

# The application of Modern Portfolio Theory to an unbalanced bidding model

Published as...

**Cattell, D.W., Bowen, P.A. and Kaka, A.P.** (2010) The risks of unbalanced bidding. *Construction Management and Economics*. Vol.28, No. 4, pp. 333-344.

Unbalanced bidding models have largely ignored the risk aspect of item pricing. Many researchers have acknowledged that there are considerable risks associated with unbalancing a bid but little has been done to describe these risks, let alone model them. This research now proposes a framework by which all of these risks can be assessed. It identifies that these risks comprise the risk of rejection, the risk of reaction, and the risk of being wrong. It has identified that the Value at Risk ('VaR') method of measuring risk is a convenient way by which to combine all of these risks into one composite number. This number then serves to describe the extent of risk generated by each level of each item's price. The authors have previously proposed an unbalanced bidding model that has likewise provided a measurement of the expected reward generated by each level of each item's price. By doing a summation of these, keeping in mind that the prices applied to all of a project's component items must add up to the overall bid price, the contractor is now able to assess both the risk as well as the rewards of all possible item price combinations. It is proposed that the contractor apply Modern Portfolio Theory ('MPT') to identify all *efficient* item price combinations and hence discard giving any further consideration to all item pricing that is instead found, by comparison, to be *inefficient*.

*Keywords:* Bidding, portfolio selection, risk analysis, risk management, mark-up.

## Introduction

Unbalanced bidding models are mathematical tools by which to determine the optimum distribution of prices to be applied to a project's component items when engaging in competitive bidding.

In the past, unbalanced bidding models have been largely focused on the optimization of the expected profits for a contractor rather than give too much consideration to the risks involved. Researchers have, however, often acknowledged these risks but, despite this, they have made little effort to properly incorporate these risks into their models (see Stark, 1968, 1972 and 1974; Cattell *et al.* 2004 and 2007; Diekmann *et al.* 1982; Teicholz and Ashley, 1978; and Tong and Lu, 1992).

Moreover, besides the work of Diekmann *et al.* (1982) little has been done by which to structure these models so that they recognize the inherent nature of the trade-off that exists between unbalanced bidding's contributions to these risks and that of the prospective gains. Diekmann *et al.*'s (1982) efforts in this regard, whilst somewhat limited from the point of view that they only recognized one type of risk and then required that a contractor arbitrarily decide on the value of a constant by which to shift the objective from the maximization of profit to the minimization of risk, were nevertheless pioneering.

All models have otherwise given recognition to risk by constraining prices using an imposition of lower and upper bounds to each and every item price (see Stark, 1968,

1972, 1974; Diekmann *et al.*, 1982; Teicholz and Ashley, 1978; and Tong and Lu, 1992). These bounds ended up becoming the single most important aspect of these models, with the effect that all items are priced at either their upper price limit or else their lower price limit, with the exception of only one item. This one item price would serve to ensure that the summation of all the priced items equaled the tender price. The effect of these models was therefore reduced to only serving to split all the items into these two groups: those priced high and those priced low.

Prior research (see Cattell *et al.*, 2009) has already identified that, whilst it is heuristically appropriate to constraint the prices for all items, it is not appropriate that these constraints be imposed as fixed, non-negotiable limits.

Previous unbalanced bidding models (see Stark, 1968, 1972, 1974; Diekmann *et al.*, 1982; Teicholz and Ashley, 1978; and Tong and Lu, 1992) have not quantified the risks that are generated by way of unbalancing a bid albeit that all of them have vaguely acknowledged some of the risks and attempted to avoid them. The authors' research proposes a model that quantifies and manages these risks and provides the contractor with a means by which they can weigh up their goals of pursuing a maximum return relative to a minimum risk. It effectively gives consideration to all possible item price combinations for a project, and from all of these it sifts out those that are *efficient* and discards all those that are instead *inefficient*. It is not logical that a contractor might prefer an *inefficient* item price scenario in preference to any of the *efficient* ones. The result is a unbalanced bidding model that fully embraces risk and that manages it to the

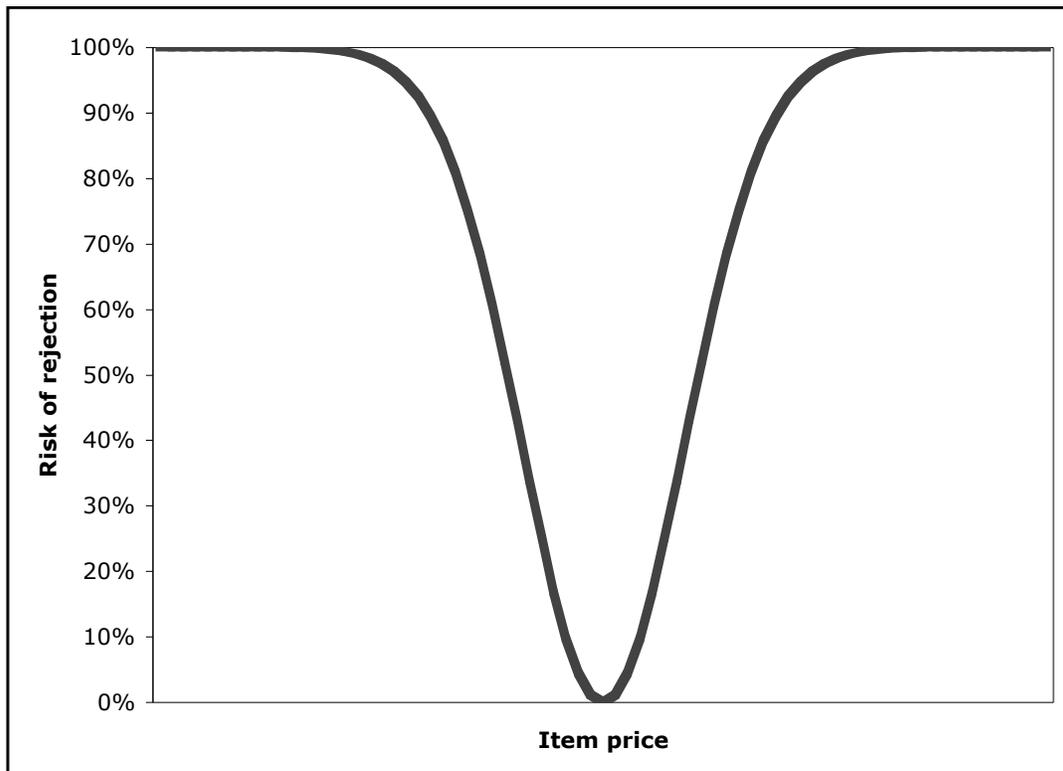
same extent and with the same priority as that of the expected returns. This model facilitates that contractors can both maximize these returns and minimize their risks, being able to fully assess and manage the trade-off between these two objectives.

### **The risks of item pricing**

There are many risks that are either the direct result of item pricing or that are affected by item pricing.

#### **The risk of rejection**

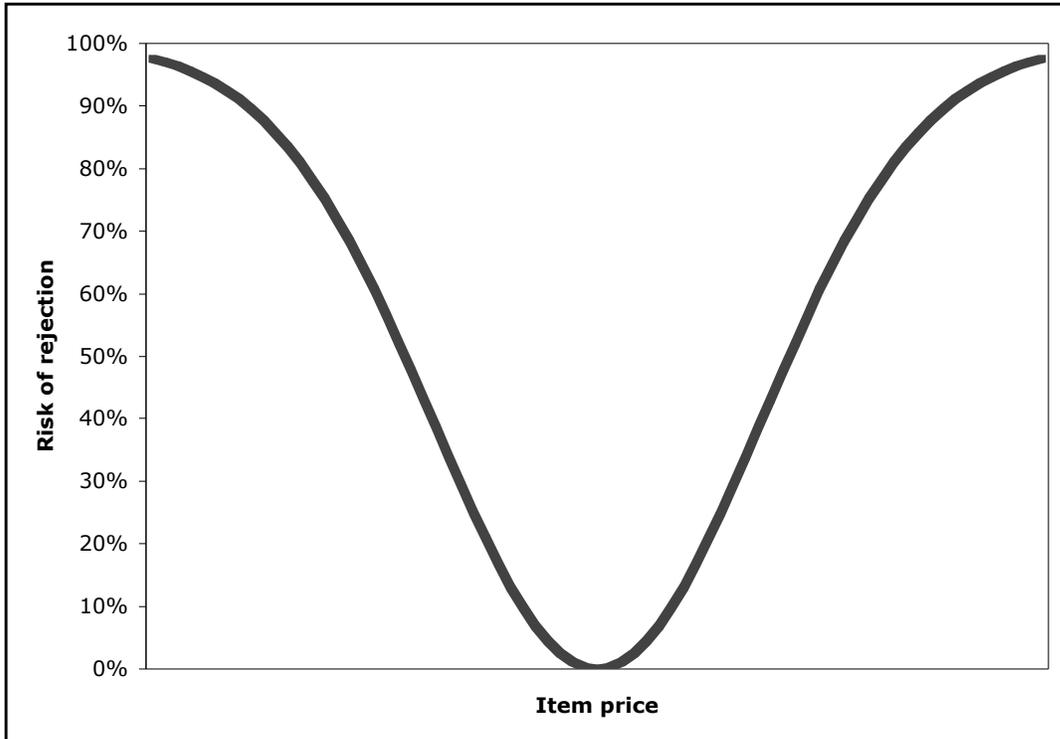
A priced Bill of Quantities stands the risk of being rejected, particularly for reason of being unbalanced. If any single item price is either too high or too low it could serve to trigger this rejection. If any one item price is either extremely low or else extremely high, the contractor can be almost 100% certain that the client will reject their bid.



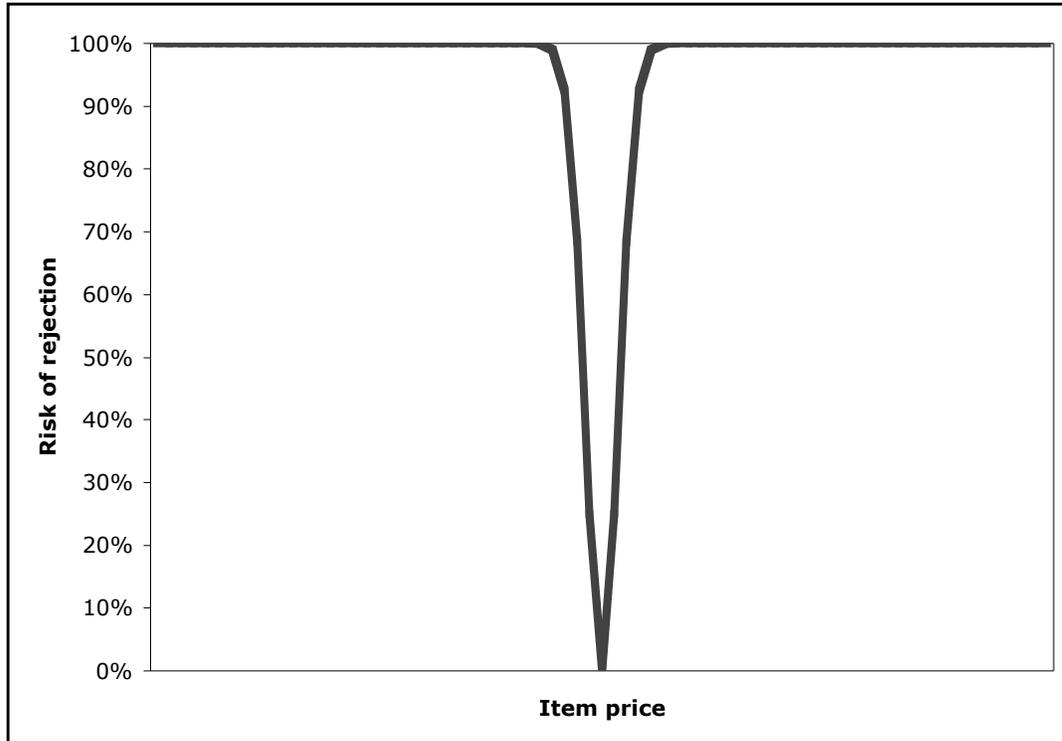
This is shown in Chart 1 above, resembling an inverted normal distribution. This function has the notable characteristics that the risk of rejection at any extreme price is almost 100% and the risk around some “reasonable price” is almost nil. The difference between this function for each item is particular to two aspects:

- the point along the x-axis (being the mean) where the price is least likely to be objectionable, and
- the variance (or ‘spread’ or ‘width’) of this curve.

Heuristics suggest that different items have risk curves with different ‘spreads’. For example, items of excavation are likely to have a ‘wider’ range of tolerance than items that have a greater certainty.



An item that has characteristics that is inherently less certain, such as any of those related to earthworks, is likely to ensure that the client is more tolerant of a wider range of prices, before they are likely to reject the overall bid. This is shown in Chart 2 above.



On the other hand, there are some items where the contractor can be almost certain that the client will not be accepting of any price other than one possibility, and where any other price will likely arouse suspicions and rejection. This type of situation is graphically depicted in Chart 3 above.

Various efforts have been made (see Wang *et al.*, 2006) to develop models to assist clients with identifying unbalanced bids. Typically, these are dependent on comparing a contractor's prices against a benchmark and then concluding that if these differ by too much, then the bid is likely unbalanced. The problem with this approach rests largely on their dependence on these benchmarks that are themselves potentially (and likely) unbalanced, thus making any such comparison less than objective. The reason for these benchmarks being likely unbalanced is their dependence, in turn, on previous bids which themselves are, quite possibly, unbalanced. These models might therefore serve to assess

only whether a bid is extraordinarily unbalanced and less so serve to determine conclusively that any bid has some (more normal) degree of unbalancing.

In practice, the use of such a model is unlikely. Instead, it is almost certain that clients (and their professional agents) will be looking out, using less formal methods, for any item prices that appear extraordinarily high or low. For this reason, two methods have become popular as techniques by which to constrain unbalanced bidding models, namely:

1. upper and lower price limits for each item

$$L_j \leq P_j \leq U_j \quad \text{Eq.1}$$

where  $j$  = item numbers

$L_j$  &  $U_j$  = lower and upper bounds, respectively,

for the price of item  $j$

Previous research (Cattell *et al.*, 2009) has shown that some such nature of constraints are necessary but that this particular formulation of these constraints is simplistic:

*“The use of  $L_j$  and  $U_j$  as fixed constants suggests that a contractor would never consider pricing item  $j$  at the price  $L_j - 0.01$  or at  $U_j + 0.01$  regardless of the merits of doing so. This is patently unrealistic. It is surely more practical to imagine that a contractor may consider using a price of  $L_j - a$  or  $U_j + a$  where  $a$  is a small positive number,*

*should there be sufficient cause for doing so. On this basis, it is impossible to contemplate the justification for the use of any constant values for  $L_j$  and  $U_j$  no matter how extreme, or how far apart, they are.”*

## 2. inter-item pricing limits

$$P_i - P_j \geq 0$$

where  $i$  &  $j$  = different item numbers

Similarly, Stark (1968) identified that the prices for some items should be made to be the same or higher than other items. For example, he argued that the price for the excavation of hard-rock should logically be more than the price for the excavation of soft-rock, else it shall be obvious that the bid has been unbalanced.

### **The risk of reaction**

If an item's price is very high, it might inspire an architect to redesign the project so as to avoid this item. Similarly, if an item's price is thought to be attractively low, it might give rise to a variation order in which the project uses more of this item. For example, if the price of face-brick walls is low relative to alternative finishes then the architect might take this as an opportunity to switch finishes to take advantage of this pricing. This is a risk that the contractor should factor into their item pricing strategy.

## **The risk of being wrong**

Unbalanced bidding models use many variables that require that the contractor has to make estimates, which are inevitably, to some degree, wrong. Any such errors obviously lead to the generation of item prices that might be different had the contractor not being wrong. In other words, if, at the time of pricing a bill, a contractor could enjoy the accuracy of hindsight then their item prices are likely to be different than those that they produced on the basis of their (imperfect) estimates.

In particular, the item pricing model explained in Cattell *et al.* (2009) requires that the contractor provide estimates for the following variables:

- a discounting rate
- the scheduled timing of items
- any expected variation in quantity (such as if the contractor thinks that the site contains more rock than is contained in the Bill)
- escalation rates (for each workgroup).

It is interesting to note that although variables such as the discounting rate are common to all items, the risk associated with the variability in estimating this rate is different for each item. Items scheduled at the beginning of the project are hardly discounted whilst items scheduled at the end of the project are heavily discounted. Furthermore, the later an item is scheduled to occur, the greater the chance that the estimate of the rate will be wrong.

## **Two types of risk**

If one analyses the above-mentioned risks one can notice that there are the following two types of risk involved: the risk of variability (in particular, describing all those as regards the risk of being 'wrong'); and the risk of rejection/reaction. The latter type of risk is not of the nature that it can be described in terms of variability or inaccuracy or uncertainty.

The lower that any item price is, the higher the risk that either the bid will be rejected else that the project will be redesigned so as to increase the quantity of that item. In addition, at the other extreme, the higher any item price is, again the higher the risk of rejection and the higher the risk of a variation order by which the client might avoid this item. It is proposed that this second category of risk can be described by way of an inverted normal distribution (i.e. resembling an inverted 'bell' curve). There is a central region around which the risk is least, whilst outside of this region, the risk is ever-greater. At the extremes the risk is such that the contractor can be almost 100% certain that this price will induce a 'rejection' or 'reaction'.

The position (on the x-axis) of the central region (i.e. the bottom of the bell) is not as much having to be influenced by any estimate of the cost of this item as it is having to be positioned relative to item prices that the industry in general has grown accustomed to with respect to this nature of item. For instance, if clients have become accustomed to high costs of excavation (perhaps for reason of an industry-wide systemic prevalence of front-end loading) then the bell-curve of such items should be centred around that expectation.

## **Value at Risk**

One is ideally wanting a way by which to measure all of the risk (namely, all of the above-mentioned risks, combined) generated at each level of price for each item of a project. With this, one will then know both the rewards that are to be expected for all of these price levels and also the risks. If these are structured that they 'interlock' (i.e. that they are additive) with those from other items, and if one recognises that all of the item prices must add up to the overall bid price, then one has the basis by which to measure the overall expected reward and risk from all possible item price combinations.

To reach this goal, one is needing to have one single measure of all of the risks combined. However, we have already identified that the overall risk from any item comprises two very different types of risk: the one normally suited to measurement by way of some assessment of its variance and the other not. To overcome the challenge of combining these, the authors have found a very useful, relatively new measure of risk that appears to be conveniently well suited to this purpose. This measure is known as 'Value at Risk' and is abbreviated as 'VaR'.

The VaR method is said (Kolman *et al.*, 1998) to have been developed at JP Morgan in the early 1990s for the purposes of assessing the bank's exposure to risk on its equity positions. According to Manganele and Engle (2001) the VaR measure has gone on to *"become the standard measure that financial analysts use to quantify market risk. VaR is defined as the maximum potential change in value of a portfolio of financial instruments with a given probability over a certain horizon."* Typically this time horizon is only one day and the probability used is often 1%. A common form of expressing the VaR of an

investment portfolio is therefore along the lines that it has been assessed that there is only a 1% risk that the value of a portfolio could drop by more than (say) \$1m within the next day. Expressed another way, one expects that the value of this portfolio will erode by more than \$1m within 24 hours, only as often as 1 day in 100. A lot of the appeal of this method would appear to lie in this simple method by which risk can be expressed and comprehended.

This method of expressing risk lends itself well for the purposes of unbalanced bidding, rather than the more traditional methods of expressing risk in terms of variance. As an example, it facilitates that an unbalanced bidding model might predict that there's a 50% risk being generated by way of an item being assigned a price of \$1.00 that, as a direct result of this, the contractor might lose (say) \$10,000. Furthermore, this model might predict that if this item is priced at \$1.10 that there's then a 50% risk that the contractor might lose (say) \$8,000. If the contractor can have this nature of knowledge and insight into their item pricing, they are well-equipped to price their items to not only maximize their expected reward but also to minimize their risk.

Notice that this methodology facilitates that contractors need no longer implement the item price constraints that are of the form of Equation 1.

## **Valuing these risks**

### **The risk of rejection**

To implement this model, the contractor has need to estimate the extent of their loss that they will suffer in the event that any item price of theirs leads to a rejection of their

overall bid. Also, they need to estimate the degree of variance by which the aforementioned normal distribution describes the likely response from the client – as shown in the above charts. The equation for a normal distribution takes the following form:

$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

To produce the risk function (shown in the above charts) the above equation needs to be inverted and standardised so that the range of the risk is ‘stretched’ to range from 0% to 100% (or infinitesimally close to this, to be theoretically correct). This can be done using the following equation:

$$risk = (y - y_{\min})(1 / y_{\max})$$

The variance (as described in the above equation by way of the standard deviation  $\sigma$ ) is a function of two influences: one of which is item-specific and the other which is general to the overall project. The item-specific variance relates to the nature of item as described by way of the different scenarios underlying the differences between Charts 1, 2 and 3 above. For instance, it is hypothesized that prices across the industry for earthwork items have far greater variability than the prices of, say, items of specified ironmongery or prime cost items or provisional sums. Ideally, some research needs to be done to confirm this hypothesis and variances need to be identified for different categories of items, as reflected in the variability of item pricing between contractors, across a wide range of projects.

The second above-mentioned influence on this variance is dependent on the relative competitive strength of the winning contractor. If the contractor's (lowest) bid is only marginally less than the bids of other contractors, then this places the client in a stronger negotiating position by which to reject any proposed item pricing than if the lowest bid is substantially less than any other. In an extreme situation, if the 'winning' contractor were to realize that their bid had been a mistake, it is quite easy to imagine that they will be highly motivated to then submit pricing that is considerably unbalanced. This could serve to make up for some of their loss (resulting from their mistake) and / or to encourage the client to reject their bid (thereby releasing them from suffering the consequences of this mistake). In any event, it is not difficult to imagine that the knowledge of all the bids is likely to influence the extent of both the contractor's motivation and the client's willingness-to-accept any unbalancing of the bid.

The proposed system is therefore as follows: that contractors conduct research on the variability of item prices, specific to different trades / bills across their industry. This should determine the Standard Deviation that they will apply to the analysis of the 'Risk of Rejection' of all such item prices for all projects of theirs. Secondly, that they apply some intuition to either 'dial this up' or 'dial it down', to some overall degree for any project in question, based on their perception of their relative competitive-strength for reason of knowledge of the spread between their (lowest) bid and that of the second lowest.

The next important aspect of this risk is the quantum of the loss that the contractor will suffer in the event that a client does reject their prices. It is proposed that this doesn't amount to the profit budgeted for the project. If a contractor has their pricing rejected,

the likely consequence is that they'll have to submit several more bids before winning another one. The cost of being rejected is proposed to therefore be representative of the following aspects:

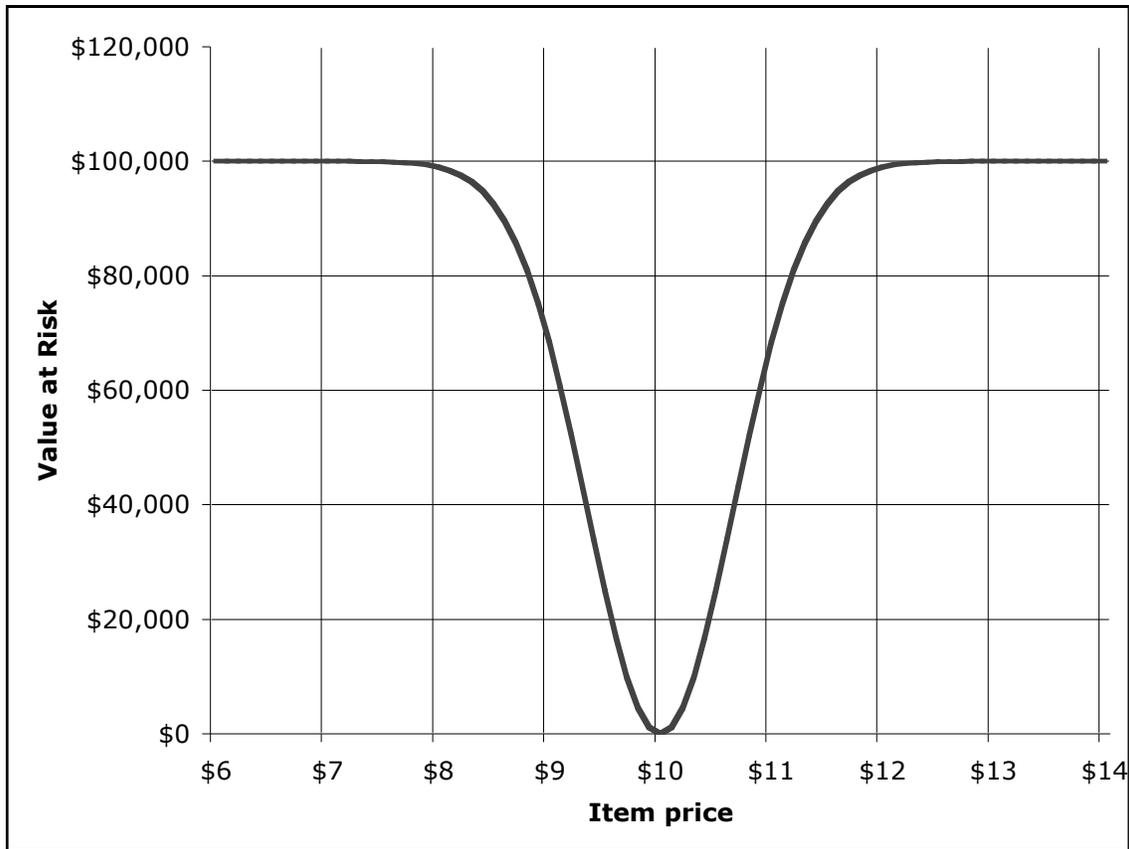
- the cost of estimating and bidding several more projects, including all those that will be lost before the contractor is again in the position where their bid is the lowest
- accounting for any extraordinary profit / loss that's expected from this project.

It is hypothesized that the latter is again a function of the spread "left on the table" between the lowest bid and the second lowest. If the contractor has regrets that they should have submitted a higher bid, they might perceive the amount of the prospective loss to be minimal. Indeed, they may even perceive that it will be to their advantage to lose this project, i.e. that the prospective loss from doing so is a negative one.

The value of 100% of the risk of rejection for any item is the same as for any other. They should all be assigned the same number in this regard, with the differences between the risk on items being reflected solely by way of different mean prices applying to different items (with this determining the position on the x-axis of the chart for each item, i.e. the same 'expected' price) and the 'width' or variance for each item, as explained above.

On this basis, if the contractor determines that the opportunity cost for them of losing this project is, say, \$100,000 and if they have determined that the Mean Price and Standard Deviation for a specific item is \$10.00 and \$1.00 respectively, then the function for

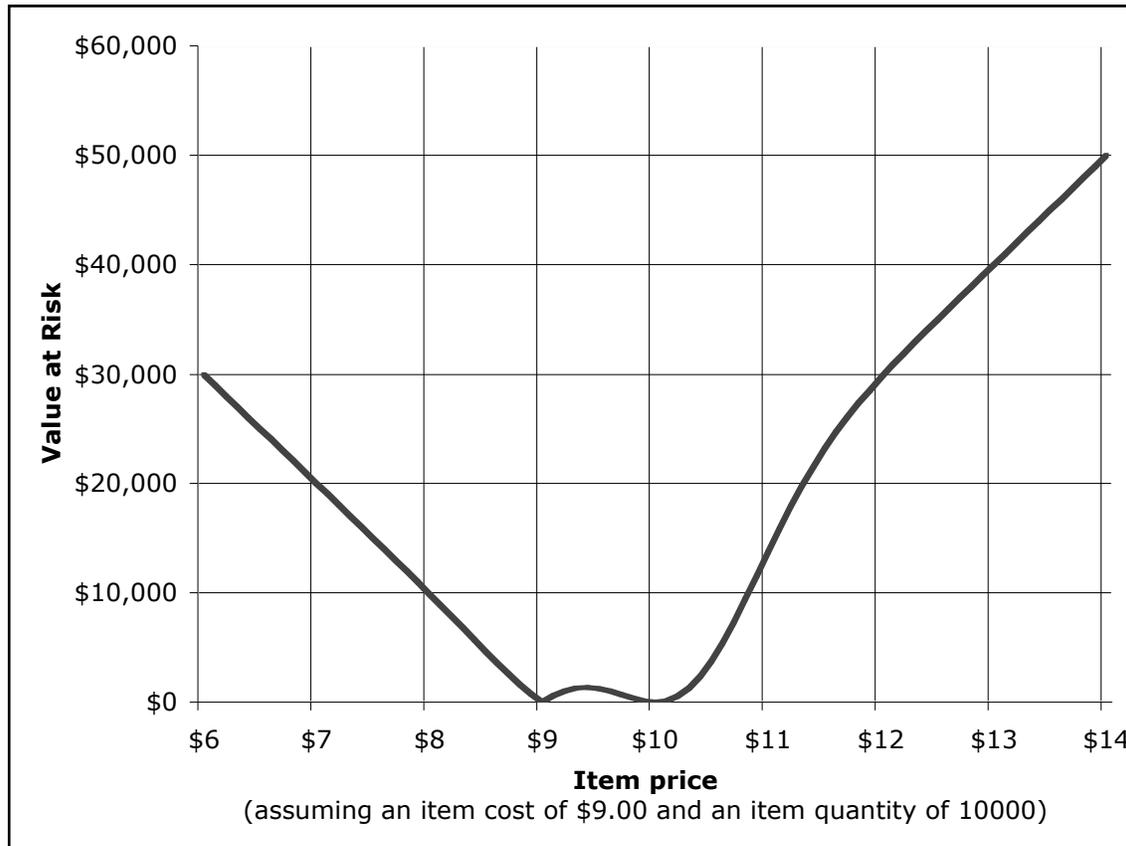
determining the risk of rejection for this specific item could be plotted that it will resemble that the following chart:



### **The risk of reaction**

The contractor has a similar need to quantify the risk of inspiring a variation order. For many items this risk may be *nil* seeing as it may not be possible for the client to redesign the project to avoid this item, else to increase its quantity. For other items, it may be considerably more.

By comparison to the (above-described) 'risk of rejection', this 'risk of reaction' doesn't plateau at any maximum monetary value. The higher the price, the higher the expected loss if the client were to react by way of a variation order.



This chart shows how if the item is priced at less than its cost, then there is a risk associated with the expectation that the quantity of this item may be increased by way of a variation order. Beyond the mean expected price, the risk becomes one of the item being replaced with another. As this risk ascends the normal distribution curve, in the case of this example reaching almost 100% at a price of around \$12.00, the quantum of the monetary risk is the loss of profit that will be suffered should this item be replaced.

## The risk of being wrong

The monetary quantum of the risk of being wrong is a function of the uncertainty in the underlying assumptions and estimates that comprise the computation of the expected return that is to be enjoyed from the activity of item pricing.

Previous research (Cattell *et al.*, 2009) has already provided the means by which to quantify the expected return that can be contributed from each item at each possible price.

This has been expressed by way of the following equation:

$$\beta_j = \sum_{n=1}^N \left( \frac{1}{1+r_j} \right)^n \lambda_{nj} (Q_j + Q'_j) (1 + Y_{nj} f) (1 - R_n)$$

where  $\beta_j$  = the sensitivity of an item's contribution  $PV_j$  to an item's price  $P_j$

$j$  = item number

$n$  = month number

$N$  = duration of project in months

$r_j$  = monthly discount rate appropriate to the risk of item  $j$

$\lambda_{nj}$  = proportion of  $Q_j$  to be built in month  $n$

$$\sum_{n=1}^N \lambda_{nj} = 1.0 \text{ for any item } j$$

$Q_j$  = bill quantity of item  $j$

$Q'_j$  = additional quantity of item  $j$  due to variation

$R_n$  = proportion retained in month  $n$

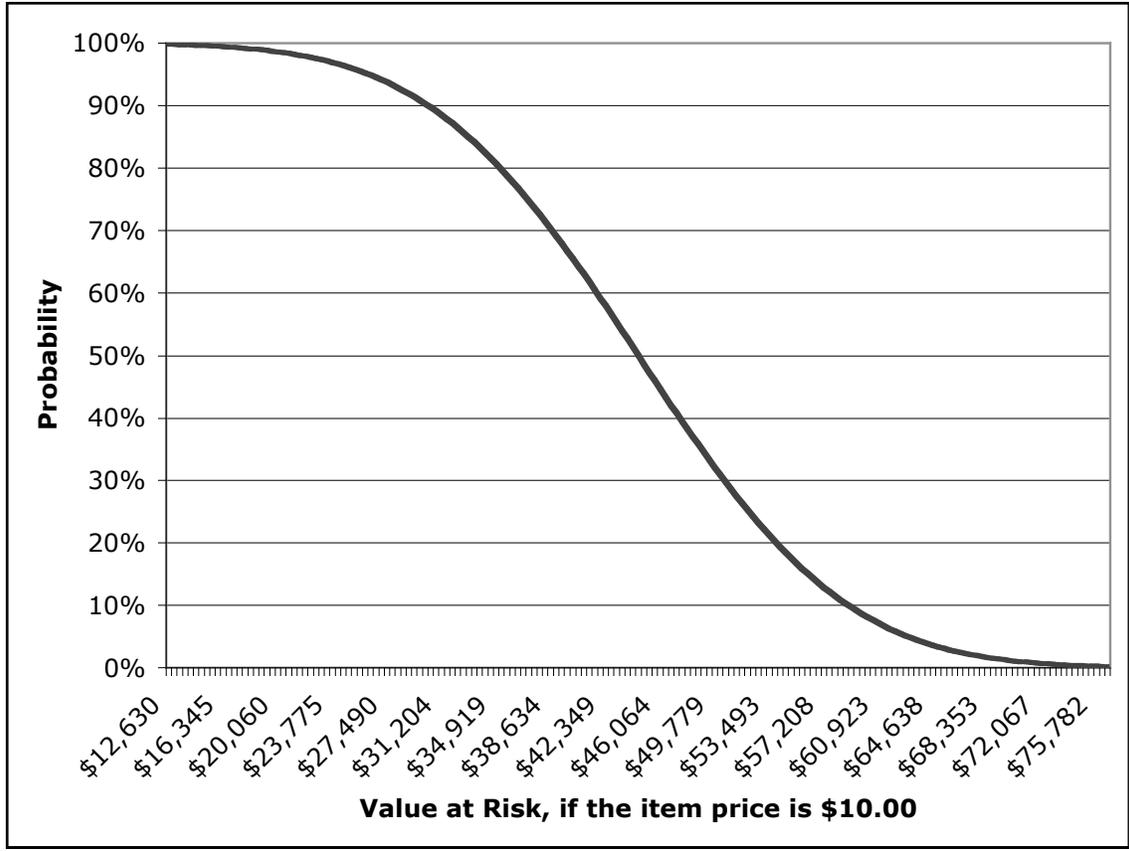
$f$  = adjustable factor (e.g. 0.85 for "Haylett" contracts)

$$Y_{nj} = \text{adjustment for escalation} = \frac{\text{index}_n - \text{index}_0}{\text{index}_0}$$

This formulation provides the measure of expected reward from each item  $j$  for each unit of currency applied to its price. To determine the risk associated with this, the contractor needs to consider the underlying uncertainty in each of the following estimates that have been utilized:

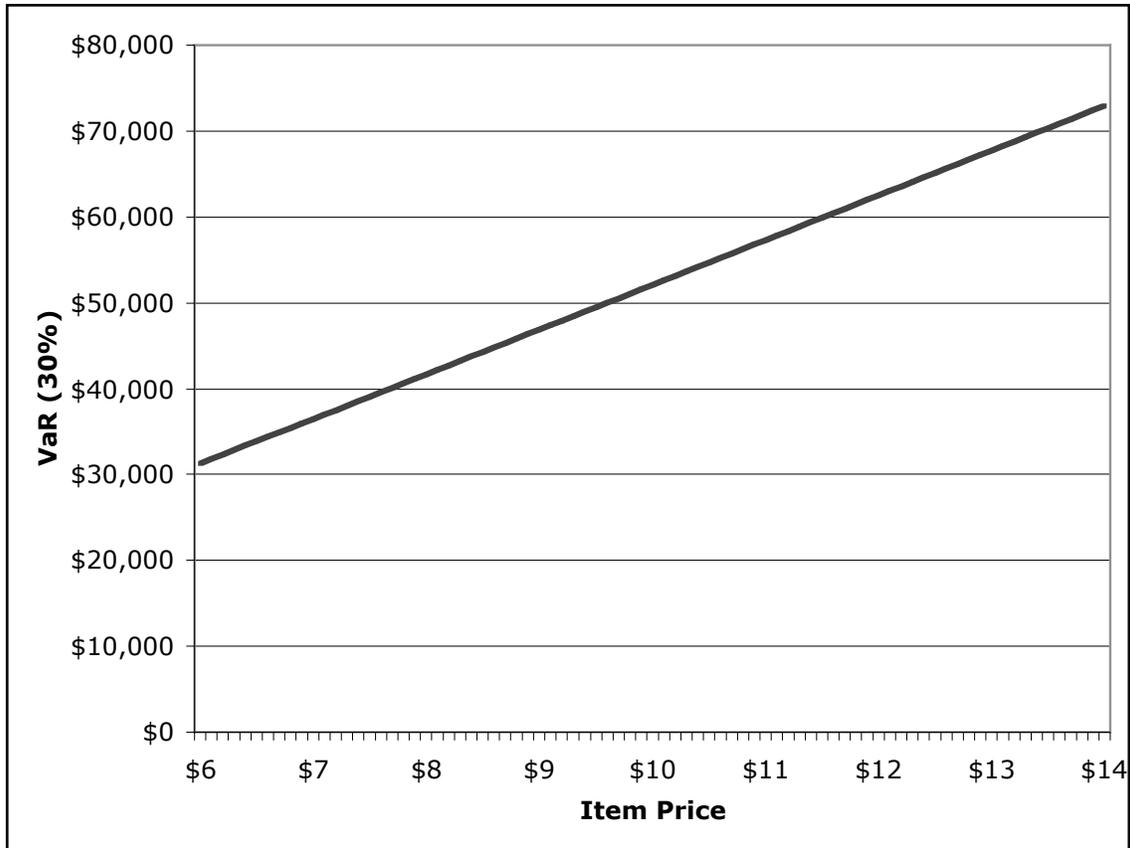
- a discounting rate
- the scheduled timing of items
- any expected variation in quantity
- escalation rates (for each workgroup).

By using Monte Carlo simulation, the contractor can determine the Value at Risk by way of these factors combined. The authors have used a spreadsheet analysis to do this to generate the following chart representing this risk. Various assumptions have been made to accomplish this illustration, as regards the certainty / variance in the above-mentioned variables. These will all vary from project to project depending on the prevailing circumstances – which has to be estimated by the contractor in each instance.



This chart of the inverse cumulative distribution function shows that, for instance, there's a 30% probability that, if this item is priced at \$10.00, the (present-day) Value at Risk, for reason of uncertainty in the above variables, is around \$50,000. In other words, it has been calculated that there is a 30% chance that the return from having assigned a \$10.00 price to this item will be \$50,000 less than that which is expected.

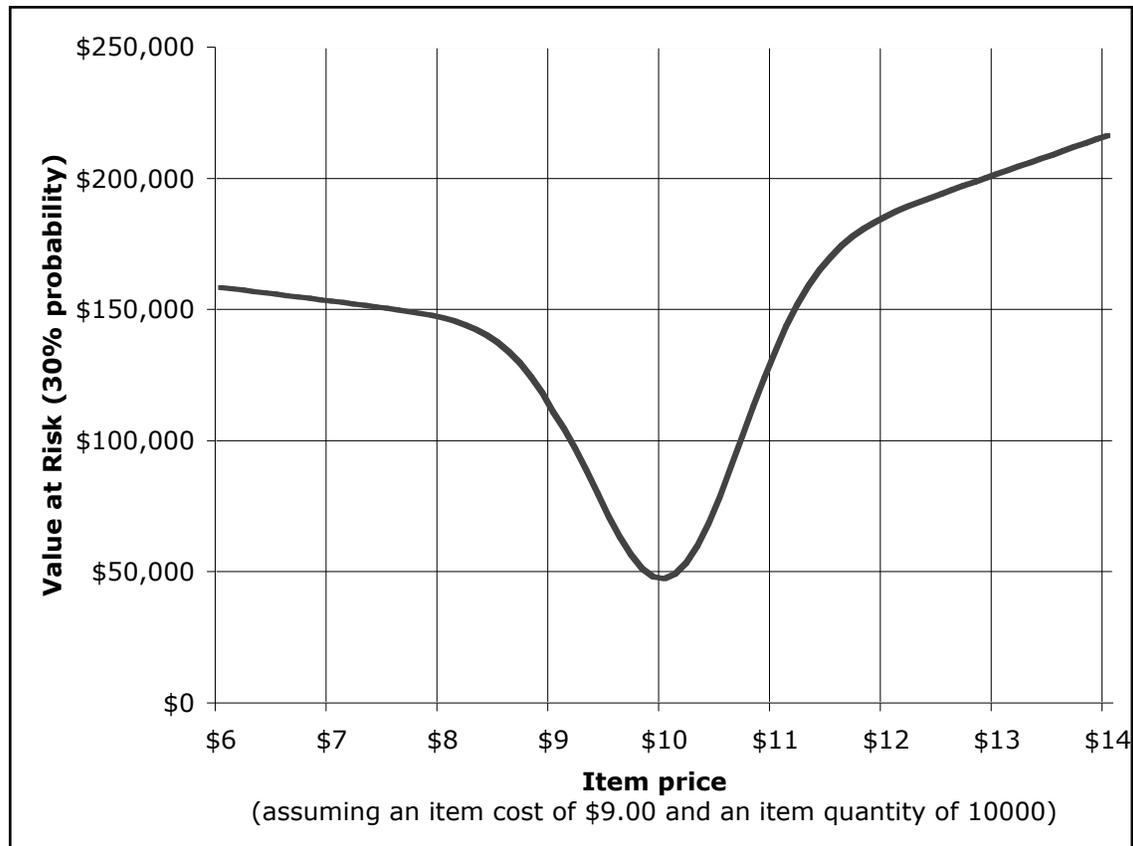
Higher prices for any item result in more risk, the extent to which depends on the inherent uncertainty associated with this item.



When this risk is expressed in this form it gives a basis by which it can be combined with the risks contributed by way of ‘rejection’ and ‘reaction’.

### **Illustration of the combined risks**

By deciding on a rate of risk for the purposes of this analysis, the contractor can then add up the three risks that will be generated by each item at each price point. An example is shown in the chart below.

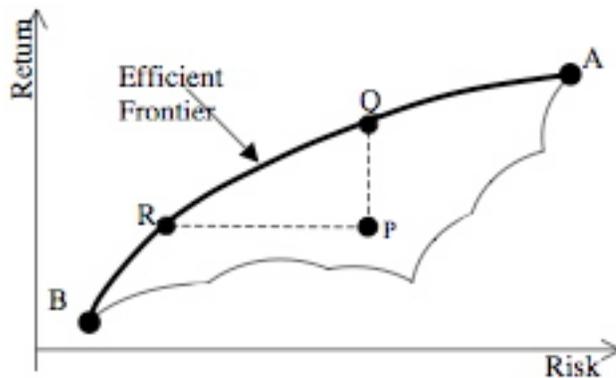


## Markowitz and Modern Portfolio Theory

The above-mentioned techniques provide measures of both the expected rewards as well as the risks of all possible item price combinations for a project. Modern Portfolio Theory ('MPT') provides a basis by which to assess these relative to each other.

MPT was developed by Markowitz (1952) and is well described in Markowitz's Nobel lecture (1990). It was intended purely as a basis by which to identify *efficient frontiers* of investment portfolios. These are defined as the combinations of a given set of investments, such that no other combination could accomplish any higher expected rate of return without an increase in risk. Markowitz hence found the method by which to account for risk such that one should logically discard any consideration for any other

combination of portfolio that is not found to be *efficient*. For any choice of inefficient portfolio, there is logically a better choice of an efficient one available: one that offers either a better expected return (for the same or lesser amount of risk) or else a lower rate of risk (for the same or better rate of expected return).



In the above chart (taken from Cattell *et al.*, 2004), this is illustrated by way of options Q and R being both preferable to option P. Option Q is a higher risk, higher return alternative to option R, but different contractors, with different risk profiles, may justifiably choose either of these in preference to the other.

MPT served as a precursor to VaR and later, both methods came to be used extensively in combination. Markowitz's own research measured risk by way of variance (or what is often called 'volatility') (often using the Standard Deviation as the specific statistical indicator) whereas, in more recent years, this has typically been replaced by the use of VaR as the indicator of risk.

As with VaR, MPT was intended solely for application with investments, and more particular so, equity investments. Nevertheless, Vergara (1977) pioneered the application of MPT to the field of cost estimating in construction. Additionally, it was first proposed by Cattell (1985) that it suits the purposes of unbalanced bidding models. The latter are

also in need of a method by which returns and risk are considered in unison: with contractors having very similar goals in this regard as does a portfolio investment manager. Contractors are also wishing to maximize their returns at the same time as minimize their risks.

## **Conclusion**

This paper has provided a new framework by which unbalanced bidding models can account for risk. Furthermore, it has proposed that the VaR measure of risk is better suited than any of those that express risk in terms of variability or uncertainty. It has then also proposed that these risks can best be weighed up against the expected prospects of unbalanced bidding by way of the use of Modern Portfolio Theory. This paper is therefore drawing on the field of mainstream financial economics, as most often employed in investment finance, so as to make the field of unbalanced bidding of adequate effect and sophistication that it might become of practical value to contractors. It is suggested that this value could be substantial.

Notice that this model is the first to successfully avoid having to impose upper and lower price limits on each item. It succeeds in constraining prices by way of a more ‘fuzzy’ nature of boundary recognizing that extreme prices not only have the prospect of generating high expected returns but that these also generate considerably more risk than prices that more-closely resemble those that the industry is more accustomed to. This model succeeds in quantifying this in a manner that provides contractors with a tool by which they can weigh up the allure of high returns relative to the risks that are involved.

## **Suggestions for further research**

It is proposed that unbalanced bidding is comparable to some degree to the practice, in retail marketing, of using loss-leaders to attract customers to a store, knowing that these same customers are likely to stock up their trolleys, whilst they are there, with many other items that have been priced with far higher markups. Unbalanced bidding models may therefore be able to contribute something to the science of consumer retail marketing, and visa versa.

Further research is being done to apply Expected Utility Theory and Indifference Mapping to add further to the sophistication of this model. This combined model then needs testing to determine its likely practical worth to contractors.

## **References**

- Cattell, D.W. (1985) An introduction to bidding models. *Planning and Building Developments*, No.74, June/July, pp.61-63.
- Cattell, D.W., Bowen, P.A. and Kaka, A.P. (2004) A model to distribute mark-up amongst quotation component items: an outline, in *2<sup>nd</sup> Postgraduate Conference on Construction Industry Development proceedings*, Cape Town, Oct, CIDB, pp. 154-165.
- Cattell, D.W., Bowen, P.A. and Kaka, A.P. (2007) Review of unbalanced bidding models in construction. *American Society of Civil Engineers, Journal of the Construction Division*, Vol. 133, No. 8, pp. 562-573.

- Cattell, D.W., Bowen, P.A. and Kaka, A.P. (2009) A simplified unbalanced bidding model. *Construction Management and Economics*, Vol. xx, No.x, pp. xxx-yyy.
- Diekmann, J.E., Mayer, R.H. Jr. and Stark, R.M. (1982) Coping with uncertainty in unit price contracting. *American Society of Civil Engineers, Journal of the Construction Division*, Vol. 108, No. C03, pp. 379-389.
- Kolman, J., Onak, M., Jorion, P., Taleb, N., Derman, E., Putnam, B., Sandor, R., Jonas, S., Dembo, R., Holt, G., Tanenbaum, R., Margrabe, W., Mudge, D., Lam J. and Rozsypal, J. (1998) Roundtable: The Limits of VaR. *Derivatives Strategy*, April.
- Manganele, S. and Engle, R.F. (2001) Value at risk models in finance. European Central Bank: *Working Paper Series*, No. 75, August.
- Markowitz, H.M. (1952). Portfolio selection. *Journal of Finance* 7 (1), pp. 77–91.
- Markowitz, H.M. (1990) Foundations of portfolio theory. *Nobel lectures in economic sciences 1981-1990*. Ed. Karl-Göran Mäler, World Scientific, pp.279-288.
- Stark, R.M. (1968) Unbalanced bidding models – theory. *American Society of Civil Engineers, Journal of the Construction Division*, Vol. 94, No. C02, pp. 197-209.
- Stark, R.M. (1972) Unbalancing of tenders. *Proceedings of the Institute of Civil Engineers*, Technical Note 59, Vol. 51, pp. 391-392.
- Stark, R.M. (1974) Unbalanced highway contract tendering. *Operational Research Quarterly*, 25(3), pp. 373-388.
- Teicholz, P.M. and Ashley, D.B. (1978) Optimal bid prices for unit price contract. *American Society of Civil Engineers, Journal of the Construction Division*, Vol. 104, No. 1, pp. 57-67.

Tong, Y. and Lu, Y. (1992) Unbalanced bidding on contracts with variation trends in client-provided quantities. *Construction Management and Economics*, **10**, pp. 69-80.

Vergara, A.J. (1977) Probabilistic estimating and applications of portfolio theory in construction. Unpublished PhD thesis, Department of Civil Engineering, University of Illinois at Urbana-Champaign.

Wang, W.C., Wang, H.H., Lai, Y.T. and Li, J.C.C. (2006) Unit price based model for evaluating competitive bids. *International Journal of Project Management*, Vol. 24, Issue 2, pp. 156-166.