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# Analysis of the Problem of Prediction in Physics from a Computational and Foundational Approach

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**Abstract**

In this paper, we analyze the problem of prediction in physics from a computational and foundational viewpoint. Besides showing that assumptions like Laplace determinism, statistical determinism, etc., can be naturally explained by this computational analysis, we use the concepts of Kolmogorov complexity and algorithmic randomness from Algorithmic Information Theory, as well as the novel more physics-oriented variants of these notions, to show that the dynamical evolution of a system can be practically predicted with the dynamical laws, the initial, *and* final conditions of the system.

## 1 Introduction

### 1.1 Computability in theoretical computer science: traditional approach and practical problems.

Traditionally, the theoretical analysis of computations is mainly concentrated on analyzing which *mathematical* functions can be computed, which can be efficiently computed, etc. Furthermore, from the practical viewpoint of computability, computing mathematical functions and solving precisely formulated mathematical problems are auxiliary tasks.

One of the main objectives of computations is to predict the behavior of different processes in the real (*physical*) world. In view of this objective, it is

important to distinguish between:

- general *data* and
- data that helps with prediction–i.e., that represents *knowledge*

It is also important to analyze what can be computably predicted and what cannot be predicted.

## 1.2 A Brief History of the Problem of Predictability

**Predictability in physics: different paradigms.** The understanding of what can be predicted has changed throughout the history of physics; see, e.g., [**?**]. Usually, a new paradigm appears when for some real-life processes, the previous paradigm turns out to be too optimistic; examples will be given below.

**First paradigm: Full Cognition.** At the beginning of the modern scientific era, the understanding was that science could potentially describe and predict all the details of the process "from scratch" and for all types of experiences. For example, Kepler was not only predicting how the planets move, he was also trying to predict the fate of princes by looking at the positions of the stars.

At present, we understand that it is possible to predict how the stars and the planets move, but it does not seem to be possible to predict human behavior and humanity's future. However, in Kepler's time, the promise of natural philosophy was high and there seems to have been no understanding of this difference: science was supposed to be able to explain and predict everything. Yes, it is easier to predict how the planets move, it is more difficult to predict how people will behave, but both were considered legitimate science.

Similarly, now we know that it is not possible to make long-term (even yearly) weather predictions. However, even in the early 18th century, many weather almanacs were published that tried to predict weather for one (or more) years ahead – and these almanacs were actively bought by practicing farmers.

**Limitations of the Full Cognition (FG) paradigm.** The inability to predict such things as weather and human behavior led scientists to realize that a universal prediction "from scratch" is not always possible.

**Laplace Determinism (LD).** It turned out that while we cannot predict these phenomena based on *general principles* and ideas for all types of experiences, we can often get reasonable predictions if we know the current state of the system.

For example, it is impossible to predict the fate of the newborn child based on the locations of stars and planets at the time when the child was born. However, when we observe the behavior of a person, when we find out his or her strengths, weaknesses, and behavior patterns, we can often reasonably well predict this person's future especially if we assume that no life-changing events

occur. Similarly, when we observe the weather pattern in a given location, we can usually predict, e.g., that May will be dry and warm, that August will be rainy, etc. These examples illustrate that prediction of future states of a system depends on including on the system the relevant 'forces' and influences affecting it. The stars will not exert relevant influence on the child's future, while behavior patterns probably will. The general weather pattern of a region can be predicted if we don't care about the detailed patterns, which is extremely difficult to do.

The idea that if we know the current state ("initial conditions") then we can predict the future behavior of the system was first explicitly formulated by Pierre-Simon Laplace, the 19th century mathematician and physicist. Because of this, the corresponding paradigm is usually called *Laplace Determinism*. Early on in *A Philosophical Essay on Probabilities"* Laplace states his idea of determinism where an intelligent agent, often referred as Laplace's Demon, given initial conditions and the equations of motion, would be able to compute all future states of the system:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it–an intelligence sufficiently vast to submit these data to analysis–it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes. ([**?**], p. 4)

**Laplace Determinism: successes.** Laplace Determinism has been the major paradigm of 19th century physics. Newton's mechanics, Maxwell's electrodynamics, many other physical theories – they are all formulated in terms of systems of differential equations, equations that enable us to go from the initial conditions to the predictions of the future states.

Even now, Laplace Determinism covers a significant portion of practical applications of physics.

**Laplace Determinism: limitations.** At the end of the 19th century, several new physical processes were discovered in which prediction turned out to be impossible. The most well known of these processes was radioactivity, i.e., the ability of atoms to spontaneously turn into other atoms.

All atoms of a radioactive material are absolutely identical, i.e., have practically identical initial conditions. So, from the viewpoint of the Laplace Determinism, they should all behave in exactly the same way – in particular, they should all undergo radioactive decay at the same time.

In reality, different atoms decay at different times, and all attempts to predict these times were unsuccessful.

Another limitation of Laplace Determinism comes from the analysis of decisions made by humans (and by other living beings). The famous Buridan's ass paradox considers a donkey placed between two absolutely identical bales of hay. Since these bales are absolutely identical, there is no reason for select one of them and not the other. Thus, a deterministic decision-making donkey will not be able to select one of these bales – and thus, the poor donkey may eventually starve to death. In practice, this will never happen – because a donkey would select one of the bales at random.

The need to take into account such random selections makes deterministic predictions impossible. Furthermore, suspicions arose that such a position challenged the possibility of a meaningful view of free will and therefore ethics and laws. This enhanced an already developing debate on the role and nature of chance, which Laplace himself developed a position which is now regarded as a classical formulation of the theory of probability, which in turn contributed to the further development and teasing out the concepts of chance, probability, determinism, and prediction. This historical development is rich, complex, ongoing, and relevant to our proposal but also beyond the scope of this paper. A decent review on the topic is found in [**?**].

**Statistical Determinism (SD).** The physicists started collecting data about individual radioactive decay events. While they were unable to predict the time of individual events, they found out that the *frequencies* with which different atoms decay at different moments of time can be predicted.

In other words, while we cannot predict the exact future states, we can predict the *probabilities* of different future states.

At first, it was assumed that this need to restrict ourselves to statistical predictions was only typical of *some* exotic events like radioactivity. However, the successes of quantum physics have shown that a similar phenomenon holds for *all* physical predictions – at least those predictions in which we must take into account the properties of microobjects like atoms or elementary particles. Equations of quantum mechanics do not predict the values of the future location or velocity of a particle, they only describe the so-called *wave function* $\psi(q)$. Once we know the wave function, we can estimate the probability of finding a particle at a location $q$ (as $|\psi(q)|^2$).

From the modern viewpoint, this need to consider probabilities is a well-known textbook fact. However, in the 1920s Laplace Determinism was still the norm even though new approaches to understanding probability began to appear. Logical theories of probability attempted to generalized and axiomatize the classical theory (Keynes [**?**] and Carnap [**?**]). The Frequentist interpretations also began to gain adherents (Reichenbach [**?**] and even Einstein) by considering that probability refers to actual or possible outcomes of many trials. Bayesianism, taking probability to entail degree of belief of the subject (De Finetti [**?**], Jaynes [**?**]) and propensity interpretations, where single events also have a meaningful sense of its probability of occurrence (Popper [**?**]), also took hold to address the concerns with traditional accounts of prediction and

probability. In general, the shift away from Laplace's idea entailed treating the role of probabilities in prediction much more carefully.

The paradigm in which, once we know the initial conditions, we can predict the probabilities of different future events, is usually called *Statistical Determinism.*

Statistical Determinism is still the prevailing paradigm in most areas of fundamental physics. The debates dwell, instead, on how to interpret the meaning of probabilities and the range of determinacy.

**Limitations of Statistical Determinism and the emergence of a new paradigm.** Starting with the 1960s, it turned out that for many real-life phenomena, even predicting probabilities is not always possible.

This phenomenon is best known for *chaotic systems*, i.e., systems in which a minor (practically non-observable) deviation in the initial conditions can lead to a drastic qualitative change in the future behavior of a system.

This phenomenon was first observed on the example of a simplified system of equations for predicting weather, and it has since been observed in numerous real-life phenomena.

Since even predicting probabilities is not always possible, the modern paradigm of physics – which we would call the Modern Approach (MA) – claims simply that if we know the initial conditions, then we can predict *some information* about the probabilities of different future states.

**Physics paradigms: computational challenge.** We have described the past and current physical paradigms. We have described them in a purely empirical way. Since these paradigms are about predictions – i.e., in effect, about prediction-related *computations* – it is desirable to understand these paradigms (and their relations and transitions) from the computational viewpoint.

**What we do in this paper.** In the paper, we will show that an appropriate computational analysis of predictability naturally explains the appearance of these different physical paradigms. We will also describe how this analysis can be used in physics. As an example, we will give a recent unusual result from computational cosmology.

*Comment.* From the mathematical viewpoint, our analysis uses the notions of Algorithmic Information Theory such as Kolmogorov complexity and algorithmic randomness, as well as the novel, more physics-oriented variants of these notions.

## 2 Algorithmic Information Theory: How It Naturally Arises from Physical Paradigms

**The need to describe randomness.** As we have mentioned, in Statistical Determinism, we predict the probabilities of different future states. For example, we predict the probabilities that an atom decays at different moments of time. We are unable to predict when each individual atom decays.

For example, if we start with a given set of $n$ atoms $a_1, \ldots, a_n$, and we know that at a certain moment of time, half of these atoms decay. We can describe the final state of this system by describing which atoms decayed and which did not. This can be naturally done by describing an $n$-bit sequence in which the $i$-bit is 1 if the $i$-th atom decayed and is 0 if it did not. For example, 101... means that the 1st and the 3rd atoms decayed, but the 2nd atom did not. We cannot predict which of the $n$ atoms decayed. The actual sequence of decayed atoms is *random*, in the sense that all the information it contains is that half of the atoms decayed.

The fact that we cannot predict which atoms decay means that regular sequences like 010101... are not physically possible – because in such a sequence, we will be able to easily predict which atoms will decay and which atoms will not.

How can we describe this notion of randomness in precise terms?

**Kolmogorov's original definition of randomness.** In the 1960s, A. N. Kolmolgorov and his student P. Martin-Löf used the difficulty-to-predict as a formal definition of randomness.

They started by observing that when a binary sequence $s$ is regular, i.e., has the form 010101..., then even when this sequence is long, we can write a short (and simple) program $p$ that generates this sequence. For example, to generate a sequence consisting of 1,000 pairs 01, we can write a simple for-loop:

```
for (i = 0; i < 1000; i++)
  print("01");
```

Vice versa, if a short program can generate a long sequence, this means that this sequence $s$ is regular.

The fact that a sequence is random means that it is not regular, i.e., that it is not possible to have a short program that generates this sequence.

Of course, we are dealing with finite sequences, and a finite sequence $s =$ 11001... can always be generated by a program that prints this sequence bit by bit:

```
print("11001...");
```

However, if we use this idea to generate a sequence consisting of 1,000 symbols, we will need a program of the same length of 1,000 symbols (even a little bit longer, because we also need a print command).

Thus, the difference between regular sequences and random sequences is that

- regular sequences can be generated by programs which are much shorter, while
- random sequences can only be generated by programs of approximately the same length.

To capture this difference, Kolmogorov and Martin-Löf defined *Kolmogorov complexity* $K(s)$ of a finite sequence $s$ as the shortest length of a program that generates the sequence $s$.

We can always generate a given sequence $s$ by simply printing it bit by bit. We have already mentioned that the length of this program is equal to the length $\text{len}(s)$ of the sequence $s$ plus a small number of bits $C$ needed to describe the print statement. Thus, the Kolmogorov complexity $K(s)$ of an arbitrary sequence $s$ – which is defined as the shortest length of the program that generates $s$ – cannot exceed the length $\text{len}(s)+C$ of this bit-by-bit program: $K(s) \le \text{len}(s) + C$.

For random sequences, no significantly shorter programs are possible. Thus, for a random sequence $s$, its Kolmogorov complexity $K(s)$ – the length of the shortest program for generating $s$ – cannot be much smaller than its length. This idea is behind the definition of randomness of a binary sequence.

Let $C$ be an integer. A finite binary sequenced is called *$C$-random* if $K(s) \ge \text{len}(s) - C$.

*Comment.* Strictly speaking, to make this definition precise, we need to fix a (universal) programming language, i.e., a programming language in which every algorithm can be programmed (like C or Java). However, it turned out that all the languages are, in some reasonable sense, equivalent: namely, the choice of a language only changes the length by an additive constant.

**Beyond regular and random sequences: a useful consequence of Kolmogorov's formalization of randomness.** In the previous paragraphs, we only considered two types of sequences: regular sequences and random sequences.

What about generic sequences? For an arbitrary sequence $s$, we can also describe its Kolmogorov complexity as the length of the shortest program $p$ that generates this sequence $s$. Once we know the program $p$, we can easily generate $s$ – by simply running a compiler from the corresponding programming language.

The important point is that, as we will show, the program $p$ is itself random in the Kolmogorov-Martin-Löf sense. Indeed, if we could generate $p$ by running a much shorter program $q$ with $\text{len}(q) \ll \text{len}(p)$, then we would be able to generate $x$ from $q$ as well: by first generating the code $p$, and then by running this code to generate $x$. The fact that $p$ is the shortest program for generating $x$ means that $q$ cannot be much shorter than $p$ – i.e., that the Kolmogorov complexity $K(p)$ must be approximately equal to the length $\text{len}(p)$ of this sequence $p$.

By definition, sequence for which $K(p) \approx \text{len}(p)$ are called random. Thus, $p$ is indeed a random sequence.

So, we arrive at a somewhat non-intuitive conclusion: that an arbitrary binary sequence $s$ can be obtained by applying a simple algorithm to some auxiliary random sequence $p$.

*Comment.* For a mathematician or for a physicist, the fact that an arbitrary sequence can be obtained by applying an appropriate algorithmic process to a random sequence may sound novel. While mathematics behind Kolmogorov's complexity is indeed reasonably novel, the possibility to represent an arbitrary signal as a combination of deterministic and random effects is well known and well established in statistical data and signal processing. There, the main objective – usually attainable – is to to find an approximate deterministic dependence for which the approximation error is purely random.

**What we do in this paper.** In this paper, we use the Kolmogorov complexity ideas to show that many physicists paradigms can be explained by the need for algorithmically feasible predictions.

# 3 Explanation of Main Cognitive Paradigms of Physics

**Need for prediction: reminder.** Our objective is to predict the state $s(t)$ of the Universe at future moments.

To be more precise, we would like to predict *as much as possible* about the future states of the Universe.

**Ideal situation.** In the ideal situation, we should have an algorithm that, given a system with an external reference frame for localization $q$, a dynamical evolution $p$, and the quantity $f$, would predict the value $f(q, p)$ of the quantity $f$ of a future state of the system.

This ideal situation corresponds to what we called *full cognition.*

**Algorithm that uses measurement results.** Since it turned out that no such universal algorithm is possible, the next natural idea is to use the initial conditions – i.e., the current values of the physical quantities – to predict the future state of the Universe. This is what we called *Laplace Determinism.*

**Where is the beef?** At this point, a reader may rightfully ask: what is the point of simply repeating the paradigms that we have already covered in the introduction? Up to now, there was indeed no point yet, but non-so-trivial consequences will start in the next paragraph.

**Interesting consequence: initial conditions should be random.** What does this mean that we are unable to predict the state of the universe without using initial conditions? It simply means that initial conditions cannot be predicted "from scratch" – otherwise, we would be able to combine the two algorithms:

- algorithm for computing these initial conditions, and
- the algorithm for predicting the future state of the world based on these initial conditions

and thus, get an (assumed impossible) algorithm for full cognition.

In algorithmic terms, what does it mean for a sequence to be unpredictable? In Kolmolgorov's formalization, unpredictability simply means that the initial conditions are *random*.

**Why is this consequence physically interesting?** The fact that initial conditions are random may be an interesting mathematical observation, but more importantly is that it has physical consequences.

These physical consequences are related to the seeming contradiction between the physical equations and common sense about the direction of time.

From the purely mathematical viewpoint and assuming the existence of an external parameter $t$, Newton's laws do not change if we simply replace the original time coordinate $t$ with a new coordinate $t' = -t$ – i.e., if we change the direction of time. If we videotape a purely mechanical motion and then show the motion backwards, the resulting motion will still make perfect sense.

However, when we turn from a system consisting of a few bodies to a system consisting of a large number of molecules, this time symmetry somehow disappears. Examples of this obvious (and mysterious) time asymmetry are well known and often illustrated with Maxwell's demon.

For example, it is easy to observe that if we prepare a vessel with two separate compartments, place smoke in one compartment, leave clean air in another compartment, and then remove the separation, the gases will mix. The equations all remain time-invariant. Strictly speaking, if at the moment when the two gases have mixed, we simply invert the directions of all the velocities, we should see the same process reverting itself – and gases separating. A mathematician may say that this reversion is still physically possible but its probability is very low. However, a physicist would usually make a much stronger statement: the separation is simply not possible.

Let us give another example in which this impossibility is even more clear. If I drop a breakable cup on the hard floor, it will break into pieces. Theoretically, it may seem possible to imaging the reverse phenomenon: that the broken pieces magically fall back into place – but from the common sense viewpoint, everyone will agree that this magical recovery of the whole cup from pieces is not possible.

How do physicists explain this impossibility? Clearly this impossibility does not come from equations – since all the equations do not change if we simply replace $t$ with $-t$. A usual physicists' explanation is that the hypothetic cup

recovery process is not stable. We never know the exact values of the initial locations and initial velocities, and if we change these initial conditions a little bit, the magic recovery of a broken cup will simply not occur.

In precise terms, what the physicists are saying is that for a cup to recover, we need to have very special, very well-defined initial conditions. However, since there are many unpredictable effects, the actual initial conditions cannot be predictable and exact.

In other words, what the physicists are saying is that it is not sufficient to describe the equations describing physical dynamics, we must also postulate that the initial conditions are *random*.

Our claim is that there is actually no need for a special new postulate – the very fact that we went from Full Cognition to Laplace Determinism means, in effect, that the initial conditions should be random.

**Towards explaining other physical ideas within the Laplace Determinism paradigm.** The fact that the randomness of initial conditions seems to follow naturally from our analysis of cognition makes us wonder that maybe other physical ideas within Laplace Determinism can be also be naturally explained by a similar analysis.

Indeed, the general idea of Laplace Determinism is that we predict the values $f(q_i, p_i)$ of different physical quantities $f$ at difference locations $q_i$ dynamically based on the initial conditions $g(q_0, p_0)$. In the worst-case scenario,

- to predict each value $f(q_i, p_i)$, we need to have full information about all the initial conditions, i.e., to have all the values $f(q_j, p_0)$ corresponding to all possible spatial locations $q_j$;
- each quantity can attain all possible numbers as its values, so, e.g., to get the value with accuracy $2^{-k}$, we need to actually generate all $k$ bits from this sequence;
- finally, once we have succeeded in predicting the value of one physical quantity $f$ at a certain location, this prediction will not in any way help in predicting the values of other quantities at different spatial locations.

The resulting computations require a large amount of input (all the initial conditions), produce the large size output (all bits need to be produced), and cannot utilize other previous predictive computations. In this worst case, the success of these predictive computations is thus in doubt. To make successful predictions, it is therefore reasonable to look for physical systems in which at least some of these complexities are relaxed.

Let us describe possible relaxations and let us show that they indeed correspond to fundamental physical ideas.

**Relaxing input complexities: the notion of causality.** Let us first consider what will happen is we relax the need for the value $f(q_i, p_i)$ to depend on the initial values $g(q_j, p_0)$ for *all* possible spatial locations $q_j$. Relaxing this need

means that we assume that the value $f(x,t)$ of a physical field only depends on the values $g(q_j, p_0)$ for *some* spatial locations.

The values of $g(q_j, p_0)$ at other spatial locations $q_j$ do not affect the values $f(q_i, p_i)$ at all. In other words, there may be systems defined by $g(q_j, p_j)$ that do not affect in reality the system defined by $f(q_i, p_i)$. The simplest example would be to imagine the lightcone of system $f$ where the system $g$ lies outside of it. We are conceiving here that the system is the physical field.

Since physical fields are interconnected, it is reasonable to expect that since the value $f(q_i, p_i)$ of the physical field $f$ does not depend on $g(q_j, p_0)$, the values of all other physical fields at the location $q_i$ at future moments are also not affected by the values $g(q_j, p_0)$. Thus, this limitation does not depend on the specific fields, it depends only on the choice of coordinates for $q$ and $p$. That is, using our earlier example, we could choose a value of the variable $q$ and $p$ so that $g$ would be part of the lightcone of $f$.

Thus, we have systems described by $f(q_i, p_i)$ which are affected by other systems $g(q_j, p_0)$ and systems which are nor affected. In other words, we naturally arrive at the notion of *causality*, the notion that – in this space-time form – only appeared in the mainstream physics in the early 20 century, with Einstein's Special Relativity Theory. Reichenbach is often credited with this insight. [**?**]

What we have shown is that from the algorithmic viewpoint, this notion can be very naturally derived.

**Relaxing output limitations: bounds on physical quantities, quantization, conservation laws.** The output complexity comes from the fact that, in general, all real values are possible for each physical quantity. In the worst case scenario,

- arbitrarily large and arbitrarily small values are possible, and (no lower or upper bounds)
- once two real values are possible, all intermediate values are possible too (continuity).

Thus, to relax these limitations, it is reasonable to consider the possibility that

- there are bounds on the values of physical properties – from above and/or from below, and
- not all intermediate values are possible, there is only a limited set of possible values (discreteness).

Such possibilities are well known in physics. For example, there are bounds on the possible values of the velocities (bounded by the speed of light), distance (bounded by the size of the Universe), entropy's rate of change (bounded by 0 from above), etc.

It is also well known that some quantities like electric charge can only take values from a certain discrete of values.

For some quantities, possible future values are limited to a single value – these are properties described by *conservation laws* such as energy conservation if certain conditions are met.

**Relaxing the independence limitations: the emergence of symmetry.** Finally, the relaxation of the third complication – that predicting one values $f(q_i, p_i)$ does not help in predicting other values – means that once we know the value $f(q_i, p_i)$, this will help us predict the values of some other quantities dynamically at some other spatial locations.

The possibility to use the values at some points to make predictions about the values at other points is related to the notion of *symmetry*, one of the most fundamental notions of modern physics.

**These physical consequences are not always independent.** In our informal description, all these physical consequences seem independent from each other. However, a detailed analysis has shown that they these consequences are actually interrelated.

For example, an accurate Kolmogorov complexity-based formalization of the idea that initial conditions should be random leads to the need to bound the values of physical quantities; see, e.g., [**?**].

To be more precise, this interrelation is based not only on the original notion of Kolmogorov randomness but also on it physics-related modification. This modification [**?**, **?**] is based on the fact that according to physicists, events with sufficiently small probabilities cannot occur. This means, in particular, that once we have a definable sequence of events $A_1 \supseteq A_2 \supseteq \ldots \supseteq A_n \supseteq \ldots$ with an empty intersection $\cap A_n = \emptyset$, we have $p(A_n) \to 0$ and therefore, for some $N$, this probability becomes so small that this event does not occur.

We can thus require that the set $E$ of actually possible events has the following property: for every definable monotonic sequence $A_n \supseteq A_{n+1}$ with the empty intersection, there exists an integer $N$ for which $A_N \cap E = \emptyset$.

To show that this definition leads to boundedness of a physical quantity $q$, we can simply take, as $A_n$, the set of all the events in which the value of $q$ is outside the interval $[-n, n]$: $|q| > n$. Clearly, this sequence is monotonic, and the intersection of all the sets $A_n$ is empty: since a real number cannot be larger than all possible integers $n$.

Thus, there exists a value $N$ for which no events from the set $A_N$ can occur. By definition of the event $A_N$, this means that values $q$ with $|q| > N$ are impossible – and thus, that all the physical values of the quantity $q$ are bounded by the bound $N$.

Another way of expressing this result is that the initial conditions are not sufficient for determining the dynamical evolution of the system, and that other dynamical conditions (some may refer to them as final conditions, see [**?**]), are also necessary. For an analysis of this idea, see [**?**].

*Comment.* This argument sounds *simple* – and it is simple. However, this simplicity is based on a rather *complex* proof that the above requirement on the set $E$ of actual events is indeed physically possible; for details of this proof, see, e.g., [**?**] and references therein.

**What if we still cannot predict everything?** In the previous paragraphs, we describe possible physical ideas that can be helpful in predicting the future state of the physical world – i.e., in effect, the future values of different physical quantities.

But what if, even with all these physical ideas, we are still unable to make predictions? According to our Kolmogorov complexity analysis, this would simply mean that the resulting values $f(q, p)$ cannot be algorithmically determined based on the initial conditions – i.e., that there are some random events that also determine these future values.

In such situation, we cannot predict the actual values of the future quantities, but we can predict the *probabilities* of different values. This is the situation that we called *Statistical Determinism.*

**Beyond Statistical Determinism.** At first glance, according to our analysis, the algorithms operating on random sequences cover all possible measurement results. Since this representation corresponds to Statistical Determinism, why do we need to go beyond this physical paradigm?

The main reason is that in the original Kolmogorov's analysis, he only took into account the *length* of the corresponding program, but not its *running time*. For physical predictions, running time can be crucial: if we can predict the future, but these predictions will continue way after these events occur, then these predictions are useless. Alas, this possibility is not purely theoretical: there are many important real-life prediction problems – such a predicting where a tornado will go – for which the only known predicting algorithms require running time which goes way past the actual tornado motion.

For such algorithms, even probabilities are difficult to predict. A typical example of such situations are chaotic systems. Chaotic systems are usually deterministic, there is a simple program generating this sequence – but the actual predictions are nevertheless not computationally possible.

In mathematical terms, these situations are described by *resource-bounded Kolmogorov complexity*, versions of Kolmogorov complexity that take into account not only the length of the program, but also its running time – and maybe other characteristics like the number of processors needed to run the corresponding algorithms. At present, applications of resource-bounded Kolmogorov complexity to physics are much less developed that for the traditional Kolmogorov complexity. We hope that future developments in this area will pour some light on what we called Modern Approach paradigm of physics.

# 4 Case Study: A Recent Cosmological Breakthrough as an Example of a Successful Prediction

**Prediction in cosmology: a brief description of the problem.** Cosmology studies how the Universe as a whole evolves, it studies the large-scale structure of the Universe.

For cosmology, we can also formulate a usual prediction problem: how can we predict the future state of the Universe? What will happen in the next ten billion years? These questions are of great fundamental importance. However, because of the large-scale structure of the corresponding processes, these predictions cannot be directly verified – unless we are willing to wait for several more billion years.

What we can try to "predict" – and where we can check our predictions – is what happened in the past. We have numerous observations about the past, for two reasons:

- first, observations from distant galaxies actually come from the distant past, simply due to the fact that, e.g., to traverse a distance of one billion light years, light has to travel one billion years;
- second, we often observe physical processes that are in a steady state, remain largely unchanged since the distant past; the most well known phenomenon of this type is 3K radiation that comes from the early stage of the Universe; there are other such phenomena as well.

**Prediction in cosmology: challenges.** One of the main problems with prediction in general is that for many physical systems, small deviations increase exponentially with the evolution of the system. Thus, in a dynamical system, even the small indeterminacy in the initial condition translates into an enormous indeterminacy for future predictions, so enormous that the prediction becomes practically useless.

Such a situation occurs in meteorology, where, because of this exponential growth, we are unable to predict weather for periods exceeding a few days or weeks. This problem is especially serious in cosmology, where dynamics involves time periods of more than ten billion years.

The fact that in cosmology, fluctuations grow exponentially, is well known. Indeed, from observing the 3K radiation, the remainder of the original state of the world, we can conclude that at that time, the state was almost perfectly isotropic and homogeneous, inhomogeneities were extremely small, barely distinguishable. However, these small inhomogeneities have led to the whole modern structure of the Universe, with all the rich structure of galaxies, clusters, stars, planets, etc.

From the purely theoretical viewpoint, in cosmology, and assuming the existence of a free parameter $t$ for the purpose of illustration, we are in an enviable

situation: in addition to knowing the state $s(t_0)$ of the Universe at the current moment of time $t_0$, we also know (via the 3K radiation) the state at a moment $t_r$ close to the beginning of the Universe. We know how the Universe evolves, so theoretically, we can use both known states as initial conditions to predict the phenomena at some past times $t$:

- we can start with the current state $s(t_0)$ and integrate the known dynamical equations back into the past, or
- we can start with the original state of the Universe $s(t_r)$, and integrate the same dynamical equations into the future.

However, in practice, due to the exponential growth of fluctuations, both methods do not lead to reasonable predictions.

**A recent breakthrough: description.** A few years ago, a group of cosmologists and mathematicians came up with a novel idea: to use *both* the current state $s(t_0)$ and the original state $s(t_r)$ when predicting the past states $s(t)$. This idea has led to successful predictions: specifically, at the large-scale level, they succeeded in using the 3K state and current locations of the galaxy clusters to "predict" (reconstruct) the velocities of these clusters (see, e.g., [**?**, **?**, **?**]; see also [**?**]).

**A recent breakthrough: methodological challenge.** From the physical viewpoint, the above result is spectacular. However, this result raises a natural methodological question: how did the above idea overcome the challenge of exponential growth of fluctuations, a challenge that seems to make all long-term predictions impossible?

**What we do in this section.** In this section, we show that the above novel approach can indeed lead to successful predictions, even in situations with exponentially growing fluctuations.

Thus, in other physical areas, exponential growth of fluctuations does not necessarily means that predictions are completely impossible: when we supplement the current state $s(t_0)$ with the additional knowledge about the past, we can indeed get very good predictions.

**Exponential growth of fluctuations: reminder.** Before we present our explanations, let us recall where the exponential growth of fluctuations comes from.

We want to trace how small deviations $x(t)$ from a current state change with time. In general, the change in these deviations is described by a dynamical equation $\dfrac{dx}{dt} = f(x)$ for an appropriate function $f(x)$. Since the deviations are small, we can expand the function $f(x)$ in Taylor series and keep only linear

terms in this expansion. Thus, we get a system of equations $\dfrac{dx}{dt} = a + Ax$ for some appropriate vector $a$ and a matrix $A$.

Since $x$ are deviations from the given state, if we start with the current state (i.e., with $x(t_0) = 0$), we should end up with the current state as well, i.e., with $x(t) = 0$ for all $t$. Thus, the value $x(t) = 0$ corresponding to the unperturbed state must satisfy the above equations. Substituting $x(t) = 0$ into these equations, we conclude that $a = 0$. Thus, the dynamical equations take the following form

$$\frac{dx}{dt} = Ax.$$

Solutions to these equations are well known. To describe these solutions, we can use the eigenvectors of the matrix $A$. In the non-degenerate case, when all the eigenvalues of the matrix $A$ are different, we can simplify the above system if we use the eigenvectors $e_i$ of the matrix $A$, i.e., unit vectors for which $Ae_i = \lambda_i \cdot e_i$ for the corresponding eigenvalues $\lambda_i$.

It is well known that the eigenvalues form a basic, in the sense that an arbitrary vector $x(t)$ can be described as a linear combination of these eigenvectors:

$$x(t) = c_1(t) \cdot e_1 + \ldots + c_n(t) \cdot e_n.$$

Substituting this expression into the above dynamical system and using the fact that $Ax_i = \lambda_i \cdot x_i$, we conclude that

$$\sum_{i=1}^{n} \frac{dc_i(t)}{dt} \cdot e_i = \sum_{i=1}^{n} \lambda_i \cdot c_i(t) \cdot e_i.$$

Since the vectors $e_i$ form a basis, the fact that the vectors on the left-hand side and on the right-hand side of the above equation coincide means that the coefficients at $e_i$ at both sides must coincide too, i.e., that for every $i$, we must have

$$\frac{dc_i(t)}{dt} = \lambda_i \cdot c_i(t).$$

This simple differential equation has a known solution $c_i(t) = C_i \cdot \exp(\lambda_i \cdot t)$ for some constant $C_i$. Thus, the general solution to the above dynamical system has the form

$$x(t) = \sum_{i=1}^{n} C_i \cdot \exp(\lambda_i \cdot t) \cdot e_i.$$

When we know the state $x(t_0)$, we thus know the coefficients $c_i(t_0)$. Thus, in principle, from the value $c_i(t_0) = C_i \cdot \exp(\lambda_i \cdot t_0)$, we can conclude that $C_i = c_i(t_0) \cdot \exp(-\lambda_i \cdot t_0)$ and thus, that for every other moment $t$, we have

$$c_i(t) = C_i \cdot \exp(\lambda_i \cdot t) = c_i(t_0) \cdot \exp(\lambda_i \cdot (t - t_0)).$$

Theoretically, this is the desired predicting formula, but in practice, predicting is difficult:

- when we predict from the moment $t_0$ into the past, the factor

$$\exp(\lambda_i \cdot (t - t_0))$$

  grows exponentially for eigenvalues with negative real part; thus, negligible indeterminacy in the current values $c_i(t_0)$ lead to huge indeterminacy in the past state;

- on the other hand, when we predict from the moment $t_r$ into the future, the factor $\exp(\lambda_i \cdot (t - t_r))$ grows exponentially for eigenvalues with positive real part; thus, negligible indeterminacy in the current values $c_i(t_r)$ lead to huge indeterminacy in the future state.

For cosmological systems it is well known that such an exponential increase indeed occurs – meaning that the corresponding matrix has both eigenvalues with positive real parts and eigenvalues with negative real parts.

*Comment.* In the degenerate case, the general solution is similar, with the exception that we can also have additional solutions of the type $t^k \cdot \exp(\lambda_i \cdot t)$. In this case, the dynamic equations are slightly different, but the exponential growth of fluctuations remains.

**What happens when we use both the current state and the past state for predictions.** If we use both the current state $x(t_0)$ and the past state $x(t_r)$ for predictions, then, for each $i$, we have both values $c_i(t_0)$ and $c_i(t_r)$. In this case, for a given moment of $t$ between $t_r$ and $t_0$, we can predict the values $c_i(t)$ as follows:

- for the eigenvalues $\lambda_i$ with negative real parts, we can predict the values $c_i(t)$ as $c_i(t) = c_i(t_r) \cdot \exp(\lambda_i \cdot (t - t_r))$;

- on the other hand, for the eigenvalues $\lambda_i$ with non-negative real parts, we can predict the values $c_i(t)$ as $c_i(t) = c_i(t_0) \cdot \exp(\lambda_i \cdot (t - t_0))$.

Once we know the values $c_i(t)$, we can reconstruct the state $x(t) = \sum\limits_{i=1}^{n} c_i(t) \cdot e_i$.

In both cases (of eigenvalues with negative real parts and of eigenvalues with non-negative real parts), the absolute values of the coefficients at $c_i(t_0)$ and $c_i(t_r)$ remain bounded by 1. Thus, fluctuations do not grow with time, and we can predict the values at all intermediate moments of time $t$ with the same accuracy with which we know the current and the past states.

Thus, prediction based on past and future values is indeed possible. This reaffirms the idea in the previous section where a complete predictive account of the state of the system depends on the initial conditions and the dynamical or future conditions of the system.

*Comment.* The difference between predicting based only on the current state or only on the past state – and the prediction based on both states – is similar to the known difference between extrapolation and interpolation:

- extrapolation means predicting values outside the interval on which the values are known, while
- interpolation means predicting values inside the interval on which the values are known – filling he gaps.

It is known that interpolation is often practically possible, while extrapolation is often not possible – especially if we want to extrapolate far away from the known values. This is exactly what we observe now:

- prediction only from the present or only from the past – an analogue of extrapolation – is, for cosmology, not practically possible;
- on the other hand, prediction of the values $x(t)$ based on the values $x(t_r)$ and $x(t_0)$ for which $t_r < t < t_0$ – an analogue of interpolation – is practically possible.

In sum, from an analysis of Kolmogorov complexity and an evaluation of the role of prediction of the dynamical evolution of systems in physics, we conclude that a general predictive model for physics does not entail only needing the initial conditions and the dynamical evolution, but rather an initial and a final condition along with the dynamical evolution. It could, in fact, be argued that physics already operates in this fashion, e.g. [**?**]. So our goal with this paper is manly to recognize the actual role of prediction in the foundations of physics, as well as proposing the computational means to do this effectively and practically in physics.

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