THE USE OF PREDICTION IN MATHEMATICS CLASSROOMS

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The role of prediction in the teaching and learning of mathematics has not received much attention in the field of mathematics education. In this paper three reasons for conducting research related to use of prediction in mathematics classrooms are discussed: students' prediction can reveal their conception, prediction plays an important role in reasoning, and prediction facilitates mathematical learning. Three perspectives on prediction are offered as an initial framework for research on prediction: prediction as a mathematical task, prediction as a mental act, and prediction as a social practice. A discussion group is formed to provide an avenue for researchers who are interested in the role of prediction in teaching and learning mathematics to exchange perspectives, discuss theoretical and methodological issues, identify potential areas of research, and consequently form collaborative teams.

In the analysis of grade-level expectations (GLEs), that require reasoning, in state mathematics curriculum standards, Kim and Kasmer (2006) found that GLEs pertaining to prediction were the most prevalent across grades and across content areas. Two examples of such GLEs are *predict what comes next in an established pattern and justify thinking* and *predict the effect on the graph of a linear equation when the slope changes*. Prediction, however, has received far less attention in mathematics education research as compared to other aspects of mathematical reasoning such as justification and generalization.

Why do we investigate prediction in mathematics education? In this paper, we offer three reasons: (a) prediction can reveal students' conceptions, (b) prediction plays an important role in reasoning, and (c) prediction facilitates learning. We then present three perspectives on prediction for research purposes. Next we outline the objectives of this discussion group and the activities planned for the scheduled sessions. We conclude by recapitulating the main points.

Students' Predictions Can Reveal Their Conceptions

Prediction can be used to uncover students' prior knowledge, schemes, misconceptions, and intuitions. Studies on stochastic misconceptions (see Shaugnessy, 1992) typically require subjects to predict or estimate the probability of an event in a given scenario. For example, most students predicted in a family of six children the sequence BGBBBB is less likely than the sequence GBGBBG although both sequences are equally likely (Kahneman & Tversky, 1972). This is because a three-boy-and-three-girl combination is more representative of the population than a five-boy-and-one-girl combination. This example reveals that students make predictions based on their judgment of *representativeness*—an event is more probable if it has some significant characteristics of its parent population. This example illustrates how we can infer from student prediction the scheme that governs the student's thinking.

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Some research in physics identifies students' misconceptions by asking participants to make predictions (e.g., Champagne et al., 1985; Gunstone & White, 1981; McCloskey et al., 1980). For example, 51% of undergraduates in a study (McCloskey et al., 1980) predicted that the path of a ball would be curvilinear (see Figure 1) when the metal ball is shot out of a curved tube at a high speed. The conceptual basis underlying their prediction is similar to the medieval theory of impetus, which claimed that "an object set in motion acquires an impetus that serves to maintain the motion" (p. 1140). Tasks that require students to imagine a scenario and then predict the motion are also found in the *Force Concept Inventory* (Hestenes et al., 1992)—a multiple-choice test for assessing students' understanding of basic concepts in Newtonian mechanics. To probe students' conceptions via interviews, Champagne et al. (1985) modified the demonstrate-observe-explain (DOE) interviews by requiring the interviewees to predict the outcome of a demonstration prior to observing the demonstration, instead of observing a demonstration and then describing their observation. White and Gunstone (1992) called these modified tasks prediction-observation-explanation (POE) tasks, although according to Gunstone and White (1981) the prediction-demonstration-observation-explanation procedure was developed in 1979 by Champagne and his colleagues.

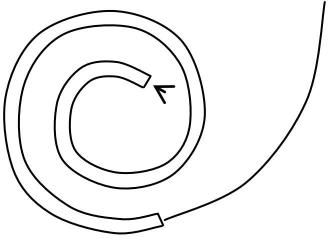

Figure 1. A curvilinear trajectory of a ball leaving a curved tube

Prediction as a tool to probe students' conceptions is more germane to physics education than to mathematics education because students tend to form naïve conceptions from their experiential world prior to receiving formal physics instructions. Nevertheless, mathematical tasks that require students to make prediction can still be designed to uncover students' mathematical conceptions in certain topics. For example, asking students to predict the largest fraction from a set of three fractions may reveal certain misconceptions about the value of a fraction. Predictions such as "all three fractions (99/100, 6/7, and 15/16) are the same because they are only one number away from being a whole" suggest a conception that disregards the denominator. Similarly, predictions such as "because 100's pieces is a smaller amount than 7's pieces you get a bigger chuck with 7 than with 100" suggest a conception that disregards the numerator. If students were not asked to predict, they could obtain the correct answer instrumentally by converting each fraction into a decimal or into its equivalent fraction with a common denominator, and then comparing the adjusted numerators. Doing so, however, might remove the opportunity for some students to reason with fraction-sense; for example a student reasoned that "6/7 < 15/16 < 99/100 because in 6/7 you just have to divide the whole into 7 pieces while in 99/100 you have to divide your whole into 100 pieces and you get more of 99/100 than 6/7, [since] 1/100 is smaller area than 1/7."

Prediction Plays an Important Role in Reasoning

To predict is to foretell an outcome prior to observing the unfolding of events. To predict can also mean to conceive an expectation of a certain result prior to performing detailed operations (Lim, 2006a). Our ability to predict or anticipate allows us to engage in thought experiments and make conjectures. An attribute of prediction is that when students predict, as opposed to meticulously working through the steps, they are psychologically relieved from the need for precision and certitude. Hence, in certain situations they can temporarily disregard the details and focus on essential features and structures. For example, a student may predict that 13^{89} is greater than 89^{13} by relying on his familiarity with base-ten structure: 10^{100} has 100 zeroes trailing the 1 whereas 100^{10} has only 20 zeroes trailing the 1 because $100^{10} = (10^2)^{10} = 10^{20}$.

Predicting complements other forms of reasoning such as generalizing, conjecturing, and abducting. Consider generalizing as an example, the process of generalizing typically involves activities like identifying commonalities, finding a pattern, checking to see if the pattern holds for "all" cases, and formulating a general statement, and in some cases identifying the process underlying the pattern. The act of testing whether a generalized pattern holds for other cases involves predicting, based on the generalized patterns, the results for those cases. Similarly, the examining and testing of conjectures for contradicting cases generally involves prediction. The testing role of prediction is inevitable in the construction of new knowledge, as expressed in Peirce's writings (1998) where abduction is differentiated from induction and deduction.

> Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value and deduction merely evolves the necessary consequences of a pure hypothesis. Deduction proves that something *must* be, Induction shows that something *actually is* operative, Abduction merely suggests that something *may* be. … Its only justification is that from its suggestion deduction can draw a *prediction* (emphasis added) which can be tested by induction … (p. 216).

Prediction Facilitates Learning

Prediction can be a pedagogical means to aid student learning. Student comprehension in reading is found to be improved when they are asked to make predictions using questions such as 'what do you know about this character that helps you predict what he or she will do next?' and 'given the situation in the story, what will possibly happen next? (Block et al., 2004; Palincsar & Klenk, 1991). In physics education, the *predict-observe-explain* (POE) instructional approach (e.g., Gunstone et al., 1992; Palmer, 1995; P. Laws, 1997) requires students to predict prior to observing a demonstration or performing an experiment, observe the outcome, and account for the discrepancy between their prediction and their observation. In biology education, Lavoie (1999) found that the addition of *prediction-discussion phase* to a three-phase learning cycle (exploration, term introduction, and concept application) could improve students' process skills, logical-thinking skills, science concepts, and scientific attitudes. In addition, prediction can increase students' level of engagement because "the commitment involved in deciding on a prediction can have powerful motivation effects" (White & Gunstone, 1992, p. 63).

In many computer-based curricula, predicting is an important element in the learning process. Tversky, Morrison and Betrancourt (2002) commented that prediction can facilitate learning that involves animation. This is because, according to Hegarty, Kriz and Cate (2003), prediction induces people to activate their prior knowledge and to articulate their understanding of the phenomenon under investigation. Effective use of certain applets (e.g. Paper-Pool applet in NCTM Illuminations and Frog-versus-Clown applet in SimCalc) would require students to make predictions before they observe the animation. Bowers, Nickerson, and Kenehan (2003) propose an instructional sequence that requires students to play with dynamic graphs, predict, and then test their predictions.

Prediction may be used to help students notice certain structural features. For example, Buendía and Cordero (2005) use prediction as a practice for students to recognize the periodical aspects of certain functions. According to Fischbein and Grossman (1997), having students predict may improve their understanding of the underlying principle in a solution.

> Encouraging the learner to guess intuitively, one creates a challenging situation. Another way of achieving this is through facing the student with a conflict between a personal guess and a mathematically accepted solution. Such a conflict may stimulate the interest of the learner and may help him or her to overcome his or her intuitive obstacles. Moreover this understanding may contribute to the understanding of the mechanisms that shape the answer. (p. 43)

When students make a prediction prior to performing calculations, they are more likely to notice certain relationships, generalize from specific cases, and expand the assimilatory range of a particular conception. For example, having students predict prior to computing whether the result of multiplying 9.29 by 7/6, or by 0.64, is greater or less than 9.29 tends to draw their attention to the effect of the multiplier. Upon further reflection, students may even advance their understanding of multiplication, moving from viewing multiplication as an algorithm-to-follow and/or multiplication as repeated-addition to viewing multiplication as enlargement or amplification. Lim (2006a) observed that prediction tasks such as, "Plugging $x = 127$ into $4x - 20 > 3x - 20$, we get 361 for the right hand side. What is the value on the left hand side?", could help students attend to the structure of algebraic expressions.

From a Piagetian's perspective, learning involves cycles of experiencing disequilibrium, resolving cognitive conflicts, and re-establishing a new equilibrium (Piaget, 1975/1985). Prediction, together with reflection, can facilitate the learning process. In the analysis of students' construction of 3D arrays of cubes in an inquiry-based learning environment to determine the number of cubes in a rectangular prism, Battista's (1999) apprehended the power of predictions.

> Having students first predict then check their predictions with cubes was an essential component in their establishing the viability of their mental models and enumeration schemes and was thus crucial for the recursive development of these models. Because students' predictions were based on their mental models, making predictions encouraged them to reflect on and refine those mental models. Strategies were refined as students reflectively reorganized their models and schemes to better fit their experiences. Indeed, the goal of the instructional tasks was to develop and refine students' mental models. Having students merely make boxes and determine how many cubes fill them would have been unlikely to have promoted nearly as much student reflection as having students make and check predictions because (a) opportunities for perturbations arising from discrepancies between predicted and actual answers would have been greatly reduced and (b) students' attention would have been focused on physical activity instead of on their own thinking. (p. 442)

In certain mathematical domains, prediction occurs rather spontaneously among students in a problem-centered inquiry-based environment. However, in other learning environments, students unless prompted may not make predictions. In those situations, explicitly asking students to predict may facilitate student learning. Kim and Kasmer (2007a) employed prediction-type questions in middle-school algebra classrooms. For example, prior to exploring and comparing two linear relationships in a real-world context, students were asked to predict whether the graphs, tables, and equations of the problem would be similar to those they had experienced before, and to explain why. Students explained that the graphs, tables, and equations would be similar because of the linear relationships between the quantities in the problem situation. This prediction task allowed students to reinforce previously-learned concepts. In particular, when predicting which of the two linear relationships would yield a steeper line, students had to consider both the starting values and growth rates in order to visualize the problem situation in terms of the graphs of the algebraic relationships. Making predictions afforded students an opportunity to relate the mathematics they experience to the real-world situation.

Kim and Kasmer (2007b) identify the following roles of prediction in promoting student learning of mathematical concepts: (a) prediction stimulates students' interests, (b) prediction can help students make sense of the problem situation and the related concept, (c) discussions of predictions can offer students new perspectives of the problem, (d) prediction can promote making connections among related concepts, (e) prediction can provoke visualization of a problem situation, and (f) prediction can be a useful means for assessing students' thinking. Research studies can be designed specifically to test some of these hypotheses.

Conceptualizing Prediction for Research in Mathematics Education

Prediction can be conceptualized from various perspectives, depending on one's research objectives. As a starting point, we offer three views of prediction: prediction as a mathematical task, prediction as a mental act, and prediction as a social practice.

Prediction as a Mathematical Task

Kim and Kasmer (2007a, 2007b) present a list of prediction expectations from State Standards in the United States. While those expectations can be categorized in terms of grade-levels or content strands, they also can be categorized in terms of the nature of prediction that is involved. In order to make a prediction, one usually has to engage in various mathematical processes, such as visualization, estimation, and generalization. Table 1 presents various types of prediction tasks, grade-level expectations, and examples. These examples suggest that the kinds of mathematical processes involved in making a prediction depend on the type of prediction tasks. For example, in extrapolation-prediction task, one has to establish a relationship and use it to predict. We also expect certain types of prediction task to be appropriate for certain mathematical topics. For example, visualization-prediction tasks, which tend to involve construction and transformation of mental images, are more common in geometry than in arithmetic or symbolic algebra.

Prediction Task	Grade-level Expectation	Example
Visualization-prediction task	Predict the position and orientation of simple geometric shapes under transformations such as reflections, rotations, and translations.	Predict whether a given scalene triangle would be an identical mapping when it's rotated by 120° around one of the vertices.
Estimation-prediction task	Predict the relative size of solutions in addition, subtraction, multiplication, and division of whole numbers.	Predict which of the following has the larger relative size of the operations: $23 + 18 + 45$ or $588/102$.
Generalization-prediction task	Make reasonable predictions using generalizations about patterns.	Jenny put sand into a clear graduated cylinder with a small opening in the bottom. She let the sand run out of the opening, and measured the height of the sand remaining in the cylinder at 10-second intervals. The height of the sand was 80 mm at beginning of this

		experiment. Use the data recorded in the table to predict the height of sand in the cylinder after 25 seconds had elapsed.
Extrapolation-prediction task	Make predictions and justify conclusions based on data.	Predict the population of rabbits in ten years using the data in the table that shows the rabbit population of the first five years.
Concept-application-prediction task	Predict and evaluate how adding data to a set of data affects measures of center.	The average height of a group of 10 students is 150cm. Predict the new average height when the 11th student whose height is 170cm joins the group.

Table 1. Examples to illustrate the different types of prediction tasks

These categories of prediction task are in early phases of development. They are by no means exhaustive. Within each category, there may be sub-categories. For example, prediction tasks in the generalization-prediction category may be aimed at (a) noticing a local commonality such as the relation between successive terms, (b) establishing a general rule, or (c) applying the established rule. An example for each sub-category is as follows: (a) Predict the next three terms in the sequence 2, 3, 5, 9, 17, 33, ___, ___, ___; (b) Predict the 30th term, and then the *n*th in the sequence 2, 3, 5, 9, 17, 33, …; and (c) Predict whether 1025 is a term in the sequence 2, 3, 5, 9, 17, 33, ….

Will students respond differently if the term "predict" in the examples in Table 1 is replaced by the term "find" or "determine"? We do not have a definitive answer; this is an open question for research. Nevertheless, the term "predict" seems to suggest to students that they can use less rigorous methods, such as relying on intuition or making an educated guess, to foretell what the answer might be. The lack of rigor usually results in a certain degree of uncertainty because making a prediction does not require a thorough solution of the problem. However, making a prediction provides students an opportunity to develop and refine their mental models because it provokes them to utilize various reasoning processes, such as testing and reflecting. The term "find", on the other hand, seems to suggest to students that there is a standard way for solving the problem. In contrast, when students are asked to predict they are encouraged to consider other ways. The act of predicting appears to have the following characteristics: speediness (less rigorous but stimulates reasoning), flexibility (uses non-routine options), and some degree of uncertainty.

Prediction as a Mental Act

Prediction as an instruction specified in a task may not correspond to prediction as a mental act performed by a student. For example, a student when asked to predict which fraction, 98/99 or 147/149, is greater may use the cross-multiplication algorithm to compare 98×149 with 147×99. The student is not considered to have predicted because the student performed detailed operations to obtain the answer. Conversely, a student may predict in tasks that do not ask for prediction. Consider this task: Solve $6x+(4x-3)^2 > 9x+(4x-3)^2$. A student may notice that both sides have the same quadratic term and predict that there is no solution because the student mistakenly thought that $6x$ is always smaller than $9x$. In summary, students may "perform" when asked to predict and predict when are asked to perform.

To solve a mathematical problem, one engages in a combination of mental acts such as interpreting, comparing, predicting, computing, representing, transforming, conjecturing, justifying, generalizing, and so forth. Lim (2006a, 2006b) explores students' mental act of anticipating as a means to characterize student thinking. Anticipating refers to the act of conceiving a certain expectation without performing a sequence of detailed operations to arrive at the expectation. Two types of anticipating acts were identified: *foreseeing an action* and *predicting a result*. Foreseeing an action refers to the act of conceiving an expectation that leads to an action. Predicting a result refers to the act of conceiving an expectation for the result of an event. An event could be a probabilistic experiment, an arithmetic computation, an algebraic manipulation, a geometrical transformation, and so forth.

Lim (2006a, 2006b) identifies three levels of sophistication in students' prediction in the context of solving non-routine problems involving algebraic inequalities and equations. In *association-based prediction*, one predicts by associating two ideas without establishing the basis for making such an association. For example, a student predicted that there is no solution when she obtained $6 > 0$ from simplifying $(6x - 8 - 15x) + 12 > (6x - 8 - 15x) + 6$; she associated the disappearance of x with the inequality not having a solution. In *comparison-based prediction*, one predicts by comparing two elements or situations in a static manner. For example, a student predicted that there was no value for x that will make $1.2x + 3456 < 7 + 8.9x$ true because "you are always going to add 3456, and this is higher than … 7." In *coordination-based prediction*, one predicts by coordinating changes or attending to relationships among quantities. For example, another student predicted "yes, eventually" there would be a value of x that would make $1.2x + 3456 < 7 + 8.9x$ true, and reasoned "8.9 is so much bigger than [1.2], as long as it's bigger than the [coefficient] of this ($1.2x$) … eventually some numbers will make it larger than the left side." These categories are pertinent to the domain of inequalities and equations. Research in other mathematical domains may uncover other categories that characterize student prediction.

Prediction as a Social Practice

Buendía and Cordero (2005) propose the use of prediction inside the mathematics classroom as an intentional activity that motivates a redefinition of the periodicity of functions. A teaching situation was designed to deal with this mathematical issue with consideration for its epistemological, cognitive and didactic aspects. In their epistemological research, they found a relation between the meaningful use of periodicity in physical phenomena like movement and the intentional use of prediction. Their awareness of this relation resulted from their search in both the mathematical structure of periodicity and its social construction.

In engineering a didactic situation for students to recognize periodicity, Buendía and Cordero (2005) used the social practice of prediction. For example, students were given eight distance-time graph for the interval from $t = 0$ to $t = 12$ and were asked to predict the position of the moving object at $t = 231$ for each graph. The students were then asked to classify the graphs and decide which graphs are periodical. These activities facilitated students to progress from "anything that repeats itself in anyway" to "how it is repeated". The discussion on the periodic property was enriched by the prediction categories found by students. Each of these prediction categories was formed by the meanings of repetitive behavior of a function, the procedures students developed, and the students' cognitive structures. The authors believe that "the practice of prediction can provide the definition with a context in which it can be mobilized as a decisive argument when discussing that which is periodical" (p. 319).

Cordero (2006) applies the social practice of prediction in the domain of Calculus. He proposes an epistemological framework in which the "final" state can be predicted using its initial function state and its "variation". For example, at any particular time the position of an object moving at a constant acceleration can be predicted using its initial position and its velocity. In a general variation situation, the variation is expressed as a succession of simultaneous derivatives according to the Taylor series $f(x+h) = f(x) + f'(x)h + f''(x)\frac{h^2}{2!} + \ldots$. For physical motions, the variation involves only *f*, *f*’, and *f*’’, which correspond to the displacement function, velocity function, and acceleration function respectively.

We have highlighted two models of prediction as a social practice. The first model involves local predictions of positions as well as global predictions of periodic movements, from which new characterizations, such as period of repetition, are defined. The second model involves local prediction of the state of a function using information related to properties of local characterizations, namely prior position and variation. We believe that the social practice of prediction can be investigated in other mathematical domains as well.

Objectives of this Discussion Group

We aim to accomplish these objectives:

- To provide a floor for researchers whose work are related to prediction (e.g., animation, conceptual change, anticipatory schemes, intuition, statistics education, motivation) to discuss various aspects of role of prediction in teaching and learning
- To promote and stimulate research on prediction in mathematics education as well as interdisciplinary research on prediction among mathematics educators and psychologists
- To form a network among researchers interested in investigating the use of prediction in mathematics classrooms.

Proposed Sequence of Activities for this Discussion Group

Meeting 1

- A brief overview on the three reasons for conducting research on prediction in mathematics education
- An activity to engage participants in making predictions and reflecting on their predicting experience
- An activity in which participants will analyze some mathematical tasks that involve prediction, identify desirable task characteristics, and create tasks that they could later use in their research or teaching.

Meeting 2

- An open forum for individuals to share their research on prediction
- A brainstorming session to identify potential areas for research on prediction
- Working in collaborative groups to formulate research questions, design research methods, and identify issues related to methodology, implementation, or analysis.

Meeting 3

- Presentation of small group work
- Discussion for the next steps

Conclusion

In this paper, we have presented three reasons for conducting research involving prediction in the field of mathematics education: prediction can reveal students’ conceptions, prediction plays an important role in reasoning, and prediction facilitates learning. We have offered three perspectives on prediction: prediction as a mathematical task, prediction as a mental act, and prediction as a social practice. We hope more researchers will conduct studies that investigate various aspects of prediction in the teaching and learning of mathematics. We also hope that mathematics educators will use prediction effectively to engage students in mathematical reasoning in their classrooms.

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