Teacher Supplement
to
Operation Comics, Issue #4

The purpose of this supplement is to provide content support for the mathematics embedded into the fourth issue of Operation Comics, and to show how the mathematics addresses the content standards provided by the National Council of Teachers of Mathematics (NCTM) for grades 3–5 and 6–8. The mathematics all takes place in preparation for Wonderguy to begin using his new jet packs, and involves a bit of physics as well as mathematics. The mathematics used in this issue addresses both the NCTM standard for problem solving and the NCTM standard for reasoning and proof, for both grade ranges, in addition to the standards addressed below.

Force, Mass, and Acceleration

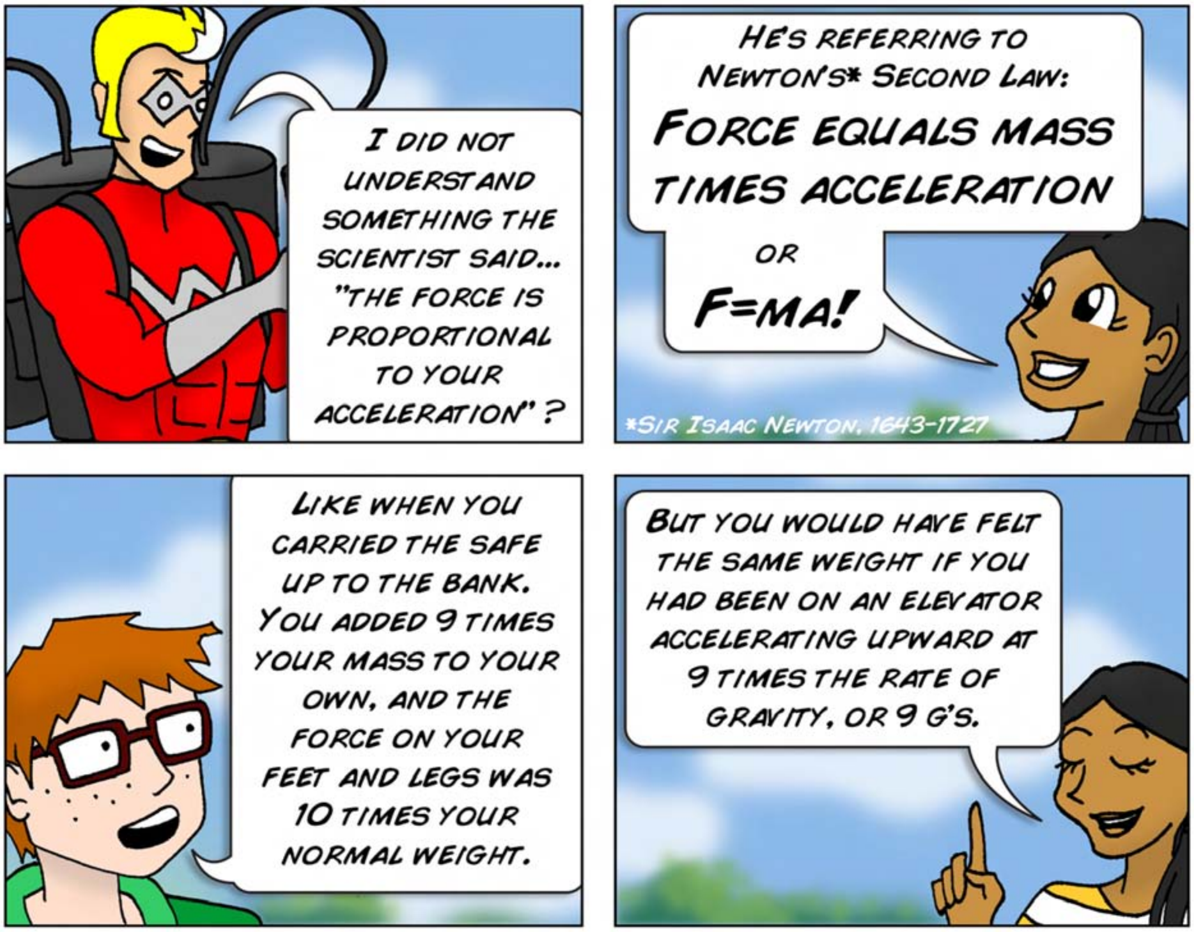
In order to explain why Wonderguy is the only person strong enough to actually use the jet packs, we have a short discussion on the relationship between the force on his arms and his acceleration when using the jet packs. Claire mentions "Newton's Second Law," which is generally expressed with the formula $F = ma$, where F represents force, m represents mass, and a represents acceleration. This equation is really only true if all of these quantities are represented in the correct units (force in Newtons, mass in kilograms, and acceleration in meters per seconds squared, for example.) In general, though, we can always say that force is directly proportional to mass when we consider acceleration to be constant, and likewise, force is directly proportional to acceleration when we consider mass to be constant. Dillon and Claire point this out to Wonderguy in the panels included on the next page. We attempt to help the students understand this concept better with the illustration on page 3 of the person on the Olympic rings with first a twin and then a baby elephant holding on to him.

Gravity vs. Wonderguy's Jet Packs

The first student supplement that accompanies this issue has the students answering questions regarding the force that Wonderguy feels on his arms when flying under certain conditions, and the limitation on how fast he can accelerate upward. Solutions are given below.

1) 150 pounds. Wonderguy's 300 pounds of weight come from his mass and the acceleration due to gravity. If we increase the acceleration from "1" gravity to "1.5", then the weight that he would feel on his arms would be 1.5 times his weight, 450 pounds. This is 150 pounds more than he would feel when simply hovering with the jet packs.

2) 600 pounds. Wonderguy's 300 pounds of weight come from his mass and the acceleration due to gravity. If we increase the acceleration from "1" gravity to "3", then the weight that he would feel on his arms would be 3 times his weight, 900 pounds. This is 600 pounds more than he would feel when simply hovering with the jet packs.



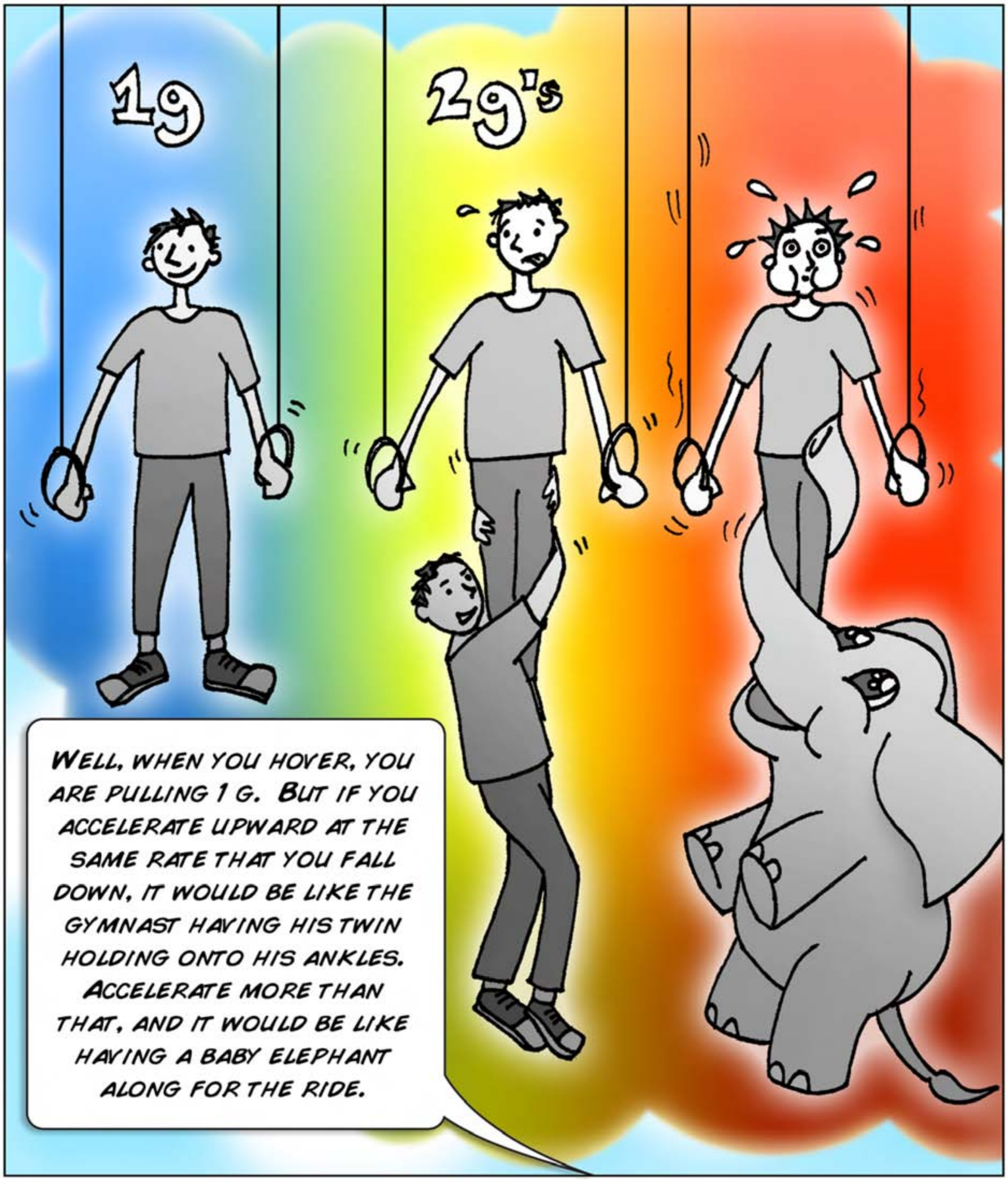
3) 4 G's. Since he now has a total of 420 pounds to carry when he is merely hovering, that will leave 1680 pounds of thrust that can be used to accelerate upward, and $\frac{1680}{420} = 4$ G's. This means that he will not be able to accelerate upward as fast with the additional passengers. However, at maximum thrust, he will feel the same stress on his arms – 2100 pounds.

Super-Tricky! This is an open-ended question that could have a number of valid answers, including the issue of balancing the additional passengers, or in having to hold them with one arm and only being able to use one jet, etc.

NCTM Standards

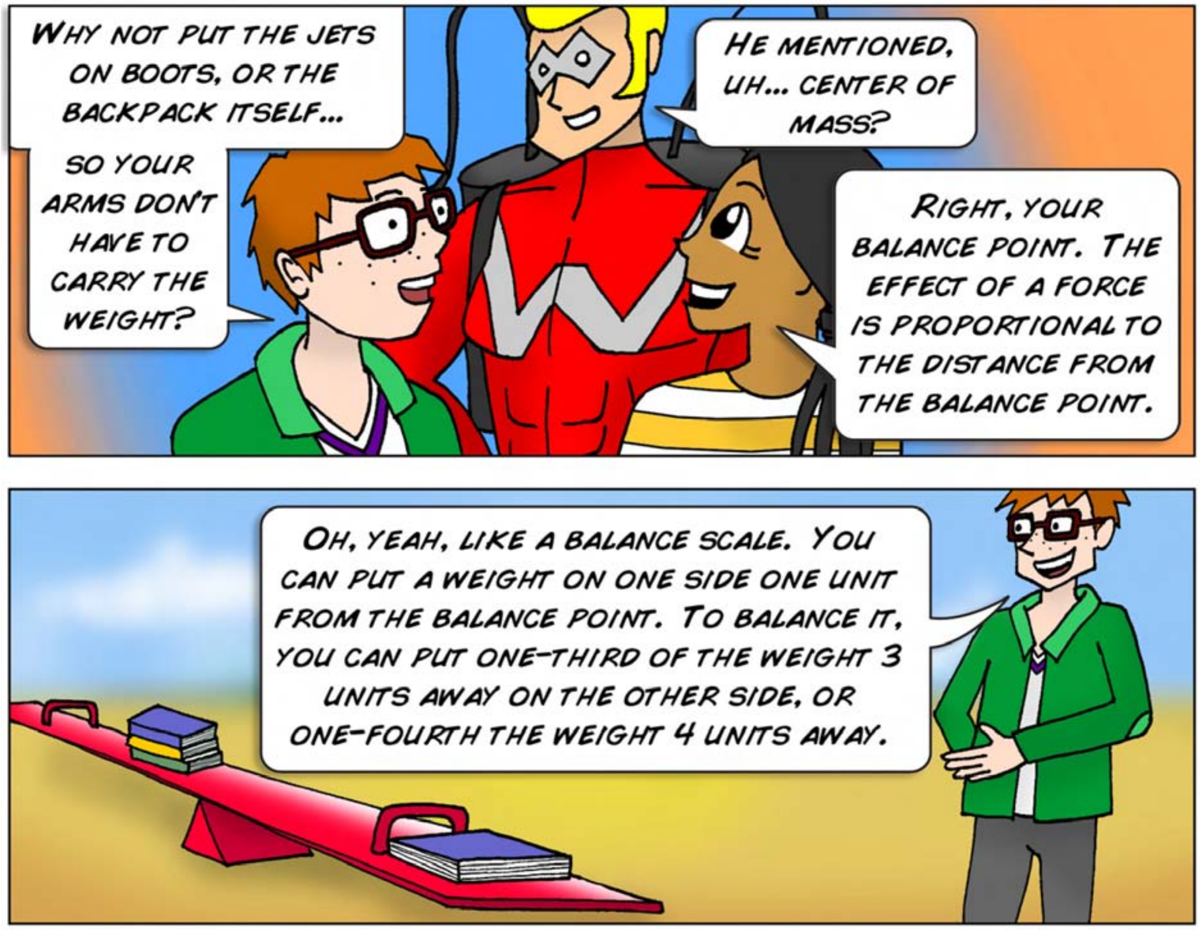
The connections between force, mass, and acceleration address the NCTM standard for algebra for grades 3–5, which says that "in grades 3–5, all students should investigate how a change in one variable relates to a change in a second variable" and "identify and describe situations with constant or varying rates of change and compare them." The worksheets with different shape data sets address the NCTM standard for data analysis and probability for grades 3–5, which says that "in grades 3–5, all students should compare different representations of the same data and evaluate how well each representation shows important aspects of the data."

In addition to the NCTM standards, the use of concepts like force and its relationship to mass and acceleration help meet core standards in the science curriculum, but we will restrict our comments to just the math standards.



Center of Mass and Torque (or Moments)

In order to explain why Wonderguy is the only person strong enough to actually use the jet packs, we have a short discussion on the effect of distance on the force applied to one side of a balance point. This physical effect, called torque or moment (but not mentioned by name in the comic,) is correctly described by Claire at the top of the next page. The torque caused by a weight located on one side of a balance point is proportional to the distance from the balance point, thus giving us another opportunity to discuss proportionality. When a weight is moved to twice the original distance from the balance point, the torque is doubled. Then to keep the system balanced (or get the same amount of torque) with the doubled distance, we would have to



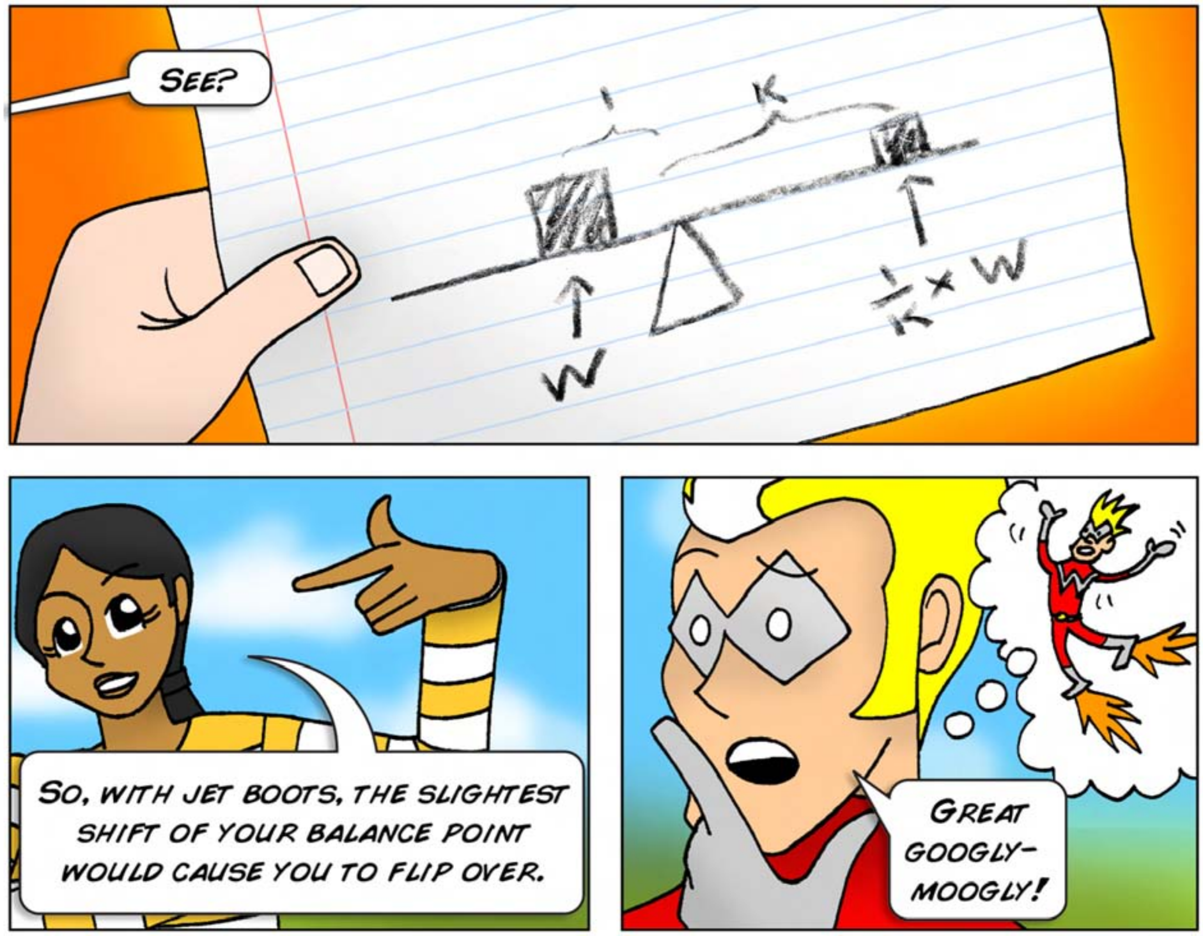
reduce the weight by half. Dillon demonstrates this concept in the second panel above, and even generalizes the idea in his pencil illustration in the panel at the top of the next page, by pointing out that $w \cdot 1 = \frac{w}{k} \cdot k$. In the left panel just underneath that, Claire makes the point that greater distance from the balance point, or center of mass, means an equal magnification of the unbalancing effect of the same force.

<u>Center of Mass and Balancing</u>

The second student supplement that accompanies this issue has the students determining whether a series of weights placed on a teeter-totter would cause it to teeter to the left, right, or would it stay balanced. Solutions are given below.

1) Teeter down on the left. The weights are equidistant from the balance point, but the weight on the left is heavier than the one on the right.

2) Stays balanced. A 10-pound weight 4 ft away from the balance point would have twice the effect as the 10-pound weight 2 ft to the left of the balance point, so half the weight at that distance would have an equal effect. Students may notice that $10 \times 2 = 5 \times 4$.



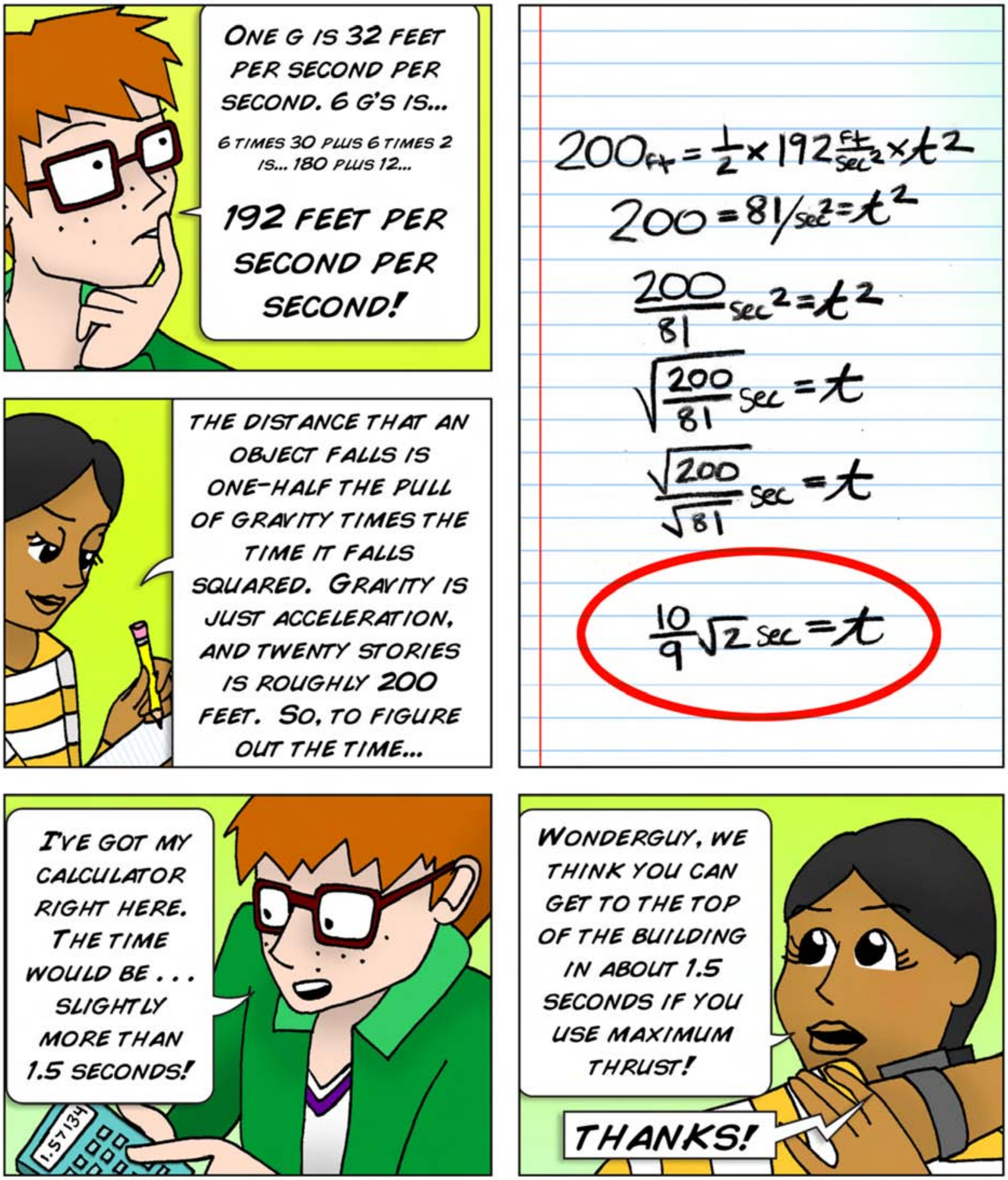
3) Teeter down on the right. The distance of 1.5 ft is $\frac{3}{8}$ of 4 ft, but 3 pounds is more than $\frac{8}{3}$ of one pound. Students may notice that $1 \times 4 < 3 \times 1.5$.

Super-Tricky! Teeters down on the right. Students may reason that to be balanced against the blue weights on the left, you would need both a 5-pound weight at 4 ft to the right and another equal 15-pound weight 4 ft to the right, for a total of 20 pounds at that distance. Having the 20-pound weight located at 4.5 ft instead of 4 ft, means that you have more force pulling down on the right than is needed to balance the teeter-totter. Students may also notice that $(15 \times 4) + (10 \times 2) = 60 + 20 = 80$, which is less than $20 \times 4.5 = 90$.

NCTM Standards

The connections between weight, distance, and torque address the NCTM standard for algebra for grades 3–5, which says that "in grades 3–5, all students should investigate how a change in one variable relates to a change in a second variable" and "identify and describe situations with constant or varying rates of change and compare them." The worksheets with different shape data sets address the NCTM standard for data analysis and probability for grades 3–5, which says that "in grades 3–5, all students should compare different representations of the same data and evaluate how well each representation shows important aspects of the data."

In addition to the NCTM standards, the use of concepts like torque and its relationship to weight and distance from a balance point help meet core standards in the science curriculum, but we will restrict our comments to just the math standards.



Solving for a Squared Variable

Near the end of the story (shown above), Claire contributes one other fact that serves to help Wonderguy time his mid-air capture of Aerial. No doubt having seen the formula in a science book somewhere, she mentions that "the distance that an object falls is one-half the pull of gravity times the time it falls squared." This is a simplified version of the formula for vertical distance in an object affected by gravity, which is a result from integral calculus: if g is the acceleration due to gravity, then the vertical velocity v is

$$v = \int g\, dt = gt + v_0, \text{ where } v_0 \text{ is the velocity at } t = 0,$$

and the vertical position y is

$$y = \int \left(gt + v_0\right) dt = \frac{1}{2}gt^2 + v_0 t + y_0, \text{ where } y_0 \text{ is the position at } t = 0.$$

This formula appears in many algebra books (without proof, of course), and the simplified version $d = \frac{1}{2} \times g \times t^2$ will precede that in some science books as a way to measure free-fall distance for a given time, or time for a given freefall distance. Claire has correctly deduced that gravity is just a specific acceleration downward, and if the acceleration is changed in magnitude and direction, the formula still gives the distance traveled.

At the top of the previous page, Claire and Dillon have correctly deduced that at 2100 pounds of thrust, a 300-pound man could expect to shoot into the air at 6 G's, $\frac{2100}{300} = 7$ G's minus the pull of the earth's one gravity. Note that Dillon uses the distributive property of multiplication over addition to calculate that $6 \times 32 = 6\left(30 + 2\right) = 180 + 12 = 192$ feet per second squared. Then, estimating 20 stories as $20 \times 10 = 200$ feet, Claire starts to use linear-equation solving techniques to solve for t^2, when, in their haste, Claire and Dillon make a mathematical error in their calculations. (Yeah, that's the ticket! It was all Claire and Dillon's fault. I wish I could really blame it on them, but the mistake is mine and mine alone, caught after the comic went to press. However, in the spirit of turning lemons into lemonade, I have used the occasion of the error as the subject of the student worksheet for this section.)

Claire correctly forms the right equation for calculating the time needed for Wonderguy to reach his target:

$$200 \text{ ft} = \frac{1}{2} \times 192 \frac{\text{ft}}{\text{sec}^2} \times t^2 .$$

However, then an arithmetic error is made: $\frac{192}{2} = 96$, not 81. I will continue with the steps with the incorrect number included, and show the correct steps in the solution to the worksheet.

$$200 = \frac{81}{\text{sec}^2} \times t^2 \text{ (cancelling "ft" and simplifying } \frac{192}{2} \text{ to 81, note extra "=" in comic panel)}$$

$$\frac{200}{81}\text{sec}^2 = t^2 \text{ (multiplying both sides by } \frac{\text{sec}^2}{81}\text{)}$$

Claire then demonstrates that she understands the inverse operation to squaring a number, finding the square root. Taking the square root of both sides, she gets

$$\sqrt{\frac{200}{81}}\text{sec} = t .$$

She also demonstrates her knowledge of square roots by pointing out that $\sqrt{\frac{200}{81}} = \frac{\sqrt{200}}{\sqrt{81}}$, and that $\sqrt{200} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$ and $\sqrt{81} = 9$. Dillon gets a decimal approximation of roughly 1.5 seconds by entering the amounts into his calculator. (He did not catch the arithmetic error, either. Luckily, the difference between their answer and the correct answer was pretty small.)

<u>One Serious Calculation, Gone Wrong!</u>

The third student supplement that accompanies this issue has the students performing algebraic steps to reach a correct solution to the problem that is explained above. The steps to the correct solution are given below.

Steps in the Comic	Correct Steps
$200\text{ ft} = \frac{1}{2} \times 192 \frac{\text{ft}}{\text{sec}^2} \times t^2$	$200\text{ ft} = \frac{1}{2} \times 192 \frac{\text{ft}}{\text{sec}^2} \times t^2$
$200\text{ ft} = 81 \frac{\text{ft}}{\text{sec}^2} \times t^2$	$200\text{ ft} = 96 \frac{\text{ft}}{\text{sec}^2} \times t^2$
$200\text{ ft} \times \frac{\text{sec}^2}{81\text{ ft}} = \frac{\text{sec}^2}{81\text{ ft}} \times \frac{81\text{ ft}}{\text{sec}^2} \times t^2$	$200\text{ ft} \times \frac{\text{sec}^2}{96\text{ ft}} = \frac{\text{sec}^2}{96\text{ ft}} \times \frac{96\text{ ft}}{\text{sec}^2} \times t^2$
$\frac{200}{81}\text{ sec}^2 = t^2$	$\frac{200}{96}\text{ sec}^2 = t^2$ or $\frac{25}{12}\text{ sec}^2 = t^2$
$\sqrt{\frac{200}{81}\text{ sec}^2} = \sqrt{t^2}$	$\sqrt{\frac{200}{96}\text{ sec}^2} = \sqrt{t^2}$ or $\sqrt{\frac{25}{12}\text{ sec}^2} = \sqrt{t^2}$
$\frac{\sqrt{100 \times 2}}{\sqrt{81}}\text{ sec} = t$	$\frac{\sqrt{100 \times 2}}{\sqrt{16 \times 6}}\text{ sec} = t$ or $\frac{\sqrt{25}}{\sqrt{4 \times 3}}\text{ sec} = t$
$t = \frac{10\sqrt{2}}{9}\text{ sec} \approx 1.5\text{ sec}$	$t = \frac{10\sqrt{2}}{4\sqrt{6}}\text{ sec} = \frac{5}{2\sqrt{3}}\text{ sec} \approx 1.44\text{ sec}$

Super-Tricky! Recreating the correct steps above with “192” replaced with “32” will generate the answer $t = \frac{5}{\sqrt{2}}\text{ sec} \approx 3.54\text{ sec}$, which is $\sqrt{6}$ times longer than the correct answer above. (Recall that the acceleration is cut by one-sixth, which raises the time-squared by a factor of 6, and then we take the square root of t^2.) This is also the time that it would take an aerodynamic object to fall, under earth’s gravity, from a height of 20 stories.

NCTM Standards

The solving of the problem addresses the NCTM standard for algebra for grades 6-8, which says that “in grades 6–8, all students should use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.” (The t^2-term is involved linearly with the rest of the equation.) It also addresses the NCTM standard for number and operations for grades 6–8, which says that “in grades 6–8, all students should understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.” Dillon’s mental steps in multiplying 6×32 also address the NCTM standard for number and operation for grades 6–8, which says that “in grades 6–8, all students should use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals." The worksheet correcting Claire and Dillon's work in the comic addresses the NCTM standard for communication for grades Pre-K through 12, which says that "all students should be able to communicate their mathematical thinking coherently and clearly to peers, teachers, and others" and "analyze and evaluate the mathematical thinking and strategies of others."

In addition to the NCTM standards, the use of concepts like gravity and its relationship to distances traveled by an object subjected to it and time in the air help meet core standards in the science curriculum, but we will restrict our comments to just the math standards.

One last note . . .

While my purpose here is to produce a comic book with embedded mathematical content, and mathematics is my background, I did endeavor to write a good story, and hopefully, the comics can be used for their literary elements as well. The following are a few examples of subtle things at work in the story.

- As in the previous issue, Operation Comics #3: Not Your Average Cat, with the villain The Cheetah, this issue's villain, Aerial, is someone with enormous talent, who decides for whatever reason to disregard the laws that apply to others in order to accumulate things for themselves. If students have also read the third comic, one could pose the question of similarities and differences in the two villains, and which is better: being talented or being a good person?
- We see both Principal Willoughby and Wonderguy's alter ego, "Big Guy", working hard at maintaining their relationship despite both of their busy schedules. One could pose the question "Who do you think would be busier: a superhero or an elementary school principal?"

This document, as with the student supplements, is a work in progress. Please contact me with corrections or suggestions, and I will make the needed changes. Thanks for inviting Wonderguy into your classrooms, and please encourage your students to contact me with their comments and suggestions for future episodes.