\alpha(s) \in \{0, 1\}$ denote the investment rule implemented following announcement s from the agent. As we will see below, in a common-priors setting, the information asymmetry does not pose an incentive problem, so that the efficient investment rule $\alpha^*(s)$ (which is optimal from the principal's perspective) is implemented in equilibrium. Investment is efficient if and only if the project yields positive expected profits conditional on the realization of the signal. That is, $\alpha^*(s) = 1$ if and only if

$$\mathbb{E}[y \mid s] \geq 0,$$

which implies

$$s + \int_{\Theta} \theta \, dF \geq 0,$$

and thus

$$s + \mu \geq 0.$$

The efficient investment rule can also be expressed as a “hurdle rate” $s^*(\mu)$, so that the project is undertaken if and only if $s \geq s^*(\mu)$. $s^*(\mu)$ satisfies

$$s^*(\mu) + \mu = 0.$$

It follows that the equilibrium contract solves

$$\begin{aligned} & \max_{s^*(\mu), w} \mathbb{E}[u(w)] \\ \text{s.t. } & \mathbb{E}[w] = (\mathbb{E}[s \mid s \geq s^*(\mu)] + \mu) \cdot \Pr(s \geq s^*(\mu)). \end{aligned}$$

When principal and agent agree on their beliefs regarding the distribution of θ , a fixed-wage contract is the equilibrium contract. The agent receives the full expected productivity of his private information, and, given that he is fully insured, from his point of view the setting of investment rule is irrelevant.

We will subsequently allow for the possibility that principal and agent hold heterogeneous beliefs about the distribution of θ . It may be useful to keep in mind the case in which the agent overestimates his expected productivity, as we should expect given the research in psychology regarding self-serving biases (see Taylor and Brown (1988)). Let a tilde denote the agent's beliefs: the agent holds the prior belief that θ has a cumulative distribution function \tilde{F} with mean $\tilde{\mu}$. We will say that the agent is *overconfident* in this setting if

$$\tilde{\mu} > \mu,$$

which means that an overconfident agent overestimates the expected return of an investment project, conditional on any given signal, relative to the principals' beliefs.⁴

3 Compensation

To isolate the consequences of heterogeneous beliefs on the contracted payment schedule from the issue of control (the negotiated investment rule), assume for now that the principal always implements the investment rule which is efficient according to her beliefs. The investment rule $\alpha^*(s)$ and respective hurdle rate $s^*(\mu)$ thus remain unchanged from the common-priors setting discussed in Section 2 above. Note that this investment rule seems too conservative from the agent's point of view: for some realizations of s , the principal chooses not to invest even though the project is profitable according to the agent's beliefs. If the agent's remuneration depends on the profitability of projects that are undertaken in equilibrium, it is possible that he would be willing to "pay" the principal in exchange for some decision power. We will address this question in Section 4.

Recall that the equilibrium contract solves

$$\begin{aligned} \max \tilde{\mathbb{E}}[u(w)] \\ \text{s.t. } \mathbb{E}[y - w] \geq 0, \end{aligned}$$

where the tilde over the first expectations operator points to the fact that the agent is interested in maximizing his perceived expected utility, which depends on his beliefs rather than the principal's. As noted before, the non-negative expected profits constraint binds, so we can write

$$\mathbb{E}[w] = (\mathbb{E}[s \mid s \geq s^*(\mu)] + \mu) \cdot \Pr(s \geq s^*(\mu)).$$

In general, a contract offer will be characterized by an investment rule $\alpha(s)$ and payment to the agent $w(\alpha(s), s, \theta)$. We can restrict attention, without loss of generality, to a wage function of

⁴There are other dimensions over which the beliefs of principal and agent could differ. One could think about an agent who is "overconfident about the precision of θ ." The effects of this type of overconfidence will become apparent when we characterize the equilibrium contract allowing for agent and principal to evaluate the likelihood of a given realization of θ according to different distribution functions in Section 3. The agent could also be overconfident about the distribution of s . In this case the agent's payment would depend on s in equilibrium. He would be willing to trade a higher payment under those realizations of s that he believes to be more likely than the principal does, in return for lower payment under those realizations of s that he believes to be relatively less likely. Absent heterogeneous beliefs about the distribution of θ , full insurance with respect to the realization of θ will still be optimal. I abstract from the possibility of disagreement about the distribution of s because it would necessarily distort the principal's objective function away from efficiency when choosing $\alpha(s)$, adding one more source of distortion to the two we uncover in Section 4.

the form

$$w = \beta + \sigma(\theta),$$

noting that θ will be realized only if the project is undertaken (if $\alpha(s) = 1$). Given that θ cannot be observed if no investment is made, $w(\alpha(s), s, \theta) = w(\alpha(s), s', \theta)$ whenever $\alpha(s) = \alpha(s') = 0$, so we can write $w = \beta$ whenever no investment is made. It is also true that $w(\alpha(s), s, \theta) = w(\alpha(s), s', \theta)$ for all θ (almost everywhere) whenever $\alpha(s) = \alpha(s') = 1$. This follows from the agent's risk aversion, independence between s and θ , and the fact that principal and agent agree about the distribution of s . Intuitively, it is costless for the principal to insure the agent against variability in s , and given that principal and agent disagree about the distribution of profits only insofar as they disagree about the distribution of θ , it is optimal for the risk-neutral principal to absorb all the risk from variability in s following investment.⁵

We can then rewrite the principal's zero-expected-profits condition as

$$\begin{aligned} \beta &= \int_{s^*(\mu)}^{\infty} \int_{\Theta} (s + \theta - \sigma(\theta)) dF dG \\ &= \left(\mathbb{E}[s \mid s \geq s^*(\mu)] + \mu - \int_{\Theta} \sigma(\theta) dF \right) \cdot \Pr(s \geq s^*(\mu)). \end{aligned}$$

In equilibrium, $\sigma(\cdot)$ is Pareto optimal from the perspective of securities trading under uncertainty given heterogeneous beliefs:

$$u'(\beta + \sigma(\theta)) \frac{\tilde{f}(\theta)}{f(\theta)} = \lambda \quad \forall \theta,$$

where λ is some constant. This is simply an application of the Borch (1962) rule for optimal risk-sharing, allowing for heterogeneous beliefs. The agent receives a higher payment following realizations of θ that he believes are more likely than the principal does. In the extreme case of a risk-neutral agent (and principal), the agent would hold an infinitely "long" position in those states he believes are strictly more likely than the principal does, and an infinitely "short" position in those states he believes are strictly less likely than the principal does. The agent's risk aversion, however, limits the extent to which he is willing to wager against the principal. It is interesting to note that if the agent believes that high realizations of θ are more likely than the principal does, then the equilibrium contract will resemble an incentive contract (in the sense that the agent is paid

⁵To see why, consider the following example. For simplicity, imagine that s and θ are discretely distributed. Consider two realizations of the signal, s' and s'' , such that $\Pr(s') > 0$, $\Pr(s'') > 0$. Assume that $w(s'', 1, \theta') > w(s', 1, \theta')$ for some realization θ' of θ such that $\Pr(\theta') > 0$ according to both the principal's and the agent's beliefs. If the principal offers more insurance to the agent in an actuarially fair manner (reducing $w(s'', 1, \theta')$ and increasing $w(s', 1, \theta')$ marginally so that expected profits to the principal remain constant), the risk-averse agent's perceived expected utility will increase, independent of what principal and agent believe to be the actual likelihood of θ' .

more for higher realizations of profit) even though there is no moral hazard or adverse selection in our model.

We can further state that

$$u'(\beta + \sigma(\theta)) \frac{\tilde{f}(\theta)}{f(\theta)} = u'(\beta).$$

Rearranging this expression and integrating with respect to θ , we find that

$$\int u'(\beta + \sigma(\theta)) \tilde{f}(\theta) d\theta = u'(\beta),$$

or

$$\tilde{\mathbb{E}}[u'(\beta + \sigma(\theta))] |_{\alpha(s)=1} = u'(\beta).$$

Because agent and principal do not disagree about the distribution of the signal s , the agent is offered as much insurance as possible (in terms of his marginal utility of consumption) across the “invest” and “do not invest” events, which depend solely on the realization of s .⁶

These results are summarized in the following proposition.

Proposition 1 *Under heterogeneous beliefs, and assuming that $s^*(\mu)$ is the hurdle rate guiding investment decisions, in equilibrium the agent receives θ -contingent payment $w = \beta + \sigma(\theta)$, where*

1. $\beta = \int_{s^*(\mu)}^{\infty} \int_{\Theta} (s + \theta - \sigma(\theta)) dF dG$, and
2. $u'(\beta + \sigma(\theta)) \frac{\tilde{f}(\theta)}{f(\theta)} = u'(\beta) \quad \forall \theta$, so that $\tilde{\mathbb{E}}[u'(\beta + \sigma(\theta))] |_{\alpha(s)=1} = u'(\beta)$.

The proof follows immediately from constrained maximization of the agent’s perceived expected utility subject to zero expected profits for the principal.

4 Control

Throughout the previous section, we assumed that the implemented investment rule was the efficient rule according to the principal’s beliefs, $\alpha^*(s)$. From the agent’s point of view, this investment rule is suboptimal. Given that the agent’s payment depends only on the realization of θ , the realization of s does not directly affect his utility. The investment rule that is implemented, however, affects the principal’s evaluation of expected revenue and thus expected payment to the agent. The investment rule therefore affects the agent’s perceived expected utility associated with a given contract offer, so $\alpha^*(s)$ might not be implemented in equilibrium.

⁶If principal and agent disagreed about the ex-ante distribution of the signal s , they would wager on the realization of s just like they wager on the realization of θ . Since s is observed at some point, however, disagreement about θ is what matters in terms of the investment rule.

If principal and agent disagree about expected revenue following investment conditional on a given signal s , they will disagree about which is the efficient investment rule. As discussed before, after observing signal s it is efficient to invest in the project whenever expected revenue from investing, conditional on the available information, is non-negative. This condition is met whenever

$$s + \mu \geq 0$$

according to the principal's beliefs, and whenever

$$s + \tilde{\mu} \geq 0$$

according to the agent's beliefs. In the case of agent overconfidence, so that $\tilde{\mu} > \mu$, there will be a range of signals over which principal and agent disagree about whether or not it is efficient to invest. Principal and agent agree that it is efficient to invest whenever $s \geq -\mu$, and that it is efficient not to invest whenever $s < -\tilde{\mu}$. For those realizations of s such that $-\tilde{\mu} \leq s < \mu$, however, the principal believes that it is best not to invest, while the agent believes that it is efficient to invest. Under heterogeneous beliefs, payment to the agent depends on the realization of θ following investment. It follows that the implemented investment rule may affect the agent's evaluation of his expected utility. We characterize the equilibrium contract when the investment rule is contractible in the following proposition.

Proposition 2 *The equilibrium investment rule can be characterized by a "hurdle rate" $\hat{s}(\mu)$, so that $\alpha(s) = 1$ if and only if $s \geq \hat{s}(\mu)$ and $\hat{s}(\mu)$ is implicitly defined by*

$$\hat{s}(\mu) + \mu - \int_{\Theta} \sigma(\theta) dF + \frac{1}{u'(\beta)} \left\{ \int_{\Theta} u(\beta + \sigma(\theta)) d\tilde{F} - u(\beta) \right\} = 0,$$

where β and $\sigma(\theta)$ are defined as in Proposition 1 substituting $\hat{s}(\mu)$ for $s^*(\mu)$.

The proof follows immediately from including the hurdle rate $\hat{s}(\mu)$ as a control variable in the problem of maximizing the agent's perceived expected utility, subject to zero expected profits for the principal and the full-revelation constraint.

First, note that the principal finds it in her best interest to implement what she believes to be the efficient rule only if she retains full stake (on average) in the project. In the case of common priors, given that the agent is offered full insurance in equilibrium, the principal receives the full marginal benefit from investing. Because of this, and given that the principal is risk neutral, her objective function is aligned with efficiency. If the principal relinquishes some stake in the project to the agent, the optimal investment rule from the principal's point of view will no longer be the one that she considers socially efficient.

Consider the investment rule that the principal would implement if she had full control over the investment decision, and she could not commit *ex-ante* to implementing any given investment rule. This would imply that the principal could not negotiate with the agent about the investment rule; absent commitment, the principal will implement the investment rule that maximizes her expected profits conditional on s . Because equilibrium requires zero expected profits for the principal, this investment rule maximizes the fixed payment β to the agent. Recall that zero expected profits for the principal implies

$$\beta = \int_{s^*(\mu)}^{\infty} \int_{\Theta} (s + \theta - \sigma(\theta)) dF dG$$

given hurdle rate $\hat{s}(\mu)$. Let $s^{*P}(\mu)$ denote the optimal hurdle rate from the principal's point of view—which allows her to maximize the fixed payment β to the agent—implicitly defined by

$$\frac{d\beta}{d\hat{s}(\mu)} = - \int_{\Theta} (s^{*P}(\mu) + \theta - \sigma(\theta)) dF = 0.$$

In the case of common priors, the agent is fully insured so that $\sigma(\theta) = 0$ for all θ , and the above expression simply yields $s^{*P}(\mu) = s^*(\mu) = -\mu$. If the principal gives some positive- or negative-expected value stake in the project to the agent, so that $\int_{\Theta} \sigma(\theta) dF \neq 0$, it follows that $s^{*P}(\mu) \neq s^*(\mu)$. Intuitively, when the principal relinquishes some of her claim over the project's outcome, the optimal investment rule according to her beliefs skews away from efficiency. Imagine, for instance, the extreme case in which the principal gives all the marginal benefits from the project to the agent and simply insures him against variation in s , so that $\sigma(\theta) = \theta$. Ex-post, the principal would like to invest only whenever her share in the project yields positive profits, so she would set $s^{*P}(\mu) = 0$. This distortion will be of particular interest in settings in which the agent's payment is bounded below (e.g. limited liability, or if the agent's remuneration includes an option on firm value).

There is a second source of distortion in the equilibrium investment rule, which depends on the agent's evaluation of the events "investment" and "no investment." If $\int_{\Theta} u(\beta + \sigma(\theta)) d\tilde{F} > u(\beta)$ in equilibrium the agent believes he is strictly better off following a decision to invest than when the contract calls for no investment. As a consequence, the agent *de facto* gives up some of his fixed payment β in return for a more liberal investment decision rule (compared to $s^{*P}(\mu)$). Consider the particular case in which the principal retains full stake in the project, so that $\int_{\Theta} \sigma(\theta) dF = 0$ in equilibrium and $s^{*P}(\mu) = s^*(\mu)$; there is no distortion of the principal's objective away from efficiency. If the principal can commit to implementing a given investment rule and $\int_{\Theta} u(\beta + \sigma(\theta)) d\tilde{F} > u(\beta)$, however the equilibrium investment rule will be $\hat{s}(\mu) < s^*(\mu)$. The opposite, of course, will hold if $\int_{\Theta} u(\beta + \sigma(\theta)) d\tilde{F} < u(\beta)$ in equilibrium; however, one should be able to show that under fairly

general assumptions overconfidence implies $\int_{\Theta} u(\beta + \sigma(\theta)) d\tilde{F} > u(\beta)$.⁷

A risk-averse overconfident agent who is not fully insured believes that he *does* hold positive stake in the project. Equilibrium should require this to be the case: given that the agent is not fully insured, we should expect $\int_{\Theta} \sigma(\theta) d\tilde{F} > 0$. Without giving more structure to the model, we cannot say anything about the sign of $\int_{\Theta} \sigma(\theta) dF$: given that the agent is paid more (a higher $\sigma(\theta)$) under those realizations that he believes more likely relative to the principal's beliefs, it follows that $\int_{\Theta} \sigma(\theta) d\tilde{F} > \int_{\Theta} \sigma(\theta) dF$, so $\int_{\Theta} \sigma(\theta) dF$ may be positive or negative. Along the lines of the speculation in the previous paragraph, we should expect that $\int_{\Theta} \sigma(\theta) dF < 0$ when the principal is pessimistic relative to the agent; wagering is good to both principal and agent from judging from their respective beliefs, and as such should make investment more attractive to both.

In the case of common priors, risk aversion implies that $\int_{\Theta} u(\beta + \sigma(\theta)) dF < u(\beta)$ whenever $\int_{\Theta} \sigma(\theta) dF = 0$ and $\sigma(\theta) \neq 0$ for some non-measure-zero subset of possible realizations of θ . This is why full insurance is optimal in that case, and the efficient investment rule—which is optimal from the principal's point of view—is implemented in equilibrium. If the agent is sufficiently overconfident (or has enough risk tolerance), we should expect that $\int u(\beta + \sigma(\theta)) d\tilde{F}(\theta) > u(\beta)$ in equilibrium. The principal will then give some degree of control to the agent, in the sense that the investment rule implemented in equilibrium will be skewed away from what the principal would unilaterally choose towards a more liberal investment rule, which is preferred by the agent.

5 Example

Consider the particular case in which the agent has a negative-exponential utility function, so that his utility can be expressed as

$$u(w) = -\exp^{-rw},$$

and normally-distributed productivity shocks θ . According to the principal's beliefs, θ has mean μ_{θ} , and mean $\tilde{\mu}_{\theta} > \mu_{\theta}$ according to the agent's beliefs; assume that both participants believe that θ is distributed with precision h_{θ} .

Recall that because of the insurance motive

$$\int_{\Theta} u'(\beta + \sigma(\theta)) d\tilde{F} = u'(\beta).$$

The agent wishes to smooth his (expected) marginal utility of consumption across “invest” and “do

⁷Defining overconfidence with the stronger statement that \tilde{F} first-order stochastically dominates F (and not just $\tilde{\mu} > \mu$) should be sufficient for this to be the case.

not invest” events.⁸ When the agent’s utility function is negative exponential, we have that

$$u'(w) = r \exp^{-rw},$$

which is simply a linear transformation of $u(w)$. Smoothing marginal utility thus also implies smoothing total utility for the agent:

$$\int_{\Theta} u(\beta + \sigma(\theta)) d\tilde{F} = u(\beta)$$

where β and $\sigma(\theta)$ maximize the agent’s perceived expected utility subject only to non-negative expected profits for the principal.

The equilibrium contract is thus characterized by

$$u'(\beta + \sigma(\theta)) \frac{\tilde{f}(\theta)}{f(\theta)} = u'(\beta).$$

Solving for $\sigma(\theta)$ we find

$$\sigma(\theta) = \frac{1}{r} (\tilde{\mu}_{\theta} - \mu_{\theta}) h_{\theta} \left[\theta - \frac{\tilde{\mu}_{\theta} + \mu_{\theta}}{2} \right],$$

a linear, increasing function of θ . As should be expected, the degree of “wagering” on the realization of θ increases with the degree of disagreement between principal and agent regarding the mean of θ ($\tilde{\mu}_{\theta} - \mu_{\theta}$), with the precision of θ (h_{θ}), and also with the agent’s tolerance for risk (measured in this case by $\frac{1}{r}$). The average of the two means, $\frac{\tilde{\mu}_{\theta} + \mu_{\theta}}{2}$, is the one realization of θ for which $\tilde{f}(\theta) = f(\theta)$; $\tilde{f}(\theta) > f(\theta)$ for any realization of θ above that, and $\tilde{f}(\theta) < f(\theta)$ for any realization below that.

In this particular case, we also find that

$$\int_{\Theta} \sigma(\theta) dF = - \int_{\Theta} \sigma(\theta) d\tilde{F} = - \frac{1}{r} h_{\theta} \frac{(\tilde{\mu}_{\theta} - \mu_{\theta})^2}{2} < 0.$$

Given that $\int_{\Theta} \sigma(\theta) dF < 0$, we know that the equilibrium investment rule is more liberal than what the socially efficient rule would be according to the principal’s beliefs. Even though the agent is indifferent in equilibrium between “invest” and “do not invest” events, the contract is such that the principal expects to pay less to the agent (on average) in the event that an investment is made than if no investment is made. She will therefore choose to invest in some instances when she expects the financial returns to investment to be negative, because they are overturned by savings in terms of lower expected payment to the agent.

Consider now the case of agent overconfidence regarding the precision of θ rather than its mean. Assume that the principal believes θ to be normally distributed with mean μ_{θ} and precision h_{θ} , while the agent believes it to have mean μ_{θ} and precision $\tilde{h}_{\theta} > h_{\theta}$. In this case, in equilibrium

$$\sigma(\theta) = \frac{1}{r} \left(\ln \tilde{h}_{\theta} - \ln h_{\theta} \right) - \frac{1}{2r} \left(\tilde{h}_{\theta} - h_{\theta} \right) (\theta - \mu_{\theta})^2.$$

⁸As noted before, this result follows from agent and principal holding identical beliefs regarding the distribution of the signal s .

Intuitively, an agent who overestimates the precision of θ relative to the principal will be willing to accept a lower payment following extreme realizations of θ in exchange for higher payment following realizations of θ close to μ_θ .

6 Concluding Remarks

When principal and agent disagree about the distribution of profits following investment, the risk-averse agent bears project risk in equilibrium. Because the agent is not fully insured in equilibrium, there are two sources of distortion of the implemented investment rule away from what the principal considers the efficient investment rule. First, if payments to the agent are higher (or lower) in expectation following a decision to invest than when no investment is made according to the principal's beliefs, the principal's objective function is skewed away from efficiency. In the example in Section 5, the principal expected to pay less to the agent under those events in which investment was undertaken. As a consequence, the optimal investment rule from the principal's point of view dictated for investment in some situations in which she believed investment to be inefficient. The second source of distortion comes from the agent's evaluation of his payments. If the agent believes his expected utility following investment is strictly higher than when no investment is made, then he will be willing to "pay" the principal to implement investment more often. This could be interpreted as the principal giving the agent some degree of control over the investment decision, even though they disagree about the best course of action.

This result is of interest in the context of previous literature. It has similar implications to those pointed out by Gervais, Heaton, and Odean (2003) and Van den Steen (2005): in the presence of agent overconfidence, pay-for-performance incentive contracts create distortions in the allocation rules implemented in an agency relationship. One important point of this paper, however, is that pay-for-performance is an equilibrium consequence of the disagreement between principal and agent, as shown in Section 3. One way to approach the intuition is that the principal, when designing the contract, is faced with a dilemma: higher-powered incentives introduce a costly distortion in terms of efficiency, but they result in savings in terms of expected payment to the agent. So even though it is true that incentive contracts imply inefficiency under some measures, to take the further step and suggest that incentives should be less powerful because of overconfidence would be naïve.

The model has some testable implications in terms of outsourcing of project implementation. Advice is obtained by firms both from outsiders (e.g. a consultant) and insiders (e.g. an in-house expert). Because of the type of contracts that tend to characterize consulting relationships (for one, consultants tend to be hired on a case-by-case basis; employees tend to make a career in a given

firm), there are more constraints on the pay to outsiders as to the extent that pay-for-performance can depend on outcomes, both in terms of size and time period. The results outlined in this paper imply that the expected profit (before subtracting payment to the agent) of projects undertaken following outsiders' expert advice should be higher than the expected profit following insiders' expert advice. In first light, one could (erroneously) infer from such asymmetry that outsiders tend to be more knowledgeable than insiders in implementing projects. If some firms do make such an inference, outsiders would be hired even in some situations in which an insider would be the better choice. Further research in this area could yield insight into corporate decision-making structures.

A natural extension to the model is to consider the effects of heterogeneous beliefs in the presence of career concerns. In a multi-period setting, and when the agent cannot bind himself to the agency relationship (i.e. when the agent cannot commit not to quit and be employed by another principal in subsequent periods), he will necessarily bear some reputational risk: investment outcome is informative about the agent's ability, and this information will be incorporated into competing principals' contract offers after the first period. A risk-averse agent might therefore be reluctant to share his private information in early periods. An agent who is overconfident about his ability relative to the principals' beliefs, on the other hand, believes that principals are likely to update their beliefs upwards following investment. An overconfident agent therefore overestimates the benefits of first-period investment in terms of his second-period remuneration. Because of this, overconfidence is likely to mitigate the agency problems related to agent risk aversion in a career-concerns setting, even though an overconfident agent bears more risk in equilibrium. This is the focus of de la Rosa (2007).

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