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Aaron S. Edlin
Mario Epelbaum

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TWO-PART MARGINAL COST PRICING EQUILIBRIA WITH n FIRMS: SUFFICIENT CONDITIONS FOR EXISTENCE AND OPTIMALITY*

BY AARON S. EDLIN AND MARIO EPELBAUM[1]

We explore the interactions among firms with increasing returns regulated to break even by pricing with two-part tariffs. We provide conditions for existence and for efficiency of general equilibria with n-firms. This involves finding hookup fees that are voluntarily paid and cover the firms' losses from marginal cost pricing—a problem that because of both substitution and income effects is complicated by multiple firms using two-part tariffs, but that must be solved to ensure the continuity of demands necessary to prove break-even equilibria exist.

1. INTRODUCTION

Increasing returns are found in many sectors of the economy. Unfortunately, they present economists with serious problems. While we like to advocate competition, economies of scale provide good arguments for other market structures. Calls for regulation of natural monopolies are justified both because competitive price-taking behavior is implausible when monopoly obtains, and because even under such behavior, equilibria frequently do not exist when firms' technologies are not convex.[2] Regulation, however, has problems of its own. Pricing at marginal cost, the economist's panacea, is *no longer* sufficient for efficiency when there are increasing returns. Compounding this, a regulated firm that has decreasing average costs will make losses, if it prices at marginal cost.

Much attention has been given to the problems associated with increasing returns by the partial equilibrium literature on public utility regulation. Telephone, electricity, and gas services are all produced or distributed under increasing returns to scale, and companies providing these services are often regulated monopolists. One image of the regulator's problem is to find an efficient pricing structure which makes the monopoly viable, covering its costs. If the monopolist has decreasing average costs, this cannot be accomplished with a linear price.[3] By using two-part tariffs (TPT's), charging each customer a fixed fee for the right to buy the good at a per-unit price, the regulator can potentially solve this problem (at least when resale

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[2] Supply correspondences may be either not defined or not convex-valued.

[3] A linear price of marginal cost might not be efficient for reasons documented at least as far back as Coase (1946); also, marginal cost pricing leaves losses which must be covered. Arguing for lump sum taxation to cover losses not only leaves the regulator vulnerable to the attacks of Coase concerning efficiency, but disregards the institutional reality that lump sum taxes do not exist.

is impossible).[4] The per-unit price is set equal to marginal cost and the fixed part of the tariff, the hookup, is used to cover the losses from the difference between average and marginal cost.

TPT's may be uniform, with the same fixed fee for each buyer, or discriminatory with different fixed fees. If there is enough willingness-to-pay, uniform two-part tariffs may allow a firm with decreasing average costs to set its marginal price equal to marginal cost and still cover its losses. Nonetheless, such tariffs can still be inefficient. Some consumers, though willing to pay the marginal cost for the good, may be discouraged by the fixed fee. If enough information were available, this problem might be remedied by using discriminatory TPT's, tailoring each household's hookup to be less than its willingness-to-pay. It must be noted though, that while discriminatory TPT's avoid the inefficiencies arising from exclusion and from running the firm when there is not enough willingness-to-pay, they do not guarantee efficiency for all kinds of increasing returns (for examples of inefficient equilibria see Vohra 1990a or Brown, Heller, and Starr 1990).

Partially discriminatory TPT's are common to public utility pricing. For example, telephone companies charge a monthly fixed fee and then a charge roughly proportional to the time of calls if we hold distance constant. The fixed fee may differ for business and home phones and for homes with different incomes. Universities often have even more discriminatory pricing schemes, charging a hookup called tuition which is determined after careful examination of a family's financial position and then frequently charging a per-unit charge of zero for each course (for details and ramifications, see Edlin 1993b).

Despite the prevalence of such pricing, there has been scant study in the partial equilibrium literature of the workings of either individual rationality or incentive compatibility constraints when more than one firm sells to a given agent using nonlinear or two-part tariffs (Stole 1991 and Epelbaum 1993a, 1993b are notable recent exceptions). This paper, because of its informational assumptions, will not address the impact of one firm's nonlinear price structure on another's incentive compatibility constraints, but it does shed light on the impact upon individual rationality constraints. Although incentive compatibility is not directly studied here, Epelbaum (1993b) showed that much of this work is useful to consider incentive compatibility.

Increasing returns has also received a fair amount of attention in the general equilibrium literature. Showing existence of a general equilibrium in an economy with increasing returns has proven to be elusive. Such results exist today, but none are completely satisfactory since they all make use of intermediate assumptions,[5] as discussed in Vohra (1992). One of the controversies in the literature has been which equilibrium concept to use. The chosen concept is more often related to the possibility of proof than to institutional realities. Most of the literature has converged on marginal cost pricing, but there is no consensus as to how to cover the losses of increasing returns firms. Brown, Heller and Starr (1990) suggest

[4] For assorted discussions of TPT's in the partial equilibrium literature, see Willig (1978), Feldstein (1972) or Vogelsang (1989).

[5] Assumptions that are not placed directly on the primitives of the model.

covering the losses of a single increasing returns firm producing a single good through discriminatory TPT's. Kamiya (1990) suggests a different nonlinear discriminatory pricing rule which actually guarantees efficiency, but to prove existence requires that the goods sold by the monopolists be necessities; others in the literature suggest covering losses via full liability shares. Bonnisseau and Cornet (1990a, b) have a different approach. They avoid the issue of covering losses by requiring in a set containing all possible equilibria the existence of abstract household income functions which (1) are positive continuous functions of prices and productions and (2) add up to the value of supply (more on this in Section 3). Requiring that household incomes add up to the value of supply implicitly assumes the monopolists' losses are somehow covered. There is no general existence proof for a model with a reasonable and realistic manner to cover losses, or equivalently, no conditions which make such a model consistent.[6]

We argue that when goods are not easily transferable, a reasonable equilibrium concept is the Two-Part Marginal Cost Pricing Equilibrium introduced in Brown, Heller, and Starr (1990). As mentioned above, in their model a monopolist is regulated to break even and uses discriminatory TPT's so as to be able to charge marginal prices of marginal cost. Unfortunately, in their model there is only one increasing returns firm and hookups are only charged to households. Our paper provides sufficient conditions for existence of such an equilibrium for an economy with any number of monopolists which are regulated to break even and use discriminatory TPT's. Our purpose is not to eliminate the necessity of an intermediate assumption, but merely to provide the appropriate assumption for the n-firm case. We extend the results of Brown, Heller and Starr (1990) by providing a surplus condition that allows the design of continuous individually rational hookups when many firms charge TPT's; the surplus condition would be more easily satisfied than Brown, Heller and Starr's (1990) in the one firm case because hookups are also charged to competitive firms. Following Edlin and Epelbaum (1993) we show how the continuity achieved by the surplus condition allows an easy and short proof of the existence of a general equilibrium.

In the one firm case, the most surplus that a firm can extract via hookups is well defined; it is the sum of all buyers' willingness-to-pay. Because of this, it is a simple exercise to determine when a firm can be run without subsidies: it can be if aggregate willingness-to-pay exceeds the firm's losses. When this condition is met, each buyer's hookup can be defined less than its willingness-to-pay, but still large enough to cover losses. Such hookups will be voluntarily paid and can be defined as continuous functions of prices and household incomes because willingness-to-pay is continuous in those variables. Finally, since continuous hookups that are voluntarily paid lead to continuous demands, equilibria can be shown to exist if aggregate willingness-to-pay covers losses.[7]

With more than one firm charging TPT's, however, the maximum extractable surplus is no longer well defined. A buyer's willingness-to-pay for a good depends

[6] Moriguchi (1991) identifies the problem as finding continuous hookup charges that are individually rational and will be voluntarily paid, but her focus is not on finding conditions under which this is possible.

[7] The condition that willingness-to-pay covers losses, assumed over the set of production equilibria, is precisely the surplus condition Brown, Heller and Starr (1990) used to show the existence of TPMCPE.

on the hookups for the other goods. This makes it significantly more difficult to find hookups which are individually rational, but are still large enough to cover losses.[8] Finding voluntary hookups is not by itself difficult: trivially, if all hookups were zero they would be voluntarily paid. The difficulty lies in finding voluntary hookups that allow the increasing returns firms to be viable, covering their losses in the marketplace. This involves careful consideration of the impact of one firm's TPT's upon the other firms' individual-rationality constraints—i.e. upon the willingness of buyers to pay for other firms' goods. Gathering more surplus for one firm reduces the amount that may be collected by other firms.

Our paper provides a nontrivial definition of extractable surpluses when many firms use TPT's. As long as hookups are less than the extractable surpluses we define, the hookups will all be voluntarily paid. The condition that these surpluses exceed the firms' losses is sufficient so that each firm can be run independently without subsidies. While it is *sufficient* that the extractable surpluses we define exceed losses, this is not *necessary*; in some cases other definitions of extractable surplus work better at covering losses, but these alternate definitions are much harder to write down in the general case. Our definition does quite well in most cases and dominates other nontrivial definitions that we contrast with it.

Since the extractable surpluses we define are continuous functions of prices and household incomes, whenever these surpluses are sufficient to cover losses, we can find continuous hookups greater than losses, but less than these surpluses so that they are voluntarily paid. Because these hookups are continuous and all voluntarily paid, the resulting demands are continuous. Hence, the condition that surpluses exceed losses allows us to cover losses with hookups while leaving demands continuous. This condition thereby guarantees the consistency (existence) of a general equilibrium.

Although discriminatory TPT's do not always guarantee efficiency when the monopolists are required to break even, if the nonconvexities in the monopolists' technologies arise solely from the presence of fixed costs, discriminatory TPT's are efficient.[9] And, while a general version of the second welfare theorem fails, a restricted version holds. The requirement for decentralization is that there be enough willingness-to-pay for the monopoly goods at the Pareto optimum in question, so that individually rational hookups that cover losses can be constructed.

Finally, we make some remarks about this paper's strong informational assumptions. Real firms do not have such detailed information about their consumers. And, the pricing rules we develop will not generally be implementable by an informationally-decentralized incentive-compatible mechanism. A model in which firms discriminated *imperfectly* would be preferred, yet we are not aware of any models that solve that problem.[10] We view this model as a necessary step in that direction and believe many of the lessons learned here about individual-rationality

[8] For a treatment of the two-firm case see Edlin and Epelbaum (1993) or Clay (1990).

[9] This was observed by Vohra (1990a) in an economy with only two goods and was subsequently and contemporaneously observed by Edlin (1993), Kamiya (1990) and Moriguchi (1991) in more general cases.

[10] In particular, in order to get the continuity required of aggregate demand to show equilibria exist, we would require that no households jump from paying their hookups to not paying them, or vice-versa.

constraints are helpful in cases where types are unknown. As is common in economic theory, progress begins with the simpler less realistic cases.

2. VIABILITY AND WILLINGNESS-TO-PAY CONDITIONS FOR EXISTENCE OF GENERAL EQUILIBRIA

We consider an economy with F firms, H households and c goods; each of the first n ($n < c$, $n < F$) goods is exclusively produced by one of the first n firms whose technologies may exhibit increasing returns to scale. We refer to the first n firms as monopoly firms and to the goods they produce as monopoly goods. The remaining goods are produced with decreasing returns, and called competitive goods.

2.1. *Households.* Each household h has a continuous, quasiconcave and monotonic utility function defined on the positive quadrant, a closed convex set bounded below: $U^h: \mathfrak{R}^c_+ \to \mathfrak{R}$. Household h is endowed with some $\boldsymbol{\omega}^h \in \mathfrak{R}^c_+$ and owns shares $(\theta^{fh}) \equiv (\theta^{1h}, \dots, \theta^{Fh})$ in the firms $1, \dots, F$ described below. In order for the restricted utility functions defined in 2.3 below to be appropriate, households are not endowed with any of the monopoly goods.

2.2. *Firms.* Following Beato and Mas-Colell (1985), each firm f has a production set formed by the free-disposal of a compact set $\mathbf{K}^f$ containing the origin:[11] $\mathbf{Y}^f = \mathbf{K}^f - \mathfrak{R}^c_+, f = 1, \dots, F$. We assume $\mathbf{K}^f$'s are convex for $f > n$, though the n monopoly firms are allowed technologies with increasing returns. Importantly, the competitive firms $f > n$ with convex technologies cannot produce monopoly goods: i.e., $\mathbf{K}^f \subset \mathfrak{R}^n_- \times \mathfrak{R}^{n-c}$. Finally, each monopoly firm f, $f \leq n$, is the exclusive producer of monopoly good f: i.e., $\mathbf{K}^f \subset \mathfrak{R}^{n-1}_- \times \mathfrak{R}^{n-c+1}$.[12]

2.3. *Viability Conditions.* We now derive conditions under which losses can be covered in the market and the productions of the n increasing returns firms are viable. In order to provide foundation for the existence of a general equilibrium, we will observe continuity as we proceed. Temporarily we assume prices are strictly positive, i.e. $\mathbf{p} \in \mathfrak{R}^c_{++}$.

Define $\mathbf{M} = \{1, \dots, n\}$ to be the indices of monopoly goods produced by the n monopoly firms. To find out the most a competitive firm will pay for the opportunity to buy some monopoly goods, we consider its profit possibilities without those goods. Profits for a firm $f > n$ unable to buy monopoly goods outside some set $\mathbf{M}_1 \subseteq \mathbf{M}$ are,[13]

[11] $\mathbf{K}^f$'s may be viewed as proxies for the attainable set. The assumption of compactness is convenient to show existence by mapping the boundary of the production set into the simplex.

[12] The restriction that each monopolist produce only one good is strictly for notational simplicity: no logical steps in the proofs would need to be changed for multiproduct firms which charge one hookup to sell all their goods.

[13] The fact that $\Pi^f_{\mathbf{M}_1}(\mathbf{p})$ exists follows from the compactness of $\mathbf{K}^f$ and the observation that the firm might just as well maximize over $\mathbf{y} \in \mathbf{K}^f$ instead of $\mathbf{y} \in \mathbf{Y}^f$. The continuity of $\Pi^f_{\mathbf{M}_1}(\mathbf{p})$ follows from Berge's Maximum Theorem.

$$\Pi^f_{\mathbf{M}_1}(\mathbf{p}) = \max_{\mathbf{y}} \mathbf{p} \cdot \mathbf{y} \quad \text{s.t.} \quad \mathbf{y} \in \mathbf{Y}^f,\ y_i = 0 \ \ \forall\, i \in \mathbf{M} \backslash \mathbf{M}_1. \tag{1}$$

Then, the willingness-to-pay or surplus from adding the set $\mathbf{M}_2$ to the permissible monopoly good set $\mathbf{M}_1$ is

$$S^f_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}) = \Pi^f_{\mathbf{M}_1 \cup \mathbf{M}_2} - \Pi^f_{\mathbf{M}_1}. \tag{2}$$

The situation with households is almost perfectly analogous, except that households' utilities are also functions of income. Let $\bar{\mathbf{V}}^h_{\mathbf{M}_1}(\mathbf{p}, I)$ be household h's restricted indirect utility function which gives the utility level of household h at prices $\mathbf{p}$ and income I when it is restricted in its purchases of monopoly goods to buy only those in set $\mathbf{M}_1$. $\bar{V}^h_{\mathbf{M}_1}(\mathbf{p}, I)$ is continuous by Berge and given by

$$\bar{V}^h_{\mathbf{M}_1}(\mathbf{p}, I) = \max_{\mathbf{x} \in \Re^c_+} U^h(\mathbf{x}) \quad \text{s.t.} \quad \mathbf{p} \cdot \mathbf{x} \le I,\ x_i = 0,\ \ \forall\, i \in \mathbf{M} \backslash \mathbf{M}_1. \tag{3}$$

We often call this measure of the household's outside opportunities its reservation utility. We now calculate the expenditure necessary to achieve a given utility level when household h cannot consume monopoly goods outside the set $\mathbf{M}_1$. This conditional expenditure function is

$$E^h_{\mathbf{M}_1}(\mathbf{p}, \bar{V}) = \min_{\mathbf{x} \in \Re^c_+} \mathbf{p} \cdot \mathbf{x} \quad \text{s.t.} \quad U^h(\mathbf{x}) \ge \bar{V},\ x_i = 0 \ \ \forall\, i \in \mathbf{M} \backslash \mathbf{M}_1. \tag{4}$$

Using this expenditure function we may calculate the most a household would pay for the right to add $\mathbf{M}_2$ to its monopoly choice set when it could only buy competitive goods and those monopoly goods in $\mathbf{M}_1$. We call this quantity the household's surplus and it is defined by

$$S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}, I) \equiv I - E^h_{\mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}, \bar{V}^h_{\mathbf{M}_1}(\mathbf{p}, I)). \tag{5}$$

These surpluses are continuous even when income equals 0 because $0 \le S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}, I) \le 1$.[14] We note finally, that surpluses are no different in principle from ordinary compensating variations; after all, permitting consumption in $\mathbf{M}_2$ is equivalent to lowering the prices of goods in $\mathbf{M}_2$ from ∞ to the prices given by $\mathbf{p}$.

2.3.1. *A single monopolist.* When could a single firm which produced all the monopoly goods sell a given production with individualized two-part tariffs without losing money? A condition for *weak viability* of a production $\mathbf{y} \in \mathbf{Y}^1$ given incomes (I^h) $h = 1, \ldots, H$ and prices $\mathbf{p} \in \Re^c$ is

[14] These surpluses are obviously not defined for negative incomes.

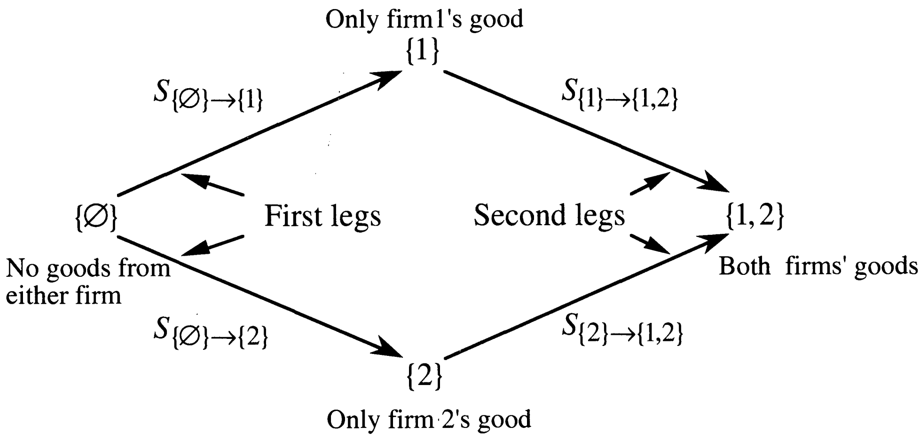

FIGURE 1

$$\sum_h S^h_{\varnothing\to\mathbf{M}}(I^h) + \sum_{f>1} S^f_{\varnothing\to\mathbf{M}} > -\mathbf{p}\cdot\mathbf{y}. \tag{6}$$

The quantity $-\mathbf{p}\cdot\mathbf{y}$ represents the losses an increasing returns firm would suffer from the production $\mathbf{y}$ being sold at linear prices $\mathbf{p}$. For a weakly viable production, we can find hookups large enough to cover these losses, but still less than the surplus from adding the monopoly goods; such hookups will be voluntarily paid, i.e., they are *individually rational.* This viability condition is *weak* in the sense that markets need not clear; it does not preclude there being too little demand for the monopoly good, nor households being willing to pay their hookups only under the expectation that they can buy more than the production allows.

2.3.2. *n-Monopoly case.* When will a set of productions for n monopolists be weakly viable? With only one firm selling the monopoly goods, the surplus from adding new goods to the economy is well defined. The situation is more complex with more firms: the surplus from adding one monopolist's good to the economy changes with the availability of other monopolists' goods. Therefore, we need a notion of independent viability which tells us when firms can be run separately without subsidies.

We begin by illustrating the difficulties with two firms which we first discussed in Edlin and Epelbaum (1993). Identity (7) and Figure 1 relate the various surpluses when there are two firms $i = 1, 2$, each producing one monopoly good.

$$S^h_{\varnothing\to 2}(I) + S^h_{2\to 1,2}(I - S^h_{\varnothing\to 2}(I)) \equiv S^h_{\varnothing\to 1,2}(I) \equiv S^h_{\varnothing\to 1}(I) + S^h_{1\to 1,2}(I - S^h_{\varnothing\to 1}(I)).\text{[15]} \tag{7}$$

$S^h_{\varnothing\to 1,2}(I)$ is the most money household h would give up for the opportunity to buy both monopoly goods. If the household paid this amount for that opportunity, its utility would be the reservation utility $\bar{V}^h_{\varnothing}(\mathbf{p}, I)$—the same utility as if it had its initial income I but could not consume any monopoly goods. $S^h_{\varnothing\to 2}(I)$ is likewise

[15] For simplicity of notation we have dropped brackets, writing $S^h_{\varnothing\to 1,2}(I)$ instead of $S^h_{\{\varnothing\}\to\{1,2\}}(I)$.

the most h would forfeit to be able to add the opportunity to buy good 2 when it could only buy competitive goods; if this trade were made, utility would again be $\bar{V}_{\emptyset}^{h}(\mathbf{p}, I)$. $S_{2\to 1,2}^{h}(I - S_{\emptyset\to 2}^{h}(I))$ is the most h would pay to add the opportunity to buy good 1 if it had just enough income to achieve utility $\bar{V}_{\emptyset}^{h}(\mathbf{p}, I)$ when restricted not to buy good 1. Therefore if h paid $S_{\emptyset\to 2}^{h}(I) + S_{2\to 1,2}^{h}(I - S_{\emptyset\to 2}^{h}(I))$ to be able to consume without restriction, its utility would be $\bar{V}_{\emptyset}^{h}(\mathbf{p}, I)$. This shows that $S_{\emptyset\to 2}^{h}(I) + S_{2\to 1,2}^{h}(I - S_{\emptyset\to 2}^{h}(I)) \equiv S_{\emptyset\to 1,2}^{h}(I)$. By analogous argument, the reader will see $S_{\emptyset\to 1,2}^{h}(I) \equiv S_{\emptyset\to 1}^{h}(I) + S_{1\to 1,2}^{h}(I - S_{\emptyset\to 1}^{h}(I))$.

With two firms, there are two measures of willingness-to-pay for each good—one before and one after the other good is added. With three goods, there are four measures and with n monopoly goods there are 2^{n-1} measures of willingness-to-pay. Hookups will be based on one or more of those measures. For example, in Figure 1, the hookup for firm 1's good can be based on the first or the second leg; the same with the hookup for firm 2's good. The right choice will allow enough surplus to be gathered in an individually rational way to run a maximal number of monopoly projects. This will require a bit of care.

On these various legs, surpluses differ for two reasons: income and substitution. We will restrict our attention to preferences such that surpluses increase with income; therefore, we assume all monopoly goods will be normal in the following sense.

DEFINITION 1. *Good i is* Surplus Normal, *if for all prices, the greater is income, the greater is the surplus derived from introducing it to the choice set. That is, good i is surplus normal if,*

$$S_{\mathbf{M}_j \to \mathbf{M}_j \cup i}^{h}(I) \geq S_{\mathbf{M}_j \to \mathbf{M}_j \cup i}^{h}(I - q), \quad \forall\, \mathbf{M}_j \subseteq \mathbf{M}\backslash i, \quad \forall\, I,\ q \ni I > q \geq 0.$$

This notion of surplus normal was first discussed in Edlin and Epelbaum (1993), where it is shown that a particular good is surplus normal if it is normal in the usual sense. Assuming surplus normality will be central in allowing us to find the right notion of extractable surpluses.

We say two goods are value substitutes if the availability of one makes the other less valuable, controlling for income effects. That is, they are value substitutes if the first legs in Figure 1 are larger than the second legs.[16] Otherwise they are value complements. It can be shown by integrating the area under the Hicksian demand curve, see Edlin and Epelbaum (1993), that if two goods are Hicksian substitutes (respectively complements) then they are value substitutes (respectively complements)—although the converse does not hold.

If hookups are designed to be some proportion of the first legs in Figure 1, then households will not necessarily choose to pay both hookups. In particular, if the hookup for good 1 is equal to its first leg, then any hookup for good 2 larger than its second leg will not be paid. If the hookup for good 2 is tailored after its first leg, it might be too large if the goods are value substitutes so that the first leg is larger than the second leg. In contrast hookups which are tailored after the small legs are

[16] It can be checked from identity (7) that if the first leg is greater (or smaller) than the second for one of the goods, the same is true for the other good.

always paid voluntarily if the goods are surplus normal, regardless of whether they are substitutes or complements. This immediately suggests making hookups proportional to the smaller pair of legs. This *minimum* solution, taking the smallest legs in the two-firm problem, is easily generalizable to the case of many goods. We can define extractable surplus for good i as

$$\bar{S}_i^h = \min_{\mathbf{W} \in 2^{\mathbf{M} \setminus i}} [S^h_{\mathbf{W} \to \mathbf{W} \cup i}(I - S^h_{\emptyset \to \mathbf{W}}(I))], \tag{8}$$

where $2^{\mathbf{M} \setminus i}$ is the power set of $\mathbf{M} \setminus i$. Hookups may be tailored to be less than these surpluses. Notice that the $\bar{S}_i^h$'s are continuous since they are each the minimum of a finite number of continuous functions.

THEOREM 1. MINIMUM HOOKUP THEOREM. *If household h's hookups (q_i^h) for consuming the monopoly goods are such that, $q_i^h \leq \bar{S}_i^h$ $\forall i \in \{1, \dots, n\}$, and all the monopoly goods are surplus normal, then paying all the hookups is individually rational.*

PROOF. Consider the largest set of goods $\bar{\mathbf{M}} \subseteq \{1, \dots, n\}$ for which it is individually rational for household h to pay hookups. If there existed a $j \leq n$ such that $j \notin \bar{\mathbf{M}}$, it would follow that $q_j^h > S^h{}_{\bar{\mathbf{M}} \to \bar{\mathbf{M}} \cup j}(I - \Sigma_{t \in \bar{\mathbf{M}}}\, q_t^h)$ since $\bar{\mathbf{M}}$ is the largest set of hookups it is rational to pay. Also, because it is individually rational for h to pay the hookups for the set $\bar{\mathbf{M}}$, $\Sigma_{t \in \bar{\mathbf{M}}}\, q_t^h \leq S^h_{\emptyset \to \bar{\mathbf{M}}}(I)$. Together with surplus normality these facts would imply $q_j^h > S^h{}_{\bar{\mathbf{M}} \to \bar{\mathbf{M}} \cup 2}(I - S^h_{\emptyset \to \bar{\mathbf{M}}}(I))$. This would contradict the theorem's predicate that $q_i^h \leq \bar{S}_i^h$ $\forall i \in \{1, \dots, n\}$. Hence, there does not exist a $j \leq n$ such that $j \notin \bar{\mathbf{M}}$; $\bar{\mathbf{M}}$ must equal $\mathbf{M}$, so it is individually rational to pay hookups. □

The $\bar{S}_i^h$'s provide definitions of extractable surplus that will continuously yield individually rational hookups. They are however unsatisfactory. A lot of available surplus which could be gathered is wasted, permitting fewer equilibria or projects to exist. For instance, these definitions would not allow an equilibrium in which a cable television company and an electric company can coexist; after all, the first leg for cable television will be zero because these goods are such strong value complements that cable television has no value without electricity.

A better solution for the two-firm case is to allow firms to extract as hookups:

$$\bar{S}_1^h = \begin{cases} S^h_{\emptyset \to 1}(I) & \textit{if} \text{ value complements} \\ S^h_{2 \to 1,2}(I - S^h_{\emptyset \to 2}(I)) & \textit{if} \text{ value substitutes,} \end{cases} \tag{9}$$

$$\bar{S}_2^h = S^h_{1 \to 1,2}(I - S^h_{\emptyset \to 1}(I)). \tag{10}$$

When goods are surplus normal, hookups less than these surpluses in the two-firm case will be voluntarily paid. This definition can permit equilibria where one of the goods has no value without the other, such as the cable television and electricity example. Such cases can be handled because these extractable surpluses depend upon the ordering of the firms, and we are free to choose either firm to be firm 1. In the cable and electricity example, if cable is firm 1, then it will be unable to gather any surplus, while if electricity is firm 1, the cable television company can take advantage of the complementarity and gather some surplus.

This solution can be generalized to the n-firm case. It will guarantee that for every strict subset of monopoly goods for which a household has chosen to pay hookups, there will always be another good for which the household would be willing to pay the hookup.

Define hookups as follows: First order the n monopoly firms in any arbitrary order, so each firm is associated with a number between 1 and n. Then let

$$\tilde{S}_i^h(I) = \min_{\mathbf{W} \in 2^{\mathbf{M}} \text{ s.t. } i = \max\{j \in \mathbf{M} \backslash \mathbf{W}\}} [S^h_{\mathbf{W} \to \mathbf{W} \cup \{i\}}(I - S^h_{\emptyset \to \mathbf{W}})]. \tag{11}$$

Again the $\tilde{S}_i^h$'s are continuous functions of prices and income since they are the minimum of a finite number of continuous functions. While the extractable surplus for each good in the minimum solution was the minimum of 2^{n-1} different quantities, the surpluses defined in (11) are minimums over fewer of the same quantities so this new definition yields a weakly larger and possibly much larger amount of surplus for each firm. Also notice that changing the order of the monopoly goods will change the value of the $\tilde{S}_i^h$'s just as in the cable and electricity example (this will be important when defining viability). The surplus for the good produced by the firm labeled 1, $\tilde{S}_1^h$, is the surplus from adding the good after all other goods have been added, $S^h_{\mathbf{M}\backslash 1 \to \mathbf{M}}(I - S^h_{\emptyset \to \mathbf{M}\backslash 1}(I))$. The surplus for the firm labeled 2 is the minimum of the surplus from adding the good last, $S^h_{\mathbf{M}\backslash 2 \to \mathbf{M}}(I - S^h_{\emptyset \to \mathbf{M}\backslash 2}(I))$, and the surplus from adding the good when all other goods except good 1 may be purchased, $S^h_{\mathbf{M}\backslash\{1,2\} \to \mathbf{M}\backslash 1}(I - S^h_{\emptyset \to \mathbf{M}\backslash\{1,2\}}(I))$. As the index increases the number of quantities included in the minimum operator grows exponentially.

As with the $\bar{S}_i^h$'s, we show below that if each of household h's hookups is less than the corresponding $\tilde{S}_i^h$, then it will be individually rational for household h to pay all hookups.

THEOREM 2. n-FIRM HOOKUP THEOREM. *If household h's hookups* (q_i^h) *for consuming monopoly goods are such that* $q_i^h \leq \tilde{S}_i^h$, $\forall i \in \{1, \dots, n\}$, *and all the monopoly goods are surplus normal, then it is individually rational for household h to pay all of the hookups in order to consume the monopoly goods.*

PROOF. Consider the largest set of goods $\bar{\mathbf{M}} \subseteq \{1, \dots, n\}$, for which it is individually rational for household h to pay hookups. If $\bar{\mathbf{M}} = \mathbf{M}$, we are done. If $\bar{\mathbf{M}} \neq \mathbf{M}$ then letting $i = \max\{j \in \mathbf{M}\backslash\bar{\mathbf{M}}\}$ implies that $q_i^h > S_{\bar{\mathbf{M}} \to \bar{\mathbf{M}} \cup i}(I - \Sigma_{t \in \bar{\mathbf{M}}} q_t^h)$. Otherwise, the household would be willing to also pay hookup q_i^h. But $\Sigma_{t \in \bar{\mathbf{M}}} q_t^h \leq S_{\emptyset \to \bar{\mathbf{M}}}(I)$, so surplus normality implies that $q_i^h > S_{\bar{\mathbf{M}} \to \bar{\mathbf{M}} \cup i}(I - S_{\emptyset \to \bar{\mathbf{M}}}(I))$, which contradicts the assumption that $q_i^h \leq \tilde{S}_i^h$ $\forall i \in \{1, \dots, n\}$. □

Unfortunately, these definitions for extractable surpluses do not always work well at covering the losses of all firms. They would fail for example if each of two firms' products were worth very little without the other (as would be the case if one firm made right shoes and the other left shoes). When the first leg is very small for both firms, changing the order doesn't help. A more general construction of the two-firm solution which involves surplus sharing and deals adequately with this case can be found in Edlin and Epelbaum (1993). Letting $\alpha \in [0, 1]$ be a surplus sharing parameter, we define extractable surpluses

$$(12)\quad \hat{S}_1^h = \begin{cases} S_{\emptyset \to 1}^h(I) + \alpha(S_{\emptyset \to 1,2}^h(I) - S_{\emptyset \to 1}^h(I) - S_{\emptyset \to 2}^h(I)) \\ \hfill \textit{if } \text{value complements} \\ S_{2 \to 1,2}^h(I - S_{\emptyset \to 2}^h(I)) \qquad \textit{if } \text{value complements}, \end{cases}$$

and

$$(13)\quad \hat{S}_2^h = \begin{cases} S_{\emptyset \to 2}^h(I) + (1-\alpha)(S_{\emptyset \to 1,2}^h(I) - S_{\emptyset \to 1}^h(I) - S_{\emptyset \to 2}^h(I)) \\ \hfill \textit{if } \text{value complements} \\ S_{1 \to 1,2}^h(I - S_{\emptyset \to 1}^h(I)) \qquad \textit{if } \text{value complements}. \end{cases}$$

When goods are surplus normal, hookups less than these surpluses in the two-firm case will be paid. The surplus sharing parameter α may be varied to increase the surplus either firm can gather. The $\tilde{S}_i^h$'s we previously presented are the special case where $\alpha = 0$ (reversing the firms' ordering corresponds to $\alpha = 1$). If neither firm's good is valuable without the other good, some intermediate value of α would be appropriate, depending on the relative size of the two firms' losses. In the two-firm case the extractable surplus definition with the α's is very nearly as good as can be done. It is almost necessary for two firms to cover their losses that for some α these extractable surpluses exceed losses (because of income effects slightly more productions actually might be viable, e.g., with slightly negative α's). Regrettably, a generalization of this definition to many firms is difficult and is left for further research. We envision it involving definitions of complements and substitutes which make sense with more than two firms.

From here on we will restrict our attention to the $\tilde{S}_i^h$'s. Analogous definitions of extractable surplus can be made for the competitive firms and a hookup theorem for competitive firms can be constructed in exactly the same way except that normality is not needed because of the absence of income effects. It is left for the reader to verify this. We call the extractable surplus from competitive firm f for monopoly good i $\tilde{S}_i^f$.

$\Sigma_{f>n} \tilde{S}_i^f + \Sigma_h \tilde{S}_i^h$ represents the total extractable surplus for monopoly firm i. As explained above changing the ordering of the monopoly firms changes the value of the surpluses. If a firm moves earlier in any given ordering, it can gather more surplus since this shrinks the set over which the minimum in (11) is taken. Notice that the monopolists' losses will depend on prices and productions and the surpluses on prices and incomes. This motivates the following definition.

DEFINITION 2. *A vector of productions* $(\mathbf{y}^f)$, $\mathbf{y}^f \in \mathbf{Y}^f$, $f \in \{1, \ldots, n\}$ *is* weakly independently viable with respect to some ordering *of the monopoly firms at prices* $\mathbf{p}$ *and incomes* (I^h) *if the sum of each firm's extractable surplus is larger than losses, i.e.,* $\Sigma_{f>n} \tilde{S}_i^f + \Sigma_h \tilde{S}_i^h > \max[0, -\mathbf{p} \cdot \mathbf{y}^i]$, $i = 1, \ldots, n$.

Without entering into extravagant complexities, we have tried to give sufficient conditions for firms to be independently viable that are not too distant from necessary conditions. In fact, in the two-firm case, except for income effects, the extractable surpluses we define give each firm the most surplus it can extract given the other firm's extractable surplus (however, as noted above, if the firms share surplus they can be viable in more circumstances).

Throughout this section, in order to make household choice sets compact, we have assumed that the prices of all goods were strictly positive, $\mathbf{p} \in \Re^c_{++}$. For the proof of existence in general equilibrium, we need to remove that restriction. To do this we use the standard trick of defining a very large compact box $\mathbf{B}$ that contains the attainable set, and we temporarily restrict household demands to be inside this box. This ensures that the surplus functions are defined at all prices. The reader should keep in mind that "weak independent viability" will refer to the condition in Definition 2 being satisfied by the surplus functions that have the extra demand restriction (as usual after the existence proof, the restriction can be removed).

3. GENERAL EQUILIBRIUM: EXISTENCE

In this section, we present the independent viability condition (IVC) which requires that the n monopoly firms be weakly independently viable over the set of production equilibrium. This allows us to find continuous hookup rules that are voluntarily paid by households and competitive firms, and which exactly cover the losses of the monopoly firms. Hookups exactly covering losses means that the nonhookup value of aggregate household demand is equal to the value of the supply of competitive goods (net of the monopolists' productions). That is, Walras' law holds, so we can show the existence of a general equilibrium with firms using two-part tariffs to cover losses in the market without subsidies.

Below, we describe the economy and two-part marginal cost pricing equilibria. Before though, we must introduce the Clarke normal cone. It is a natural generalization for nonsmooth production technologies with increasing returns of marginal cost pricing and the normal cone for convex sets. Pioneered by Clarke (1975), and discussed in Brown (1991) and Bonnisseau and Cornet (1990a, b), it is a convenient choice because it is always closed and convex and its intersection with the simplex Δ^c is an upper-hemicontinuous map $\zeta^f\colon \partial \mathbf{Y}^f \to \Delta^c$. When the production set is differentiable, the Clarke normal cone collapses to simply the normal vector.

3.1. *Defining an Equilibrium.* Define the competitive firms' continuous distributable profit functions as their profits after paying hookups to the monopoly firms. Using the definitions of Section 2, for prices $\mathbf{p}$ and hookups (q_i^f), $i = 1, \dots, n$ this yields $\Pi^f = \Pi^f_{\mathbf{M}} - \Sigma_{i=1}^n q_i^f$, $f = n + 1, \dots, F$. Importantly, $\Pi^f \geq 0$ provided $q_i^f \leq \bar{S}_i^f$. Because when the monopoly firm f makes losses, it charges hookups to exactly cover them, its distributable profits at output $\mathbf{y}^f$ and prices $\mathbf{p}$ are $\Pi^f = \max\,[0, \mathbf{p} \cdot \mathbf{y}^f]$, $f = 1, \dots, n$.

These profit definitions allow us to calculate households' shares of profits and hence incomes: at prices $\mathbf{p}$, hookups (q_i^f) and production $\mathbf{y} \equiv (\mathbf{y}^1, \dots, \mathbf{y}^F)$, household h's share of profits are $\Pi^h = \Sigma_f\, \theta^{fh} \Pi^f$, and total income is

$$I^h = \mathbf{p} \cdot \boldsymbol{\omega}^h + \Pi^h. \tag{14}$$

DEFINITION 3. *A Two Part Marginal Cost Pricing Equilibrium (TPMCPE) for the economy* $\mathscr{E} \equiv ((U^h), (\theta^{hf}), (\boldsymbol{\omega}^h), (\mathbf{Y}^f))$ *described above is a list of productions* $\mathbf{y} \equiv (\mathbf{y}^1, \ldots, \mathbf{y}^F)$, $\mathbf{y}^i \in \Re^c$, *prices* $\mathbf{p} \in \Delta^c$, *consumptions* ($\mathbf{x}^h \in \Re^c_+$), *and nonnegative individualized hookups* (q_i^h), (q_i^f) $i = 1, \ldots, n$, $f = n+1, \ldots, F$, $h = 1, \ldots, H$ *such that:*

(1) *All monopolies are marginal cost pricing and all convex firms are maximizing profits:* $\mathbf{y}^f \in \partial \mathbf{Y}^f$ *and* $\mathbf{p} \in \zeta^f(\mathbf{y}^f)$ $\forall f$.

(2) *It is rational for competitive firms to pay all hookups:* $\Pi_{\mathbf{M}}^f - \Sigma_{i \in \mathbf{M}}\, q_i^f \geq \max_{\bar{\mathbf{M}} \in 2^{\mathbf{M}}} [\Pi_{\bar{\mathbf{M}}}^f - \Sigma_{i \in \bar{\mathbf{M}}}\, q_i^f]$.

(3) *It is rational for each household to pay all hookups and consume* $\mathbf{x}^h$:

$$\mathbf{x}^h \in \arg\max \left\{ U^h(x) | \mathbf{p} \cdot \mathbf{x} \leq I^h - \sum_{i \in \mathbf{M}} q_i^h \right\} \text{ and}$$

$$U^h(\mathbf{x}^h) \geq \max_{\bar{\mathbf{M}} \in 2^{\mathbf{M}}} \left[\bar{V}^h_{\bar{\mathbf{M}}} \left(I^h - \sum_{i \in \bar{\mathbf{M}}} q_i^h \right) \right]$$

where I^h *is given by* (14).

(4) *There is positive supply of and demand for all monopoly goods.*

(5) *Markets clear, i.e.,* $\Sigma\, \mathbf{x}^h - \boldsymbol{\omega}^h \leq \Sigma\, \mathbf{y}^f$, *and if for any* i $\Sigma\, x_i^h - \omega_i^h < \Sigma\, y_i^f$ *then* $p_i = 0$.

(6) *Hookups are zero when there are no losses, but otherwise exactly cover losses:*

$$\sum_h q_j^h + \sum_{f > n} q_j^f + \min\,[0, \mathbf{p} \cdot \mathbf{y}^j] = 0 \quad j = 1, \ldots, n.$$

Notice that the equilibrium concept, following the general equilibrium tradition, requires that the monopolists take prices of other goods as fixed so they do not take into account any general equilibrium effects when pricing their own goods.

3.2. *Conditions for Existence.* The following conditions ensure that an equilibrium exists.

IVC. For some ordering of the monopoly firms, at all production equilibria—i.e., $(\mathbf{p}, \mathbf{y})$, for which $\mathbf{y}^f \in \partial \mathbf{Y}^f$ and $\mathbf{p} \in \zeta^f(\mathbf{y}^f) \forall f$—$(\mathbf{y}^f)$ are weakly independently viable with respect to that ordering at prices $\mathbf{p}$ and incomes

$$I^h = \mathbf{p} \cdot \boldsymbol{\omega}^h + \sum_{f > n} \theta^{hf} \left[\mathbf{p} \cdot \mathbf{y}^f - \sum_{i \leq n} \bar{S}_i^f \right] + \sum_{f \leq n} \max\,[0, \mathbf{p} \cdot \mathbf{y}^f]. \tag{15}$$

SC. At all production equilibria, the incomes defined in (15) are positive.

Some condition akin to the survival condition (SC) is standard in the literature (see Vohra 1992 for a discussion and an alternative). Instead of assuming SC, we might have let $\boldsymbol{\omega}^h \gg 0$ (as in Debreu 1959) and suitably modified the restricted utility and surplus definitions in Section 2. In fact something akin to that would be required for a one-firm economy with only a monopoly firm with fixed costs; at zero production the technology is horizontal, so a price of 1 for the monopoly good and

0 for all other goods constitutes a production equilibrium. Unless households are endowed with some of the monopoly good, incomes would be zero at such a production equilibrium.[17]

We assume the independent viability condition (IVC) to guarantee surplus is sufficient to make voluntary hookups cover losses so that Walras law holds and a fixed point of the map constructed in the Appendix corresponds to an equilibrium. IVC requires that the monopoly firms be weakly independently viable at any candidate equilibrium. The income at which weak viability is considered in (15) makes the viability condition most likely to be satisfied because it envisions each competitive firm's hookups being equal to its entire surplus. If any competitive firm's surplus were distributed to households instead of being taken as hookups, only a fraction of it would appear as higher surplus on the part of households.[18]

We could bring condition IVC closer to the primitives by assuming that the indifference surfaces do not touch the axes: i.e. $\forall x^h, \bar{x}^h$ s.t. $x^h \in \partial \Re_+^c$ and $\bar{x}^h \in \Re_{++}^c$, $U^h(x^h) < U^h(\bar{x}^h)$. While such assumptions have been made in the literature, e.g., Kamiya (1990),[19] they would make independent viability a trivial matter because each household would be willing to pay its entire income as hookups divided in any way among the monopolists.[20]

In contrast to IVC, Bonnisseau and Cornet (1988, 1990a, 1990b) have abstract income rules without reference to share ownership. They thus smooth covering the monopolies' losses into a survival assumption and so have nothing to combat the individual-rationality or willingness-to-pay problem. When these abstract rules are taken to be the reduced form of full liability share holdings, as they suggest at one point, then a household's payment for the monopoly firms' losses does not vary with changes in its income from other firms (this would make their survival assumption less implausible than ours and in some ways less implausible than the union of IVC and SC). Finally, it should be noted that IVC could be weakened as Vohra (1990b, pp. 20–21) does in the one-firm case, requiring that IVC hold for all of what Vohra terms marginal valuation equilibria instead of production equilibria. What we need is for IVC to hold in any set that contains all the points that might possibly be an equilibrium.

These comments notwithstanding, IVC is regrettably still quite a strong condition. Even if we took Vohra's (1990b) approach, surplus must exceed losses at a great many production and price combinations. If the production technology is not

[17] Vohra (1990b) showed that $\boldsymbol{\omega}^h \gg 0$ could be replaced by an assumption of positive but bounded marginal returns to show existence of marginal cost pricing equilibria.

[18] For more discussion of this sort of viability assumption, see Brown, Heller and Starr (1990) who discuss their viability assumption in their context and argue that it holds under weak assumptions with a large number of households.

[19] See assumption A5 in Kamiya (1990), a very creative paper which explores efficiency but not the separate viability of firms.

[20] These preferences together with SC imply that each monopoly firm can cover its losses at every production equilibria. Given SC, there is enough income to cover losses since $\mathbf{p} \cdot (\boldsymbol{\omega} + \Sigma_f \mathbf{y}^f) > 0 \Rightarrow \mathbf{p} \cdot (\boldsymbol{\omega} + \Sigma_{f>n} \mathbf{y}^f) > -\mathbf{p} \cdot \Sigma_{f \leq n} \mathbf{y}^f$. Therefore, each monopoly firm i can cover losses by charging households hookups $q_i^h = \lambda_i I^h$, $\lambda_i = \max\,[0, -\mathbf{p} \cdot \mathbf{y}^i]/\mathbf{p} \cdot (\boldsymbol{\omega} + \Sigma_{f>n} \mathbf{y}^f)$ with no hookups to competitive firms. And paying all hookups is surely rational since the boundary is indifferent to 0 and inferior to any interior point.

smooth, many prices will pair with any production to constitute a production equilibrium. In particular, as one referee was helpful enough to point out, even when there are no losses, just having strictly positive surplus might be problematic: at zero production, strictly positive surplus would exist if and only if some household had a shadow price when it did not consume the monopoly good greater than the marginal cost of beginning production.[21] Surplus could be less than losses for some productions even though other productions are sustainable TPMCPE. Some points in the technology may have very large losses which cause IVC to fail even though for the rest of the technology—where equilibria may exist—losses are much smaller. Consider a technology with fixed costs. At low productions, the losses from production will be large because of the fixed cost, but as production increases its losses may be reduced, making it more and more plausible for weak viability to hold. In some sense, in order for us to show existence and that production is sustainable, the more dramatic are the increasing returns and the more inputs are needed before much scale is reached, the more nearly necessities the goods need to be to prove existence. It is unfortunate that the state of the art for existence proofs requires viability at low outputs, even when the desirable equilibrium may involve high output. But this is a problem generic to the TPMCPE literature.

THEOREM 3. EXISTENCE THEOREM. *If conditions IVC and SC hold and all monopoly goods are surplus normal, then there exists a TPMCPE for economy* $\mathscr{E}$.

With IVC, Theorem 3 could be proven using the methods of Brown, Heller and Starr (1990). However, by using Kakutani's fixed point theorem instead of Brouwer's, a mapping may be constructed that makes for a more straightforward proof without their extension and approximation arguments. The essential feature is the identification of n continuous individually rational hookup functions which cover the n monopoly firms' losses. These functions (again because independent viability ensures that Walras' law holds) ensure that fixed points of the convex-valued upper-hemicontinuous correspondence defined in the Appendix correspond to equilibria.

4. WELFARE

The welfare properties of equilibria are unchanged from the two-good case to the n-good case.[22] TPMCPE are not always efficient but when the nonconvexities in production arise solely from fixed costs, efficiency is guaranteed.[23] Also a completely general second welfare theorem fails, but given sufficient surplus for independent viability, all Pareto optimal allocations may be decentralized.

[21] In fact, in the extreme case of a fixed cost production technology, this would require that indifference surfaces actually become tangent to the hyperplane of zero consumption of the monopoly good. This would be almost equivalent to assuming that indifference curves do not touch that plane at all.

[22] The two-good case is discussed in Edlin and Epelbaum (1993).

[23] Efficiency results in the fixed cost case are proven in recent papers by Edlin (1993), Kamiya (1990), and Moriguchi (1991).

The intuition for the efficiency of TPMCPE with fixed costs is the following. Efficiency depends upon whether the production possibility set is cut by the social indifference surface defined by the equilibrium allocation. The marginal cost concept guarantees that locally these sets are tangent; however, two problems with increasing returns are that tangency need not imply local optimality (we might be at a local minimum) and even if the tangency is at a local optimum, it does not guarantee global optimality. The latter problem is the pertinent one when increasing returns arise solely from fixed costs. In an attempt to solve the latter problem, TPMCPE introduce voluntary hookups and the viability condition which guarantee that the total value of each monopoly good exceeds its production cost. While not being enough to guarantee efficiency in all circumstances, this is exactly the sort of condition needed for global optimality if the increasing returns arise solely from fixed costs. To see this, characterize such production sets as $\mathbf{O} \cup \mathbf{C}^f$ for some closed convex set $\mathbf{C}^f$ which is the translation of an ordinary production set from the origin $\mathbf{O}$ by fixed costs. Then, in a TPMCPE, the same prices $\mathbf{p}$ which support the better than set also support the convex portions of production $\Sigma\, \mathbf{C}^f$ (the set of deviations in production such that no monopoly firms shut down). Thus, production and better than sets are almost separated by a hyperplane. It remains to consider the nonconvex portions of the production set; these involve shutting down the production of some or all of the monopoly firms. The social indifference surface, however, cannot cut the aggregate production set in the hyperplane where some group of the monopoly goods is not produced, because that would contradict the fact that households are willing to pay the full production costs of moving from such a situation, where some of the productions are shut down, to the equilibrium.

Finally, we state a restricted second welfare theorem involving independent viability. We omit the proof because it is an easy extension of the proof in the two-good case found in Edlin and Epelbaum (1993).

THEOREM 4. SECOND WELFARE THEOREM. *For any Pareto optimum* $\{(\mathbf{x}^h), (\mathbf{y}^f)\}$, *let* $\mathbf{p}^{po} \in \Re^c$ *be a vector of prices that supports consumptions* $(\mathbf{x}^h)$ *and competitive firms' productions* $(\mathbf{y}^f)_{f>n}$:[24] *if* $(\mathbf{y}^f)_{f\leq n}$ *are independently viable with respect to some ordering at prices* $\mathbf{p}^{po}$ *and incomes* $(I^h = \mathbf{p}^{po} \cdot \mathbf{x}^h)$, *than* $\{(\mathbf{x}^h), (\mathbf{y}^f)\}$ *can be turned into a TPMCPE with appropriate transfers if the monopoly goods are surplus normal.*

Stanford University, U.S.A.

APPENDIX

Here we will define the map whose fixed points will correspond to equilibria.

From the compactness of the $\mathbf{K}^f$'s we know that $\forall f\, \exists\, r_f \ni \mathbf{K}^f \subset \{-r_f e^c + \Re^c_+\}$, where $e^c = (1, \dots, 1)$. Choose $r \ni r > \max\{r_f\}_{f=1}^F$ so that $\forall f, \mathbf{K}^f \subset \{-re^c + \Re^c_+\}$. There exist (see Beato and Mas-Colell 1985) homeomorphisms η^f mapping the simplex into the boundary of the truncated production set, preserving the natural orientation of faces: $\eta^f : \mathbf{\Delta}^c \to \partial \mathbf{Y}^f \cap \{-re^c + \Re^c_+\}$. Figure 2 illustrates these homeomorphisms.

[24] Since these sets are convex, such prices must exist by Minkowski's supporting hyperplane theorem.

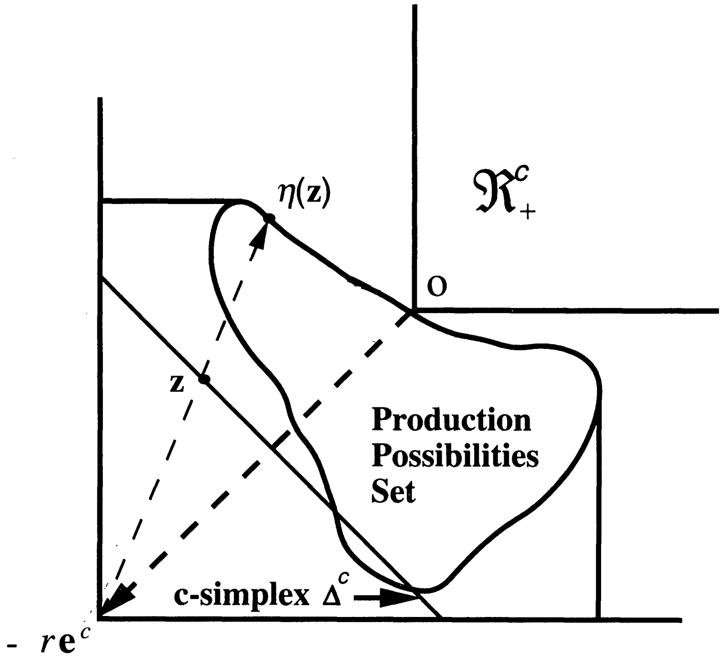

FIGURE 2

In order to define household demand and surplus functions when prices are zero, we use the standard trick throughout this section of restricting the household's maximizing choices of goods in all problems to be in a convex and compact box $\mathbf{B} \subset \Re^c_+$ that contains the attainable set $\Re^c_+ \cap \Sigma\, \mathbf{Y}^f + \Sigma\, \omega^h$. By standard methods, see Debreu (1959), section 5.7, equation (6), it can be shown that at a fixed point each household's maximizing choice would be in $\mathbf{B}$ without the restriction.

We proceed to extend the map in Edlin and Epelbaum (1993) to a convex-valued upper-hemicontinuous correspondence $\mathbf{\Phi}$: $\mathbf{D} \rightarrow \mathbf{D}$, where $\mathbf{D} \equiv (\Delta^c)^{2F+1} \times \mathbf{B}^H \times [0, 1]^n$. $\mathbf{\Phi}$ thus has $2F + H + n + 1$ components. A typical member of the domain $\mathbf{D}$ is $d = ((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i))$ where $(\mathbf{z}^f) \in (\mathbf{\Delta}^c)^F$, $(\mathbf{g}^f) \in (\mathbf{\Delta}^c)^F$, $\mathbf{p} \in \mathbf{\Delta}^c$, $(\mathbf{x}^h) \in \mathbf{B}^H$, $(\lambda_i) \in [0, 1]^n$. Since $\mathbf{D}$ is the finite Cartesian product of convex compact sets, it itself is convex and compact, and $\mathbf{\Phi}$ has a fixed point. This fixed point defines an equilibrium.

The prices $\mathbf{p}$ allow definitions of continuous surpluses for competitive firms as in Section 2. Interpreting λ's as the benefit taxation rate to cover the losses of the monopoly firms we are motivated to define continuous hookup functions $q_i^f = \lambda_i \tilde{S}_i^f$ $i = 1, \dots, n$ for the competitive firms $f > n$ and define continuous distributable profits $\Pi^f = \Pi^f_{\mathrm{M}} - \Sigma_{i \in \mathrm{M}}\, q_i^f$, $f > n$ and $\Pi^f = \max\,[0, \mathbf{p} \cdot \mathbf{y}^f]$, $f = 1, \dots, n$. These profit functions allow us to define continuous income functions $I^h = \mathbf{p} \cdot \boldsymbol{\omega}^h + \Pi^h$, where $\Pi^h = \Sigma_f\, \theta^{fh} \Pi^f$. These parameterize the household problems described in Section 2 so that for each monopoly firm i continuous extractable surpluses $(\tilde{S}_i^h)$ are defined. From these we define hookups $q_i^h = \lambda_i \tilde{S}_i^h$ $i = 1, \dots, n$ which are continuous functions of λ's and $(\tilde{S}_i^h)$.

Define the map $\mathbf{\Phi}$ in five parts, i.e. $\mathbf{\Phi} = (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5)$, where $\Phi_1 \in (\Delta^c)^F$, $\Phi_2 \in (\Delta^c)^F$, $\Phi_3 \in \Delta^c$, $\Phi_4 \in \mathbf{B}^H$, $\Phi_5 \in [0, 1]^n$. Letting $\mathbf{y}^f = \boldsymbol{\eta}^f(\mathbf{z}^f)$, define Marshallian output adjustment,

$$\Phi_{1,s}^t((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i)) \equiv \frac{z_t^s + \max[0, p_t - g_t^s]}{1 + \sum_{j=1}^{c} \max[0, p_j - g_j^s]},$$

$$s = 1, \dots, F, \quad t = 1, \dots, c;$$

marginal cost pricing,

$$\Phi_{2,s}((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i)) \equiv \zeta^s(\mathbf{y}^s), \; s = 1, \dots, F;$$

Walrasian price adjustment,

$$\Phi_3^t((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i)) \equiv \frac{p_t + \max\left[0, \sum_h (x_t^h - \omega_t) - \sum_f y_t^f\right]}{1 + \sum_{j=1}^{c} \max\left[0, \sum_h (x_j^h - \omega_j) - \sum_f y_j^f\right]},$$

$$t = 1, \dots, c;$$

utility maximizing demands,

$$\Phi_{4,r}((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i))$$
$$\equiv \begin{cases} \arg\max \left\{ U^r(\mathbf{x}) | \mathbf{x} \in \mathbf{B}, \; \mathbf{p} \cdot \mathbf{x} \le I^r - \sum_{i=1}^{n} q_i^r \right\} & \text{when } I^r - \sum_{i=1}^{n} q_i^r > 0 \\ \mathbf{B} & \text{when } I^r - \sum_{i=1}^{n} q_i^r \le 0, \end{cases}$$

$$r = 1, \dots, H;$$

break-even benefit taxation (or covering losses),

$$\Phi_5^m((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i))$$
$$\equiv \begin{cases} \min\left\{1, \dfrac{-\min[0, \mathbf{p} \cdot \mathbf{y}^m]}{\sum_h \bar{S}_m^h + \sum_{f>n} \bar{S}_m^f}\right\} & \text{when } \sum_h \bar{S}_m^h + \sum_{f>n} \bar{S}_m^f \neq 0 \\ [0, 1] & \text{when } \sum_h \bar{S}_m^h + \sum_{f>n} \bar{S}_m^f = 0, \end{cases}$$

$$m = 1, \dots, n.$$

Notice that: Φ_1 is a single-valued continuous function; Φ_2 is the Clarke normal cone which is convex-valued and upper hemicontinuous; Φ_3 is a single-valued continuous function; Φ_4 is convex-valued and upper hemicontinuous because by Berge it is convex-valued and upper hemicontinuous when $I^h - \Sigma_{i=1}^n q_i^h \geq 0$, and it is the whole domain **B** if $I^h - \Sigma_{i=1}^n q_i^h \leq 0$; finally Φ_5 is also convex-valued and upper hemicontinuous because it is continuous and single-valued if extractable surplus is greater than zero and the whole domain if surplus is zero.

By Kakutani then, this map has a fixed point, and it is fairly standard to show that if $((\mathbf{z}^f), (\mathbf{g}^f), \mathbf{p}, (\mathbf{x}^h), (\lambda_i))$ is a fixed point of $\mathbf{\Phi}$, then an equilibrium is given by consumptions $(\mathbf{x}^h)$, outputs $(\mathbf{y}^f = \eta^f(\mathbf{z}^f))$, prices $\mathbf{p}$, hookups $(q_i{}^f = \lambda_i \bar{S}_i{}^f)$, $i = 1, \ldots, n$ for competitive firms $f > n$ and $(q_i^h = \lambda_i \bar{S}_i^h)$ for households, where surpluses are measured at incomes $I^h = \mathbf{p} \cdot \boldsymbol{\omega}^h + \Pi^h$.

The critical fact in showing that a fixed point is an equilibrium is that Φ_1 guarantees that we are at a production equilibrium and so because of IVC, $\Phi_5^i = -\min\,[0, \mathbf{p} \cdot \mathbf{y}^i]/(\Sigma\, \bar{S}_i^h + \Sigma\, \bar{S}_i{}^f)$ at that fixed point. Individually rational hookups cover losses, income flows circularly, and Walras law holds. Details in the case of two firms can be found in Edlin and Epelbaum (1993).

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